A two-thermocouple probe technique for estimating thermocouple time constants in flows with combustion: in situ parameter identification of a first-order lag system

Masato Tagawa, T. Shimoji, Y. Ota

REVIEW OF SCIENTIFIC INSTRUMENTS

Volume 69, Issue 9, Pages 3370-3378, September 1998

doi: 10.1063/1.1149103
A two-thermocouple probe technique for estimating thermocouple time constants in flows with combustion: \textit{In situ} parameter identification of a first-order lag system

M. Tagawa, a) T. Shimoji, and Y. Ohta
Department of Mechanical Engineering, Nagoya Institute of Technology, Nagoya 466-8555, Japan
(Received 16 March 1998; accepted for publication 17 June 1998)

A two-thermocouple probe, composed of two fine-wire thermocouples of unequal diameters, is a novel technique for estimating thermocouple time constants without any dynamic calibration of the thermocouple response. This technique is most suitable for measuring fluctuating temperatures in turbulent combustion. In the present study, the reliability and applicability of this technique are appraised in a turbulent wake of a heated cylinder (without combustion). A fine-wire resistance thermometer (cold wire) of fast response is simultaneously used to provide a reference temperature.

The frequency response of a fine-wire thermocouple can be described as a first-order lag system. In turbulent combustion fields, velocity, temperature, density and gas composition will vary spatially and temporally, and so does the time constant of the thermocouple response. Hence, for accurate measurement of fluctuating temperature, we need a correct time constant together with a reliable and robust compensation method for the thermocouple response. So far, the thermocouple time constant is almost always measured/calibrated by internal or external heating of the thermocouple wire. The time constant thus obtained is effective only in the calibration condition, so it would be next to impossible to perform such a dynamic calibration of the thermocouple to cover all experimental conditions. We have an easier alternative than the dynamic calibration, which may predict the time-constant value with the aid of a correlation equation of the heat transfer coefficient of a fine wire. However, there is a serious problem in its application to a combustion experiment, since every available correlation equation is effective only for flows without combustion. Thus, we should not rely on this alternative method, unless we are confident of the universality of the correlation equation applied.

A two-thermocouple probe, which is composed of two fine thermocouples of unequal diameters, is a unique tool for measuring temperature fluctuations, and the dynamic calibration of the thermocouple is not needed. If a good data-reduction scheme for the two-thermocouple probe technique is available, we will be free of the very demanding task of dynamic calibration of the thermocouple response, i.e., measurement of a time constant. The two-thermocouple probe technique provides \textit{in situ} measurement of the time constant, and the spatial and temporal variations in the time-constant

\textsuperscript{a)Electronic mail: tagawa@megw.mech.nitech.ac.jp
value can naturally be reflected in the compensation of the thermocouple response. In short, the two-thermocouple probe technique makes no distinction, hitherto existing, between the dynamic calibration of the thermocouple and the data collection.

In general, if a measurement system can be approximated by a linear dynamic system of appropriate order, the system is fully described by a set of system parameters. These parameters can usually be determined from a calibration experiment, in which a “known” input such as a sinusoidal or a step-wise signal is given to the system, and the system output is then measured. On the contrary, we are proposing a different approach which enables us to determine the system (the thermocouple time constant) from the “unknown” input (fluid temperature). In short, a kind of in situ parameter identification of the system is realized and is applicable not only to the two-thermocouple probe technique but also to a general first-order lag system.

In the present study, the reliability and applicability of the two-thermocouple probe technique are evaluated using a fine-wire resistance thermometer of fast response (cold wire). So far, a few studies investigating the response characteristics of a thermocouple are available, where a fine cold wire was used as a reference thermometer which can provide a “true” temperature. It should be noted here that the main subject of the studies was to show how accurately the temperature fluctuation can be reproduced by compensating the thermocouple response with a given time constant. In this study, on the other hand, we are aiming at establishing a reliable scheme for estimating/measuring the thermocouple time constant, and are dealing with the cold-wire measurement from a different point of view. First, we appraise a previous scheme for the two-thermocouple probe technique and then improve the scheme to give it wider applicability and higher reliability.

II. METHOD FOR ESTIMATING TIME CONSTANT AND ITS IMPROVEMENT

Two kinds of schemes have so far been proposed for estimating the thermocouple time constants using the two-thermocouple probe technique: one for determining the time constants from a cross spectrum of two temperature signals and the other for obtaining the time constants by assuming the ratio of two time constants is a known constant. Recently, we have proposed an alternative scheme for estimating the time constants, in which there is no need for sophisticated data reduction and introduction of any a priori assumption. In its practical applications, however, we come to realize that the proposed scheme is sensitive to the changes in the signal-to-noise ratio of thermocouple signals and in the spatial resolution of the two-thermocouple probe. As a result, the time-constant values tend to be underestimated. In the following, we outline briefly the previous scheme and then propose an improved version for the two-thermocouple probe technique.

The fluid temperatures sensed by the two thermocouples, \( T_{g1} \) and \( T_{g2} \), can be given by the following first-order lag systems:

\[
\begin{align*}
T_{g1} &= T_1 + r_1 G_1 \\
T_{g2} &= T_2 + r_2 G_2,
\end{align*}
\]

where \( T_1, \tau \), and \( G \) are a hot-junction temperature (raw/uncompensated temperature), a time constant and the derivative of \( T \), respectively. The subscripts “1” and “2” denote the thermocouples of \( d_1 \) and \( d_2 \) in diameter, respectively.

As for the two adjacent thermocouples of the two-thermocouple probe, the relation \( T_{g1} = T_{g2} \) holds. Thus, the unknown time constants \( r_1 \) and \( r_2 \) can be determined from the minimization of the difference between \( T_{g1} \) and \( T_{g2} \):

\[
e = (T_{g2} - T_{g1})^2.
\]

If \( r_1 \) and \( r_2 \) are kept constant through time averaging in Eq. (2), we obtain time constants that will minimize \( e \):

\[
\begin{align*}
\tau_1 &= [G_1^2 G_1 \Delta T_{21} - G_1 G_2 \Delta T_{21}]/D, \\
\tau_2 &= [G_1 G_2 \Delta T_{21} - G_1^2 G_2]/D,
\end{align*}
\]

where \( \Delta T_{21} = T_2 - T_1 \) and \( D = G_1^2 G_2 - (G_1 G_2)^2 \). The physical meaning of the derivation of Eq. (3) is clear. However, as mentioned above, this scheme tends to underestimate the time constants, unless the noise in the thermocouple signals is below a negligible level and/or the measurement volume formed by the two thermocouples is sufficiently small compared with a characteristic length scale of a turbulent flow measured. The reason for this underestimation may be explained as follows: compensation of the thermocouple response amplifies not only a temperature signal itself but also noises and/or high-frequency signal components of no correlation between the two thermocouples; therefore, Eq. (3) will give time constants smaller than correct ones to minimize the difference \( e \) as a whole.

To diminish the above effect of the noise and the signal components of no correlation, we introduce a new criterion instead of \( e \) for estimating the thermocouple time constants. The criterion is based on a “similarity” between the two compensated temperatures. The similarity can be quantified by a cross-correlation coefficient between the fluid-temperature fluctuations measured by the two compensated thermocouples. We can therefore obtain the highest similarity between \( T_{g1} \) and \( T_{g2} \) by maximizing the correlation coefficient \( R \):

\[
R = \frac{T'_{g1} T'_{g2}}{\sqrt{T_{g1}^2} \sqrt{T_{g2}^2}},
\]

where the prime denotes a fluctuating component. By expressing \( T_g \) and \( G \) as \( T_g = T_g^* + T_g' \) and \( G = G^* + G' \) and using Eq. (1), we obtain the relations \( T_{g1}' = T_{g1} - T_{g1}' = T_1' + r_1 G_1' \) and \( T_{g2}' = T_{g2} - T_{g2}' = T_2' + r_2 G_2' \). Now, we can rewrite Eq. (4) using the following set of equations:

\[
\begin{align*}
\overline{T_{g1}^2} &= a_{11} + 2 \beta_{11} \tau_1 + \gamma_{11} \tau_1^2, \\
\overline{T_{g2}^2} &= a_{22} + 2 \beta_{22} \tau_2 + \gamma_{22} \tau_2^2, \\
\overline{T_{g1}' T_{g2}'} &= a_{12} + 2 \beta_{12} \tau_1 + 2 \gamma_{12} \tau_1 \tau_2 + \gamma_{12} \tau_1 \tau_2,
\end{align*}
\]
where every element of the coefficients $\alpha$, $\beta$ and $\gamma$ is defined by

\[
\begin{align*}
\alpha_{11} &= T_1^2, & \alpha_{22} &= T_2^2, & \alpha_{12} &= T_1 T_2, \\
\beta_{11} &= G_1 T_1, & \beta_{12} &= G_1 T_2, & \beta_{21} &= G_2 T_1, & \beta_{22} &= G_2 T_2, \\
\gamma_{11} &= G_1^2, & \gamma_{22} &= G_2^2, & \gamma_{12} &= G_1 G_2. \\
\end{align*}
\]

(6)

The time constants to maximize $R$ will satisfy the following conditions:

\[
\begin{align*}
\frac{\partial R}{\partial \tau_1} &= 0, \\
\frac{\partial R}{\partial \tau_2} &= 0. \\
\end{align*}
\]

(7)

The substitution of Eqs. (4) and (5) into Eq. (7) yields the quadratic equations for $\tau_1$ and $\tau_2$:

\[
\begin{align*}
\tau_1^2 + \frac{(a,d) + (b,c)}{(b,d)} \tau_1 + \frac{(a,c)}{(b,d)} &= 0, \\
\tau_2^2 + \frac{(a,d) + (c,b)}{(c,d)} \tau_2 + \frac{(a,b)}{(c,d)} &= 0, \\
\end{align*}
\]

(8)

where the bracket $\langle \cdot \rangle$ denotes a mathematical operator defined as $\langle p,q \rangle = p_1 q_2 - p_2 q_1$, and the elements are given by

\[
\begin{align*}
a_1 &= \alpha_{11} \beta_{12} - \alpha_{12} \beta_{11}, & a_2 &= \alpha_{22} \beta_{21} - \alpha_{12} \beta_{22}, \\
b_1 &= \beta_{11} \alpha_{12} - \alpha_{12} \beta_{11}, & b_2 &= \alpha_{22} \gamma_{12} - \beta_{12} \gamma_{12}, \\
c_1 &= \alpha_{11} \gamma_{12} - \beta_{11} \gamma_{12}, & c_2 &= \beta_{21} \gamma_{22} - \alpha_{12} \gamma_{22}, \\
d_1 &= \beta_{11} \gamma_{12} - \beta_{21} \gamma_{11}, & d_2 &= \beta_{22} \gamma_{12} - \alpha_{12} \gamma_{22}. \\
\end{align*}
\]

(9)

Rewriting Eq. (8) as $\tau_1^2 + B_1 \tau_1 + C_1 = 0$ and $\tau_2^2 + B_2 \tau_2 + C_2 = 0$ and solving these equations on condition that $\tau_1 > 0$ and $\tau_2 > 0$, we finally obtain

\[
\begin{align*}
\tau_1 &= (-B_1 + \sqrt{B_1^2 - 4C_1})/2, \\
\tau_2 &= (-B_2 + \sqrt{B_2^2 - 4C_2})/2. \\
\end{align*}
\]

(10)

Compensation of the thermocouple response using the time constants thus obtained will give a maximum correlation coefficient between the two thermocouples. It is noted here, however, that Eq. (10) becomes mathematically trivial when a fluid temperature $T_g$ fluctuates in an exactly sinusoidal form of a single frequency. This is because the correlation coefficient $R$ becomes maximum ($R = 1$) whenever no phase difference exists between the two compensated temperatures $T_g$ and $T_g$ irrespective of their amplitudes. Fortunately, there is no such case where real turbulent temperature field is composed strictly of a single sinusoidal fluctuation, and therefore this defect in Eq. (10) causes hardly any problem in the application of the present scheme.

Before a practical application of the two-thermocouple probe technique, we need to examine whether the frequency response of the thermocouples can be represented by the first-order lag system [Eq. (1)]. Equation (1) holds if a thermal inertia term of the local heat balance equation of a fine wire is balanced with a convective heat transfer term. This means that all other terms, i.e., conductive (axial and radial) and radiative heat transfer, internal heating due to electric current, and surface reaction (catalytic heating), should be negligible. As for a fine-wire thermocouple, we may neglect the conductive heat transfer in the radial direction and the internal heating. In the present experiment, the thermal radiation and the surface reaction contribute little to the heat transfer, and thus the assessment of the axial-direction heat conduction becomes a key issue. It is widely recognized that one can neglect the axial-direction heat conduction when $l/d > 200$ ($l$ is wire length; $d$ is wire diameter) or $l/l_c > 10$ [$l_c$ is cold length = $(a_s \tau)^{1/2}$; $a_s$ is thermal diffusivity of wire; $\tau$ is time constant] for a fine-wire thermocouple, and when $l/d > 1000$ for a cold wire. The difference in the requirement between a thermocouple and a cold wire results from the principles of operation of these thermometers. A cold wire will sense the temperature spatially averaged over the entire wire length, while a fine-wire thermocouple can detect the temperature at the hot junction (normally located at the midpoint between the supports). Petit et al. have shown that the first-order lag system can represent the frequency response of a fine-wire thermocouple which meets the above requirement. We therefore make the two-thermocouple probe along the above guideline.

The present scheme is applicable to various temperature sensors as far as their frequency responses are expressed in Eq. (1). If this is not the case, when for example, a second- or a higher-order system applies well rather than the first-order system, we may not obtain reliable results. Thus, we have to confirm prior to the application that the first-order lag system is an appropriate model for a sensor used.

From now on, we term the estimation scheme of Eq. (3) an $e_{\text{min}}$ method and that of Eq. (10) an $R_{\text{max}}$ method, respectively, and test these methods.

**III. EXPERIMENTAL APPARATUS AND PROCEDURE**

The experimental apparatus and the coordinate system are shown in Fig. 1. The origin of the coordinate axes is located in the center of a wind-tunnel exit (some details of the wind tunnel are described in a previous article). An air flow behind a heated cylinder, i.e., the wake, forms turbulent velocity and thermal fields. The cylinder is a quartz–glass pipe 11 mm in diameter and 230 mm in length, which contains an electric heater of power consumption 400 W. In the

![FIG. 1. Experimental apparatus and coordinate system.](image-url)
The two-thermocouple probe is composed of two ports 100 mm upstream from the exit.

The end of each wire is cut 20 mm in length, and are soldered on steel needles (prongs) for support. Since the time constant of the cold wire remains 0.3–0.4 ms under the present experimental condition, its frequency response is far faster than that of the thermocouples. In the cold-wire measurement, however, the crucial problem is that the frequency response of the cold wire can deteriorate in a low-frequency region due to heat conduction along its axis. Hence, we need to take into account not only the ordinary response delay to high-frequency temperature fluctuations but also the deterioration in response at the low-frequency region (see the Appendix). After compensating the cold-wire response in the manner shown in the Appendix, we may obtain a reference (true) fluid temperature $T_g$.

Here, we have some reason to use a somewhat long cold wire ($l=1.5$ mm). In the cold-wire techniques, the two contradictory properties of the cold wire should be taken into account: one is the attenuation of the low-frequency components of temperature fluctuations due to the axial-direction heat conduction, and the other the attenuation of high-frequencies due to eddy averaging, resulting from the finite spatial resolution of the cold wire. We can compensate the attenuation due to the heat conduction as shown in the Appendix. On the one hand, the eddy-averaging effect can be estimated by the Wyngaard’s analysis. From the analysis for the cold wire used ($l=1.5$ mm), we find that the root-mean-square (rms) value of temperature fluctuations will diminish by 4% at most. If we apply the same analysis to a much shorter cold wire, $l=0.8$ mm as an example, we have 1.6% decrement in the rms value. This shows that the use of a shorter cold wire improves slightly the accuracy in the rms measurements. However, the attenuation due to the axial-direction heat conduction becomes more pronounced when $l=0.8$ mm, and the gain in the low-frequency region decreases by about 15% from that shown in Fig. 8. Consequently, the use of a short cold wire will need large compensation for reproducing fluid (true) temperature. In addition, a short cold wire is likely to cause fluid-dynamical interference between the cold-wire stubs and the thermocouple junctions. We thus use the somewhat long cold wire of $l=1.5$ mm to give a sufficient gain to the low-frequency components which make a key contribution to the power spectrum density of fluid temperature fluctuations.

The electromotive forces (emfs) of the thermocouples are amplified by a factor of 1000 with high-precision instrumentation amplifiers. An electric current driving the cold wire is 0.27 mA, and internal heating of the wire due to this

![Diagram](image_url)

**FIG. 2.** Details of two-thermocouple probe with cold wire.
current is negligible. The voltage across the wire is amplified by a factor of 500. These outputs are digitized by a 12-bit analog to digital (A/D) converter (Canopus ADXM) at a sampling frequency of 5 kHz with a range 0–5 V, then stored and processed on a personal computer (CPU: Intel DX4, 100 MHz). The number of samples is 30,000 for each wire. The noise in the instrumentation-amplifier output and the quantized errors due to the A/D conversion, when evaluated in terms of temperature, are 0.03 and 0.03 K for the thermocouples, and 0.15 and 0.20 K for the cold wire, respectively.

IV. RESULTS AND DISCUSSION

A. Preliminary appraisal of $e_{\text{min}}$ and $R_{\text{max}}$ methods in combustion flow

To test the performance of the $e_{\text{min}}$ [Eq. (3)] and the $R_{\text{max}}$ [Eq. (10)] methods, we applied these schemes to a combustion flow case, where two kinds of two-thermocouple probes were used depending on the flow conditions: one is composed of 40 and 100 $\mu$m R-type thermocouples and the other of 25 and 60 $\mu$m thermocouples. Table I shows a comparison of the time constants estimated by the $e_{\text{min}}$ and $R_{\text{max}}$ methods. These two different schemes give virtually the same results. The previous experiment was carefully arranged and the measurement conditions were controlled to be suitable for the appraisal of the two-thermocouple probe technique. Consequently, these two schemes may work equally well in the previous combustion experiment.

B. Mean characteristics of velocity and thermal fields in the wake of a heated cylinder

To outline the velocity and thermal fields of the wake, we have measured the mean and rms velocities using a hot-wire probe and a mean temperature with a $K$-type thermocouple probe. In the present experiment, the mean flow velocity is $\bar{U}$=4 m/s at the wind-tunnel exit. We have performed the velocity measurement under the isothermal condition in order to utilize the hot-wire technique. This will be justified because the temperature can be regarded as a passive scalar in this experiment. Figure 3 shows the profiles of these measurements. The mean velocity profile takes a minimum at $y=0$ mm where the cylinder is located, and approaches the free stream velocity in the $y>20$ mm region. The rms velocity has a gentle profile ranging from 0.4 to 0.8 m/s in the $y$ direction. From spectral analysis of the velocity fluctuations, it is found that the power spectrum densities become maximum around 20–60 Hz, and the frequency components higher than 1 kHz indicate little contribution to the power spectra. The mean temperature profile becomes symmetrical at $y=0$ mm naturally and is similar to a Gaussian distribution. The maximum difference in the mean temperature profile is at most 22 K. The temperature gradient becomes steepest at $y=15$ mm, where the rms temperature peaks (Fig. 6).

C. Efficient calculation of derivative of hot-junction temperature

The $e_{\text{min}}$ and the $R_{\text{max}}$ methods need the accurate derivative values of the hot-junction temperature (raw/uncompensated temperature). Thus, a polynomial curve-fitting method should be useful. Although the method is intrinsically suitable for smoothing noisy data, it also provides efficient computation of the derivative $G$. Here, we use the second-order polynomial for the curve fitting. The concrete procedure follows below.

Now, let $\Delta t$ and $y_{n+i}$ be a sampling time interval of the A/D conversion and discrete temperature data at the time $(n+i)\Delta t$, respectively, where $i$ denotes the offset from the time $n\Delta t$. Then, a curve-fitted value $\tilde{y}_{n+i}$ for the data $y_{n+i}$ will be expressed as a function of $i$:

$$\tilde{y}_{n+i} = a_n i^2 + b_n i + c_n,$$

(11)

where the coefficients $a_n$, $b_n$ and $c_n$ can be determined through the minimization of the following mean square value:

$$e = \sum_{i=-m}^{m} (y_{n+i} - \tilde{y}_{n+i})^2,$$

(12)

where $m$ is a length of the data window. Differentiation of Eq. (11) on $i$ leads to $d\tilde{y}_{n+i}/di|_{i=0} = b_n$. Thus, the first-order derivative at the time $n$, $G_n$, is given by $G_n = b_n/\Delta t$, and we derive the following formula for $b_n$:

$$b_n = -\frac{3}{m(m+1)(2m+1)} \sum_{i=-m}^{m} i y_{n+i}.$$

(13)

As seen from Eq. (13), we can compute $b_n$ very efficiently in the same manner as the moving-average method. Since the polynomial curve-fitting method works as a kind of low-pass filter, high-frequency fluctuations will be attenuated by increasing the window length $m$ in Eq. (12). Because of this.

### Table I. Comparison of the time constants estimated by the $e_{\text{min}}$ and $R_{\text{max}}$ methods using previous combustion measurements by Tagawa and Ohta.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\tau_1$ (d1:40 $\mu$m)</th>
<th>$\tau_2$ (d1:100 $\mu$m)</th>
<th>$\tau_1$ (d2:25 $\mu$m)</th>
<th>$\tau_2$ (d2:60 $\mu$m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\text{min}}$ method</td>
<td>98.4 ms</td>
<td>31.4 ms</td>
<td>8.3 ms</td>
<td>31.9 ms</td>
</tr>
<tr>
<td>$R_{\text{max}}$ method</td>
<td>101.5 ms</td>
<td>34.9 ms</td>
<td>9.9 ms</td>
<td>34.0 ms</td>
</tr>
</tbody>
</table>

*See Ref. 12.*
we need to find what is the most adequate value for \( m \). Figure 4 may give guidelines on this matter, where the variations of the correlation coefficient \( R_m \) and the time constant \( \tau_1 \) (both obtained by the \( R_{\text{max}} \) method) are indicated as a function of \( m \). As seen in Fig. 4, \( R_m \) increases gradually with increasing \( m \). This is because both the noise and the high-frequency temperature fluctuations, which show no correlation between the two thermocouples, are increasingly diminished as \( m \) becomes large. For the further large \( m \) value, however, attenuation of the high-frequency components becomes too strong due to the low-pass filtering effect of Eq. (13) to reproduce the fluid temperature, and will cause a gentle decrease in the \( R_m \) value (somewhat difficult to distinguish from the decrease in Fig. 4). The time constant of the 25 \( \mu \)m thermocouple, \( \tau_1 \), also indicates only a slight variation over a wide range of \( m \). The same applies to the 51 \( \mu \)m thermocouple case. On the basis of the results, we have determined to use \( m = 15 \) in the stage of estimating the time constants to maximize \( R_m \), because both thermocouples are then participating. When compensating each thermocouple response, on the other hand, we have used \( m = 3 \) so as to reproduce high-frequency components of temperature fluctuation within a permissible level of noise.

D. Estimation of time constants in a nonisothermal turbulent wake

In this section, we estimate the thermocouple time constants \( \tau_1 \) and \( \tau_2 \) using the \( \epsilon_{\text{min}} \) [Eq. (3)] and the \( R_{\text{max}} \) [Eq. (10)] methods with the aid of the above scheme for calculating \( G \). The results are shown in Fig. 5. The upper part of the figure shows the distributions of \( \tau_1 \) (\( d_1 = 25 \mu \)m) and the lower one those of \( \tau_2 \) (\( d_2 = 51 \mu \)m). When the time constants estimated by the \( \epsilon_{\text{min}} \) method (\( \bigtriangleup \)) are compared with those by the \( R_{\text{max}} \) method (\( \bigcirc \)), we find that the \( \epsilon_{\text{min}} \) method gives consistently smaller values. The ratio ranges from 0.6 to 0.8. Needless to say, the use of a thermocouple instead of a cold wire would be inappropriate for measuring ordinary temperature with small amplitude fluctuation. Hence, the present measurement may suffer from a much lower signal-to-noise ratio than the previous combustion case in Sec. IV A. In addition, the cold wire placed between the two thermocouples (Fig. 2) makes the measurement volume of the probe, i.e., size of the sensing part, larger than that of an original two-thermocouple probe. These two factors may cause the discrepancy in the results between the \( \epsilon_{\text{min}} \) and the \( R_{\text{max}} \) methods. It is noted here that this discrepancy is not always linearly reflected in the measurements of turbulence quantities.

Apart from the above estimation, we can obtain a reference/true time constant by minimizing the time-averaged difference between the cold-wire measurement \( T_g \) and the compensated-thermocouple one \( T_{g1} \) or \( T_{g2} \), which is mathematically expressed as \((T_g - T_{g1})^2\) or \((T_g - T_{g2})^2\). In short, this procedure will fit the compensated-thermocouple temperature \( T_{g1} \) or \( T_{g2} \) to the fluid temperature \( T_g \). The result is included in Fig. 5 (\( \Diamond \)). A comparison between the reference time constant thus estimated and those obtained using the two-thermocouple probe technique shows that the \( R_{\text{max}} \) method gives better results than the \( \epsilon_{\text{min}} \) method. From Table I and Fig. 5, we may conclude that the \( R_{\text{max}} \) method is robust and has higher reliability than the \( \epsilon_{\text{min}} \) method.

As stated in Sec. I, if the heat transfer coefficient of a fine wire is given, we can predict a time constant of the fine wire. We have performed this prediction using the well-known Collis-Williams law$^{34}$ for the heat transfer coefficient together with the mean velocity and temperature distributions shown in Fig. 3. The physical properties of the \( K \)-type (chromel–alumel) thermocouple wires were replaced by those of nickel, since both chromel and alumel consist mainly of nickel (the nickel content is more than 90%). As for a flow without combustion, as in the present experiment, it is widely accepted that the above approach can predict the time constant of a fine wire fairly accurately. The time-constant values thus predicted are also included in Fig. 5. A comparison between this prediction and the \( R_{\text{max}} \) result for the 51 \( \mu \)m thermocouple shows that both are in good agreement for all but the regions around \( y = 0 \) mm and the
thermal-boundary-layer edge where the temperature fluctuations become small (Fig. 6). As for the 25 μm thermocouple case, on the other hand, the discrepancy between them becomes a bit large into the y > 10 mm region. In the present stage, we cannot unequivocally state the reason for the discrepancy, but it is legitimate to ascribe it to limited applicability of the Collis–Williams law. In addition, as for the 25 μm thermocouple, we can hardly make a sufficiently small bead size relative to the wire diameter, and this may lead to the larger discrepancy compared with the 51 μm thermocouple case.

From the results of Fig. 5, we use the $R_{\text{max}}$ method for further compilation of the data.

E. Compensation of thermocouple response using the $R_{\text{max}}$ method

Figure 6 shows the rms values of the uncompensated temperatures $T_1$ and $T_2$ (○, △) and those of the compensated ones $T_{g1}$ and $T_{g2}$ (●, ▲). The compensated rms values become 3–4 times as large as the uncompensated ones in the 25 μm thermocouple case, and 5–6 times as large as those in the 51 μm thermocouple case. In reality, a thermocouple is not suitable for measurement of ordinary temperature fluctuations with small amplitude. In this sense, the present experiment should be regarded as a rigorous test for the compensated-thermocouple technique. It is clear from Fig. 6 that the compensated rms temperatures are in good agreement with the cold-wire measurement (□), and therefore we may expect that the $R_{\text{max}}$ method will work well even under demanding experimental conditions.

The good agreement of the compensated rms temperatures does not always guarantee correct reproduction of instantaneous temperature signals. Thus, we compare the instantaneous signal traces of the compensated thermocouples with that of the cold wire at $y = 14$ mm as an example. The result is shown in Fig. 7. The cold wire provides a reference signal. The uncompensated signals are presented in the upper part of Fig. 7, which cannot trace the reference signal at all. This means that the uncompensated thermocouple does not allow us to obtain turbulence characteristics such as probability density function or power spectrum density. The compensated signals shown in the lower part of Fig. 7, on the other hand, can reproduce the instantaneous fluid temperature with satisfactory accuracy. The slight deviation seen in the result can principally be attributed to inherent differences in both the spatial position and the region for sensing between the thermocouple and the cold wire.

On the basis of our experience, we may conclude that the time constants thus estimated will be reliable when $R > 0.99$.

ACKNOWLEDGMENTS

The authors would like to thank S. Nagaya for his assistance in constructing the experimental apparatus. This work was partially supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan (Grant No. 09650235).

APPENDIX: FREQUENCY RESPONSE AND COMPENSATION OF A FINE-WIRE RESISTANCE THERMOMETER

The frequency response of a fine-wire resistance thermometer (cold wire) is given by an analytical formula. Figure 8 shows a bode diagram—gain $\eta$ and phase lag $\phi$ ($\phi > 0$) versus frequency—for the cold wire in Fig. 2. The cold wire is placed in an air flow whose mean velocity and mean temperature are 4 m/s and 300 K, respectively. In Fig. 8, the time constant of the prong (supporting needle) is assumed to be 1 s. The gain $\eta$ indicates a stepwise decrease in a low-frequency region around 0.1 Hz because of the thermal inertia of the prong, and then holds a constant value lower than unity, which is due to the heat conduction in the axial direction of the cold wire. In a frequency region higher than 20 Hz, $\eta$ begins to fall again because of the thermal inertia of both the stub and the wire itself. In reality, the bode diagram will be influenced by velocity fluctuation and uncer-
tainty in the time-constant value of the prong. However, both effects are negligible in the present experiment.

On the basis of the bode diagram shown in Fig. 8, we may compensate the cold-wire response. The procedure can be summarized as follows:

(1) The time-series data of $2^N$ samples from the cold wire are transformed into frequency components by the fast Fourier transform (FFT). In the following, the real and imaginary components thus obtained are denoted by $x_R$ and $x_I$, respectively. The total number of samples was 8192 ($N = 13$) in this study. To avoid obtaining a false spectrum due to discontinuity at both ends of the time-series data, we have applied a cosine-type data window to fluctuating parts of the 100 samples from the ends.

(2) The measurements of mean velocity and mean temperature (Fig. 3) permit us to calculate the gain $\eta$ and the phase $\phi$ in the same way as in Fig. 8. Then, a compensation function for the cold-wire response, $A$, is given by

$$A = \exp(j \phi)/\eta,$$  \hspace{1cm} (A1)

where $j$ denotes the imaginary unit.

(3) Using Eq. (A1), response compensation in the frequency domain is given by $(x_R + j x_I)A$. Then, the inverse FFT of this will output the compensated cold-wire measurement, i.e., a reference fluid temperature, $T_g$. After this stage, we exclude the 100 samples at both ends from further data compilation so that we can diminish the influence of the data window. Since the present cold-wire measurement is somewhat affected by the electric noise and the quantized error due to the A/D conversion, we have damped frequency components higher than 1 kHz using a finite impulse response (FIR) digital low-pass filter. This operation has little influence on the rms temperatures shown in Fig. 6. It is noted here that a compensation scheme for the thermocouple response by Bradley et al. provided useful information on a FFT-based compensation technique.

Procedures (1)–(3) enable us to obtain the reference fluid temperature. In the present experiment, the compensation of the cold-wire response increased the rms temperature by 20% in comparison with the uncompensated one. This is primarily due to restoration of the low-frequency components of the cold-wire signal. As seen from Fig. 8, the systematic (bias) error will be introduced into the cold-wire measurement because of the deterioration in the response to low-frequency temperature fluctuations, unless the length-to-diameter ratio $l/d$ ($l$ is the length of the sensing part) is larger than at least 1000. On the other hand, temperature measurement by a thermocouple of $l/d > 200$ is almost free from this kind of error, since the thermocouple will inherently provide point measurement and hardly suffer from the heat loss due to heat conduction along the wire axis. Thus, the frequency response of a fine-wire thermocouple can be simply expressed as the first-order lag system given by Eq. (1). Compensation of the response only to high-frequency temperature fluctuations is therefore necessary. A fine-wire resistance thermometer (cold wire) generally shows a much faster response than a thermocouple. However, great caution is needed with regard to its quantitative accuracy in the fluctuating temperature measurement, since a cold wire does not always show a proper response to every frequency component of fluctuating temperature.

**NOMENCLATURE**

- $d$: wire diameter
- $e$: mean square value of the difference between two compensated temperatures, $T_{g1}$ and $T_{g2}$ [Eq. (2)]
- $G$: time derivative of hot-junction temperature $=dT/dt$
- $m$: length of data window [Eq. (12)]
- $R$: correlation coefficient between $T_{g1}$ and $T_{g2}$ [Eq. (4)]
- $T$: hot-junction (raw/uncompensated) temperature
- $T_g$: compensated temperature
- $t$: time
- $U$: flow velocity
- $y$: coordinate axis (Fig. 1)

**Greek symbols and others**

- $\Delta T_{21}$: difference between two hot-junction temperatures $=T_{2} - T_{1}$
- $\tau$: thermocouple time constant [Eq. (1)]
- $\langle \cdot \rangle$: time average
- $\hat{(\cdot)}$: fluctuation component
- $(p,q)$: mathematical operator $=p_1q_2 - p_2q_1$
- 1,2: thermocouple 1, thermocouple 2
- $i,n$: $i$th, $n$th in time-series data

---