Power Consumption of Mixing Impellers in Non-Newtonian Fluids

——Simple estimation of average shear rate——

Setsuro HIRAOKA, Ikuho YAMADA, Kazutoshi SUZUMURA, Akira KAWAI, Akira ISONO and Ryuzo ITO*.

Department of Engineering Chemistry

(Received September 2, 1975)

Based on the assumption that the flow behaviour in agitated vessel is specified by the impeller tip condition, the average shear rate is derived from the correlation curve for the friction factor in laminar range. This average shear rate gives a good correlation to frication factor in pseudo-plastic fluid with the modified Reynolds number in both laminar and turbulent ranges. And this result points out that the character istic length defined by $(D/2)\ ln\ (D/d)$ should be corrected by the factor η for the proximity impeller, which is determined experimentally. Finally the friction factor for both Newtonian and non-Newtonian fluids is well correlated with the single curve for the wide range of d/D.

Introduction

Agitation of non-Newtonian fluid is very important in chemical processes, and many investigators have proposed various design equations for the power input. Ito[9] has introduced the concept of "energy similarity law", i.e., the over-all energy consumption of mixer in non-Newtonian fluid is similar to that in Newtonian fluid, and proposed the calculation method of modified Reynolds number. Metzner-Otto[11] and others [1,2,5,12,15–18,20] have introduced the concept of "apparent viscosity" into the correlation of power input in non-Newtonian fluid, which is estimated with the average shear rate, $\dot{\gamma}_{av}$, in agitated vessel, as follows;

$$\mu_a = \frac{\tau(\dot{\gamma}_{av})}{\dot{\gamma}_{av}} \tag{1}$$

where $\tau=\tau(\dot{\gamma})$ is the flow curve of the fluid. Then the estimation of apparent viscosity is replaced with that of average shear rate. Metzner-Otto has assumed that the average shear rate is proportional to the rotational speed of impeller

$$\dot{\gamma}_{av} = \kappa N \tag{2}$$

and obtained that $\kappa=13$ for some impellers. After that, Calderback-MooYoung[2] and others[1,15,16] have shown experimentally that κ depends on the impeller dimensions, and have proposed the empirical formula for κ. But no general estimation of κ has yet been proposed, because being not obvious the physical meaning of average shear rate in agitated vessel. Schilo[19], Chavan-Ulbrecht[3,4] and Mitsuishi-Hirai[13] have proposed the direct correlation method of power consumption in non-Newtonian fluid, based on the analysis of flow pattern in coaxial rotating cylinders. This analytical method gives the complicated correlation equations.

This paper deals with the simple estimation of the average shear rate based on the concept of friction factor instead of power input.

Experimental Apparatus and Procedures

The friction factor at the wall of agitated vessel

*Dept. of Chem. Eng., Fac. of Eng. Sci., Osaka Univ., Toyonaka, Japan. 560.

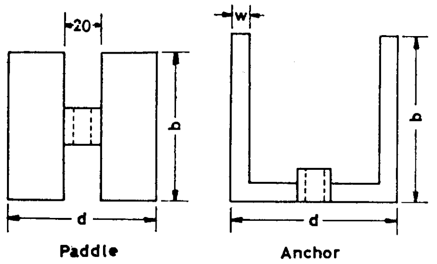

Fig. 1 Impeller types.

have been measured for paddle impellers and anchors, shown in **Fig. 1**. The agitated vessel used is 10cm inner diameter and 10cm height without free surface. The geometries of paddle impellers are characterized by $d/D=0.42$, 0.52, 0.62, 0.72, 0.82, 0.92 and $b/D=0.80$, and that of anchors by $d/D=0.88$, $b/D=0.92$ and $w/D=0.10$ and $d/D=0.95$, $b/D=0.95$ and $w/D=0.10$, respectively. The impeller is set in the vessel centre.

The fluids used are glycerol aqueous solutions for Newtonian fluid and CMC aqueous solutions for non-Newtonian fluid. The flow behaviour index, n, varies in the range 0.63 to 0.96, and the consistency index, K, varies in the range 0.26 to $29.9\,\mathrm{g/cm\cdot sec^{n-2}}$.

The detail of the experimental apparatus and procedures are given in the previous paper [14].

Estimation of Average Shear Rate

Derivation of average shear rate in agitated vessel is based on the assumption that the flow behaviour is specified by the impeller tip condition [8].

In laminar range the shear stress at the vessel wall for the large size impellers is expressed by the following equation [6].

$$\frac{f}{2}=\frac{\tau_w}{\rho v_\theta^2}=\frac{1.2\,(b/H)}{Re_G} \tag{3}$$

From the force balance the apparent shear stress at the impeller tip, τ_d, is expressed as;

$$\left(\frac{\pi}{2}d^2 b\right)\cdot\tau_d=\left(\frac{\pi}{2}D^2 H\right)\cdot\tau_w \tag{4}$$

Based on the assumption mentioned above, the average shear rate, $\dot{\gamma}_{av}$, is equal to the apparent shear rate at the impeller tip. The combination of Eqs. (3) and (4) gives the following equation to the average shear rate in agitated vessel.

$$\dot{\gamma}_{av}=\frac{\tau_d}{\mu_a}=\left\{7.5\left(\frac{D}{d}\right)\Big/\left(\frac{D}{d}\right)-\left(\frac{d}{D}\right)\right\}\cdot N \tag{5}$$

Where it is assumed that the average shear rate for non-Newtonian fluid is equal to that for Newtonian fluid. This formula is very simple compared with the previous empirical ones and similar in the form to that by Calderbank-MooYoung [3].

The experimental values of κ measured by many investigators are plotted to compare with each other in **Fig. 2**. Eq. (5) represents well the experimental

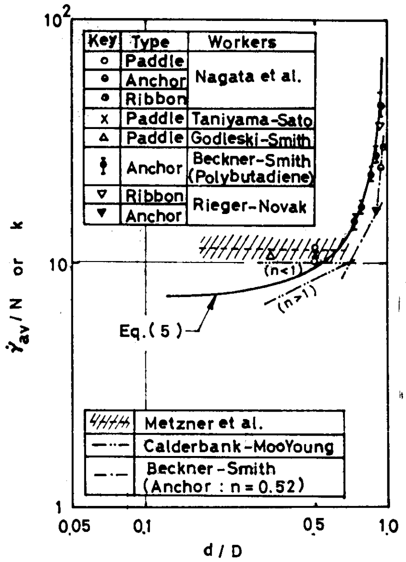

Fig. 2 Comparison of the average shear rate calculated from Eq. (5)with the experimental results.

values in the range of $d/D=0.4\sim0.8$, but for the proximity impellers of $d/D>0.8$ Eq. (5) seems to overestimate the value of κ. This discrepancy may be caused by the uncertainty of Eq. (3) in this range of d/D.

Correlation of Friction Factor

The friction factor and the modified Reynolds number are obtained from the following definitions [7].

$$f=\frac{\tau_w}{(\rho v_\theta^2/2)} \tag{6}$$

$$Re_G=\frac{L v_\theta \rho}{\mu_a} \tag{7}$$

where L and v_θ are the characteristic length and velocity, respectively. And the apparent viscosity, μ_a, is given by the combination of Eqs. (1) and (5).

$$\mu_a=K\left\{7.5\frac{(D/d)}{(D/d)-(d/D)}\cdot N\right\}^{n-1} \tag{8}$$

The correlation of friction factor are shown in **Fig. 3**, for two different size paddle impellers. It is clear

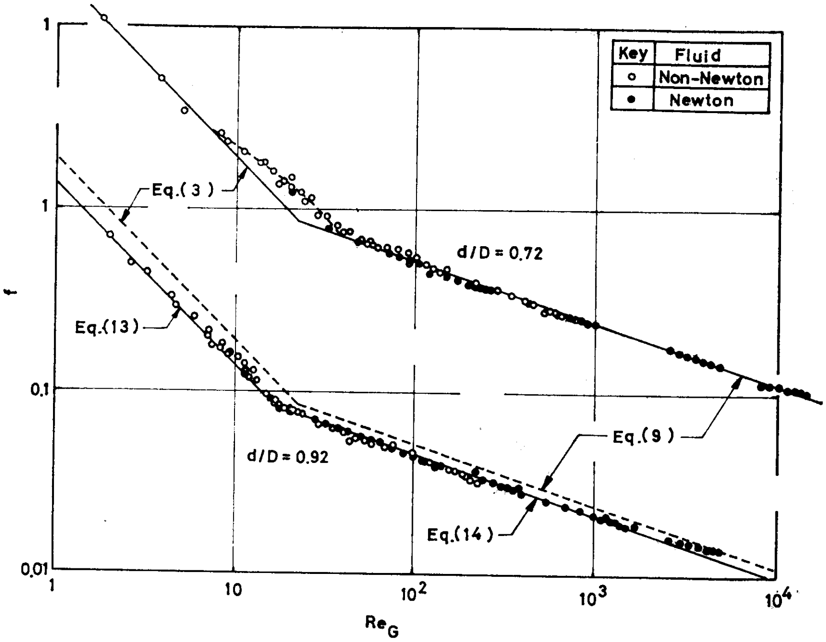

Fig. 3 Correlation of friction factor in Newtonian and non-Newtonian fluids for paddle impellers. (d/D=0.72 and 0.92)

that the experimental values for both Newtonian and non-Newtonian fluids are well correlated with the single curve not only for the laminar range, but also for the turbulent range. And the data on d/D=0.72 are expressed by Eq. (3) in laminar range and by the following equation in turbulent range.

$$\frac{f}{2}=0.121 Re_G^{-1/3} \tag{9}$$

But the data on d/D=0.92 shift slightly from Eqs. (3) and (9).

The coeiffcients in correlation equations are shown in Fig. 4 for laminar range and in Fig. 5 for turbulent range, respectively, for different sizes of d/D. It is also obvious in these figures that Eqs. (3) and (9) do not satisfy the experimental data for the proximity impellers.

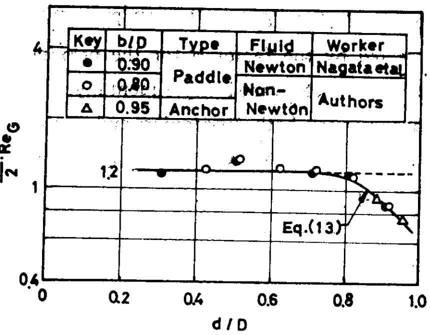

Fig. 4 Correlation for laminar range.

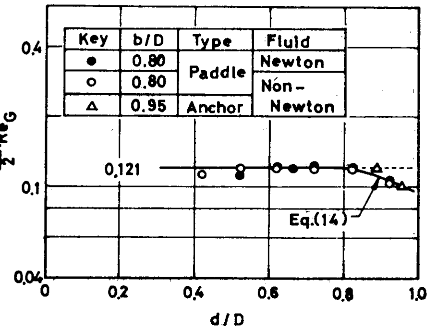

Fig. 5 Correlation for turbulent range.

The limit of applicability seems to be attributed not to the estimation of μ_a, but to the characteristic length in the modified Reynolds number, because the data for both Newtonian and non-Newtonian fluids are well represented by the same curve in the full range of d/D. Then the characteristic length should be corrected for the proximity impeller.

Correction of Characteristic Length

For the proximity impeller the characteristic length corrected by the experimental results is described by using the correction factor, η.

$$L'=L\cdot\eta=\left\{\frac{D}{2}\,ln\left(\frac{D}{d}\right)\right\}\cdot\eta \tag{10}$$

where η is determined as follows. (see Fig. 6)

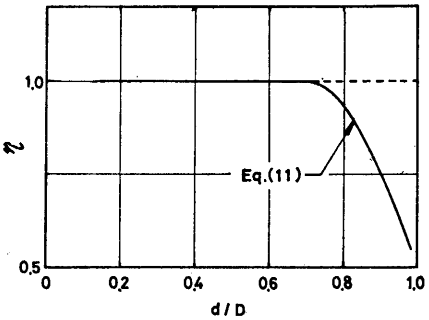

Fig. 6 Correction factor for characteristic length

$$\eta=1+\exp[-10\{(D/d)-1\}] \tag{11}$$

It is noted that the form of the function η does not unique.

The modified Reynolds number is also corrected with η, as follows.

$$Re'_G=\frac{L'v_\theta\rho}{\mu_a}=\eta\cdot Re_G \tag{12}$$

Then the friction factor for the proximity impeller can be expressed straightly by using this Reynolds number in both laminar and turbulent ranges, as

$$\frac{f}{2}=\frac{1.2\,(b/H)}{\eta\cdot Re_G}\quad\text{(laminar)} \tag{13*}$$

$$\frac{f}{2}=0.121\,(\eta\cdot Re_G)^{-1/3}\quad\text{(turbulent)} \tag{14*}$$

It is shown in Fig. 3 that Eqs. (13) and (14) satisfy well the experimental data on the proximity impeller of $d/D=0.92$. And it is also clear in Figs. 4 and 5 that the coefficients are well corrected by the factor, η, for the range of $d/D>0.8$.

Application to Bingham Plastic Fluid

The flow curve of Bingham plastic fluid is expressed as;

$$\tau=\tau_y+\mu_o\dot{\gamma} \tag{15}$$

where τ_y is yield stress. In the same manner as that for the pseudo-plastic fluid it is assumed that the apparent viscosity of Bingham plastic fluid is described with Eqs. (1) and (15).

$$\mu_a=\frac{\tau_y}{\dot{\gamma}_{av}}+\mu_o \tag{16}$$

Eq. (16) is rearranged by using Eq. (2) to express the dimensionless form.

$$\frac{\mu_a}{\mu_o}-1=\frac{1}{\kappa}\left(\frac{\tau_y}{\mu_oN}\right) \tag{17}$$

This equation has already been derived by Ito[9] and Nagata et al.[15] independently, and compared with experimental results. And Nagata et al. confirmed that the experimental value of κ for Bingham plastic fluid was about twice of that for pseudo-plastic fluid[16]. This is also confirmed with Ito's data on $d/D=0.33$, i.e.[10],

$$\frac{\mu_a}{\mu_o}-1=0.123\left(\frac{\tau_y}{\mu_oN}\right)^{0.915}\qquad(10^3<(\tau_y/\mu_oN)<10^5) \tag{18}$$

$$\approx\frac{1}{(2)\,(8.8)}\left(\frac{\tau_y}{\mu_oN}\right)\quad\text{(Max. error}\pm20\%)$$

where the value of 8.8 in the right-hand side is nearly equal to the calculated value of $\kappa=7.8$ by Eq. (5).

Conclusive Remarks

Based on the assumption that the flow behaviour in agitated vessel is specified by the impeller tip condition, the average shear rate is derived from the correlation curve for the friction factor in laminar range. This average shear rate gives a good correlation to friction factor in pseudo-plastic fluid with the modified Reynolds number in both laminar and turbulent ranges. And this result points out that the characteristic length defined by $(D/2)\,ln\,(D/d)$ should be corrected by the factor η for the proximity impeller, which is determined experimentally. Finally the friction factor for both Newtonian and non-Newtonian fluids is well correlated with the single curve for the wide range of d/D.

---

*Eqs. (13) and (14) are rearranged with N_p and Re_d, respectively, as;

$$N_p\cdot Re_d=36\left(\frac{n_pb}{D}\right)\left(\frac{D}{d}\right)^2\Big/\left(\frac{D}{d}-\frac{d}{D}\right)\cdot\eta \tag{13a}$$

$$N_p\cdot Re_d^{1/3}=10.1\left[\left\{ln\left(\frac{D}{d}\right)\right\}^4\Big/\eta\left\{1-\left(\frac{D}{d}\right)^2\right\}^5\right]^{1/3} \tag{14a}$$

Nomenclature

b = impeller height [cm]
d = impeller diameter [cm]
D = vessel diameter [cm]
f = friction factor [—]
H = vessel height [cm]
K = fluid consistency [$g/cm \cdot sec^{n-2}$]
L = characteristic length [cm]
L' = corrected characteristic length [cm]
n = flow behaviour index [—]
n_p = number of impeller blade [—]
N = rotational speed [sec^{-1}]
N_p = power number [—]
Re_d = impeller Reynolds number [—]
Re_G = modified Reynolds number [—]
v_θ = characteristic velocity [cm/sec]
w = impeller width [cm]
$\dot{\gamma}$ = shear rate [sec^{-1}]
$\dot{\gamma}_{av}$ = average shear rate [sec^{-1}]
η = correction factor defined by Eq. (11) [—]
κ = proportional constant in Eq. (2) [—]
μ_a = apparent viscosity [$g/cm \cdot sec$]
μ_o = plastic viscosity [$g/cm \cdot sec$]
ρ = density [g/cm^3]
τ = shear stress [$g/cm \cdot sec^2$]
τ_d = apparent shear stress at impeller tip [$g/cm \cdot sec^2$]
τ_w = shear stress at wall [$g/cm \cdot sec^2$]
τ_y = yield stress [$g/cm \cdot sec^2$]

Literature cited

1) Beckner, J. L. and J.M. Smith; *Trans. Instn. Chem. Engrs.*, **44**, 224 (1966)
2) Calderbank, P.H. and M.B. Moo-Young; *ibid*, **37**, 26 (1959)
3) Chavan, V.V., A.S. Thaveri and J. Ulbrecht; *ibid*, **50**, 147 (1972)
4) Chavan, V.V. and J. Ulbrecht; *Ind. Eng. Chem. Process Design Develop.*, **12**, 472 (1973)
5) Godleski, E.S. and J.C. Smith; *A. I. Ch. E. Jounal*, **8**, 617 (1962)
6) Hiraoka, S. and R. Ito; *J. Chem. Eng. Japan*, **5**, 464 (1973)
7) Hiraoka, S., I. Yamada, N. Doi, H. Takeda, A. Kawai, Y. Usui and R. Ito; *Bull. Nagoya Inst. of Tech.*, **26**, 239 (1974)
8) Hiraoka, S., R. Ito, I. Yamada, K. Sawada, M. Ishiguro and S. Kawamura; *J. Chem. Eng. Japan*, **8**, 156 (1975)
9) Ito, S.; *"Fluid Dynamics"* (Ryutai Kogaku) Kagaku Gijutsusha Press (1972)
10) Ito, S.; Preprint at the Mixing Research Seminar in Kyoto (1973)
11) Metzner, A.B. and R.E. Otto; *A.I. Ch. E. Journal*, **3**, 3 (1957)
12) Metzner, A.B., R.H. Feehs, H.L. Ramos, R.E. Otto and J.D. Tuthill; *ibid*, **7**, 3 (1961)
13) Mitsuishi, N. and N. Hirai; *J. Chem. Eng. Japan*, **2**, 136 (1969)
14) Mizushina, T., R. Ito, S. Hiraoka, A. Ibusuki and I. Sakaguchi; *ibid*, **2**, 89 (1969)
15) Nagata, S., M. Nishikawa, H. Tada, H. Hirabayashi and S. Gotoh; *ibid*, **3**, 237 (1970)
16) Nagata, S., M. Nishikawa, H. Tada and S. Gotoh; *ibid*, **4**, 72 (1971)
17) Pavlushenko, I.S. and M.D. Gluz; *Theor. Foundation Chem. Eng.*, **2**, 105 (1968)
18) Rieger, F. and V. Novak; *Trans. Instn. Chem. Engrs.*, **51**, 253 (1969)
19) Schilo, D.; *Chemie-Ing-Techn*, **41**, 253 (1969)
20) Taniyama, Y. and T. Sato; *Kagaku Kogaku*, **29**, 39 (1965)