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Cooperative Relaying Techniques for Wireless Networks

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Abstract

In general, the advantage of employing space-time codes in multiple-input, multiple-output (MIMO) wireless channels is an enhancement in transmission reliability or system throughput. However, this benefit cannot be implemented in some wireless systems where size or other constraints preclude the use of multiple antennas. Recently, cooperative diversity networks have shown a promising potential in wireless communication systems to overcome this issue by extending the communication range and at the same time, improving the error rate performance in wireless networks. This technique also offers an effective countermeasure against channel fading by providing the receiver with multiple versions of the same information.

To realize these performance enhancements, one of the critical aspects is the relaying algorithm at the relay nodes. The relays have to effectively use the available resources received from the source and cooperate with the source to communicate with the destination. The decisions made at the relay nodes are crucially paramount to the overall performance of cooperative relay networks. Another key aspect which affects the performance of cooperative networks is the choice of the signal combining technique at the destination node. One of the popular techniques is maximum ratio combining (MRC). Nonetheless, this conventional MRC is not optimal in a typical relay scheme since the contribution or adverse effect from the source-relay link is not considered. The occurrence of detection errors at the relays becomes one of the main limitations of multi-hop relaying networks. If the relaying is not done properly, these errors cause significant performance degradation at the destination, a problem usually associated as error propagation.

This thesis contributes to the advancement of wireless relay communications by introducing several novel relaying techniques in wireless networks which enable significant performance improvement with low-complexity relay schemes. The contributions of this thesis can be divided into three parts. The first part focuses on a new signal combining strategy at the destination node based on Maximum Likelihood (ML) criterion which accounts the potential errors at relay nodes. These errors are simply expressed as the Gaussian Q-function for each symbol error rate (SER) of the
constellations. By applying these expressions in the detection at the destination, we can accurately model the transition probabilities for the erroneous transmission from noisy relays. The proposed ML scheme is shown to be superior to the conventional schemes in many channel setups. We also extend this work into multiple relay schemes and generalize it to higher modulation constellations, thereby providing a simple solution for combining noisy relayed signals with arbitrary modulation levels. The second part deals with the relaying protocols at the relay nodes. Our proposal gives further enhancement to the popular Soft Forwarding strategy. We propose Symbol-based Soft Forwarding (SSF) protocol which is based on the symbol-wise detection in a coded cooperative communications. We employ a unified framework which provides a simple method of forwarding soft information. Essentially, a relay node, usually behaves like a repeater, can avoid unnecessary computation complexity but with a significant performance improvement. This strategy avoids severe impact of decoding errors at relays and hence, expected values can be accurately computed for subsequent re-transmissions.

The final part of this thesis studies reliability-threshold relaying techniques to reduce error propagation. A set of optimal thresholds are proposed and their performance for various channel setups is evaluated. By using a simple threshold strategy based on signal reliability computations, we achieve a significant error rate performance compared to the baseline scheme with reduced system complexity. Importantly, our scheme strikes an interesting trade-off between the error rate performance and system complexity since no source-relay channel knowledge is required for detection at the destination node.
Acknowledgement

First and foremost, I am grateful to Allah (God) for His infinite blessings and mercy upon me. This dissertation would not be possible without the help and inspirations from many people during my studies. A few words mentioned here cannot sufficiently express my appreciation.

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# Table of Contents

Abstract ................................................................. i  
Acknowledgments ....................................................... iii  
Table of Contents ...................................................... iv  
List of Figures and Tables ........................................... vii  
List of Acronyms ....................................................... x  

1. **Introduction** .................................................. 1  
   1.1 Motivation ...................................................... 1  
   1.2 Contributions .................................................. 3  
      1.2.1 Maximum Likelihood Detection for Detect-and-Forward  
      Relay Channels ............................................... 3  
      1.2.2 Maximum Likelihood Detection with Arbitrary Modulations in  
      Cooperative Relay Channels .................................. 4  
      1.2.3 A Simple Symbol Estimation for Soft Information Relaying in  
      Cooperative Relay Channels .................................. 5  
      1.2.4 Mitigating Error Propagation with An Adaptive Detect-and-Forward  
      Strategy in Cooperative Relay Channels ................. 6  
   1.3 Organization of the Dissertation ......................... 6  

2. **Background on Cooperative Wireless Communication** ........ 8  
   2.1 Overview of Cooperative Wireless Communication ........... 10  
      2.1.1 A Brief Review on Fundamental Research Work of Cooperative  
      Communications ............................................... 10  
      2.1.2 Basic System Model on Cooperative Communications ....... 12  
      2.1.3 System Assumptions and Limitations .................... 13  
   2.2 Relay Protocols ............................................. 15  
      2.2.1 Decode-and-Forward (DF) .............................. 16  
      2.2.2 Amplify-and-Forward (AF) ............................. 17
2.3 Diversity Techniques in Wireless Channels ........................................... 18
  2.3.1 Time Diversity ................................................................. 20
  2.3.2 Spatial Diversity or Antenna Diversity .................................... 21
  2.3.3 Frequency Diversity .......................................................... 21
2.4 Signal Combining Strategies ........................................................... 22
  2.4.1 Selection Combining (SC) ..................................................... 22
  2.4.2 Equal Gain Combining (EGC) ................................................. 22
  2.4.3 Maximal Ratio Combining (MRC) ............................................. 23

3. Diversity Combining of Signals in Cooperative Communication ............ 25
  3.1 Signal Combining at Destination Node ........................................... 27
    3.1.1 ML-based Combining Strategy in A Single Relay System .............. 27
    3.1.2 ML-based Combining Strategy with Arbitrary Modulations in Multiple
        Relay Systems ................................................................... 34
  3.2 Analysis of Signal Combining Schemes ............................................ 40
  3.3 Complexity Comparison ............................................................. 41
  3.4 Results .................................................................................. 42
    3.4.1 ML-based Combining Strategy in Multiple Relay Systems .......... 43
    3.4.2 ML-based Combining Strategy with Arbitrary Modulations in Multiple
        Relay Schemes ................................................................. 47
  3.5 Conclusions .............................................................................. 53

4. Symbol-based Soft Relaying Strategy .................................................... 55
  4.1 System Model ............................................................................ 57
  4.2 Soft Information Relaying ............................................................ 59
    4.2.1 Baseline System ................................................................. 59
    4.2.2 Symbol-based Soft Forwarding .............................................. 60
    4.2.3 Mean Square Error (MSE) at Relay ........................................ 63
    4.2.4 Relay Mutual Information .................................................... 64
  4.3 Results ...................................................................................... 66
    4.3.1 Multihop Setup ................................................................. 67
List of Figures and Tables

List of Figures

2.1: Relay channel system model. .......................................... 12
2.2: Block diagram of a cooperative relay channel. ..................... 15
3.1: Block diagram of the cooperative relay system with a DEF protocol. MOD denotes the modulation of the received signal at the relay. ............... 28
3.2: Block diagram of the cooperative relay system with multiple relay channels. 36
3.3: PER comparison between the proposed ML scheme and C-MRC (dashed lines) using DEF protocols for $L=1,2$ and 3 relays. ......................... 44
3.4: PER comparison between the lower bounds (dashed lines) and the proposed ML schemes (solid lines) in multiple relay schemes. The lower bounds are simulated with perfect relays. .................................. 45
3.5: PER performance when the proposed ML scheme (solid lines) using only average SER, $e_0$ against the baseline C-MRC (dashed lines). ............... 45
3.6: PER comparison between the proposed ML scheme (solid lines) and C-MRC (dashed lines) when the average SNR of R-D link, $\bar{\gamma}_{rd}$ varies at $L=1$ relay case. ................................................................. 46
3.7: PER comparison with 16QAM at the source and the relay nodes between the proposed ML scheme and C-MRC (dashed lines) using DEF protocols for $L=1,2$ and 3 relays. ................................................................. 48
3.8: PER comparison with 16QAM at the source and the relay node between the proposed ML scheme (solid lines) and C-MRC (dashed lines) when the average SNR of R-D link, $\bar{\gamma}_{rd}$ varies at $L=1$ relay case. ............... 49
3.9: PER comparison proposed ML scheme and SC with 16QAM at the source and different modulation at the relay using DEF protocols for $L=1$ relay. .... 51
3.10: PER comparison proposed ML scheme and SC with different modulations at the source and the relay using DEF protocols for $L=1$ relay. .......... 53
4.1: Cooperative relay scheme with multiple relays. ....................... 57
4.2: Block diagram of the proposed scheme. ............................. 61
4.3: Mean squared error at the output of the relay node over source input signal. 63
4.4: Average capacity of the relayed link at the destination for the proposed scheme. 66
4.5: BER comparison of SSF (blue) and BSF (red) in a multihop coded relay scheme. 68
4.6: BER comparison of SSF (blue) and DF (green) in a multihop coded relay scheme. 68
4.7: BER comparison of SSF (blue), BSF (red) and DF (green) in a multibranch topology. 70
4.8: BER comparison of SSF (blue), BSF (red) and DF (green) in a hybrid multihop multibranch relay scheme. 71
5.1: Block diagram of the cooperative relay system with multiple relay channels. 75
5.2: Block diagram of the relay for ADeF. 76
5.3: BER comparison for ADeF (threshold, \( P_{th} \): 0.0001~0.2) and DeF with BPSK under AWGN channel for relay \( L = 1 \). 80
5.4: BER comparison for ADeF (threshold, \( P_{th} \): 0.0001~0.2) and DeF with QPSK under Quasi-static Rayleigh Fading channel for relay \( L = 1 \). 81
5.5: BER comparison for ADeF (threshold, \( P_{th} = 0.01 \)) and DeF (dashed lines) with QPSK under Quasi-static Rayleigh Fading channel for relay \( L = 1, 2 \) and 3. 81
5.6: BER comparison for ADeF (threshold, \( P_{th} \): 0.001~0.2) and DeF with 16QAM Quasi-static Rayleigh Fading channel for relay \( L = 1 \). 83
5.7: BER comparison for ADeF (BER threshold: 0.01) and DeF (dashed lines) with 16QAM Quasi-static Rayleigh Fading channel for relay \( L = 1, 2 \) and 3. 83
5.8: BER dependency over various threshold values, \( P_{th} \) for BPSK under AWGN channel. 85
5.9: BER dependency over various threshold values, \( P_{th} \) for QPSK under Quasi-static Rayleigh fading channel. 86
5.10: BER dependency over various threshold values, \( P_{th} \) for 16QAM under Quasi-static Rayleigh fading channel. 86
5.11: Cooperative relay scheme in multihop setup. 87
5.12: BER comparison for ADeF (threshold, \( P_{th} \): 0.01) and DeF (dashed lines)
with BPSK under Quasi-static Rayleigh Fading channel for a multihop relay
channel with $L = 2$ and 3. ........................................ 88
5.13: BER comparison for ADeF (threshold, $P_{th}: 0.01$) and DeF (dashed lines)
with 16QAM under Quasi-static Rayleigh Fading channel for a multihop relay
channel with $L = 2$ and 3. ........................................ 89
A.1: Signal QPSK symbols and symbol error probabilities. ............... 104
A.2: 16QAM symbols and symbol error probabilities. ...................... 108

List of Tables

3.1: The number of multiplications and additions at each scheme. ......... 41
3.2: Simulation parameters. ............................................. 42
4.1: Comparison of the proposed protocols against BSF. .................... 62
4.2: Simulation parameters. ............................................. 67
5.1: Simulation parameters. ............................................. 79
5.2: Optimal threshold values for AWGN channel for BPSK modulation. ... 84
5.3: Optimal threshold values for Quasi-static Rayleigh fading channel for QPSK
modulation. ......................................................... 85
5.4: Optimal threshold values for Rayleigh fading channel for 16QAM modulation. 86
## List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACK</td>
<td>Acknowledgement</td>
</tr>
<tr>
<td>ADeF</td>
<td>Adaptive Detect-and-Forward</td>
</tr>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>ARQ</td>
<td>Automatic Repeat reQuest</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BSC</td>
<td>Binary Symmetric Channel</td>
</tr>
<tr>
<td>BSF</td>
<td>Bit Soft Forwarding</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code-Division Multiple Access</td>
</tr>
<tr>
<td>C-MRC</td>
<td>Cooperative Maximum Ratio Combining</td>
</tr>
<tr>
<td>CRC</td>
<td>Cyclic Redundancy Check</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
</tr>
<tr>
<td>DEF</td>
<td>Detect-and-Forward</td>
</tr>
<tr>
<td>DF</td>
<td>Decode-and-forward</td>
</tr>
<tr>
<td>DMF</td>
<td>Demodulate-and-forward</td>
</tr>
<tr>
<td>e2e</td>
<td>End-to-end</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal Gain Combing</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Independent and identically distributed</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-density Parity Check</td>
</tr>
<tr>
<td>LLR</td>
<td>Log-likelihood Ratio</td>
</tr>
<tr>
<td>MAC</td>
<td>Multiple Access Control</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple-Input Single-Output</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>MOD</td>
<td>Modulation</td>
</tr>
<tr>
<td>MPSK</td>
<td>$M$-ary Phase Shift Keying</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal Ratio Combining</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal-Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OSI</td>
<td>Open Systems Interconnection</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PER</td>
<td>Packet Error Rate</td>
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<tr>
<td>PHY</td>
<td>Physical layer</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase-shift Keying</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>SC</td>
<td>Selection Combining</td>
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<tr>
<td>SEP</td>
<td>Symbol Error Probability</td>
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<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>SF</td>
<td>Soft Forwarding</td>
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<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SSF</td>
<td>Symbol-based Soft Forwarding</td>
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<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
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Chapter 1

Introduction

1.1 Motivation

Modern communication systems are an important part of our daily life. Integrated connectivity is overwhelmingly increasing. With the deployment of applications like mobile video applications and multimedia real-time services, the demand for higher data rates and uninterrupted connectivity is increasing. Users require current wireless communication systems such as mobile phone, wireless local area network (WLAN), video-on-demand, etc., to provide a greater freedom for them to roam and access the media from anywhere at any time. Therefore, the next generation wireless communication systems are expected to be highly interconnected and heterogeneous. On the other hand, researchers face a number of challenges which include the limited availability of the radio frequency spectrum and a complex time-varying wireless channel environment. Moreover, meeting the increasing demand for high data rates, better quality of service (QoS), fewer dropped calls, longer battery life and higher network user capacity pave the way for innovative techniques and further expand the growth of the present wireless communication systems which improve spectral efficiency and link reliability.

Multiple-input multiple output (MIMO) communication schemes have long been
proposed because it improves reliability through diversity, provides high capacity and interference suppression [1]-[3]. However, these advantages come with great cost as the transceiver design has to cope with higher system complexity, for example, in terms of multiple antennas and high power consumption particularly for applications in sensor networks and cellular phones. In fact, it is inconceivable for small-size mobile nodes to fit an array of antennas with large number without experiencing highly correlated signals. These limitations have triggered a flurry of research about cooperative communications.

With the evolution of rapid communication breakthrough, cooperative communication inherits outstanding features to help us realize future wireless communication. The appeal in cooperative communications is evident. Cooperating nodes can pool their resources by forming virtual antenna array, increase reliability and energy-efficiency without the need of bulky and many antennas at each node. Due to the broadcast nature of wireless communication networks, cooperative communication can bring many opportunities for channel diversity. This strategy tries to exploit idle radio nodes in the vicinity of the source node by relaying the source’s signal to the destination node to further improve the system performance. With this technique, it is possible to gain spatial diversity of the conventional MIMO techniques without each node necessarily being equipped with multiple antennas.

Cooperative diversity platform brings much promise as a practical idea to enhance the performance of many wireless networks like wireless cellular, ad-hoc and sensor networks. In particular, various theoretical and empirical studies suggest that significant gains can be obtained by using cooperative strategies in these wireless networks. In particular, this approach has showed to provide an effective diversity gain, even under the constraint of limited resources [4]-[6]. This technique has also proven to provide significant diversity gain which can effectively tackle the fading channel effects. Under constraint resources, this technique provides inherent capabilities to perform effectively and efficiently than that of the classical wireless communication schemes have to offer. For these reasons, cooperative communication is highly appealing for future communication networks. However, there exist numerous design challenges for implementing these networks into practice. For example, [4]-[6] have studied several methods on how the cooperation should be carried out.
The system performance is influenced by a number of factors that should be taken into account. One of the essential design aspects is the relaying strategy at the relay nodes. If the channel between the transmitter and the receiver is large or has poor channel condition, the probability of errors at the receiver increases, thus resulting into low performance gain. Therefore, the choice of cooperation between the source and relays is crucial such that the end-to-end performance is improved. Two prominent relaying techniques that have been studied extensively are amplify-and-forward (AF) and decode-and-forward (DF). With AF, the relay forwards a scaled version of the received signal based on a power constraint to the destination. However, a major problem with this protocol is the amplification of noisy in the forwarded signal which gives rise to serious error propagation to the destination. For DF schemes, when there are decoding errors at the relay, these errors will also propagate to the destination hence, harming the system performance. Each scheme has got some merits and yet, lacks in other aspects.

Motivated by these problems and findings, it is desirable to investigate other relaying options which can bring benefits to the cooperative networks. Another design issue of such cooperative networks is the signal combining technique at the destination. Since the destination node receives multiple noisy copies of the same messages, employing traditional combining methods like maximal ratio combing (MRC) is no longer giving optimal performance and subject to deleterious effects of error propagations. For this reason, it is also of interest to investigate a new method which can tackle this issue effective to reap the benefits of cooperative communications.

The objective of this dissertation is to investigate and analyze new relaying protocols and signal combining strategies which can bring performance improvement compared to the conventional schemes. The primary contributions of this dissertation can be summarized as below.

1.2 Contributions

1.2.1 Maximum Likelihood Detection for Detect-and-Forward Relay Channels

In practical relay communications, relay architecture is designed simple which requires
less computational complexity and time-delay. However, relays are subject to detection errors and eventually, the detection at the destination can be erroneous. This phenomenon is called error propagation. Error propagation can limit the end-to-end performance of the relaying scheme if it is not tackled effectively. However, if the destination is provided with the channel knowledge of source-relay (S-R) link, the detection can be optimally done. Typically, for example in wireless mobile networks, the base station is not mobile and has higher degree of flexibility in computational complexity and energy consumption compared to the mobile users (or nodes). Having extra channel state information (CSI) at the destination, thus, provides a good trade-off between the system-complexity and performance while leaving the relay simple.

We introduce a simple combining technique for cooperative relay scheme which is based on a Detect-and-Forward (DeF) relay protocol. Cooperative relay schemes have been introduced in earlier works but most of them ignore the quality of the S-R channel in the detection at the destination, although this channel can contribute heavily to the performance of cooperation schemes. For optimal detection, the destination has to account all possible error events at the relay as well. Here we present a Maximum Likelihood criterion (ML) at the destination which considers closed-form expressions for each symbol error rate (SER) to facilitate the detection. Computer simulations show that significant diversity gain and Packet Error Rate (PER) performance can be achieved by the proposed scheme with good tolerance to propagation errors from noisy relays. In fact, diversity gain is increased with additional relay nodes. We compare this scheme against the baseline Cooperative-Maximum Ratio Combining (C-MRC).

1.2.2 Maximum Likelihood Detection with Arbitrary Modulations in Cooperative Relay Channels

In digital cooperative communications, signals from the source and relay nodes may not be necessarily the same. In poor channel conditions, for example, lower modulations are more preferable since they are more resilient to noise and fading. This poses a basic problem if one needs to combine the signals at the destination in such cooperative networks. Conventional combining schemes like MRC just cannot work under this
condition and we are left only with more basic strategies like selection combining (SC) which is far than optimal.

Here, we propose a novel combining strategy with different modulations for cooperative relay networks. We generalize our previous work on Maximum Likelihood (ML) detection to multiple relays with higher order modulations such as quadrature amplitude modulation (QAM) which accounts individual SER to facilitate the detection. Our proposed algorithm is flexible to signals with different modulations as detection is done symbol-by-symbol basis. If different modulations are used at the source and the relays, we propose that lower modulation is used at the source. By computer simulations, significant PER performance can be obtained by the proposed scheme against C-MRC.

1.2.3 A Simple Symbol Estimation for Soft Information Relaying in Cooperative Relay Channels

Diversity gain at the destination can also be realized if the relay nodes can detect and correct the errors in the received signals. Therefore, it is of paramount need that the relaying protocol at the relay nodes is carefully chosen such that the error propagation is minimized. Conventional relaying protocols like AF and DF can be good candidates but they inherit major problems under lossy networks. For example, AF scheme is simple but amplification of the received signals also means the amplification in the noise. For DF, if the source-relay channel is poor, the relay may be vulnerable to decoding errors and then, this poses a risk in serious error propagation. One way to avoid this problem is to compute and forward the soft information instead of making a decision based on the transmitted symbols at the relays. Therefore, we can provide the destination with additional information for detection. Symbol-based Soft Forwarding (SSF) protocol for coded transmissions is introduced which is based on a newly proposed soft symbol estimation (but no decoding) at relay nodes.

We derive a unified framework which provides a simple strategy of forwarding soft information based on a simple linear summation of likelihood functions of each modulated symbol. Relay nodes can avoid unnecessary computation complexity and above all, are simple in structure relative to the relay scheme with Soft Forwarding (SF)
protocols. Specifically, with SSF, we demonstrate that exclusion of decoding at the relays costs no significant performance loss. To validate our claims, we examine bit error rate (BER) performance for the proposed scheme against the baseline SF scheme through computer simulations for various channel setups. We find that the proposed scheme can obtain considerable performance gains compared to the conventional relaying protocol.

1.2.4 Mitigating Error Propagation with an Adaptive Detect-and-Forward Strategy in Cooperative Relay Channels

Another simple way to reduce the error propagation is to selectively forward signals that contain no errors or signals within the limit of the required error performance. In such schemes, relay nodes use a threshold to decide when to re-transmit or keep silent (or simply discard the signals). We name this strategy as reliability threshold which is based on the computation of bit log-likelihood ratio (LLR) values at relay nodes. The relays calculate the LLR values for bits received from the source. Then, these values are compared against some predetermined threshold values and they are selectively chosen for re-transmission if the criterion is met and discarded if otherwise. This strategy is particularly useful for relaying among sensor devices that perform detection, but cannot accommodate channel decoding for each link due to energy constraints. In this dissertation, we name this simple relaying protocol as Adaptive Detect-and-Forward (ADeF). Essentially, this simple strategy can adaptively select the bits that are most reliable to the destination, and thus mitigate error propagation from noisy relays.

We investigate two-hop relay schemes which implement only a hard-decision detect-and-forward (DeF) at relays. We examine the BER performance for the proposed strategies against the baseline DeF through Monte Carlo simulations. We also suggest a list of threshold values which provide optimal BER performance improvements. We find that the proposed schemes can obtain a considerable performance gain against the baseline scheme by using this simple reliability threshold at relays.

1.3 Organization of the Dissertation

This dissertation is organized in a manuscript-style. Thus, the writing centers on the
published, accepted or submitted manuscripts. This main body of this dissertation is organized into three chapters, preceded by a chapter on the background studies and introduction, finally concluded by summary of contributions and a discussion of future research directions. An overview of these chapters is provided below. Chapter 2 provides some background studies of this dissertation. Chapter 3 focuses on the proposed diversity combining signals in cooperative communication. Chapter 4 investigates a new relaying strategy which is based on soft relaying information strategy. Chapter 5 is dedicated to the new ADeF protocol. In the beginning of each chapter, the literature review is provided that is relevant to the subject. Finally, summary of contributions and future works are presented in Chapter 6. We also present some derivations related to our research works in the appendices preceded by a list of our published and submitted papers and a list of references for this dissertation.
Chapter 2

Background on Cooperative Wireless Communication

The recent exploration of wireless communication systems for higher data rate and high-quality wireless communication in an indoor environment has been one of the new interesting subjects in the telecommunication research area. What’s more, the fast growing market demand for broadband multimedia services, gigabit networking and Internet are pushing the development of modem architectures for high bit rate transmission to the limit. Undoubtedly, these services require reliable wireless channels with large capacities.

It is shown that reliability and achievable data rate of wireless communication systems increase dramatically by employing multiple transmit and receive antennas. Transmit diversity is a powerful technique for combating multipath fading in wireless communications. However, employing multiple antennas in a mobile terminal to achieve the transmit diversity in the uplink is not feasible due to the limited size of the mobile unit. In fact, for the conventional mobile wireless network, wireless terminals usually communicate directly with a fixed base station. More explicitly, at times these terminals may be outside the coverage area resulting in frequent dropped calls and loss of communication. These problems can be aggravated further when the scheme has to support future generation wireless networks which require ubiquitous communication.
To ease the burden on the link budget, we can replace long and weaker links with shorter and stronger links. By having a few alternative routes between the terminals and the base station, the network is resilient against shadowing and multi-path fading, and introduces new design options for scheduling and routing.

In recent years, there has been a growing interest in cooperative wireless communication which is a promising candidate to solve these problems. The basic idea of cooperative communications is to assist the destination node through the multiple replicas of the same information transmitted from the source. Since the relayed signals can be poorer in quality than that of the direct transmission, the signal processing at the cooperative nodes (relays) requires careful treatment such that end-to-end diversity can be achieved at the destination. Various relaying schemes have been proposed to explore the benefits of cooperative communication, mainly divided into three categories, including Decode-and-Forward (DF), Amplify-and-Forward (AF) [4]-[6] and Detect-and-Forward (DeF). Among these protocols, DeF is attractively simple in complexity where the relay detects the signals (hard-decision detection) and modulates before forwarding to the destination.

In cooperative communications, multiple nodes in a wireless network work together to form a virtual antenna array. Using cooperation, it is possible to exploit the spatial diversity of the traditional MIMO techniques without each node necessarily having multiple antennas [1]-[3]. Due to the nature of broadcast transmission, signals from the source to the destination are overheard by some nodes within the vicinity. The destination receives multiple versions of the same message from the source and the relays and combines these signals to obtain a more reliable estimate of the transmitted signal as well as higher data rates. Once there are diversity signals in the system, there are multiple ways such that diversity gain can be realized. The choice of combining these signals is based on the level of complexity sustainable in the schemes and also the amount of CSI available at the receiver side. Some conventional methods are selection combining (SC), equal gain combining (EGC) and maximal ratio combining (MRC) which will be briefly discussed in the following few pages [7], [8].
2.1 Overview of Cooperative Wireless Communication

2.1.1 A Brief Review on Fundamental Research Work of Cooperative Communications

The study of cooperative communications dates back to the pioneering work of [9] in the 1970's. It laid a ground work of the theoretical formulations of the capacity for relay channels which was investigated for the case of three-node scheme. Then the channel capacity of relay networks over non-fading channel has been studied in [10]. The authors derive the maximum achievable rate for cases with or without feedback to the source or relay node for Gaussian channels but with no apparent application to wireless relay networks. Their main assumption is that relays can receive and transmit at the same time in the same band, and this is difficult to achieve in practical networks. Nevertheless, this work provides the fundamental concepts for the cooperative schemes and protocols [4], [5] that recently have gained much attention.

Moreover, [5] present a simple idea of user cooperation diversity scheme when two users form a cooperation to transmit each other's signals to the same base station. First, each of the partners exchanges cooperative information. Each partner detects this cooperative signal and transmits it along with its (own) signal to the destination. With this type of cooperation, the capacity for users and the user achievable rate will increase, i.e, rate that is less susceptible to channel variations. The authors also show the practical aspect of their design which fits nicely in present wireless platforms such as Code-Division Multiple Access (CDMA). In particular, the effectiveness of this cooperative scheme largely depends on the inter-user channels. The inter-user channel is the link from the output of each encoder to the input of its partner.

The work in [4] is credited with a similar approach but in a conceptual manner, describing the user cooperation in a mathematical framework. It discusses a cooperative protocol for combating multipath fading effects of wireless networks by exploiting the spatial diversity available among nodes. The proposed protocol helps remove some of the
practical constraints of earlier work. Due to the limitation of the existing radio frequency (RF) hardware, the nodes simply cannot receive and transmit simultaneously on the same channel referred to half-duplex transmission as opposed to full duplex communication as generally assumed in the early relay work [10]. Though the cooperating nodes transmit and receive at distinct timeslots, the overall transmit time for each node creates a basis for comparing cooperative cases. This raises an exciting fact that this protocol could fit into the existing time division multiple access (TDMA)-based systems. The literature in [4]-[6] demonstrates that cooperation achieves full spatial diversity gain coupled with significant power savings at a given performance criterion, conditioned on all things being equal.

Numerous aspects of relay networks have been extensively studied. For example, the capacity of relay networks over Rayleigh fading channels has been investigated in [11]–[13]. Moreover, relay networks with distributed space-time codes also have been investigated in [14], [15]. User cooperation which is the generalization of relay networks to multiple sources has been proposed in [5]. The diversity-multiplexing trade off of DF and AF relays has been studied in [4], [14]. In terms of the capacity, cooperative communication is shown to offer a performance enhancement as proposed in [16], [17]. Although the problems of cooperative methods have been studied for years in many aspects such as communications, networking, and information theory, they still attract the research community as a new paradigm for future wireless and mobile networks. With more emerging challenges, recently, the relaying method also has been employed in the WiMAX standards and its use is looming to be adopted inclusively by many other commercial standards [18].

In addition, we also envision a situation in which all partners take turns in creating spatial diversity for each other, turning the cooperative scheme into a virtual antenna array. Further benefits of this virtual antenna array include higher throughput and extended battery life for nodes, leading to a higher network life in the case of ad-hoc networks. However, the elements of this array are not co-located. They belong to different terminals which are connected via noisy channels. Hence one has to carefully study the conditions under which cooperation is useful and investigate practical schemes to get the desired benefits.

11
2.1.2 Basic System Model on Cooperative Communications

![Wireless relay network model](image)

Figure 2.1: Wireless relay network model.

For the most part, a generic cooperative communication system model focuses on a simple model with one source, a relay and a destination node. This setup of investigation is motivated by the hope that the fundamental results achieved in this simple model will provide plain insights and concrete ground work for more general wireless networks. Let us consider the specific half-duplex cooperative relay-based wireless system with three terminals as shown in Fig. 2.1, where the direct communication between the source (S) and the destination (D) is assisted by a relay node (R). The basic premise of cooperative communications is to exploit the broadcast nature of wireless networks where the neighboring nodes overhear the source’s signals and relay the information to the destination. The relay does not have its own information to transmit and it only helps relay the communication from the source to the destination based on any arbitrary function of its past received signals.

The source and the relay cooperate in time-division manner to transmit a signal to the destination and transmission is assumed half-duplex. This is a realistic assumption in view of existing RF hardware limitations. Therefore, there are two hops of transmission. During the first-hop transmission (broadcasting stage) with one symbol-interval, the source node, S sends the symbol to R and D simultaneously. As can be seen from Fig. 2.1, after receiving the signals resulting from S, R forwards the received signal to D resulting
in the increase of the capacity and reliability of the direct communication.

In brief, the end-to-end transmission is clearly divided into two unique stages in the TDMA mode: broadcasting and relaying phase. In the broad-casting phase, i.e., broadcasting channel as seen from the source’s view point, all receivers including the relays and destination work in the same channel (time or frequency). In the contrary, in the relaying phase, i.e., multiple access channels as seen from the destination’s viewpoint, the transmitting terminals (relay nodes) may operate in different channels to avoid co-channel interference.

2.1.3 System Assumptions and Limitations

The results of this work are solely based on simulation works using MATLAB which means that all channel models and parameters are not characterized by real-world measurements. In this dissertation which looks into physical layer protocol design (PHY) of the seven-layer Open Systems Interconnection (OSI) model, our primary focus is to present the error rate performance and diversity gain which can be achieved by using cooperative communications based on the typical assumptions and limitations below. Furthermore, we provide additional assumptions in the respective chapters which are relevant to the particular schemes.

1. The statistical model for the fading channel is Rayleigh fading channel based on the assumption that there is always a number of statistically and independently paths with random amplitudes and in fact, all delays are within a single tap duration. Each channel coefficient is a sum of independent circularly symmetric random variables.
2. The channels are assumed quasi-static which really means that each channel coefficient is constant over the duration of transmission interval but independent and identically distributed (i.i.d) in different transmission interval.
3. It is assumed that all of the nodes in the network are sufficiently separated that fading envelope of all channels are independent.
4. The nodes are assumed to be half-duplex systems in order to get round the technical difficulty of receiving and transmitting at the same time in the same frequency band.
5. A scheme is comprised of a source, a number of relays and a common destination. There is a presence of source-destination link (direct link).

6. There is a maximum number of $L$ relays available for cooperation which represents the bandwidth and time delay constraint. As a result, there is a factor of $L+1$ bandwidth expansion or rate reduction for the schemes under consideration. Our main focus in this dissertation is on the potential diversity gain of our simple proposed schemes. This knowledge becomes a priori to the system prior to the actual data transmission.

7. All transmissions are orthogonal and no multiple access interference is considered.

8. All relay nodes have identical average signal-to-noise ratio (SNR) to the source and the destination.

9. We assume a memoryless scheme which means that the transmission from relays depends only on its last received signals and no channel coding is used. Therefore, we can drop the time index in our equations.

10. Synchronization among nodes and channel estimation at the receiving nodes are perfect and done in practice through preambles and headers before the transmission of user data. In this dissertation, channel modeling, simulation and estimation are not touched. Issue like the temporal, spatial and spectral correlation functions for a cooperative communication scheme which can be different from the non-cooperative scheme are not discussed here.

11. All nodes are equipped with only an antenna. Future studies will cover multiple antennas which can be of great benefits for better data throughput and link efficiency.

There are also other practical issues that should be taken into consideration in implementing wireless cooperative communications like signaling design, user coordination, resource management and interference. Some earlier works have been proposed to solve some of the individual problems. For instance, some adaptive resource (power, time slots) allocations and partner selection schemes were developed for efficient resource management and effective user coordination. However, different
schemes depend on different system assumptions and were proposed under different system models. It is desirable that to implement cooperative communication, one needs to design a unified system model which solves all the issues. Our dissertation is simply far from meeting this practical requirement and yet, we present practical solutions to tackle part of the problems for future implementation.

2.2 Relay Protocols

The primary signal processing for cooperative communications is the technique of cooperation. The operation of signal detection and re-transmission is key element which determines the improvement in such networks which is widely known as relaying protocols. In general, the protocols are classified in two broad categories: Decode-and-forward (DF) and amplify-and-forward (AF). However, there have been extensive works on the extensions of these protocols in the literature and some of them will be described in the following chapters.

![Block diagram of a cooperative relay channel](image)

Figure 2.2: Block diagram of a cooperative relay channel.

In the first timeslot, the received signal at D, \( y_{sd} \), and at R, \( y_{sr} \), can be shown in the following relation

\[
\begin{align*}
y_{sd} &= h_{sd}x_s + n_{sd} \\
y_{sr} &= h_{sr}x_s + n_{sr}
\end{align*}
\]
where the subscripts indicate the node relation such that $h_{sd}$ and $h_{sr}$ are independent complex-valued channel gains for the S-D and S-R links respectively. For simplicity, all channels are Quasi-static Rayleigh fading channels i.e., $h_{sd} \sim \mathcal{CN}\left(0, \sigma_{sd}^2\right)$ and $h_{sr} \sim \mathcal{CN}\left(0, \sigma_{sr}^2\right)$, where $\mathcal{CN}\left(\mu, \sigma^2\right)$ denotes a complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$; $n_{sd}$ and $n_{sr}$ are modeled as independent additive white Gaussian noise with zero mean and equal variance $N_0$ at the destination and relay respectively. We assume that the average signal-to-noise ratio (SNR) for all links are the same denoted as $\bar{\gamma} = E_s / N_0$, while the instantaneous SNR is represented as $\gamma_{sd} = \sigma_{sd}^2 \bar{\gamma}$ and $\gamma_{sr} = \sigma_{sr}^2 \bar{\gamma}$ respectively. In the second timeslot, depending on the relaying protocol at the relay, R forwards the signal back to D. Next, we describe the input-output model descriptions for DF and AF protocols.

### 2.2.1 Decode-and-Forward (DF)

This relaying protocol is considered to be the earliest method of traditional cooperative communications which appeared in [19]. The basic idea for this protocol is that the relay will attempt to decode the received signal first, re-encode it and then re-transmit it to the destination. Ideally, if decoding is successful, the re-transmission is just another copy of the original message from the source. Using this strategy which is also regarded as regenerative method, the relay detects and corrects the errors before forwarding it to the destination.

The equivalent detection rule at the relay node is

$$\hat{x}_s = \arg \min_{x_s \in \chi} \left| y_{sr} - h_{sr} x_s \right|^2$$

(2.2)

where $\chi$ denotes the finite set of the constellation. However, in reality, due to the error propagation, the potentially erroneous decoded message at the relay can significantly degrade the system performance. Hence, it is typically assumed that the relays only assist the source if the original signal is correctly decoded at the relays [20], [21]. To ensure this
condition, a cyclic redundancy check (CRC) code can be employed. While assuming a perfect capability of CRC functionalities, the relay can be considered as adaptive DF. However, in practice, it is not always possible for the relay to know exactly the detection quality. In a simpler version, a fixed DF, has been aggressively proposed in [16], [22] to provide some degree of freedom in complexity. In fixed DF protocol, the relay always re-transmits the decoded signal to the destination regardless of the quality of received signals. It has been demonstrated that the instantaneous received signal-to-noise ratio (SNR) is asymptotically approximated as the minimum SNR from two hops. Moreover, the conventional adaptive DF protocol requires a longer time delay to do decoding at the relay. In particular, if the channel quality of the source-relay (S-R) link is good, the relay can decode the received signal quickly. Therefore, this situation brings the entire system to the inefficient use of the resources. Interestingly, a remedy for this problem is found in the dynamic DF protocol, proposed in [23], [24] where the decision time is a random variable. This strategy can overcome the problem of the adaptive DF scheme.

Another variation of DF is known as Detect-and-Forward (DeF) particularly associated to uncoded cooperative schemes where there is no use of channel decoding either due to constraints in transceiver capabilities or due to lack of knowledge of the channel codebook. In DeF, the relay simply detects the signals (hard-decision detection), modulates before forwarding to the destination. One obvious benefits from DeF are the reduction in receive power consumption and complexity since no channel decoding is employed and the minimization of overall processing delay at the destination. For DeF, the system is subject to serious error propagations. DeF is extensively studied in [25]-[30] and it is also referred to as Demodulate-and-forward (DMF).

### 2.2.2 Amplify-and-Forward (AF)

This approach was first proposed by Laneman et al. [4]. In Amplify-and-Forward (AF) relaying schemes which is regarded as a non regenerative system, the relay amplifies the signal from the source without performing any sort of decoding. Another salient feature of AF is that it preserves all of the soft information content of the received signals. However, the relay also multiplies the noisy version of the source’s signal with the
amplifying gain, $\beta$, under a certain constraint, e.g., power constraint, and then transmits the resulting signal to the destination as $x_r = \beta y_{sr}$. This is the major setback of this protocol.

The amplification factor is used so that the power constraint at the relay node is met and if we assume a normalized average transmit power at the relay node, we can express it as $\beta = \frac{1}{\sqrt{|h_{sr}|^2 + N_0^2}}$. As the relay simply retransmits the received signal from the source without any processing to the received signal, this non-regenerative method makes the hardware complexity of relay simpler than that of DF at the expense of no error correction possibility at the relay nodes. To realize the cooperative diversity in AF scheme, the destination can skillfully combine two independently faded signals from source and relay nodes, despite with the presence of the amplified noise in the relayed signal. With this channel setup, the second order diversity can be realized and proved in [4]. In AF schemes, the destination typically knows the amplification factor between cooperating nodes to obtain optimal decoding, which demands a high requirement of data exchanging of some channel parameters. AF protocol is a simple strategy which is subject to straight-forward analysis. This feature is useful to researchers to better understand cooperative communication systems.

The amount of CSI in the amplification factor determines the many classes of AF protocols. For instance, if the relay has the perfect knowledge of CSI of all links, the amplifying gain can be varied and is called as CSI-assisted AF or variable-gain AF protocol. On the other hand, if only the statistical property for the S-R channel is known, it is called as the semi-blind AF or fixed-gain AF protocol. It is of no surprise that the CSI-assisted AF is more superior to the semi-blind protocol with the expense of higher complexity [31].

### 2.3 Diversity Techniques in Wireless Channels

The key strategy to reliable and energy-efficient communication in fading channels is diversity. There are many types of diversity including time, antenna, frequency, multipath,
and angle diversity. This section presents the general concept of diversity used in many of the current and emerging wireless communication systems. The idea behind this concept is to exploit the low probability of having poor channels such that the error and outage probabilities can be minimized.

As shown in Figure 2.1, the received signal in the channel model may experience from abrupt reduction in power. Due to the destructive effects of multipath fading, the channel coefficients fluctuate in magnitude which brings the channels to be in deep fade. As a result, any transmission in this kind of environment may cause a failure. A direct remedy to this problem is to increase the overall SNR by providing some copies of the same transmitted signal through independent channels and to combine all of the signals effectively at the receiver.

Let $x(t)$ be the symbol transmitted in the $t$-th symbol period corrupted by AWGN noise at the $l$-th branch (or antenna). We simply can generalize the relation for any branch as

$$y_l(t) = h_l x(t) + n_l(t), \quad \{l = 1, 2, ..., L\}$$  \quad (2.3)

where $h_l$ is the channel coefficient observed at $l$-th branch and $n_l$ model AWGN with one-sided power spectral density of $N_0$. Thus, the instantaneous SNR at the $l$-th branch can be represented as

$$\gamma_l = \frac{E[h_l x(t)^2]}{E[n_l(t)^2]} = \frac{E[|x|^2]}{E[|n_l|^2]} = |h_l|^2 \frac{E_S}{N_0} = |h_l|^2 \bar{\gamma}$$  \quad (2.4)

where $E[\cdot]$ is the expectation operator, $E_S$ is the average transmit power and $\bar{\gamma}$ as the average SNR of the link.

Generally, diversity can be achieved when a receiver obtains two or more copies of the same signal through different independent fading channels. Let us assume that $l$ denotes the number of copies of the message received through independent and identically distributed fading channels and $p$ is the probability that the magnitude of the fading channel falls below a particular limit. Precisely, we conclude that the probability that all $l$ channels fall below this limit is $p^l$. From [32], the diversity gain from any diversity technique is defined asymptotically in terms of the rate of decay (slope) of the
error probability at sufficiently large SNR. Specially, a diversity technique is said to have diversity gain of $l$ if the error probability scales according to

$$P_e(\overline{\gamma}) \propto \overline{\gamma}^{-l}$$  \hspace{1cm} (2.5)$$

for large $\overline{\gamma}$ where $\overline{\gamma}$ denotes the average signal to noise ratio of the system. For example, a system with diversity gain of $l = 2$ will significantly outperform a system with no diversity gain ($l = 1$) at large enough SNR. Due to transmit power constraints and regulatory requirements, increasing the diversity gain of the system is considered an attractive way of reducing the error probability in such systems. Basically, the diversity order is based on the slope of the error performance curves. The steeper the curve, then the more reliable the communications system is. If properly designed, a wireless communications system can theoretically achieve diversity order based on the number of diversity paths available at the receiver, what can be termed as the maximal diversity order. There are three major ways of diversity which have been studied extensively in the literature [7], [8]:

- Time Diversity, e.g., Channel coding
- Spatial Diversity, e.g., MIMO
- Frequency Diversity, e.g., Orthogonal frequency division multiplexing (OFDM).

2.3.1 Time Diversity

Time diversity can be achieved by transmitting the same information over different time. The simplest form of coding in time diversity is repetition coding, in which the signal is repeated exactly the same over a number of time intervals. Time diversity is usually associated with error control coding and time interleaving. Coded signal is dispersed over time in different periods of the coherent time so that different parts of codewords can experience independent fades [7]. The key for this form of diversity to be useful is that the channel must provide sufficient variations in time. Recent developments in robust error correcting codes have resulted in not only the diversity gain but also coding gain like in Turbo and LDPC codes.
2.3.2 Spatial Diversity or Antenna Diversity

Spatial diversity schemes employ multiple antennas at the transmitter and/or the receiver which have to be sufficiently placed such that the coherence distance (i.e., the antenna separation distance above which the channel coefficients are assumed to be spatially uncorrelated) is exceeded. In this case, independent diversity branches can be reached by repeating the same symbol (or some form of redundancy) over multiple transmit antennas using only a single antenna for reception. This multipoint-to-point method of transmission is called spatial transmit diversity which is beyond the scope of this dissertation. However, if only a single transmit antenna is used but multiple antennas receive independently faded signal paths, spatial receive diversity is exploited [7]. This method is preferred over time and frequency diversity as it does not incur more expenditure in transmission time or bandwidth. Redundancy is generated by employing an array of antennas, with a minimum separation of half the wavelength between neighboring antennas. Such a topology is closely emulated by cooperative communications where each node is far apart and undergoes independent fading channels. Diversity in the cooperative communications relies on two principles:

1. Broadcast nature of wireless medium: Most transmissions can be heard by multiple nodes within the vicinity of the source node with no additional transmission power and bandwidth.
2. Different nodes have statistically independent fading channels to the destination node and the destination combines the signals from different nodes to achieve diversity.

2.3.3 Frequency Diversity

For this type of diversity, different frequencies are used to transmit the same signal which will face different multipath fading in the frequency domain. To achieve frequency diversity, the carrier frequencies are separated by more than the coherent bandwidth such that the copies of the same signal experience independent fading channels. As a result, with this criterion, then different parts of the relevant spectrum will suffer independent fades. This concept is the crust of a popular scheme like OFDM.
2.4 Signal Combining Strategies

Performance of communication systems employing the receive diversity technique depends on how the multiple signal copies are combined at the receiver. There are several ways of combining the received signals which vary in complexity and overall performance. According to the levels of CSI available at the receiver, there are three main combining methods, namely maximal-ratio combining (MRC), equal-gain combining (EGC) and selection combining (SC) [7], [8]. These combining schemes perform this task at signal level prior to channel decoding. Next, we briefly describe the conventional combining strategies for clarity.

2.4.1 Selection Combining (SC)

Selection combining is the simplest diversity combining method among the three techniques. The receiver only selects the signal from the branch with the highest SNR (or equivalently, with the strongest incoming path assuming equal noise power in all branches) for detection. Thus, unlike in MRC, the SC scheme is usually used with differentially coherent and non-coherent modulation techniques. The output of the SC combining is computed as

$$\gamma_{SC} = \max_{l=1,...,L} |h_l|^2 \overline{\gamma}$$  \hspace{1cm} (2.6)

In practice, the signal branch with the highest sum of signal and noise power is often chosen as it is more difficult to measure the SNR.

2.4.2 Equal Gain Combining

Another suboptimal method, called equal-gain combining (EGC) with coherent detection, is an attractive alternative since it does not require the channel amplitude estimation. In this method, all received signals are co-phased and simply added together. The received SNR in this method can be calculated as
\[ \gamma_{EGC} = \left( \frac{\sum_{l=1}^{L} |h_l|^2}{L} \right)^2 \frac{1}{\bar{y}} \]

Nonetheless, EGC is usually restricted to coherent modulations with the same energy symbols like \( M \)-ary phase-shift keying (MPSK) symbols.

### 2.4.3 Maximal Ratio Combining

Even though SC and EGC require CSI to compute the weighting factors, these factors are still not optimal. For this purpose, the weighting factor needs to be carefully chosen such that the receive SNR is maximized and consequently, the error rate is minimized. One of such schemes which is optimal for this purpose is MRC at the expense of complexity due to the requirement of perfect CSI of all links. In this method, each individual received signal must be co-phased, weighted with its respective amplitude and then added up. The method is called optimum combining (regardless of the fading statistics) in the sense that it maximizes the received signal-to-noise ratio, \( \gamma \), of the system under Gaussian noise. The maximal SNR is equal to the sum of all the instantaneous SNRs of the individual signals, i.e.,

\[ \gamma_{MRC} = \sum_{l=1}^{L} |h_l|^2 \bar{y} = \sum_{l=1}^{L} \gamma_l \]

Since it requires all the fading channel parameters at the receiver, this technique is not practical for differentially coherent and non-coherent detection. Most of the research work in cooperative networks has employed MRC technique to combine all the coming signals from source and relays based on the assumption that the destination has the perfect knowledge of all links. Conventionally for the cooperative communications, the MRC technique is the same as the maximum likelihood (ML) decoding for AF relays. However, this notion is not true for DF relays. A new weighted MRC has been proposed for DF relays in [27], namely, cooperative-MRC (C-MRC). This new combining approach has been shown to achieve the full diversity gain regardless of the constellation. However, deploying C-MRC at the destination also requires the full knowledge on CSI of
all links which is hard to implement in practical scenarios. Moreover, C-MRC is also not practical to be used in conjunction with different modulation constellations.
Chapter 3

Diversity Combining of Signals in Cooperative Communication

Cooperative communication has been developed as a promising technique to realize spatial diversity through user cooperation [4]. Various relaying schemes have been proposed to explore the benefits of cooperative communication, mainly divided into three categories, including Decode-and-Forward (DF), Amplify-and-Forward (AF) [4]-[6] and Detect-and-Forward (DeF). In DF, the relay always decodes, re-encodes and re-transmits the decoded signal to the destination. That is, any errors at the relay can be corrected and thus, error propagation can eventually be avoided. On the other hand, AF simply amplifies the received signals and forwards them to the destination after power scaling. The disadvantage of AF strategy is that it will also forward noises which received at the relay. Another relaying protocol which is simple in complexity is DeF where the relay simply detects the signals (hard-decision detection), modulates before forwarding to the destination. With DeF, in [25] the author has shown that the diversity gain can also be achieved provided the destination knows the relay probability of error.

Recently, many works were devoted to improve the relay complexity and yet strive for better error rate performance. For example, our earlier works in coded cooperative schemes [26] adopted DeF at the relay node with further enhancements at the destination. The destination employs Maximum Likelihood criterion detection (ML) to
combine the direct and relayed transmissions. However, this detection at the destination does not account sufficiently the error probability of making errors at the relays resulting in serious performance degradation. In coded relay schemes, the channel decoder is initialized with channel log-likelihood ratio (LLR); hence, requires high accuracy of LLR computation. The authors in [25] have developed a piece-wise linear receiver approximating the ML criterion detection that requires knowledge of the average signal-to-noise-ratio (SNR) of the first hop. However, this scheme cannot achieve full diversity for more than one relay. In [27], another combining technique namely Cooperative-Maximum Ratio Combining (C-MRC) is introduced that approximates the ML detector. Unfortunately, C-MRC results in serious propagation error under asymmetrical networks when SNR of R-D link is larger than that of S-D link (or S-R link). Another work in [28], has studied the performance of the hard-decision ML criterion detection-based combining technique under uncoded cooperative scheme. In [29] and [30], the authors have proposed a non-coherent combiner in uncoded cooperative relaying scheme using DeF protocol with limited Channel State Information (CSI). In [25]-[30] and [33]-[35], the authors have derived sub-optimal receivers but exploiting effectively perfect knowledge of all links is still an open problem, specifically the error probability at the relay. However, many of the previous works assumed that the destination only knows the average probability of symbol error at the relays. Since the detection rule in ML criterion detection at the destination has to consider every symbol error probability, this error model may substantially affect the attainable end-to-end performance. Thus, to guarantee an optimal ML criterion detection, the destination needs to know the error characteristics of the S-R link (perfect CSI) in the form of relay error probabilities.

In this study, we aim at providing the destination with a more accurate CSI in its decision criterion. While the idea itself of having perfect CSI is not new [30], [34], [36], the CSI in the previous works is calculated based on the assumption of average error probability. For binary phase-shift keying (BPSK) case, the solution is straightforward, but for quadrature phase-shift keying (QPSK) and higher modulation constellation, the Euclidean distance between symbols become no longer the same; hence the symbol error probability can also differ. To circumvent this problem, here we develop a simple
ML detection algorithm for the destination node for QPSK modulation. For simplicity, we analyze this ML performance with a simple DeF in uncoded cooperative relay networks and compare against the baseline C-MRC. Unlike in C-MRC, the instantaneous CSI in the proposed scheme involves Q-function expression for each symbol in the modulation which provides accurate knowledge of S-R link. Our scheme also outperforms C-MRC especially when SNR of R-D link is sufficiently high or low (asymmetrical network). We show through computer simulations that the performance of the proposed cooperative relay schemes can be improved significantly particularly in multiple relay nodes. We observe that there is remarkable potential to achieve increasing orders of diversity with better packet error rate (PER) performance than that of C-MRC.

The organization of this chapter is as follows: Section 3.1 is system description and the proposed scheme for a single relay scheme followed by the extension for multi relay scheme with arbitrary modulations, in Section 3.2 we present the analysis of signal combining schemes, followed by the complexity comparison in Section 3.3. Simulation Results and Discussions are given in Section 3.4, and finally in 3.5, this chapter is summarized. The derivation of the individual SER for QPSK and 16QAM in Gray mapping is presented in Appendix A.

3.1 Signal Combining at Destination Node

In the following sections, the proposed scheme is presented for a cooperative communication with one relay node and followed by the extension to multiple relay schemes with arbitrary modulations at the transmitting nodes.

3.1.1 ML-based Combining Strategy in a Single Relay System

With reference to Figure 3.1, we first consider a classical relay model in which only one relay (R) assists the source (S) to communicate with the destination (D). We assume each node has only one antenna and is not equipped with cyclic redundancy-check (CRC) codes. In this dissertation, we consider a time division multiple access (TDMA)
mode where data transmission is split into two phases, that of the source node and that of the relay node. It is assumed that all the receiving nodes have perfect CSI. Furthermore, the destination also requires CSI of all three links for detection.

### 3.1.1.1 System Model for a Single Relay Scheme

![Diagram of the cooperative relay system with a DeF protocol](image)

Figure 3.1: Block diagram of the cooperative relay system with a DeF protocol. MOD denotes the modulation of the received signal at the relay.

At timeslot 1, the source broadcasts its information, $x_s$, to the destination and the relay with the average power $E_s$. Due to the broadcast nature of wireless channel, both the destination and the relay receive a noisy observation of $x_s$ denoted by $y_{sd}$ and $y_{sr}$ respectively in the following relation

\[
\begin{align*}
y_{sd} &= h_{sd} x_s + n_{sd} \\
y_{sr} &= h_{sr} x_s + n_{sr}
\end{align*}
\]  

(3.1)

where the subscripts indicate the node relation such that $h_{sd}$ and $h_{sr}$ are independent complex-valued channel gains for the S-D and S-R links respectively. For simplicity, all channels are Quasi-static Rayleigh fading channels i.e., $h_{sd} \sim \mathcal{CN} \left(0, \sigma_{sd}^2 \right)$ and $h_{sr} \sim \mathcal{CN} \left(0, \sigma_{sr}^2 \right)$, where $\mathcal{CN} \left(\mu, \sigma^2 \right)$ denotes a complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$; $n_{sd}$ and $n_{sr}$ are modeled as independent additive white
Gaussian noise with zero mean and equal variance $N_0$ at the destination and relay respectively. We assume that the average SNR for all links are the same denoted as $\bar{\gamma} = E_s / N_0$, while the instantaneous SNR is represented as $\gamma_{sd} = \sigma_{sd}^2 \bar{\gamma}$ and $\gamma_{sr} = \sigma_{sr}^2 \bar{\gamma}$ respectively. The relay performs a hard-decision detection (DeF) and re-modulates (MOD) the detected signals as $x_r$ with the same average power $E_s$ for re-transmissions in timeslot 2. The symbol received at the destination is given as

$$y_{rd} = h_{rd} x_r + n_{rd}$$

(3.2)

where $h_{rd} \sim \mathcal{CN} \left(0, \sigma_{rd}^2 \right)$ and $n_{rd} \sim \mathcal{CN} \left(0, N_0 \right)$. The instantaneous SNR is $\gamma_{rd} = \sigma_{rd}^2 \bar{\gamma}$.

At the destination, the signals from the source and the relay node are combined to recover the original source data. The assumptions in the context of this topology are summarized in Section 2.1.3 with the exception that in this chapter, we also consider when the relay-destination (R-D) link is asymmetrical to other links. We also assume that the destination knows the S-R channel knowledge and the degree of this CSI depends upon the combining schemes i.e., our proposed scheme requires only the average SNR and the baseline schemes need instantaneous SNR for optimal combining. This information can be obtained through simple feedback (training symbols) to the destination prior to the actual data transmission.

### 3.1.1.2 Baseline Scheme

Many cooperative relay schemes use MRC to exploit spatial diversity gain. It is one of the simplest and practical approaches when S-R link is reliable. However, this conventional MRC at the destination cannot guarantee full diversity in cooperative schemes. The destination node requires perfect (instantaneous) CSI of S-R link with effective combining.

In [27], the authors have proposed an improved version of MRC termed as C-MRC. The combined signal at the destination node is given by

$$y_{cmrc} = h_{rd}^* y_{sd} + \frac{\gamma_{min}}{\gamma_{rd}} h_{rd}^* y_{rd}$$

(3.3)

where $\gamma_{min} = \min(\gamma_{sr}, \gamma_{rd})$, $\gamma_{sr}$ and $\gamma_{rd}$ are instantaneous SNR of the S-R and R-D
channels respectively. The usual intuitive meaning associated with (3.3) is that when $\gamma_{sr}$ is high, the detector places full confidence to the arriving signals from the relay. In case of low $\gamma_{sr}$, the confidence is weighted according to the ratio of both hops that is S-R-D link.

**3.2.2.3 Proposed ML-based Combining Strategy**

In this section, we first derive the proposed ML combining technique in case of one relay. In the subsequent section, we generalize it to multiple relays. The main purpose of this algorithm is to optimally combine the noisy signals received at the destination node, $y_{sd}$ and $y_{rd}$. It should be noted here that the maximum likelihood detector at the destination should also consider the effect of detection errors at the output of the relay. Such errors are mainly due to fading events in the S-R link. When this link is affected by a deep fade, the detection errors committed at the relay are propagated to the destination. To mitigate these errors which are originated from both source-relay and relay-destination links, an end-to-end ML-based detector should be employed. Thus, the basic assumption for this strategy is that the destination node makes a coherent detection for the signals from the source and the relay nodes requiring all three channels to be known at the destination node, i.e., $h_{sd}$, $h_{sr}$, $h_{rd}$ and the noise variances are available at the destination (only the received symbol $y_{sr}$ is not known). This is a standard assumption based on the fact that these parameters have been estimated as a priori. For a fair comparison, we maintain the same assumptions applicable for the baseline scheme C-MRC. The proposed ML decision rule at the destination is determined by taking all the possible symbol detection scenarios both at the destination and the relay. By applying Bayes’ rule, the decision criterion can be shown as
\[ \hat{x}_s = \arg \max_{x_s \in \mathcal{X}} P(x_s, x_r \mid y_{sd}\Delta y_{sd}, y_{rd}\Delta y_{rd}) \]

\[ = \arg \max_{x_s \in \mathcal{X}} \frac{P(x_s)P(y_{sd}\Delta y_{sd} \mid x_s)P(y_{rd}\Delta y_{rd} \mid x_r)}{P(y_{sd}\Delta y_{sd}, y_{rd}\Delta y_{rd})} \]

\[ \times [P(y_{rd}\Delta y_{rd}, x_r = x_s) + \sum_{x_s \neq x_r} P(y_{rd}\Delta y_{rd}, x_r)] \]

\[ = \arg \max_{x_s \in \mathcal{X}} \frac{P(y_{sd}\Delta y_{sd}, y_{rd}\Delta y_{rd})}{P(y_{sd}\Delta y_{sd} \mid x_s)P(y_{rd}\Delta y_{rd} \mid x_r = x_s)} \]

\[ + \sum_{x_s \neq x_r} P(y_{rd}\Delta y_{rd}, x_r) \]  \hspace{1cm} (3.4)

From (3.4), by considering the potential errors at the relay node, the decision criterion can be expanded as

\[ \hat{x}_s = \arg \max_{x_s \in \mathcal{X}} P(y_{sd}\Delta y_{sd} \mid x_s)\{P(x_r = x_s)P_{rd}(y_{rd}\Delta y_{rd} \mid x_r = x_s) \]

\[ + \sum_{x_s \neq x_r} P(x_r \neq x_s)P_{rd}(y_{rd}\Delta y_{rd} \mid x_r) \} \]  \hspace{1cm} (3.5)

where we assume \( P(x_s) = 1/M \) e.g., \( M=4 \) for QPSK; \( \mathcal{X} \) denotes the finite set of the constellation and \( \Delta y \) is the observation space for the respective received symbol. First, we assume that the transmit signals are modulated by QPSK and later, we consider a special case of BPSK. Throughout the sequel, we use capital \( P \) as the probability function. From (3.5), after some simplifications, the detector at the destination will find \( \hat{x}_s \), an estimate of \( x_s \) by using the following criterion

\[ \hat{x}_s = \arg \max_{x_s \in \mathcal{X}} p_{sd}(y_{sd} \mid x_s)\{P(x_r = x_s)p_{rd}(y_{rd} \mid x_r = x_s) \]

\[ + \sum_{x_s \neq x_r} p(x_r \neq x_s)p_{rd}(y_{rd} \mid x_r) \} \]  \hspace{1cm} (3.6)

where \( p_{sd}(y_{sd} \mid x_s) \) is the probability density function (PDF) of the source signal \( y_{sd} \) conditioned upon the transmitted data \( x_s \) and \( p_{rd}(y_{rd} \mid x_r = x_s) \) is the PDF of the relayed signal \( y_{rd} \) conditioned on the equality of both transmitted symbols \( (x_r = x_s) \).

However the bracketed terms in (3.6) has to consider the error probability of the received signals \( y_{sr} \) at the relay accounting the SER of each signal point.

For QPSK case, the solution is not straightforward due to different Euclidean
distance among symbols. In QPSK, a symbol with two bits, $x_s = [h_1, h_2]$ takes from the constellation set $\chi_s = \{s_1, s_2, s_3, s_4\}$. Assuming the Gray mapping, we represent the complex symbols of $\chi$ as $s_1 \rightarrow (0,0), s_2 \rightarrow (1,0), s_3 \rightarrow (1,1), s_4 \rightarrow (0,1)$. The detection at the destination is performed jointly by the ML criterion and we can expand (3.5) as

$$\hat{x}_s = \arg \max_{x_s \in \chi} p_{sd}(y_{sd} | x_s)$$

$$= \left\{ \begin{array}{l}
1-(\varepsilon_1 + 2\varepsilon_2) \right\} + \varepsilon_1 p_{rd}(y_{rd} | x_r = x_s) \\
+ \varepsilon_2 p_{rd}(y_{rd} | x_r = x_s e^{j\pi/2}) \\
+ \varepsilon_3 p_{rd}(y_{rd} | x_r = x_s e^{-j\pi/2})
\end{array} \right\}$$

(3.7)

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ denote the symbol error probabilities from $s_1 \rightarrow s_3, s_1 \rightarrow s_2$ and $s_1 \rightarrow s_4$ respectively; $\varepsilon_1$ and $\varepsilon_2 = \varepsilon_3$ are analytically expressed as the Gaussian $Q(\cdot)$ function where $Q(x) := \left(1/\sqrt{2\pi}\right) \int_x^{\infty} \exp(-t^2/2) dt$. In the 2nd term of (6), we include the multiplicative error term in exponential function, $e^{j\phi}$ with the following equality $x_r = x_s e^{j\phi}$ where $\phi \in \{0, \pi, \pi/2, -\pi/2\}$ denoting the phase changes that depends on the symbols transmitted from the relay. This means that (3.7) takes into account that the relay does not operate error-free. In this paper, we derive closed-form expressions for the probability of error for each constellation symbol for QPSK as

$$\varepsilon_1 = P(x_r = x_s e^{j\pi})$$

$$= \frac{1}{4} erfc^2\left[\frac{E_s}{\sqrt{2} N_0}\right] = \left\{ \frac{E_s}{\sqrt{N_0}} \right\}^2$$

(3.8)

$$\varepsilon_2 = P(x_r = x_s e^{j\pi/2}) = \varepsilon_3 = P(x_r = x_s e^{-j\pi/2})$$

$$= Q\left(\frac{E_s}{\sqrt{N_0}}\right) - \left\{ Q\left(\frac{E_s}{\sqrt{N_0}}\right) \right\}^2$$

(3.9)

where $erfc$ is the complementary error function. When $y_{sd}$ and $y_{rd}$ are received at the destination, by inserting $s_1, s_2, s_3$ or $s_4$ to $x_s$ and examining how large the argument value from the augment (3.7), we can determine the transmit signal point $x_s$ from the finite set $\chi$ in QPSK constellation. The PDF for each term corresponding to the following expressions:
\[ p_{rd}(y_{rd} \mid x_r = x_s e^{j\phi}) = \frac{1}{2\pi N_0} \exp \left\{ -\frac{|y_{rd} - h_{rd} x_s e^{j\phi}|^2}{2N_0} \right\} \] (3.10)

where \( \phi \in \{0, \pi, \frac{\pi}{2}, -\frac{\pi}{2}\} \). Detail derivations of (3.8) and (3.9) are presented in Appendix A. It is clear from (3.8) and (3.9) that each QPSK symbol takes on a different symbol error probability. The analytical results presented thus far in previous works have been derived from studies which examined the SER problem assuming that the symbol error probability of each QPSK symbol is equally likely. Thus, these results cannot be treated as offering a complete ML solution. Note that another advantage in the proposed ML over C-MRC is its flexibility of combining different modulated signals from different nodes since each link can be treated independently (symbol-wise detection). In C-MRC, if the potential errors cannot be accurately modeled, maximizing the SNR would not result in the improvement of the scheme.

For completeness, we also examine the scheme in the absence of the CSI of S-R link at the destination node. The destination ignores the error possibility at the relays i.e., \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0 \) in (3.7). Hence, we obtain

\[ \hat{x}_s = \arg \max_{x_s \in \mathbb{Z}} p_{sd}(y_{sd} \mid x_s) p_{rd}(y_{rd} \mid x_r = x_s) \] (3.11)

One can gain insight about (3.11) that it is identical with the conventional MRC. This means the combiner at the destination does not explicitly take into account the uncertainty of the relay decisions when S-R link is erroneous.

Next, we present a special case when BPSK is used, instead. For BPSK, the relay decision process can be simply modeled as a binary symmetric channel (BSC) with the probability of a binary decision error at the relay \( P(x_r \neq x_s) \). This is because the S-R link is modeled as a Quasi-static Rayleigh fading channel and each channel is equivalent to a specific AWGN channel having a complex gain of \( h_{sr} \). When the relay node \( R \) receives the signal from the source node, it does a hard decision \( x_r \) for \( y_{sr} \) and forwards it to the destination node.

The relay decision process can be characterized by a random variable \( \mu \) defined as

\[ \mu = \begin{cases} 1 & : x_r = x_s \\ 0 & : x_r \neq x_s \end{cases} \] (3.12)
with \( E\{\mu\} = 1 - \varepsilon_b \) = \( P(x_r = x_s) \) where \( \varepsilon_b \) is the probability of bit error at the relay. Therefore, \( p(y_{rd} | x_r) \) can be shown as

\[
p(y_{rd} | x_r) = \begin{bmatrix} P(x_r = x_s)p_{rd}(y_{rd} | x_r = x_s) \\ + P(x_r \neq x_s)p_{rd}(y_{rd} | x_r \neq x_s) \end{bmatrix} = P(\mu = 1)p_{rd}(y_{rd} | x_r = x_s) + P(\mu = 0)p_{rd}(y_{rd} | x_r \neq x_s)
\]

(3.13)

where the transition PDF of \( p_{rd}(y_{rd} | x_r \neq x_s) \) is represented by

\[
p_{rd}(y_{rd} | x_r \neq x_s) = \frac{1}{\sqrt{2\pi N_0}} \exp \left[ -\frac{|y_{rd} - h_{rd}x_s|^2}{2N_0} \right]
\]

(3.14)

In (3.14), \( x_s \) is the complement of \( x_s \in \{+1, -1\} \).

Inserting the conditional PDF into (3.13), the transition PDF of this relayed path for BPSK case can be shown as

\[
p(y_{rd} | x_r) = p_{srd}(y_{rd} | x_r) = (1 - \varepsilon_b)\frac{1}{\sqrt{2\pi N_0}} \exp \left[ -\frac{|y_{rd} - h_{rd}x_s|^2}{2N_0} \right] + \varepsilon_b \frac{1}{\sqrt{2\pi N_0}} \exp \left[ -\frac{|y_{rd} - h_{rd}x_s|^2}{2N_0} \right]
\]

(3.15)

In brief, the proposed scheme features

i. A simple detector which is capable of mitigating the potential errors at the noisy relays.

ii. A new error model which serves as the side information for the detection at the destination by considering individual SER of the signal points in QPSK. Since (3.7) considers individual SER in its decision, our framework also suits well for other type of modulations. Nonetheless, since having complete CSI for S-R link can be resource-exhaustive, we propose another scheme as in (A.9) in Appendix A for QPSK considering only average SER for each modulation.

### 3.1.2 ML-based Combining Strategy with Arbitrary Modulations in Multi Relay Schemes

C-MRC proposed in [27] can result in serious propagation error under asymmetrical networks when SNR of relay-destination (R-D) link is larger than that of source-destination (S-D) link or source-relay (S-R) link. In addition, unlike ML-based
strategy, C-MRC cannot be used in relay networks with arbitrary modulation [38]. In our work [39], we have proposed an ML-based combining strategy which exploits every symbol error probability for the detection at the destination in quaternary phase-shift keying (QPSK). To guarantee an optimal ML criterion detection, the destination needs to know the error characteristics of the S-R link (perfect CSI) in the form of relay error probabilities. However, [39] is not defined properly for higher modulation constellations but focusing on only binary signals with the same modulation constellation at the source and the relays.

In this section, we generalize the proposed algorithm in [39] to multiple relay schemes with $M$-quadrature amplitude modulations ($M$-QAM) e.g., 16QAM. Unlike in C-MRC, the instantaneous CSI in the proposed scheme involves Q-function expression for each symbol in the modulation which provides accurate knowledge of S-R link. For simplicity, we analyze this ML performance with a simple DeF in uncoded cooperative relay networks and compare against the baseline C-MRC under symmetrical channels where all link SNRs are the same and asymmetrical channels that is where the R-D link is different from others. We also show through channel capacity analysis that the proposed scheme is superior to C-MRC in both network setups. In addition, our work here also investigates the proposed scheme when different modulations are used at the source and the relay under noisy relay channels as opposed to the solution in [38]. Through computer simulations we observe that the proposed scheme is not only practical to the different modulated signals but also shows a remarkable potential in achieving significant diversity gains with better packet error rate (PER) performance than that of C-MRC.

### 3.1.2.1 System Model

We consider a general case shown in Figure 3.2, a source node (S) and a destination (D) with L relays $R_l$, $l \in \{1, 2, ..., L\}$ over flat Rayleigh fading channels. Assuming time division multiplexing for simplicity, the source transmits its signal $x$ in the first timeslot to the destination and the relays with the average power $E_x$. Due to the broadcast nature
of wireless channel, both the destination and all $L$ relays receive noisy symbols of $x_s$.

Figure 3.2: Block diagram of the cooperative relay system with multiple relay channels.

The received signals at the destination and at the $l$-th relay denoted by $y_{sd}$ and $y_{sr,l}$ respectively can be written as

$$
y_{sd} = h_{sd}x_s + n_{sd}$$
$$y_{sr,l} = h_{sr,l}x_s + n_{sr,l}
$$

(3.16)

where the subscripts indicate the node relation such that $h_{sd}$ and $h_{sr,l}$ are independent complex-valued channel gains for the S-D link and S-R link of the $l$-th relay respectively. For simplicity, all channels are Quasi-static Rayleigh fading channels i.e., $h_{sd} \sim \mathcal{CN}(0,1)$ and $h_{sr,l} \sim \mathcal{CN}(0,1)$, where $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$. $n_{sd}$ and $n_{sr,l}$ are independent additive white Gaussian noise at the destination and the relay respectively which are modeled as $n_{sd} \sim \mathcal{CN}(0, \sigma_{sd}^2), n_{sr,l} \sim \mathcal{CN}(0, \sigma_{sr,l}^2)$ with variance equal to $N_0 / 2$ per dimension. We
assume that the average SNR for all links are the same denoted as \( \bar{\gamma} = E_s / N_0 \), while the instantaneous SNR is represented as \( \gamma_{sd} = |h_{sd}|^2 \bar{\gamma} \) and \( \gamma_{sr,j} = |h_{sr,j}|^2 \bar{\gamma} \) respectively. The relay performs a hard-decision detection (DeF) and re-modulates the detected symbol as \( x_{r,j} \) with the same average power \( E_s \) for re-transmissions in timeslot 2. The symbol received at the destination is given as

\[
y_{rd,l} = h_{rd,l}x_{r,l} + n_{rd,l}
\]

(3.17)

where \( h_{rd,l} \sim \mathcal{CN}(0,1) \) and \( n_{rd,l} \sim \mathcal{CN}(0,\sigma_{rd,l}^2) \) with variance equal to \( N_0 / 2 \) per dimension. The instantaneous SNR is \( \gamma_{rd,l} = |h_{rd,l}|^2 \bar{\gamma} \).

At the destination, the received signals from the source and the relay node are combined in order to recover the original source data.

### 3.1.2.2 ML-based Combining Strategy with Arbitrary Modulations

In this section, we generalize our work in [39] to M-QAM with different modulations at the source and the relay nodes. To better motivate the proposed ML combining strategy, let us take a closer look at C-MRC. In [27], the authors have proposed an improved version of MRC termed as C-MRC. For multiple relay cases, the C-MRC output at the destination node is given by assuming independent relay channels

\[
y_{cmrc} = h_{sd}^*y_{sd} + \sum_{l=1}^{L} \frac{\gamma_{min,l}}{\gamma_{rd,l}} h_{rd,l}^*y_{rd,l}
\]

(3.18)

where \( \gamma_{min,l} = \min(\gamma_{sr,l},\gamma_{rd,l}) \), \( \gamma_{sr,l} \) and \( \gamma_{rd,l} \) are instantaneous SNR of the S-R and R-D channels for the \( l \)-th relay node respectively. \( \gamma_{min,l} \) is tight approximation of the equivalent SNR of the S-R-D link at high SNR [27]. As (3.3), the logic associated with (3.18) is that when \( \gamma_{sr,l} \) is high, the detector places full confidence to the arriving signals from the relay. In case of low \( \gamma_{sr,l} \), the confidence is weighted according to the ratio of
both hops that is S-R-D link. In fact, from our knowledge, like its predecessor MRC, (3.18) cannot be easily used for signals with different modulations. Thus, we compare the proposed scheme also against the conventional SC which has been widely used to combine signals from different modulation constellations [37].

The proposed algorithm in [39] optimally combines the noisy signals received at the destination node, \( y_{sd} \) and \( y_{rd,l} \) by considering the effect of detection errors at the output of the \( l \)-th relay. However, the focus is only on the combining method with the same modulation, QPSK at both the source and the relays. From [39], the corresponding joint ML decision criterion finds \( \hat{x}_y \), an estimate of \( x_y \). and is defined as

\[
\hat{x}_y = \arg \max_{x_y, x_j \in \left\{ \chi_s = \chi_{r,l} \right\}} \left\{ P(x_{r,l} = x_j) p_{rd}(y_{rd,l} \mid x_{r,l} = x_j) \times \prod_{l=1}^{L} \left[ \frac{P(x_{r,l} \neq x_j)p_{rd}(y_{rd,l} \mid x_{r,l} = x_j)}{1 + \sum_{x_{r,l} \in \chi_{r,l}} P(x_{r,l} \neq x_j)p_{rd}(y_{rd,l} \mid x_{r,l})} \right] \right\}
\]  

(3.19)

where \( \chi_s \) and \( \chi_{r,l} \) denote the finite set of the constellation at the source and the \( l \)-th relay respectively; we use capital \( P \) as the probability; \( p_{sd}(y_{sd} \mid x_j) \) is the PDF of the source signal \( y_{sd} \) conditioned upon the transmitted signal \( x_j \) and \( p_{rd}(y_{rd,l} \mid x_{r,l} = x_j) \) is the PDF of the relayed signal \( y_{rd,l} \) conditioned on the equality of both transmitted symbols \( (x_{r,l} = x_j) \). The bracketed term in (3.19) has to consider the error probability of the received signal \( y_{sr,l} \) at the \( l \)-th relay accounting the individual SER of each signal point.

In QPSK modulation, the transmit symbol \( x_y \), which is labeled by two bits, \( (b_1, b_2) \) takes from the constellation set \( \chi_s = \{s_1, s_2, s_3, s_4\} \). Assuming the source and the relays use the same QPSK modulation i.e., \( \chi_s = \chi_{r,l} \), the detection at the destination is performed jointly by the ML criterion and we can expand (3.19) as
\[ \hat{x}_s = \arg \max_{x_s, y_s \in \{s_1, s_2, \ldots, s_4\}} p_{sd}(y_{sd} \mid x_s) \prod_{i=1}^{L} \left\{ \begin{array}{l}
1 - (\varepsilon_1 + 2\varepsilon_2) p_{rd}(y_{rd,i} \mid x_{r,j} = x_s) \\
+ \varepsilon_1 p_{rd}(y_{rd,i} \mid x_{r,j} = x_s e^{j\pi/2}) \\
+ \varepsilon_2 p_{rd}(y_{rd,i} \mid x_{r,j} = x_s e^{-j\pi/2}) \\
+ \varepsilon_3 p_{rd}(y_{rd,i} \mid x_{r,j} = x_s e^{j\pi/4}) \end{array} \right\} \quad (3.20) \]

where \( \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) denote the symbol error probabilities from \( s_1 \to s_3, s_1 \to s_2 \) and \( s_1 \to s_4 \) respectively; \( \varepsilon_1 \) and \( \varepsilon_2 = \varepsilon_3 \) are analytically expressed as the Gaussian \( Q \) function where \( Q(x) = \left(1/\sqrt{2\pi}\right) \int_x^{\infty} \exp\left(-t^2/2\right) dt \). In the bracketed term of (3.20), we include the multiplicative error term in exponential function, \( e^{j\phi} \) with the following equality \( x_{r,j} = x_s e^{j\phi} \) where \( \phi \in \{0, \pi, \pi/2, -\pi/2\} \) denotes the phase changes that depends on the symbols transmitted from the relay. This means that (3.20) takes into account the fact that the relay does not operate error-free. In this work we employ closed–form expressions for the probability of error for each constellation symbol for QPSK as

\[ \varepsilon_1 = P(x_{r,j} = x_s e^{j\pi/2}) = \frac{1}{4} \text{erfc}\left[ \frac{E_s}{\sqrt{2}N_0} \right] = \left\{ Q\left( \frac{E_s}{\sqrt{N_0}} \right) \right\}^2 \quad (3.21) \]

\[ \varepsilon_2 = P(x_{r,j} = x_s e^{j\pi/4}) = \varepsilon_3 = P(x_{r,j} = x_s e^{-j\pi/4}) = \left\{ Q\left( \frac{E_s}{\sqrt{N_0}} \right) \right\}^2 \quad (3.22) \]

where \( \text{erfc} \) is the complementary error function. For higher modulation like 16QAM, we provide some expressions of the individual SER in Appendix A. When \( y_{sd} \) and \( y_{rd,i} \) are received at the destination, by inserting \( s_1, s_2, s_3 \) or \( s_4 \) to \( x_s \) and examining how large the argument value in (3.20), we can determine the transmit signal point \( x_s \) from the finite set \( \chi_s \) in QPSK constellation. The PDF expression in (3.20) can be represented by

\[ p_{rd}(y_{rd,i} \mid x_{r,j} = x_s e^{j\phi}) = \frac{1}{\sqrt{2\pi\sigma_{rd,i}^2}} \exp\left\{ -\frac{|y_{rd,i} - h_{rd,i} x_s e^{j\phi}|^2}{2\sigma_{rd,i}^2} \right\} \quad (3.23) \]

where \( \phi \in \{0, \pi/2, \pi\} \). The analytical results presented thus far in previous works
have been derived from studies which examined the SER problem assuming that the symbol error probability of each QPSK symbol is equally likely (average SER). Thus, these results cannot be treated as offering a complete ML solution. Note that another advantage in the proposed ML over C-MRC is its flexibility of combining different modulated signals from different nodes since each link can be treated independently (symbol-wise detection).

Next, we generalize (3.20) to $M$-QAM. From (3.20), we observe that there are $\chi_{r,l} - 1$ ways of making an incorrect decision and their impacts on detection at the destination are not necessarily the same. Thus, we can easily show the decision criterion for general $M$-QAM as

$$
\hat{x}_s = \arg \max_{x_s, x_s \in \{X_s \neq X_s\}} p_{sd}(y_{sd} | x_s) \times \prod_{l=1}^{L} \left\{ \left( 1 - \sum_{\kappa = 1}^{L} \epsilon_{\kappa} P_{rd}(y_{rd,l} | x_{r,l} = x_s) \right) + \sum_{\kappa = 1}^{L} \epsilon_{\kappa} P_{rd}(y_{rd,l} | x_{r,l}^\kappa) \right\}
$$

(3.24)

where $\epsilon_{\kappa}, \kappa = \{1, 2, ..., \chi_{r,l} - 1\}$ is the symbol error probability for each symbol in $M$-QAM according to the modulation size in each relayed path and are expressed in Q-function as well. For example in Appendix A, we illustrate the derivations of $\epsilon_{\kappa}$ for some 16QAM symbols.

### 3.2 Analysis of Signal Combining Schemes

In this sub-section, we analyze C-MRC and the proposed ML schemes in terms of their channel capacity. Here, we assume one relay node for simplicity. Let us denote the channel capacities of S-R, R-D and S-D links by $C_{sr}(\gamma_{sr}) = \log_2(1 + \gamma_{sr})$, $C_{rd}(\gamma_{rd}) = \log_2(1 + \gamma_{rd})$ and $C_{sd}(\gamma_{sd}) = \log_2(1 + \gamma_{sd})$ respectively, and the joint capacity of the combined signals at the destination during the cooperative phase by $C_{tol}$. The channel capacity unit is bit per channel use. The total capacities $C_{tol}$ for C-MRC and the proposed scheme are as follows [40]
Assuming the instantaneous SNR for each link $\gamma_{sr}, \gamma_{rd}, \gamma_{sd} \geq 0$, then we have

$$C^{C-MRC}_{tol} = C(\gamma_{sd} + \gamma_{rd})$$
$$C^{ML}_{tol} = C(\gamma_{sd}) + C(\gamma_{rd})$$

(3.25)

if and only if $C(\gamma_{sr} + \gamma_{rd}) + C(\gamma_{sd}) \geq C(\gamma_{sd} + \gamma_{rd}) + C(\gamma_{sr})$  

(3.26)

(3.25) and (3.26) show that the variations in the relayed link reduces the total channel capacity. Particularly, the degradation in performance of C-MRC can be worse than that of the ML i.e., $C^{ML}_{tol} \geq C^{C-MRC}_{tol}$. We also prove this claim by computer simulations in Section 3.4.

3.3 Complexity Comparison

The computational complexity of the receiver at the destination depends on the detection algorithms, the hardware architectures, and other factors. In this study, we evaluate the computational complexity for our proposed scheme, C-MRC and SC, based on the number of multiplications and additions only. For convenience, we consider the required computations for the functions of equalization, detection and signal combining at the destination in a relay node scheme ($L = 1$) only. Table 3.1 compares the number of required complex multiplications and additions for each scheme per symbol. In this case, we assume QPSK modulation is used at the source and relay node.

<table>
<thead>
<tr>
<th>Complexity</th>
<th>SC</th>
<th>C-MRC</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>46</td>
<td>60</td>
<td>224</td>
</tr>
<tr>
<td>Addition</td>
<td>16</td>
<td>24</td>
<td>76</td>
</tr>
</tbody>
</table>

Table 3.1 shows that the proposed scheme has the highest computation complexity among
the receivers. SC turns out to be the lowest but with a significant reduction in the error rate performance as shown in the following section. SC only uses one signal for detection at the receiver and hence, the computation is less. This outcome for our proposed scheme is expected since the additional complexity in the scheme is coupled with a significant error rate improvement compared against the conventional SC and C-MRC in various simulation setups as shown in the manuscript. The complexity of the scheme is high because the destination has to consider individual SER of making wrong decisions at relay nodes in the detection. However, to assist the detection at the destination, our proposed scheme only requires the average receive SNR of S-R link. Therefore, our proposed scheme still inherits an interesting trade-off between the error rate performance and the system complexity. Although C-MRC is simpler in the computational complexity, its biggest challenge is to have accurate instantaneous channel knowledge at the receiver. In practice, one needs a high signaling overhead in C-MRC scheme to feedback the channel knowledge to the destination. In fact, another disadvantage for C-MRC is that there is no practical C-MRC approach for combining different modulated signals.

### 3.4 Results

Table 3.2: Simulation parameters

<table>
<thead>
<tr>
<th>Information Bits</th>
<th>1008 per packet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulations</td>
<td>QPSK (4QAM) and 16QAM</td>
</tr>
<tr>
<td>Channel Model</td>
<td>Quasi-static Rayleigh Fading Channel</td>
</tr>
<tr>
<td>Relay Protocol</td>
<td>DeF</td>
</tr>
<tr>
<td>Relay Equalizer</td>
<td>Zero Forcing</td>
</tr>
<tr>
<td>Combining Protocol</td>
<td>Proposed ML / C-MRC/SC</td>
</tr>
</tbody>
</table>

We use the parameters in Table 3.2 for the following simulation works unless otherwise specified.
3.4.1 ML-based Combining Strategy in Multiple Relay Systems

We analyze Packet Error Rate (PER) against average SNR in decibel (dB). We first assume the source and relay nodes transmit QPSK symbols with the same average power $E_s$ resulting in the average SNR, $\bar{\gamma} = E_s / N_0$ (symmetrical network). For C-MRC, we also consider the destination has a perfect knowledge of S-R links (i.e., instantaneous SNR) and perfect channel estimation is assumed. In this simulation, we only consider blind cooperative relaying schemes where relay nodes always re-transmit to the destination whether the signal is correctly detected or contains errors. No automatic repeat request (ARQ) protocol is used to avoid the error propagation from the relay nodes to the destination.

First, we check the PER performance of the proposed scheme as shown in (3.20) against the baseline for multiple relay nodes i.e. $L = 1, 2$ and $3$. In Figure 3.3, as expected, the proposed schemes outperform C-MRC (3.3) in all cases with around 0.2dB, 0.5dB and 1dB gap at $\text{PER} = 10^{-3}$ for 1, 2 and 3 relay cases respectively. In particular, we can also observe that all the cooperative schemes achieve full order diversity as observed from the slopes of the curves i.e., $10^{-(L+1)/10}$ (diversity order of $L + 1$) but a significant decrease in diversity gain for No CSI cases (3.11). This result demonstrates that the proposed algorithm has better accuracy of symbol detection due to the sufficient statistics of the received signals $y_{sd}$ and $y_{rd}$. For this reason, the conditional probability $p_{rd}(y_{rd} | x_r)$ can be computed using the observations $p_{rd}(y_{rd} | x_r \neq x_s)$. As for the No CSI case, the diversity order of 1 is achieved for all cases. PER performance further degrades from $L = 3, 2$ and 1 relay case. We note that when no CSI of S-R link is available at the destination, the decision by the destination are done using significantly erroneous assumptions that there are no detection errors at the relay nodes. As a result, they convey false reliability measures to the decoder and the performance is noticeably affected (more than 15dB loss at $\text{PER} = 10^{-3}$ compared to the proposed ML schemes for all cases). In practice therefore, a cooperative scheme with DeF which does not account for the S-R CSI may not be very effective.
Second, we simulate the lower bounds corresponding to each case where a perfect relay is employed i.e., the same signal from the source is transmitted to the destination (complete MRC). As shown in Figure 3.4, for PER=10^{-3} the gap between the lower bound and the proposed scheme is 2dB for 1 relay case and 3dB for 2 and 3 relay cases each. In perfect relay cases, the relays just replicate the same data as the source node; hence the performance can be regarded as the lower bound of the cooperative schemes. We can also notice that the gap to the lower bounds become almost constant if larger network is applied (multiple relays). This performance improvement is achieved by employing only a hard-decision protocol (DeF) at the relay nodes. Such a system design is practical for wireless networks which usually cannot compromise on the high energy consumption and longer time delay at the relays. We also plot no relay case using BPSK modulation for fair comparison. It is clear that the gain from the proposed ML scheme is significantly large due to cooperative gain.

Figure 3.3: PER comparison with QPSK between the proposed ML scheme and C-MRC (dashed lines) using DeF protocols for $L = 1, 2$ and 3 relays.
Figure 3.4: PER comparison with QPSK between the lower bounds (dashed lines) and the proposed ML schemes (solid lines) in multiple relay schemes. The lower bounds are simulated with perfect relays.

Figure 3.5: PER performance with QPSK when the proposed ML scheme (solid lines) using only average SER, $\xi_0$, against the baseline C-MRC (dashed lines).
Third, we investigate the proposed ML schemes when the detector considers the average SER of QPSK in the decision criterion (equation (A.9) is derived in Appendix A) as shown in Figure 3.5. Interestingly, their performance is almost the same in all SNR regions as C-MRC. By using average SER in the algorithm, the observation in $p_{rd}(y_{rd} | x_r \neq x_s)$ is equally weighted for all QPSK symbols. As a result, this has caused some degradation in PER performance compared to the proposed pure ML case (3.20) as shown in Figure 3.3. In fact, the convergence of the two schemes is also expected due to the approximation at high SNR of the baseline scheme [27]. Note that keeping perfect CSI of S-R link at the destination can be energy consuming and involve higher computations i.e., $\varepsilon_1$ and $\varepsilon_2$ required for perfect CSI, but only $\varepsilon_0$ is required for the average case. Thus, this proposed ML strategy poses a practical solution which constitutes a good tradeoff between the perfect CSI requirement and error rate performance.

![Figure 3.6: PER comparison with QPSK between the proposed ML scheme (solid lines) and C-MRC (dashed lines) when the average SNR of R-D link, $\gamma_{rd}$ varies at $L = 1$ relay case.](image-url)
Finally, in Figure 3.6, we simulate the proposed scheme and C-MRC when only one relay node is used but we vary the average SNR for R-D link $\bar{\gamma}_{rd}$ and we keep the average SNR for S-D link and S-R link the same, $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}$. We simulate the schemes at three different scenarios of R-D link quality: $\bar{\gamma} + 15$dB (+15dB), $\bar{\gamma} - 15$dB (-15dB) and $\bar{\gamma}_{rd} = \bar{\gamma}$ (equal). In view of this result, we infer that the proposed scheme can outperform C-MRC when R-D link has sufficiently high SNR quality (+15dB) with 1dB gap at PER=$10^{-3}$ and 2.5dB gap at PER=$10^{-2}$ for low SNR quality (-15dB). When R-D link has higher SNR compared to S-D or S-R links, the combined signal at the destination is dominated by the erroneous signal from the relayed link. Thus, the PER performance is degraded further compared to the case of equal SNR. This significant performance demonstrated by the proposed ML scheme renders it more suitable when the relayed link has poor SNR quality. In C-MRC, one can also check from (3.3) that the sub-optimality of C-MRC becomes inherently evident when the weighted signal from the relayed link becomes larger than that of the direct link. C-MRC effectiveness is largely conditioned on the link quality of R-D link over S-D link (direct path). Surprisingly, it can be easily seen that the proposed ML scheme achieves the advantages of the cooperative gain by using only simple DeF protocol with increasing improvement in multiple relay schemes. In particular, although the received signals at the relays are noisy and only DeF is used at the relays, the proposed scheme improves achievable PER performance. Furthermore, the proposed scheme also even provides better error rate performance at low and high SNR of R-D link which becomes an added advantage compared to C-MRC.

3.4.2 ML-based Combining Strategy with Arbitrary Modulations in Multiple Relay Schemes

Throughout the simulation works, we use also the stipulated parameters in Table 3.2 unless otherwise stated. We analyze the Packet Error Rate (PER) against average SNR in decibel (dB). For convenience, we restrict our simulation work to QPSK and 16QAM modulations only. To reduce the computational complexity in the proposed ML for 16QAM, we adopt the max-log approximation. We assume the source and all relay nodes
transmit with the same average power $E_s$ resulting in the average SNR, $\mathcal{P} = E_s/N_0$ (symmetrical network). For C-MRC, we also consider the destination has a perfect knowledge of S-R link (i.e., instantaneous SNR) and perfect channel estimation is assumed. In this simulation, we only consider blind cooperative relaying schemes where relay nodes always re-transmit to the destination whether the signal is correctly detected or contains errors. No automatic repeat request (ARQ) protocol is used to avoid the error propagation from the relay nodes to the destination.

Figure 3.7: PER comparison with 16QAM at the source and the relay nodes between the proposed ML scheme and C-MRC (dashed lines) using DeF protocols for $L = 1, 2$ and 3 relays.
Figure 3.7 and Figure 3.8 show the PER performance of the proposed scheme for 16QAM modulation at both the source and the relays, \( \chi_s = \chi_{r,l} \) against the baseline for multiple relay nodes i.e., \( L = 1, 2 \) and 3. As expected, the proposed schemes outperform C-MRC (3.3) in all cases with 0.5dB, 1dB and 1.5dB gap at \( \text{PER} = 10^{-3} \) for 1, 2 and 3 relay cases respectively. We can also observe that all the cooperative schemes achieve full order diversity as observed from the slopes of the curves i.e., \( 10^{-(L+1)} / 10 \) (diversity order \( \bar{\gamma} \) of \( L + 1 \)). This result demonstrates that the proposed algorithm has better accuracy of symbol detection due to the sufficient statistics of the received signals \( y_{sd} \) and \( y_{rd,l} \). For this reason, the conditional probability \( p_{rd}(y_{rd,l} | x_{r,l}) \)
can be computed using the observations $p_{rd}(y_{rd,i} | x_{r,i} \neq x_s)$ . In practice therefore, with necessary CSI, the destination can optimally combine signals received from the source and noisy relays assisted by DeF protocol only.

Next, in Figure 3.8 we simulate the proposed scheme and C-MRC with the same 16QAM in both nodes under different R-D link quality. From here onwards we only simulate for one relay node ($L = 1$). Thus, for convenience we remove the subscript $l$ in the notation. We vary the average SNR for R-D link $\bar{\gamma}_{rd}$, and we keep the average SNR for S-D link and S-R link the same, $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}$. This scenario is feasible due to the nature of broadcast transmission of the source node with relays which are typically power-constraint nodes. We simulate the schemes at three different scenarios of R-D link quality: $\bar{\gamma} + 15\text{dB} (+15\text{dB})$, $\bar{\gamma} - 15\text{dB} (-15\text{dB})$ and $\bar{\gamma}_{rd} = \bar{\gamma}$ (equal). From Figure 3.8, we find that the proposed scheme can outperform C-MRC when R-D link has sufficiently high SNR quality (+15dB) with marginal 1dB gap at PER=$10^{-3}$ and 2.5dB gap at PER=$10^{-2}$ for low SNR quality (-15dB). One way to explain this is that when R-D link has higher SNR compared to S-D link, the combined signal at the destination is dominated by the erroneous signal from the relayed link whose error is due to the detection error at the relay. Given that the relay has made a decision error and hence the source and the relay send contradicting information to the destination. As a result, when the R-D has very low SNR, the PER performance is degraded further compared to the case of equal SNR. In C-MRC, one can also refer to (3.3) that the suboptimality of C-MRC is due to the weighted signal from the relayed link which becomes larger than that of the direct link. C-MRC effectiveness is largely conditioned on the link quality of R-D link over S-D link (direct path). In particular, although the received signals at the relays are noisy and only DeF is used at the relays, the proposed scheme improves achievable PER performance which becomes an added advantage compared to C-MRC. This result also confirms the channel capacity analysis in Section 3.2.

Another feature of our proposed scheme in (3.24) is the feasibility aspect in combining arbitrary modulations. In Figure 3.9, we simulate the proposed scheme with different modulations, QPSK and 16QAM at the relay node ($L = 1$). For the comparison, we use selection combining (SC) with the same channel setup. We also simulate a scheme when no relay is used with BPSK modulation. A simulated lower bound with one perfect relay
(i.e., error-less relay detection) is also included in this simulation. The results in Figure 3.9 clearly show that the proposed scheme outperforms SC scheme with great margins. In both combining techniques, as expected, we can clearly see that there is a slight improvement in PER if lower modulation i.e., QPSK is used at the relay which is about 1dB gain at \( \text{PER} = 10^{-2} \). This result is expected due to the fact that lower modulation is less vulnerable to errors.

![PER comparison proposed ML scheme and SC with 16QAM at the source and different modulation at the relay using DeF protocols for \( L=1 \) relay.](image)

Figure 3.9: PER comparison proposed ML scheme and SC with 16QAM at the source and different modulation at the relay using DeF protocols for \( L=1 \) relay.

Notwithstanding, Figure 3.9 does not consider the same total transmission rate at the destination. In Figure 3.10, in a fixed transmission rate scheme i.e., \( \eta = ((\log_2 \chi_s)^{-1} + (\log_2 \chi_r)^{-1})^{-1} \) which means that \( \eta \) is the same for the cases in comparison, we simulate when the source and the relay use different modulation assignments, \( \chi_s \neq \chi_r \). For simplicity, the scheme uses 2 sets of modulation
combinations from QPSK and 16QAM. For case 1 when S uses QPSK, the relay employs 16QAM (S=QPSK, R=16QAM). In case 2, the source uses 16QAM and the relay uses QPSK (S=16QAM, R=QPSK) which is identical to the curves in Figure 3.9. From Figure 3.10, the result clearly shows that the proposed scheme performs better when lower modulation is used at the source which is about 3dB improvement in the proposed scheme at PER $=10^{-3}$. The proposed scheme also easily achieves the full diversity gain of 2 for both cases. The same trend occurs in SC scheme with around 3dB improvement at PER=10$^{-2}$ but with lower diversity gain due to the error propagation from the relay. Higher modulations at the relay tend to be more susceptible to noisy channels. The 1dB loss in the simulation result is the direct outcome of the error propagation of the noisy channels. It is expected that higher $M$-QAM modulation is more susceptible to noise. In addition, in our proposed scheme, the ML detector places more weight on the signals coming from the source directly, thus giving less weight on the relayed link. This reduces the effect of the error propagation from the relay node. From all cases, we can draw a conclusion that assigning lower modulation like QPSK at the source is a better strategy to bring more performance improvement in relay networks. To prevent the deleterious effect of error propagation at the relay, it is important that the source node is assigned with lower modulation.
Figure 3.10: PER comparison proposed ML scheme and SC with different modulations at the source and the relay using DeF protocols for $L=1$ relay.

### 3.5 Conclusions

In this study, we developed a new signal combining strategy based on ML criterion for cooperative relay scheme which accounts the potential errors at relays. The errors are expressed as the Gaussian Q-function for each SER in the constellations. By applying these expressions in the detection at the destination, we can accurately model the transition probabilities for the erroneous transmission from noisy relays. The proposed ML scheme is superior to the conventional C-MRC in PER performance in all cases under the same CSI requirement. The proposed ML scheme has more flexibility in implementation compared to C-MRC depending on CSI at the destination. In fact, unlike C-MRC, the proposed scheme remains resilient to propagation errors even if R-D link has different SNR quality. In addition, with DeF protocol, this proposed ML scheme has shifted the processing complexity to the destination node whilst the relay nodes can conserve the energy and simple data processing. Thus, this makes this proposed strategy
amenable to implementation especially for resource-constraint environment such as wireless sensor networks.

In this work also, an extension of ML-based combining strategy for cooperative relay scheme to arbitrary modulations is proposed. Since the proposed scheme accounts the potential errors at the relays for the detection at the destination, we can accurately model the transition probabilities for the erroneous transmission from noisy relays. Our work also investigates the PER performance of the proposed scheme when the source and the relays have different modulations. We found that it is better to use lower modulation at the source, thus reducing possible error propagation from the relays. Through computer simulation, we show that the proposed ML scheme is superior to the conventional C-MRC in PER performance for all cases under the *symmetrical* or *asymmetrical* channels with greater flexibility in implementation compared to C-MRC regardless of the modulation schemes.

There are numerous detrimental issues, however, such as resource management (power, time), relay schedulers and increased overhead which have not been discussed in this chapter. Considering the assumptions and limitations of the scheme of interest, our primary concern here is to show the performance improvement against the baseline scheme under the specific channel models.
Chapter 4

Symbol-based Soft Relaying Strategy

Soft Forwarding (SF) has been widely studied as an effective relaying protocol which combines the features of Amplify-and-Forward (AF) and Decode-and-Forward (DF) [4]. AF preserves the reliability information but ignores the channel coding. DF, however, enjoys the coding gain but suffers from the error propagation. Furthermore, it loses the reliability information which provides the degree of uncertainty in the relayed signals. Therefore, SF is presented in [41]-[42] to reap the benefits of both previous strategies with applications to various systems. For instance, SF can be represented by the bit log likelihood ratios (LLR) values which are generated by a channel decoder. With this method, optimizing relay functionalities is of essential design strategies such that the reliability information brings sufficient statistics in the signal decoding at the destination [43]. These soft values are re-transmitted by relays in different ways e.g., based on the power constraint at relays (AF strategy) as Soft-Decode-and-Forward (SDF) in [41], or transmitting the expectation values in terms of the mean squared error (MSE) namely Estimate-and-Forward (ENF) [42],[43], Decode-Estimate-Forward scheme (DENF) [44]. Transmission of expected values from unreliable decoded signals can harm the detection at the destination. While these works assume Gaussian distribution in bit LLR computations for simplicity, [6] and [45] improve LLR computations to improve the detection at the destination which are based on the actual distribution but with higher complexity. However, these methods require high search operations in bit LLR
computations, especially if higher modulations are applied.

In conventional SF schemes, channel between source-relay link has to be highly reliable enough to ensure error-free re-transmission at the relay. In [41]-[42], to have reliable re-transmissions from the relays, the authors employ error correcting codes and cyclic redundancy checks (CRC) to assist the detection at the destination. Generally, early coded cooperative relay schemes used convolutional or Turbo codes [6], [27], [44] and [45], but recently applying Low Density Parity Check (LDPC) codes have gained more attention [46]. A drawback for single-input single output (SISO) coded scheme under block fading channel is the lack of coding gain due to the constant fading coefficient in each block of codeword [47]. This becomes a challenging task for LDPC coded schemes to cope with the erroneous and unreliable decoded signals at the relays. Another strategy to solve error propagations from the relays has been proposed in [26] and [48] based on Maximum Likelihood (ML) criterion bit LLR combining at the destination. However, this strategy increases the computational complexity at the destination due to the requirement of perfect S-R channel knowledge for optimal detection.

Unlike [41], [42], [44], which require bit-wise detection and decoding at relays, we propose a simple relaying strategy which is known as Symbol-based Soft Forwarding (SSF). This method provides a novel soft symbol estimation and a simple forwarding strategy by transmitting the expectation values from a linear combination of posteriori probabilities of each symbol in the constellation set. In particular, this strategy avoids severe impact of decoding errors and hence, expected values can be accurately computed. This idea is mooted from the fact that a relay behaves like a repeater. Thus, detail processing and bit-wise analysis of receive signals are not necessary at this stage. To achieve these goals, we simplify the relay architecture by developing a soft symbol estimation technique, as opposed to [42], [44] which is based hyperbolic tangent functions. More importantly, it will be shown that the proposed schemes improve bit error rate (BER) remarkably in various simulation setups. For comparison, we re-name the SF as bit-based Soft Forwarding (BSF).

The organization of this chapter is as follows: Section 4.1 is system description followed by the proposed scheme in Section 4.2. We present simulation results in Section
4.3, and in Section 4.4 this chapter is summarized. Finally the derivation of symbol LLR and expected value of transmit signal point for the proposed relay function is described in Appendix B.

4.1 System Model

We consider a hybrid relay network as in Figure 4.1 with a source, destination and parallel/serial relays. It is assumed that there is also a direct transmission from the source (S) to the destination (D). The total number of serial relays in one link is denoted as $L_s$ and the total number of parallel relays as $L_p$. We denote $R_{p,s}$ as a relay node with the $s$-th hop and $p$-th branch. We assume that all receiving nodes have perfect channel state information (CSI). All nodes have only one antenna working in a half-duplex mode. In this system, $S$ broadcasts to $D$ and to all branch relays of the first hop $(R_{1,1},...,R_{L_p,1})$. Then, these relays forward the received signal to its respective relay nodes in the $p$-th branch as shown in Figure 4.1 above and the final $s$-th hop forwards the signal to the destination. It is assumed that each link does not disturb each other which can be implemented by using orthogonal channels.

At the source, information bits $u = \{u^{(c)}, c = 1,2,...,K\}$ of length $K$ with $u^{(c)} \in \{0,1\}$
are encoded by an LDPC encoder of rate $R = K / N$ to $b = [b^{(1)}, ..., b^{(n)}, ..., b^{(N)}]$ of length $N$, $b^{(n)} \in \{0,1\}$. These coded bits are modulated as $x_S \in \{s_1, ..., s_m, ..., s_M\}$, where $M$ is $M$-phase shift-keying (MPSK) constellation size. For example, when the source uses QPSK modulation with Gray mapping then $M = 4$. Since every symbol $s_m$ corresponds to two bits $(b_1, b_2)$, we assign $s_1 \rightarrow (0,0)$, $s_2 \rightarrow (1,0)$, $s_3 \rightarrow (1,1)$, $s_4 \rightarrow (0,1)$ accordingly. The received symbol at a relay $R_{p,s}$ can be shown as

$$y_{p,s} = h_{p,s}x_{p,s-1} + n_{p,s}$$

(4.1)

where $h_{p,s}$ is the channel gain between the relay $R_{p,s}$ and $R_{p,s-1}$ with the signal $x_{p,s-1}$ originating from the latter relay node in the $p$-th branch, and $n_{p,s}$ is additive Gaussian noise with unit variance. For simplicity, we assume that all channels are Quasi-static Rayleigh fading channels with zero mean and variance $\sigma^2_{p,s}$. Upon receiving $y_{p,s}$, this relay estimates the symbol $x_{p,s-1}$ according to its relay function $f(y_{p,s})$ which is described in the following section, and forward the signal $x_{p,s}$ to successive relays in one path i.e., $R_{p,s}$ to $R_{p,s+1}$. We assume that the source and all relays operate under the same average power constraints which means $E\left\{\left|x_S\right|^2\right\} \leq P_S$ and $E\left\{\left|x_{p,s}\right|^2\right\} \leq P_R$, for some $P_S$ and $P_R$ where $E\{\}$ denotes the expectation operator. The average transmit SNR $\bar{\gamma}_0$ for all links are assumed the same $\bar{\gamma}_0 = \bar{\gamma}_S = \bar{\gamma}_R$ such that $\bar{\gamma}_0 = P_0 / N_0$, while the instantaneous receive SNR at relay $R_{p,s}$ is represented as $\gamma_{p,s} = \left|h_{p,s}\right|^2 \bar{\gamma}_0 = \left|h_{p,s}\right|^2 P_0 / N_0$. Finally, at the destination, all signals from the relays in the final hop are combined by a maximum ratio combining (MRC) strategy.

The system assumptions of this scheme are also summarized in Section 2.1.3. Apart from this, in this scheme, we assume the transmitting nodes are assumed to satisfy a delay constant on the transmission which requires each channel codeword must be
transmitted over one fading block. For the baseline schemes which employ decoding process at the relays, we assume there are possible decoding errors at the relays and this is common for schemes in block fading channels.

4.2 Soft Information Relaying

4.2.1 Baseline System

First, we summarize some of the relaying protocols described in Section I.

*Amplify-and-Forward (AF)*: The relay amplifies the received signal by a scaling factor. The received signal is normalized so that the power constraint is satisfied. That is,

\[ x_{p,s} = f(y_{p,s}) = \beta y_{p,s} \quad (4.2) \]

where \( \beta = \sqrt{P_R / E\{|y_{p,s}|^2\}} \) is a constant chosen so that \( E\{|x_{p,s}|^2\} \leq P_R \).

*Decode-and-Forward (DF)*: The relay decides on the hypothesis that minimizes the probability of error at the relay node and forwards this decision with constant power

\[ x_{p,s} = f(y_{p,s}) = \beta \text{ arg max } p(x_S | y_{p,s}) \quad (4.3) \]

*Soft Forwarding (SF)*: The relay forwards the MSE estimate subject to the power constraint amounts to linearly scaled version of the conditional expectation

\[ x_{p,s} = f(y_{p,s}) = \beta E\{x_S | y_{p,s}\} \quad (4.4) \]

With SF, the relay forwards reliability information of the detected signal to the destination. The behavior of SF is that when receive SNR at the relay is high, it behaves like DF and when the SNR is low it behaves like AF.

Next, to better motivate the proposed scheme, we elaborate the baseline SF protocol which we rename here as BSF to avoid confusion. From (4.1) after equalizing the channel between relays \( R_{p,s} \) and \( R_{p,s-1} \) (assuming perfect CSI), \( R_{p,s} \) receives the symbol

\[ \tilde{s} = \frac{y_{p,s}}{h_{p,s}} = x_{p,s-1} + \frac{n_{p,s}}{h_{p,s}} \quad (4.5) \]
where \( n_{p,s} / h_{p,s} \) is the equivalent zero mean complex Gaussian white noise with the equivalent variance \( \bar{\sigma}_{p,s}^2 = \sigma_{p,s}^2 / | h_{p,s} |^2 \). Then, the relay will compute bit LLR values as

\[
\lambda_b = \log \left( \frac{\sum_{s \in M, b(s) = 0} f(\tilde{s}, s)}{\sum_{s \in M, b(s) = 1} f(\tilde{s}, s)} \right)
\]

(4.6)

where we define \( f(\tilde{s}, s) = \exp\left\{ -s(\tilde{s} - s)^2 / (2\bar{\sigma}_{p,s}^2) \right\} \) and from (4.6), the bit LLR values are passed through the channel decoder (i.e., LDPC decoder in this study) to generate the a-posteriori probabilities (APP) which is denoted as \( \lambda_{\text{Dec}} \).

Then, if BPSK is considered, these LLR values are forwarded to the destination based on the normalized expectation value [42], [44]

\[
\lambda_{p,s}^{\text{BSF}} = \frac{P_r}{\sqrt{E\left\{ |\tanh(\lambda_{\text{Dec}} / 2)|^2 \right\}}} \tanh(\lambda_{\text{Dec}} / 2)
\]

(4.7)

As for QPSK, odd and even LLR values are scaled and forwarded over the in-phase and quadrature axes. This result is shown to be optimal in terms of MSE at the receiver of the relay node. The normalization in (4.7) is done to meet the power constraint at the relay. Nonetheless, the problem in this scheme is that it does not consider the probability of the entire bit sequence to approximate the LLR values since it employs the expression (4.6) above. Eventually, the likelihood function in (4.6) may contain approximation errors and then, these errors would be propagated to the destination along with the errors resulted from the message passing algorithm for LDPC decoder at the relay. The impact can be higher if more relays or higher modulation schemes are employed. Furthermore, the APPs obtained from the decoder are suboptimal because bits corresponding to one symbol are not independent.

### 4.2.2 Symbol-based Soft Forwarding (SSF)

In SSF as depicted in Figure 4.2, the relay utilizes a simple maximum likelihood detection (MLD) implementing soft symbol estimation and expectation values \( \bar{s} \) are forwarded.
without decoding to the destination. At the destination, all received signals from the relays and the direct link are combined in order to recover the original source data.

In this work, we introduce a new method of signal detection and forwarding at the relays which leads to a more efficient use of the relay resources. Details of the derivations are presented in Appendix B. From equation (B.7) in the Appendix, the forwarded signal from a relay which is in the form of the expectation values for one QPSK symbol can be computed as

\[ \bar{s} = E\{s\} = \frac{s_4 e^{\lambda_4} + s_3 e^{\lambda_3} + s_2 e^{\lambda_2} + s_1}{1 + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}} \]  

(4.8)

where \( \lambda_m, \ m \in \{1, 2, 3, 4\} \) is the \( m \)-th symbol LLR in QPSK modulation. Here, we propose a simple linear combination scheme to combine all the possible constellation points together with the corresponding posterior probability as the weight. With this strategy, we can decrease the computational complexity at the relays and avoid approximation errors in (4.6) of the baseline relay detection method. Furthermore, unlike in the proposed scheme, (4.6) has to do many search operations for optimal detection and this requirement is proportional to the constellation size of the modulation. From here onwards, we drop the subscripts of node relations in (4.1) for simplicity. In Table 4.1, we summarize the comparison between the proposed scheme and the baseline.

Figure 4.2: Block diagram of the proposed scheme
Table 4.1: Comparison of the proposed protocols against BSF

<table>
<thead>
<tr>
<th></th>
<th>SSF</th>
<th>BSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decoding</td>
<td>No</td>
<td>Yes (LDPC)</td>
</tr>
<tr>
<td>Transmission</td>
<td>Eq. (4.12)</td>
<td>Eq. (4.7)</td>
</tr>
<tr>
<td>Detection</td>
<td>Symbol-wise</td>
<td>Bit-wise</td>
</tr>
</tbody>
</table>

Another benefit of our proposed method in (4.8) is that it provides an alternative method to the well known soft-bit computations in [49].

**Lemma**: For special case of BPSK, (4.8) converges to the well known \( \bar{\sigma} = \tanh(\lambda_m/2) \).

**Proof**: The proof of this convergence is easily shown as follows. From (4.8), the expected value for BPSK signal is

\[
\bar{\sigma} = s_1 P(s_1 | y) + s_2 P(s_2 | y) = \frac{s_2 e^{\lambda_2} + s_1}{1 + e^{\lambda_2}} \tag{4.9}
\]

where we ignore the 3rd and the 4th term in the equation. Then, inserting \( s_1 = -1; \ s_2 = +1; \) Eq. (4.8) simply becomes

\[
\therefore \bar{\sigma} = \left( e^{\lambda_2} - 1 \right) / \left( e^{\lambda_2} + 1 \right) = \tanh \left( \lambda_2 / 2 \right) \tag{4.10}
\]

which proves the convergence of the proposed scheme to the formulation in [49]. Therefore, for QPSK \( (M = 4) \), we can re-write (4.8) as

\[
\bar{\sigma} = \sum_{m=1}^{M} s_m e^{\lambda_m} / \sum_{m=1}^{M} e^{\lambda_m} \tag{4.11}
\]

As opposed to (4.6) in the baseline scheme, (4.11) is just a simple linear combination of the probability of each symbol in the QPSK modulation. As described above for the baseline protocol, (4.11) also needs the power restriction as in (4.7) before re-transmission. And we denote the forwarded symbol from the relay as \( x_{p,s}^{SSF} \).

\[
x_{p,s}^{SSF} = \sqrt{\frac{P_R}{E[|\bar{\sigma}|^2]}} \beta \bar{\sigma} \tag{4.12}
\]

where we define \( \beta = \sqrt{\frac{P_R}{E[|\bar{\sigma}|^2]}} \) as the amplification factor such that \( x_{p,s}^{SSF} \) obeys the
power constraint $P_R : E\left\{ |x_{p,s}^{SSF} |^2 \right\} \leq P_R$.

4.2.3 Mean Square Error (MSE) At Relay

In fact, using (4.8), we minimize the MSE of the relayed signals and preserve the soft information. It provides sufficient reliability information which amounts to maximizing SNR at the relay output. MSE is described as the variance of the equivalent noise term. Nonetheless, MSE only provides a clue on the error rate performance since BER does not rely on the variance of the error but on the whole error distribution [44]. To prove this claim, we consider the MSE of the input signal from the source and the relay output for the first relay branch of the first relay hop can be shown as

$$MSE = E\left\{ (\bar{s} - s)^2 \mid y_{1,1} \right\}$$  \hspace{2cm} (4.13)

where $\bar{s}$ is the modulated symbols from the relay output. Below we plot the MSE versus average SNR of the relay node for SSF and the baseline BSF.

![Graph showing MSE vs. Average receive SNR](image)

Figure 4.3: Mean squared error at the output of the relay node over source input signal.

From Figure 4.3, it is obvious that the proposed scheme achieves better performance in MSE especially at higher SNR region. Although the difference is significant between
these schemes, the performance of these schemes can be further improved if accurate
approximation on the equivalent noise is found. In fact, Gaussian distribution is assumed
to model the equivalent noise in both schemes. Such an accurate distribution is important
for further improvement for these schemes and is beyond the aim of this thesis. This
means the proposed scheme retains the reliability in order to reduce propagation errors to
the destination. In fact, if decoding is used at the relay, wrong decisions are likely to
happen and this may cause further error propagation to the destination due to the
approximation in the message passing algorithm of the LDPC decoder at the relay.

4.2.4 Relay Mutual Information

In this section, we analyze the channel capacity for the relayed link against that of the
baseline. For convenience, we restrict this analysis to one relay node scheme
\((L_p = 1; L_s = 1)\). First, we have to evaluate the equivalent SNR of the relayed link. The
received signal at an arbitrary relay node can be shown in the following relation

\[
y_{p,s} = h_{p,s}x_{p,s-1} + n_{p,s}
\]

\[
= h_{p,s}\beta \left(\sum_{m=1}^{M_s} s_m e^{\lambda_m} / \sum_{m=1}^{M_s} e^{\lambda_m}\right) + n_{p,s}
\]

\[
= \beta h_{p,s} \sum_{j \neq c} s_j e^{\lambda_j} + \frac{\beta h_{p,s} \sum_{j \neq c} s_j e^{\lambda_j}}{\sum_{m=1}^{M_s} e^{\lambda_m} + \sum_{m=1}^{M_s} e^{\lambda_m}} + n_{p,s}
\]

where \(s_c\) in the first term is the symbol estimate of the original symbol from the source
and the second term is the noise. After some algebraic manipulations, for one relay node
scheme \(R_{1,1}\), the instantaneous SNR at the destination can be expressed as

64
\[
\gamma_{1,1}^D = \frac{\beta^2}{\rho} \frac{h_{1,1} e^{j\lambda_1}}{\sqrt{E|s_c|^2}} \left( \sum_{j=1}^{M} \frac{h_{1,1} e^{j\lambda_j}}{\rho} E|s_j|^2 \right) + E|n_{1,1}|^2
\]

where we let \( \rho = \sum_{m=1}^{M} e^{j\lambda_m} \). Therefore, the average channel capacity for the proposed scheme considering the relayed link is found by averaging over the channel gain distribution as follows

\[
C_{SSF} = \int_{0}^{\infty} p(\gamma_{1,1}^D) \log_2 \left( 1 + \gamma_{1,1}^D \right) d\gamma_{1,1}^D
\]

(4.16)

To illustrate the performance of the proposed scheme, we simulate it through Monte-carlo simulations for one relay node case which is shown in Figure 4.4.
4.3 Results

This section presents some results of simulations undertaken to illustrate the performance of the proposed schemes in BER against the average SNR per bit in decibel (dB), $\bar{\gamma}_0 = P_0 / N_0$ for the direct link in various simulation setups. Receiving nodes are assumed to have the same average receive SNR and perfect CSI of the immediate links. All simulation works use the following parameters in Table 4.2 unless otherwise stated. In this simulation, for simplicity, we only consider blind cooperative relaying schemes where relay nodes always re-transmit to the destination whether the signal is correctly detected or contains errors. No automatic repeat request (ARQ) protocol is used to avoid the error propagation from the relay nodes to the destination.

Figure 4.4: Average capacity of the relayed link at the destination for the proposed scheme.
Table 4.2: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information Bits</td>
<td>504 bits/packet</td>
</tr>
<tr>
<td>Modulations</td>
<td>QPSK</td>
</tr>
<tr>
<td>Channel Model</td>
<td>Quasi-static Rayleigh Fading Channel</td>
</tr>
<tr>
<td>Error Correcting Code</td>
<td>Regular (3,6) LDPC (1008,504)</td>
</tr>
<tr>
<td>Sum-Product Iteration</td>
<td>20 times</td>
</tr>
</tbody>
</table>

4.3.4 Multihop Setup ($L_p = 1; L_s = 1, 2$ and 3)

In this simulation, firstly we consider one relay branch ($L_p = 1$) with multiple hops. Figure 4.5 shows the BER versus average SNR in dB for the proposed schemes against the baseline BSF. The simulation results validate the derivation of the proposed symbol LLR using MLD criterion and show the performance improvement by SSF against the baseline BSF. In Figure 4.5, we observe that the combination of the soft symbol estimation technique and the proposed forwarding strategy outperforms the rest and provides large performance improvement at no decoding cost. When inter-node channel is noisy, a relay node without a decoder is sufficient to provide BER performance improvement. SSF improves the BER curve around 2dB margin against the baseline BSF for case ($L_p = 1; L_s = 1$). The relative margin increases as the number of relay nodes slightly increases in comparison with the baseline scheme. For comparison, in Figure 4.6, we also compare the proposed schemes against the performance of DF protocol. As expected, the gap is larger in DF protocol due to the absence of reliability information and decoding errors at relays.
Figure 4.5: BER comparison of SSF (blue) and BSF (red) in a multihop coded relay scheme.

Figure 4.6: BER comparison of SSF (blue) and DF (green) in a multihop coded relay scheme.
4.3.2 Multibranch Setup ($L_p = 1, 2$ and $3; L_s = 1$)

Next, Figure 4.7 illustrates the BER performance when 1, 2, or 3 relays in parallel are available to assist the source. It is clear that the proposed SSF can also improve effectively the BER performance for multiple relays. Interestingly, the gain demonstrated in this result increases remarkably as the number of relays increases. For instance, performance gain of more than 1dB each can be achieved easily for all cases at BER of $10^{-4}$. Intuitively, we expect that the effectiveness can be more evident if this proposed strategy is applied to larger network configurations. The degradation of the overall system for the baseline BSF is due to two main factors as follows:

1. Lack of diversity gain due to the use of LDPC codes in block fading channel [47].
2. Due to the approximation error since in the baseline scheme, relays need to compute the following

$$
\lambda_s = \log \left\{ \sum_{s \in M, b(s)=0} f(s|\tilde{s}) / \sum_{s \in M, b(s)=1} f(s|\tilde{s}) \right\} \tag{4.17}
$$

BSF utilizes the expression above to approximate the bit LLR values. This expression does not consider the probability of the entire bit sequence as opposed to our proposed scheme in (4.11). Note that the performance achievement of SSF is topped with better resource efficiency which simplifies the symbol LLR calculations and forwards reliability information of the detected symbols to the destination.
Figure 4.7: BER comparison of SSF (blue), BSF (red) and DF (green) in a multibranch topology.

### 4.3.3 Hybrid Multihop and Multibranch Setup \( (L_p = 2; L_s = 2) \)

Here, we consider a more general setup with parallel and serial relays in Figure 4.8. For simplicity, we only use \( (L_s = 2; L_p = 2) \) and we name this setup as hybrid multihop and multibranch scheme. Like in the previous results, SSF outperforms BSF with considerable margin around 2dB and 4dB against DF respectively. The loss in the baseline scheme is due to the constraints explained in the previous simulation result.
These results reflect the performance improvement in the proposed scheme for all simulation setups due to better reliability information and forwarding strategy we have employed. Nonetheless, even without decoding at the relays, the proposed schemes do not lose the coding gain entirely. The proposed schemes reduce the impact of error propagation from decoding errors since decoding is only done at the destination. Such a simple strategy is beneficial for low-complexity networks like sensor network which allows the possibility to deploy a large number of low-complexity relays.

4.4 Conclusions

This study proposes a novel soft forwarding protocol in LDPC coded scheme. SSF implements symbol-wise detection (but no decoding) based on a ML criterion. This strategy minimizes the impact of propagation error at relays and thus, provides better reliability information. We introduce a unified framework which features two key
strategies in these schemes: detection based on simple symbol LLR estimation at the relay and soft-forwarding strategy based on transmission of the expected values of signal point. This strategy sums up the probabilities of each modulated symbol and hence, avoids unnecessary approximation errors. A relay can be further simplified if the signals are treated symbol-wise since the signals are not originally intended for the relay use. Our main motivation is that LDPC decoder in Quasi-static Rayleigh fading environment gives little impact in SISO scheme and bit-wise analysis requires high computation and thus, consumes many resources at the relay node. Through simulation results, we prove that our simple strategy of SSF presents significant gains than the baseline BSF scheme.

In this thesis, the equivalent noise of the S-R-D link is approximated as a Gaussian distribution which is not a very accurate approximation. A more accurate and realistic approach should be devised such that more accurate analysis and performance can be obtained. Some earlier works like in [35] present an error model for soft information relaying protocols. However, there is still a lack of an accurate mathematical representation for modeling these decoding errors which still becomes an unresolved problem.
Chapter 5

Adaptive Relaying Protocol with Reliability Threshold

Detect-and-Forward (Def) relaying protocol is simple in complexity where the relay simply detects the signals (hard-decision detection), re-modulates before forwarding to the destination. If no error correction codes are used, the forwarded signals may be incorrect and, thus, the error probability must be taken into account in the decision process at the destination. With DeF, the authors in [25] have developed a piece-wise linear receiver approximating the maximum likelihood (ML) criterion detection that requires knowledge of the average signal-to-noise-ratio (SNR) of the first hop. However, this scheme cannot achieve full diversity for more than one relay. In [27], another combining technique namely Cooperative-Maximum Ratio Combining (C-MRC) is introduced that approximates the ML detector. In [25] and [27], the authors have derived sub-optimal receivers, but exploiting effectively perfect knowledge of all links is still an open problem, specifically the error probability at the relay. However, many of the previous works assumed that the destination only knows the average probability of symbol error at the relays. In [39], we have introduced a near-optimal detector with the knowledge of individual symbol error rate (SER) at relays. However, the complexity of this detector is high and proportional to the modulation size.

Transmission in cooperative relay networks usually requires orthogonal channels.
This bandwidth requirement makes these systems inefficient. Relay selection is an attractive method to bring about bandwidth efficiency of cooperative networks. In DeF relaying protocol, relays can forward erroneous information to the destination and thus, affects the end-to-end performance of the system. One popular technique to mitigate this problem is found in selective and adaptive relaying techniques where relays use link SNRs to evaluate the received signal from the source and forwards the signal if it passes a predetermined threshold value [50] and [51]. In [52] and [53], the reliability information is used to estimate the bit error rate (BER) of the received signals such that quality of service (QoS) can be improved. [54] proposed the idea of using log-likelihood ratios (LLRs) of the received bits as the threshold in a cooperative relay scheme with error-correcting codes. The authors presented a relaying strategy which adaptively employ amplify-and-forward (AF) and decode-and-forward (DF) protocols based on the reliability information.

On the contrary, in this work, we consider an adaptive strategy for DeF to a classical uncoded two-hop relay scheme generalized to multiple relay nodes, which we term this scheme as Adaptive DeF (ADeF). This relay node can decide whether to cooperate or not based on the quality of the received signal. Most reliable symbols can be forwarded to the destination and undesired symbols are discarded to prevent error propagation. In ADeF, relays need not forward the received signals if it is below a predetermined threshold level and thus, block unreliable symbols from re-transmissions through a simple threshold strategy. This method also discards the laborious requirement for perfect channel knowledge of the source-relay link for optimal detection at the destination. From computer simulations, we reveal that this proposed scheme achieves a significant performance gain under Additive White Gaussian Noise (AWGN) and Quasi-static Rayleigh fading channels with a marginal complexity increase in the relay architecture compared to the baseline DeF (no threshold strategy). Our contribution also includes a list of optimal threshold values for some SNR values.

The organization of this chapter is as follows: Section 5.1 is system description and the proposed scheme is shown in Section 5.2. In Section 5.3, the adaption algorithm is presented, followed by simulation results in 5.4. We present the conclusion of this chapter in Section 5.5. Derivations of the proposed scheme are presented in Appendix C.
5.1 System Model

We consider a general case shown in Figure 5.1, a source node (S) and a destination (D) with \( L \) relays \( R_l, l \in \{1,2,\ldots,L\} \). Assuming time division multiplexing, the source transmits its signal \( x_s \) in timeslot 1 to the destination and the relays with the average power \( P_s \). Due to the broadcast transmission, both the destination and all \( L \) relays receive noisy symbols of \( x_s \).

The received signals at the destination and at the \( l \)-th relay can be written respectively as

\[
\begin{align*}
y_{sd} &= h_{sd}x_s + n_{sd} \\
y_{sr,l} &= h_{sr,l}x_s + n_{sr,l}
\end{align*}
\]

where the subscripts indicate the node relation such that \( h_{sd} \) and \( h_{sr,l} \) are independent complex-valued channel gains for the S-D link and S-R link of the \( l \)-th relay respectively. For simplicity, all channels are Quasi-static Rayleigh fading channels i.e., \( h_{sd} \sim \mathcal{CN}(0,1) \) and \( h_{sr,l} \sim \mathcal{CN}(0,1) \), where \( \mathcal{CN}(\mu,\sigma^2) \) denotes a complex Gaussian random variable with mean \( \mu \) and variance \( \sigma^2 \). \( n_{sd} \) and \( n_{sr,l} \) are independent additive...
white Gaussian noise at the destination and the relay respectively which are modeled as \\
\( n_{sd} \sim \mathcal{CN} \left(0, \sigma^2_{sd}\right), n_{sr,l} \sim \mathcal{CN} \left(0, \sigma^2_{sr,l}\right) \) with variance equal to \( N_0 / 2 \) per dimension. We assume that the average transmit SNR for all links are the same denoted as \( \gamma = P_s / N_0 \), while the instantaneous receive SNR is represented as \( \gamma_{sd} = |h_{sd}|^2 \gamma \) and \( \gamma_{sr,l} = |h_{sr,l}|^2 \gamma \) respectively. The relay performs a hard-decision detection (DeF) and re-modulates the detected symbol as \( x_{r,l} \) with the same average power \( P_s \) for re-transmissions in time slot 2. The symbol received at the destination is given as \\
\( y_{rd,l} = h_{rd,l} x_{r,l} + n_{rd,l} \) \hspace{1cm} (5.2) \\
where \( h_{rd,l} \sim \mathcal{CN} \left(0,1\right) \) and \( n_{rd,l} \sim \mathcal{CN} \left(0, \sigma^2_{rd,l}\right) \) with variance equal to \( N_0 / 2 \) per dimension. The instantaneous receive SNR is \( \gamma_{rd,l} = |h_{rd,l}|^2 \gamma \). At the destination, the received signals from the source and the relay node are combined in order to recover the original source data.

![Block diagram of the relay for ADeF.](image)

The main assumptions of this scheme are also summarized in Section 2.1.3. Apart from these characteristics, we also assume that the destination has the knowledge by which reliable relays will do the re-transmissions. This approach can be done via a bit transmission to the destination e.g. Automatic Repeat Request (ARQ).

76
5.2 Adaptive Detect-and-Forward (ADeF)

In this section, we propose a simple Adaptive DeF (ADeF) protocol. In time slot 1, like in DeF, the source broadcasts its information to all relays and the destination. Instead of fixed re-transmission like DeF (no threshold strategy) regardless of the quality of the received signal, the relays in ADeF schemes decide to forward or remain idle based on the reliability information of the received signals which is a good measure of the quality of the received signal [53]. No error-control scheme like Automatic Repeat Request (ARQ) or Cyclic Redundancy Codes (CRC) as the error detecting mechanism is employed in ADeF. Thus, if the criterion is met, in time slot 2, after re-modulation (MOD), the relays forward the signals again to the destination. Finally, the destination combines the signals with that of the direct link with a bit LLR addition. Unlike the schemes in [25], [27] and [39] which necessitate the knowledge of S-R link for the detection; here we only use a conventional bit LLR addition without the need of such a S-R channel knowledge.

In this study, we introduce a new method of detection and forwarding which leads to a more efficient use of the relay resources. Generally soft bit estimates are in the form of LLR which has been well defined as

\[
\lambda \{x_s\} = \tilde{x}_s = \log \frac{P(x_s = +1 | y_{sr,t})}{P(x_s = -1 | y_{sr,t})}
\]  

(5.3)

The LLR \( \lambda \) indicates the reliability measures of the bit and the sign of \( \lambda \) represents the hard decision value. The reliability of the bit increases if the magnitude of the bit LLR increases. We adopt the reliability computation shown in [53] as the parameter which is related to the average posteriori error probability of the received bits:

\[
Z = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{1 + e^{\lambda_m}}
\]  

(5.4)

where \( \lambda_m \) is the LLR value of every bit in a symbol, i.e., \( M = 1 \) for binary phase-shift keying (BPSK) and similarly, \( M = 2 \) for quaternary phase-shift keying (QPSK). The relay chooses whether to transmit or not based upon (5.4) against the predetermined threshold value \( P_{th} \) such that the following criterion is met, \( Z \leq P_{th} \). In fact, if we set
$P_{th}$ too small, the probability that the relay re-transmits is small and thereby, reducing the performance gain. This is fair since re-transmitting erroneous signals will give adverse effect in the detection stage at the destination. Likewise, similar effect may happen when $P_{th}$ is large enough. In the following section, we provide some near optimal threshold values for certain SNRs only. Other threshold values may be found from similar exhaustive search. We provide the detail derivations of (5.4) in Appendix C.

5.3 Adaptation Algorithm at Relays

First, we describe how the adaptation algorithm at the relay works. Here we focus on BPSK but it also applies to any other modulation formats in any channel. Without loss of generality, we remove the subscripts for convenience. The basis of this algorithm is (5.4) in [53] which indicates a good measure of reliability information of the received bits. Since the sum of the posteriori probabilities given $y$ is unity that is $P(x=+1|y)+P(x=-1|y)=1$; therefore, from (5.3) after some algebraic manipulations, we obtain the following relations

$$P(x=+1|y) = \frac{e^{+\lambda}}{1+e^{+\lambda}}$$

$$P(x=-1|y) = 1 - \frac{e^{+\lambda}}{1+e^{+\lambda}} = \frac{e^{-\lambda}}{1+e^{-\lambda}}$$

(5.5)

It follows from above that for $\lambda \geq 0$, the following relation holds

$$P(x=+1|y) \geq P(x=-1|y)$$

(5.6)

Then, if $P(x=-1|y) \leq P_{th}$, as a result, the relay forwards the signal. Similarly, we can also show that for $\lambda < 0$ that is

$$P(x=+1|y) < P(x=-1|y)$$

(5.7)

and if $P(x=+1|y) \leq P_{th}$ then, the relay also forwards the signal to the destination. By this strategy, we limit the erroneous transmission from the relay and thereby, reduce the harmful effect of the erroneous hard-decision detection at the relay. This adaptation strategy based on the bit reliability makes the bit LLR combining at the destination between the received signals from the source and the relay quite effective. If either (5.6)
or (5.7) is not met, then the relay stops the relaying and the source will proceed with the next transmission.

5.4 Results

Table 5.1: Simulation parameters

<table>
<thead>
<tr>
<th>Modulations</th>
<th>BPSK/QPSK/16QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Model</td>
<td>AWGN/Quasi-static Rayleigh Fading Channel</td>
</tr>
<tr>
<td>Relay</td>
<td>Adaptive Detect-and-Forward (ADeF) or Detect-and-Forward (DeF)</td>
</tr>
<tr>
<td>Combining at D</td>
<td>Bit LLR Addition</td>
</tr>
</tbody>
</table>

This section presents results of simulations undertaken to illustrate the performance of the proposed scheme, ADeF and DeF in terms of bit error rate (BER) versus average receive SNR per bit for S-D link. Receiving nodes are assumed to have the same average receive SNR and perfect channel knowledge of the immediate links. Note that the destination only uses the immediate links (S-D and R-D channels) only for detection which is far from optimal. All simulation works use the following parameters in Table 5.1 unless stated otherwise.

5.4.1 BER Performance in AWGN channel

In Figure 5.3, we simulated the BER versus receive SNR for one relay scheme \((L = 1)\) with BPSK under AWGN channel when the threshold value, \(P_{th}\) of one relay node is varied from 0.0001–0.2. Interestingly, the proposed scheme for \(P_{th} = 10^{-4}\) outperforms the baseline DeF (no threshold) about 1.8dB gap at the BER of \(10^{-6}\), with 1dB margin to the theoretical MRC (perfect cooperation) and at 2dB to non-cooperative case, i.e., single-input single output (SISO). This result marks a good feature of the proposed scheme that although in AWGN channel, there is still reasonable improvement observed.
5.4.2 BER Performance in Quasi-static Rayleigh Fading Channel

Figure 5.4 shows the simulated BER at the destination when the threshold value, $P_{th}$ of one relay node with QPSK is varied from 0.0001~0.2 under Quasi-static Rayleigh fading channels. The improvement shown by the proposed scheme with the threshold, $P_{th}$ of 0.01 is around 10dB at the BER of $10^{-4}$. The influence of $P_{th}$ on the end-to-end BER performance is clearly shown when $P_{th}$ is increased from 0.0001~0.2. This shows that at low receive SNR, smaller threshold $P_{th}$ generates higher probability of propagation errors from the relay and finally, reduces the diversity gain in the end-to-end BER performance. We also plot the theoretical MRC (perfect cooperation) and SISO case for comparison.

Figure 5.5 presents the BER performance when we apply multiple relays $L=1$, 2 and 3 with QPSK under Quasi-static Rayleigh fading channel assuming the same channel setup as the previous simulations. The threshold $P_{th}$ at the relays is fixed to 0.01 and the same for all relay nodes. As expected, the end-to-end BER performance is increased as we increase the number of relay nodes.
Figure 5.4: BER comparison for ADeF (threshold, $P_{th}: 0.0001$~$0.2$) and DeF with QPSK under Quasi-static Rayleigh Fading channel for relay $L = 1$.

Figure 5.5: BER comparison for ADeF (threshold, $P_{th}=0.01$) and DeF (dashed lines) with QPSK under Quasi-static Rayleigh Fading channel for relay $L = 1, 2$ and $3$. 
In Figure 5.6, we simulated the BER versus receive SNR for one relay scheme \((L = 1)\) with 16QAM under Rayleigh fading channel when the threshold value, \(P_{th}\), of one relay node is varied from 0.0001~0.2. Interestingly, the proposed scheme for \(P_{th} = 0.01\) outperforms the baseline DeF (no threshold) about 11dB gap at the BER of \(10^{-4}\), with 3dB margin to the theoretical MRC (perfect cooperation). This shows that at low receive SNR, smaller threshold \(P_{th}\) generates higher probability of propagation errors from the relay and finally, reduces the diversity gain in the end-to-end BER performance. We also plot the non-cooperative case, i.e., single-input single output (SISO) case for comparison. With the adaptive strategy applied at the relays, fewer errors are propagated to the destination and thus, the detection is improved.

In Figure 5.7, we simulate the scheme with multiple relay nodes. It is evident that the improvement becomes larger if multiple relay nodes are implemented with threshold BER fixed at 0.01. From these simulation results, we can show that ADeF presents the significant performance improvement in BER by using only bit LLR combining at the destination. The proposed schemes reduce the impact of error propagation from relay detection errors. Such a simple strategy is beneficial for low-complexity networks like sensor network which allows the possibility to deploy a large number of low-complexity relays. Since the threshold imposed at the relays are not optimal, we expect more improvement if optimal threshold is applied which will be dealt in the future.

From these simulation results, we can show that ADeF presents significant performance improvements in BER by using only bit LLR addition at the destination, where we should note that the MRC equals the bit LLR combining when the S-R link is perfect (contains no error). The proposed scheme reduces the impact of error propagation from the relay detection errors. Such a simple strategy is beneficial for low-complexity networks like sensor network which allows the possibility to deploy a large number of low-complexity relays but leveraging on limited resources. Re-transmissions are not done if the received signal qualities are poor.
Figure 5.6: BER comparison for ADeF (threshold, $P_{th}$: 0.001–0.2) and DeF with 16QAM Quasi-static Rayleigh Fading channel for relay $L = 1$.

Figure 5.7: BER comparison for ADeF (BER threshold: 0.01) and DeF (dashed lines) with 16QAM Quasi-static Rayleigh Fading channel for relay $L = 1, 2$ and 3.
5.4.3 BER Dependency over Threshold Values

The influence of the threshold value, $P_{th}$, over the BER performance at some fixed SNR values which are assumed the same for all links, can be seen in the following plots: Figure 5.8 in AWGN channel for BPSK; Figure 5.9 and Figure 5.10 with Quasi-static Rayleigh fading channel for QPSK and 16QAM respectively. For brevity, we mark only a few points in the plots which give sufficient details of the curves and leave other trivial information. Our work here is purely based on the exhaustive search (via simulations) using MATLAB. Two scenarios are considered for one relay scheme with various threshold values:

i- AWGN channel: As shown in Table 5.2, the optimal threshold value is found to be 0.2 at SNR=2dB, 0.01 at SNR=4dB and SNR=6dB but 0.0001 at SNR=8dB. This result confirms the BER curves in Figure 5.8, that is, at low SNR, smaller threshold value permits less re-transmissions from the relay and hence, reduces the diversity gain at low SNR. On the other hand, at higher SNR, the threshold value of 0.0001 is found to be optimal.

ii- Quasi-static Rayleigh fading channel: For QPSK modulation, the optimal threshold presented in Table 5.3, the optimal threshold is found to be 0.2 at SNR=4dB, SNR=8dB and SNR=12dB, but 0.01 at SNR=15dB. This result also confirms the BER curves in Figure 5.9. That is, when SNR is below 12dB, larger $P_{th}$= 0.2 provides larger gain but gradually decreases thereafter.

For completeness we also investigate the BER dependency of 16QAM for comparison. For 16QAM modulation as shown Table 5.4, the optimal threshold is found to be 0.2 at SNR=4dB and SNR=10dB, but 0.01 at SNR=12dB and SNR=15dB. This result also confirms the BER curves in Figure 5.10. That is, when SNR is below 8dB, larger $P_{th}$=0.2 provides larger gain but gradually decreases thereafter.

Table 5.2: Optimal threshold values for AWGN channel for BPSK modulation.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{th}$</td>
<td>0.2</td>
<td>0.01</td>
<td>0.01</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Figure 5.8: BER dependency over various threshold values, $P_{th}$ for BPSK under AWGN channel.

Table 5.3: Optimal threshold values for Quasi-static Rayleigh fading channel for QPSK modulation

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{th}$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure 5.9: BER dependency over various threshold values, $P_{th}$ for QPSK under Quasi-static Rayleigh fading channel.

Table 5.4: Optimal threshold values for Rayleigh fading channel for 16QAM.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{th}$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Figure 5.10: BER dependency over various threshold values, $P_{th}$ for 16QAM under Quasi-static Rayleigh fading channel.
Comparing Fig. 5.9 and 5.10, in Rayleigh fading channels, we found that the optimal threshold is 0.01 for all modulations at sufficiently high SNR and 0.2 at low SNR region. In AWGN channel, the optimal value is smaller due to better channel quality.

5.4.4 ADeF Performance in Multihop Relays

Next, we simulate a multihop scheme as depicted in Figure 5.11. The source transmits to $R_l$ and the destination in timeslot 1. In the following timeslot, relay-to-relay transmission occurs until the final relay node.

![Cooperative relay scheme in multihop setup.](image)

The last relay, $R_L$, eventually forwards the signal again to the destination. We also assume that the direct link from the source is available at the destination. For ADeF scheme, each relay has to satisfy the threshold ($P_{th}=0.01$) for re-transmission to the next hop. We compare the proposed scheme with the baseline DeF with relay $L=2$ and $3$ for both BPSK and 16QAM modulation respectively.
Figure 5.12: BER comparison for ADeF (threshold, $P_{th}$: 0.01) and DeF (dashed lines) with BPSK under Quasi-static Rayleigh Fading channel for a multihop relay channel with $L = 2$ and 3.

As for BPSK modulation, from the simulation result in Figure 5.12, we observe that the proposed scheme also outperforms the baseline multihop schemes close to 13dB gap for 2 and 3 multihop relays respectively. From here, the proposed strategy has proved its effectiveness not only in relay-multibranch simulation setups but also in a multihop scheme.
For higher modulations, we use 16QAM as shown in Figure 5.13. From this simulation, we observe that the proposed scheme outperforms the baseline close to 13dB gap for 2 and 3 multihop relays respectively.

From Figure 5.12 and 5.13, we can show that ADeF presents significant performance improvements in BER by using only bit LLR addition at the destination requiring no CSI of S=R link for detection. We note that MRC is no longer capable to mitigate the serious error propagation from the multihop transmissions. As in the proposed scheme, by having ADeF at each relay node, relays can minimize the error propagation by selecting the reliable bits for re-transmission. This strategy leaves the destination node so simple that the receiver at the destination needs no CSI of every link (hop) for detection but only the final hop. This simple strategy is beneficial for low-complexity networks which require multihop transmissions like wireless sensor network and ad-hoc wireless network.
5.5 Conclusions

This work proposes a novel relaying technique to mitigate the error propagation in cooperative relay networks. By using a simple threshold strategy based on the bit reliability at the relay, we achieve a significant error rate performance improvement compared to the conventional DeF scheme with moderate system complexity. Our scheme strikes an interesting trade-off between the error rate performance and system complexity since no source-relay channel knowledge is required for detection at the destination. In particular, in a multihop channel setting, the destination only requires CSI of the final hop for detection. Hence, this strategy makes the scheme simple and practical for implementation. Through simulations, we prove that the proposed strategy is better than the baseline DeF with great performance margins under AWGN and quasi-static Rayleigh fading channels.

The optimal threshold values proposed in this thesis are obtained through exhaustive search (via simulations). It would hence be desirable to obtain closed form expressions, either in exact or approximate for these open problems. In this work, we do not discuss the relay scheduling which is beyond the scope of this thesis and can be dealt in future research.
Chapter 6

Conclusions and Future Works

6.1 Summary of Contributions

The key contributions of this dissertation are the following:

- **Chapter 3**: Cooperative communication that exploits spatial diversity from relays has been shown to better than systems with collocated antennas [4], [5]. The work in [27] has gained much popularity to its direct application to cooperative communications. However, it is feasible when complete CSI of the relayed link is available and this requirement is hardly fulfilled in practice. In fact, it performs badly in asymmetric relay channels. In this dissertation, development of a novel signal combining strategy is presented which exploits the average SER of S-R channel such that near-optimal diversity gain is achieved. C-MRC, on the other hand, requires constant update of CSI at the destination based on instantaneous bit error rate and SNRs over each link. This has become a dilemma because the exchange of such information is pricey and would reduce the effective throughput of the transmission. The error rate performance has shown that the proposed ML-base strategy is superior than the baseline MRC scheme under various channel setups. We have derived a scheme which is generalized to any QAM modulations. This makes this scheme attractive for future wireless communication systems. By
comparing the proposed scheme to C-MRC, we found that our proposed scheme is superior than C-MRC in many channel setups. In fact, ML-based combining scheme is also robust in asymmetric channel setups where each link experiences different channel quality. If different modulations are used at the source and the relays, we propose that lower modulation is used at the source so that more improvement in end-to-end error rate performance.

- **Chapter 4:** A simple soft relaying information protocol has been developed which employs a simple symbol estimation technique. We propose a relaying protocol which is based on symbol-by-symbol signal detection (SSF). This technique also employs arbitrary mapping constellation which simplifies the modulation strategy at the relay node. Our main motivation is that LDPC decoder in Quasi-static fading environment does little effect in SISO scheme and bit-by-bit analysis requires higher computation and thus, consumes a lot of resources at the relay node. We began by simplifying the ML detection rule into one-dimensional rule which eases lots of computational burden at the relay and destination node. The convincing results reflected in the error rate performance of the proposed scheme are due to better reliability information and forwarding strategy we have employed. Without decoding at the relays, SSF does not lose the coding gain entirely. SSF mitigates the impact of error propagation from decoding errors since decoding is only done at the destination. SSF is simpler than the baseline protocols which is suited for low-complexity networks with energy-constrained systems like wireless sensor network. Soft information relaying is basically an analog signal. In practice, we require compression and/or quantization or other modulation techniques at the relays. Techniques like Wyner-Ziv coding can be used to quantize analog soft information into digital signals and compress the quantized signals before transmitting. Thus, these techniques can be employed into a relay protocol to compress and quantize the signals.

- **Chapter 5:** Development of a new adaptive relaying protocol based on the reliability threshold is introduced and its end-to-end error performance has been analyzed. In this chapter, we have presented a novel technique to mitigate error propagation from cooperative networks. Our proposed scheme is based on finding
the reliability bits at relays based on LLR computations, and exploits these values to prevent forwarding unreliable bits through a thresholding technique. A set of threshold values that minimize the end-to-end BER of relay channels are proposed for BPSK, QPSK and 16QAM modulations under AWGN and fading channels. The proposed scheme is simple since no CSI of S-R link is required at the destination. We compare our proposed scheme with the conventional scheme using no thresholding scheme (DeF). Through simulation works in various channel setup namely, AWGN and Rayleigh fading channels with multi-branch and multi-hop settings, a significant end-to-end error performance is achieved by using only the proposed strategy at relays.

6.2 Future Recommendations

A large number of research areas on relay communications and related topics deserve to be investigated further. Following are some of the major important directions:

- A major challenge in multiple relay communications is to devise a strategy to coordinate the relay transmissions. This conventional approach like TDMA is undesirable since it increases the system complexity, wastes the resources and degrades the performance in terms of transmission rate. Current research also discuss the implementation of a division free (full duplex) operation which can be done with some sophisticated signal processing and hardware design and this would be a significant step to resolve this open problem.

- Another problem for such cooperative networks which typically employ only single antenna system is their low throughput compared to that of MIMO schemes. To overcome this shortcoming, the techniques in MIMO can be utilized together with the cooperative communication schemes. Therefore, it is obvious that with a suitable association between MIMO and cooperative communication scheme, a higher performance can be achieved with better spectral and energy efficiencies.

- In Chapter 3, we found that ML-based combining scheme is attractive in combing signals with different modulations, assuming no error correcting codes. Future
extension to this work may include developing a new combining scheme with error correcting codes and even different codes at source and relay nodes.

- In Chapter 4, it is shown that the practical implementation of SSF scheme is relatively high due to the transmission of soft information. Thus, for SSF to be implemented in digital relaying transceiver, the soft information from the relay output has to be quantized such that the MSE between the source signal and its reconstructed version at the output of the de-quantizer at the destination is minimized.

- The proposed threshold values in Chapter 5 are based only on the exhaustive search for BPSK, QPSK and 16QAM modulations. Extension of this work to the theoretical analysis will be more comprehensive and sound for future studies.

- In broadband wireless communications, multipath fading introduces frequency selectivity, resulting in inter-symbol interference (ISI) that can severely degrade the system performance. OFDM is a promising technique to tackle frequency selectivity, mitigate ISI and deliver high spectral efficiency. In a frequency-selective fading channel, there is an extra source of diversity, multipath diversity that can be exploited to improve the performance of the system. Due to salient features in OFDM in wireless networks, OFDM can bring about additional freedom of making decisions on a subcarrier basis at relay nodes, according to channel conditions on the S-R and R-D links. The work in [55] shows that an OFDM-based AF relaying system can achieve the maximum multipath diversity considering only AF relaying protocol. However, generally AF schemes require expensive radio frequency chains to mitigate the existing coupling effects which are not desirable in some applications [27]. Thus, it is important to further investigate OFDM-based DF relay systems where decoding error at the relays is taken into account for optimal detection.

- Our system model in this dissertation which is common for all chapters, only assumes TDMA transmissions. Therefore, each relay is dedicated to estimate one user signal. No data exchange between relays is assumed. Future studies may include the possible performance improvement if relay data exchange is employed.
Throughout this dissertation, we assume that all nodes are synchronized, that is signals arrive at the destination synchronously. It is common that in practice, clock-drifts may occur and other real-world factors may be in place causing possibly asynchronisms in the systems. It would be desirable to have relaying and space-time techniques to tackle this problem in order to have robust networks regardless of such impediments.

Our focus in this thesis is on the physical layer aspects such as modulation and signal processing techniques. However, in practical systems with more complex situations when there is a need for multiple nodes to access the channel, a proper design of medium access control (MAC) protocols is essential to address the diversity gain in cooperative communications.
References


List of Publications

A. Journal Papers


B. Conference Papers


Appendix A

Derivation of Individual Symbol Error Rate (SER) of QPSK and 16QAM Signals in Gray Mapping

A.1: Derivation of Symbol Error Rate (SER) of QPSK Signals in Gray Mapping

Employing the two-dimensional (2-D) Gaussian Q-function representation, we present closed-form expressions for the individual SER of each QPSK signal. Figure A.1 illustrates the signal points for QPSK when Gray mapping is used. Let us denote $I$ and $Q$ as the in-phase and quadrature components respectively. Since each complex symbol of QPSK corresponds to two binary bits, as presented in Figure A.1 we assign the respective symbols $s_1 \rightarrow (0,0), s_2 \rightarrow (1,0), s_3 \rightarrow (1,1), s_4 \rightarrow (0,1)$ accordingly.

Figure A. 1: Signal QPSK symbols and symbol error probabilities
i. Derivation of $\varepsilon_1$

In this sub-section, we derive the symbol error probability of $s_1 \rightarrow s_3$. Figure A.1 depicts the difference of Euclidean distance between $s_1$ and other symbols. Here we assume $\varepsilon_2 = \varepsilon_3$. First, it is convenient to define several assumptions used in the analysis.

Consider the $i$-th received signal vector $s_i = (X_i, Y_i)$, $i \in \{1,2,3,4\}$ of QPSK transmitted over an AWGN channel with the channel gain $h$. Hence, $(X_i, Y_i)$ in $s_i$ is given by the following in-phase and quadrature components

\[
(X_i, Y_i) = \left[ \cos \theta(t) + n'_X, \sin \theta(t) + n'_Y \right]
\]  

(A.1)

where $n'_X$ and $n'_Y$ are jointly Gaussian with zero mean and equal variance $N_0$ such that the expected value is

\[
E\left\{n'_X^2\right\} = E\left\{n'_Y^2\right\} = (1/2)/(E_s/N_0)
\]  

(A.2)

The symbol error probability when $s_1$ is sent and $s_3$ is detected can be shown by the 2-D Gaussian probability integral as follows

\[
\varepsilon_1 = \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{1}{2\pi N_0} \exp \left(-\frac{(X - \frac{1}{\sqrt{2}})^2 + (Y - \frac{1}{\sqrt{2}})^2}{2N_0}\right) dXdY
\]  

(A.3)

(A.3) can be analytically calculated and we can re-write it as

\[
\varepsilon_1 = \frac{1}{4} \text{erfc}^2\left(\frac{E_s}{\sqrt{2N_0}}\right) = \left\{Q\left(\frac{E_s}{\sqrt{2N_0}}\right)\right\}^2
\]  

(A.4)

where $\text{erfc}(\cdot)$ is the complementary error function.

ii. Derivation of $\varepsilon_2$

In this sub-section, we derive the transition probability of a signal point that falls into an adjacent quadrant e.g., $s_1 \rightarrow s_2$. 

105
Similarly, from (A.3)

\[ \epsilon_2 = \int_{-\infty}^{0} \int_{0}^{\infty} \frac{1}{2\pi N_0} \exp \left\{ -\frac{(X - \frac{1}{\sqrt{2}})^2 + (Y - \frac{1}{\sqrt{2}})^2}{2N_0} \right\} dXdY \]  

(A.5)

Then, (A.5) can be analytically calculated and we obtain

\[ \epsilon_2 = \frac{1}{2} \text{erfc}\left[ \frac{E_s}{\sqrt{2N_0}} \right] - \frac{1}{4} \text{erfc}^2\left[ \frac{E_s}{\sqrt{2N_0}} \right] 
= Q\left( \frac{E_s}{\sqrt{N_0}} \right) - \left\{ Q\left( \frac{E_s}{\sqrt{N_0}} \right) \right\}^2 
\]  

(A.6)

**Proof:** Derivation of \( \epsilon_0 \) using \( \epsilon_1, \epsilon_2 \) and \( \epsilon_3 \) which is equivalent to the standard closed-form expression of the average probability of error for QPSK in AWGN channel [56].

In this sub-section, we provide an alternative derivation for the average SER of QPSK, \( \epsilon_0 \). We prove that the sum of all individual SER of QPSK symbols amounts to the average SER of QPSK. Since \( \epsilon_2 = \epsilon_3 \),

\[ \epsilon_2 = \epsilon_3 = \frac{1}{2} \text{erfc}\left[ \frac{E_s}{\sqrt{2N_0}} \right] - \frac{1}{4} \text{erfc}^2\left[ \frac{E_s}{\sqrt{2N_0}} \right] \]  

(A.7)

Therefore, the average SER for QPSK is simply

\[ \epsilon_0 = \epsilon_1 + \epsilon_2 + \epsilon_3 
= \text{erfc}\left[ \frac{E_s}{\sqrt{2N_0}} \right] - \frac{1}{4} \text{erfc}^2\left[ \frac{E_s}{\sqrt{2N_0}} \right] 
= 2Q\left( \frac{E_s}{\sqrt{N_0}} \right) - \left\{ Q\left( \frac{E_s}{\sqrt{N_0}} \right) \right\}^2 \]  

(A.8)

For comparison, we also investigate the effect of having only average SER in the proposed ML algorithm. From (3.20), by setting \( \epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_0 \), we have the following criterion
\[ \hat{x}_s = \arg \max_{x_s \in \mathcal{X}} p_{sd}(y_{sd} \mid x_s) \]  

\[ \left\{ \begin{array}{l}
\left[ 1 - (3\varepsilon_0) \right] p_{rd}(y_{rd} \mid x_r = x_s) \\
+ \varepsilon_0 p_{rd}(y_{rd} \mid x_r = x_s e^{j\pi}) \\
+ \varepsilon_0 p_{rd}(y_{rd} \mid x_r = x_s e^{j\pi/2}) \\
+ \varepsilon_0 p_{rd}(y_{rd} \mid x_r = x_s e^{-j\pi/2})
\end{array} \right. \]  \hspace{1cm} (A.9)

(A.9) can be viewed as suboptimal since it only considers average SER for all symbols in QPSK modulation.

**A.2: Derivation of Individual Symbol Error Rate (SER) of 16QAM Signals in Gray Mapping**

In this section, we present the derivations of individual SER of 16QAM symbols in AWGN channels which become the side information to our proposed scheme (A.9). Note that our framework in (A.9) also suits well for other modulations like QPSK having quadrature error or with I-Q gain mismatch [57], [58], since it treats the error probability in a symbol-by-symbol basis. Employing the two-dimensional (2-D) Gaussian Q-function representation, we present closed-form expressions for the individual SER of 16QAM signals. Figure A.2 depicts the signal points for 16QAM with its decision boundaries as the dashed lines when Gray mapping is used. The constellation points of 16QAM are normalized with the factor \( a = 1/\sqrt{10} \) to ensure that the average energy over all symbols is unity. Let us denote \( I \) and \( Q \) as the in-phase and quadrature components respectively. Since each complex symbol of 16QAM corresponds to four binary bits, \((b_1, b_2, b_3, b_4)\) as presented in Figure A.2 we label the respective symbols accordingly.
Using similar derivations in QPSK [26], first we consider, for instance, the symbol (01 01) is transmitted from the source assuming the perfect CSI is available at the receiver side. If the receiver wrongly detects the symbol as (00 01), the symbol error probability for this particular symbol is calculated from the following integrations

\[
\epsilon_2 = \int_{-\frac{a}{\sqrt{10}}}^{\frac{a}{\sqrt{10}}} \int_{-\frac{a}{\sqrt{10}}}^{\frac{a}{\sqrt{10}}} \exp \left\{-\frac{(x - \frac{1}{\sqrt{10}})^2 + (y - \frac{1}{\sqrt{10}})^2}{2\sigma^2} \right\} \, dx \, dy
\]

\[
= \frac{1}{2} \text{erf} \left[ \frac{1}{2\sqrt{5}\sigma} \right] \text{erf} \left[ \frac{1}{2\sqrt{5}\sigma} \right] - Q \left( \frac{E_s}{\sqrt{5N_0}} \right)^2
\]

where \( \text{erf} \) is the error function. For other symbols like (01 00) and (00 01), identical SER can be observed due to symmetry. Likewise, the calculation for \( \epsilon_3 \) for symbol (00 00) which is located on the right top corner of the quadrant, can be found from the
following integrations as

\[ \varepsilon_3 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma^2} \exp \left\{ -\frac{(x - \frac{1}{\sqrt{10}})^2 + (y - \frac{1}{\sqrt{10}})^2}{2\sigma^2} \right\} \, dx \, dy \]

\[ = \frac{1}{4} \operatorname{erfc}^2 \left[ \frac{1}{2\sqrt{5\sigma}} \right] = \left\{ Q \left( \frac{E_s}{\sqrt{5N_0}} \right) \right\}^2 \quad (A.13) \]

Similarly, for \( \varepsilon_4 \) which is the transition from the transmitted symbol (01 01) to symbol (11 11) as shown in yellow quadrant, it can be found from the following integration as

\[ \varepsilon_4 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma^2} \exp \left\{ -\frac{(x - \frac{1}{\sqrt{10}})^2 + (y - \frac{1}{\sqrt{10}})^2}{2\sigma^2} \right\} \, dx \, dy \]

\[ = \frac{1}{4} \left\{ \operatorname{erf} \left[ \frac{1}{2\sqrt{5\sigma}} \right] - \operatorname{erf} \left[ \frac{10 + \sqrt{5}}{10\sigma} \right] \right\}^2 \]

\[ = \frac{1}{4} \left\{ 2Q \left( \frac{21E_s}{\sqrt{5N_0}} \right) - 2Q \left( \frac{E_s}{\sqrt{5N_0}} \right) \right\} \quad (A.14) \]

Since some symbols like symbol (11 01) and (01 11) as shown in Figure A.2 are identical i.e., \( \varepsilon_1 \), computation of these SERs can be reduced. Finally, other SERs can be straightforward in a similar fashion and they are not shown here for brevity.
Appendix B

Derivation of Symbol Log-likelihood Ratio (LLR) and Expected Value of Transmit Signal Point for the Proposed Relay Function

In this sub-section, we present the proposed detection and forwarding strategies employed at relay nodes. Without loss of generality, we consider a scheme with one relay node only \((L_s = 1; L_p = 1)\); thus, we remove the subscripts of the node relation for simplicity. Since calculating the exact bit LLR by using the conventional MLD is excessively exorbitant, there are a few ways proposed to approximate bit LLR values like in [59]-[60] whose aim is to avoid the high computation from the exact bit LLR expression. Although by using the conventional MLD the optimum performance can be achieved, the approach requires computation which grows exponentially with the constellation size of the modulation schemes.

In Chapter 4, we propose another simple approximation technique in order to avoid such complexity. Our proposed scheme requires symbol-wise computations and thus, avoids higher computations to evaluate every bit of the received signals. We define that the capital \(P(s)\) denotes the probability, and \(p(s)\) is the Probability Density Function (PDF) denoted in the following relation \(P(s) = p(s)\Delta s\). For simplicity, QPSK modulation is considered which utilizes the Gray mapping. We represent the complex symbols, \(s_m \in (b_1, b_2)\) as \(s_1 \rightarrow (0,0), s_2 \rightarrow (1,0), s_3 \rightarrow (1,1), s_4 \rightarrow (0,1)\) where \(b_1\) and \(b_2\) correspond to the first and second bit of a symbol in the constellation. \(s_m\) is the \(m\)-th transmit signal, \(m = 1, \cdots, M\) where \(M(=4)\) is the constellation size for QPSK. For this system, the optimal detector will search the symbol such that it maximizes the \(a\)-posterior probability \(P(s_m \mid y \Delta y)\), the probability of receiving the transmit signal \(s_m\) given that \(y\) is received in the small region \(\Delta y\). We successively describe below two main steps of the proposed relaying strategy:
Step 1: Calculate Symbol LLR Values

The symbol LLRs can be shown as

\[ \lambda_1 = \log_e \left\{ P(s_1 \mid y_{\Delta}y) / P(s_1 \mid y_{\Delta}y) \right\} = 0, \]
\[ \lambda_2 = \log_e \left\{ P(s_2 \mid y_{\Delta}y) / P(s_1 \mid y_{\Delta}y) \right\}, \]
\[ \lambda_3 = \log_e \left\{ P(s_3 \mid y_{\Delta}y) / P(s_1 \mid y_{\Delta}y) \right\}, \]
\[ \lambda_4 = \log_e \left\{ P(s_4 \mid y_{\Delta}y) / P(s_1 \mid y_{\Delta}y) \right\} \]

where \( \lambda_m, m \in \{1, 2, 3, 4\} \) is the m-th symbol LLR in QPSK modulation, the transmit symbol \( s_1 \) is taken as the reference and its LLR value \( \lambda_1 \) is always equal to zero. Thus, there are three LLR values of \( \lambda_2, \lambda_3 \) and \( \lambda_4 \). When \( y \) is received, the symbol LLRs \( \lambda_2, \lambda_3 \) and \( \lambda_4 \) are calculated to be plotted on the real straight line. This method simplifies the MLD rule to one-dimensional space only. If \( \lambda_4 \) is the largest on the real straight line, then \( s_4 \) is detected and if \( \lambda_2, \lambda_3 \) and \( \lambda_4 \) all have minus values on the real straight line, then \( s_1 \) is detected. For example, \( \lambda_4 \) is evaluated as follows

\[ \lambda_4 = \log_e \left[ P(s_4 \mid y_{\Delta}y) / P(s_1 \mid y_{\Delta}y) \right] = \log_e \left[ P(s_4, y_{\Delta}y) / P(s_1, y_{\Delta}y) \right] / P(s_1) \]
\[ = \log_e \left[ P(s_4, y_{\Delta}y) / P(s_1, y_{\Delta}y) \right] = \log_e \left[ \frac{P(s_4)P(y_{\Delta}y \mid s_4)}{P(s_1)P(y_{\Delta}y \mid s_1)} \right] \]
\[ = \log_e \left[ \frac{P(s_4)}{P(s_1)} \right] + \log_e \left[ \frac{p(y \mid s_4)}{p(y \mid s_1)} \right] \]
\[ = \Lambda_4 + \log_e \left[ \frac{p(y \mid s_4)}{p(y \mid s_1)} \right] \]

where we let \( \Lambda_4 = \log_e \left[ P(s_4) / P(s_1) \right] \). If the transition probability density is defined as

\[ p(y \mid s_m) = \frac{1}{\sqrt{2\sigma^2}} \exp \left( -\frac{|y - h_{s_m}|^2}{2\sigma^2} \right), \]
then we can re-write (B.2) as the following
\[ \lambda_4 = \Lambda_4 + \log_e \left[ \frac{p(y | s_4)}{p(y | s_1)} \right] \]
\[ = \Lambda_4 + \log_e \left[ \exp \left( -\frac{|y-hs_4|^2}{2\sigma^2} \right) \right] \]
\[ = \Lambda_4 + \frac{1}{2\sigma^2} \left[ |y-hs_1|^2 - |y-hs_4|^2 \right] \] (B.3)

When the probability of the two symbols are the same \( P(s_4) = P(s_1) = 1/4 \), then
\( \Lambda_4 = \log_e [P(s_4) / P(s_1)] = 0 \). Thus, \( \lambda_2 \) and \( \lambda_3 \) are also evaluated in the same way as \( \lambda_4 \).

**Step 2: Compute Expected Values from Symbol LLR**

Next we elaborate the transmission method from the relay to the destination. Since the computation of expectation values involves the soft symbols, we term this technique as soft modulation. In this work we introduce another method of computing the expectation values which is another extension of our work in [26]. From Step 1, after the symbol LLR \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) are calculated, the expected value of transmit signal point \( \tau \) can be evaluated in what follows. From (B.1)-(B.3), after some simplifications, we can obtain that
\[ e^{\lambda_1} = \frac{P(s_4 | y)}{P(s_1 | y)} \quad e^{\lambda_3} = \frac{P(s_3 | y)}{P(s_1 | y)} \quad e^{\lambda_2} = \frac{P(s_2 | y)}{P(s_1 | y)} \] (B.4)

Since
\[ P(s_1 | y) + e^{\lambda_2} P(s_1 | y) + e^{\lambda_3} P(s_1 | y) + e^{\lambda_4} P(s_1 | y) = 1 \] (B.5)

Then,
\[ P(s_1 | y) = 1/(1 + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}) \]
\[ P(s_2 | y) = e^{\lambda_2} / (1 + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}) \] (B.6)
\[ P(s_3 | y) = e^{\lambda_3} / (1 + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}) \]
\[ P(s_4 | y) = e^{\lambda_4} / (1 + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}) \]

Therefore, the expectation values for one QPSK symbol can simply be computed as
\[ \bar{x} = s_1 P(s_1 \mid y) + s_2 P(s_2 \mid y) + s_3 P(s_3 \mid y) + s_4 P(s_4 \mid y) \]
\[ = \frac{s_4 e^{\lambda_2} + s_3 e^{\lambda_3} + s_2 e^{\lambda_4} + s_1}{1 + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}} = \sum_{m=1}^{M} s_m P(s_m \mid y) \quad \text{(B.7)} \]
\[ = \frac{\sum_{m=1}^{M} s_m e^{\lambda_m}}{\sum_{m=1}^{M} (1 + e^{\lambda_m})} = \frac{\sum_{m=1}^{M} s_m e^{\lambda_m}}{\sum_{m=1}^{M} e^{\lambda_m}} \]

As opposed to the earlier methods, we propose a simple strategy to utilize soft forwarding at the relay node. After computing the symbol LLR values by using (B.1)-(B.3), the expected value of signal point from (B.7) is sent by relays to the destination according to the power limitations at the relays.
Appendix C

Derivation of Reliability Threshold Using Bit LLR

The bit log-likelihood ratio (LLR) computation for BPSK modulation can be shown as follows:

\[
\lambda = \log_e \left( \frac{P(x = 0 \mid y)}{P(x = 1 \mid y)} \right) = \log_e \left( \frac{P(x = +1 \mid y)}{P(x = -1 \mid y)} \right) 
\]

(C.1)

The bit LLR \( \lambda \) indicates the reliability measures of the bit and the sign of \( \lambda \) represents the hard decision value. The reliability of the bit increases if the magnitude of the bit LLR increases.

From here, we further simplify (C.1) as

\[
e^\lambda = \frac{1 - P(x = +1 \mid y)}{P(x = +1 \mid y)} \left\{1 - P(x = +1 \mid y)\right\} e^\lambda = P(x = +1 \mid y) \\
e^\lambda = P(x = +1 \mid y) \left(1 + e^\lambda\right) 
\]

(C.2)

\[
P(x = +1 \mid y) = \frac{e^\lambda}{1 + e^\lambda} \\
\therefore P(x = -1 \mid y) = 1 - P(x = +1 \mid y) = 1 - \frac{e^\lambda}{1 + e^\lambda} = \frac{1}{1 + e^\lambda}
\]

Thus, we can summarize (C.1) and (C.2) as follows

\[
\begin{cases}
P(x = +1 \mid y) = \frac{e^\lambda}{1 + e^\lambda} \\
P(x = -1 \mid y) = 1 - \frac{e^\lambda}{1 + e^\lambda} = \frac{e^{-|\lambda|}}{1 + e^{-|\lambda|}} = \frac{1}{1 + e^\lambda}
\end{cases}
\]

(C.3)

When \( \lambda \geq 0 \) \( P(x = +1 \mid y) \geq P(x = -1 \mid y) \). In this case,

\[
P(x = -1 \mid y) = \frac{1}{1 + e^\lambda} = \frac{1}{1 + e^{|\lambda|}} \leq P_{th},
\]

then it holds \( P(x = +1 \mid y) = 1 - P(x = -1 \mid y) \geq 1 - P_{th} \). Thus the probability \( P(x = +1 \mid y) \)
is highly reliable.

When $\lambda < 0$, $P(x = +1|y) < P(x = -1|y)$. In this case,

$$P(x = +1|y) = \frac{e^\lambda}{1 + e^\lambda} = \frac{e^{-|\lambda|}}{1 + e^{-|\lambda|}} \leq P_{th},$$

then it holds

$$P(x = -1|y) = 1 - P(x = +1|y) \geq 1 - P_{th}.$$ Thus the probability $P(x = -1|y)$ is highly reliable.

Accordingly, regardless of the $\pm$ sign of $\lambda$, if the condition $\frac{1}{1 + e^{|\lambda|}} \leq P_{th}$ is satisfied, then the probability of $P(x = +1|y)$ or $P(x = -1|y)$ is highly reliable. Thus, we can say that $\frac{1}{1 + e^{|\lambda|}_h} = P_{th}$ is the reliability threshold value.