著者（英） | Kenichi Abe  
---|---
学位名 | 博士 工学  
学位授与番号 | 甲第 2882号  
学位授与年月日 | 1995.03.23  
URL | http://doi.org/10.11501/3082439
TURBULENCE MODELS FOR
PREDICTING COMPLEX
FLUID FLOW AND HEAT TRANSFER

by

Ken-ichi Abe

B.S., Kyoto University, 1985
M.S., Kyoto University, 1987

Doctoral Dissertation

Submitted to
Nagoya Institute of Technology
in partial fulfillment of
the requirements for the degree of
Doctor of Engineering

January 1995
Acknowledgments

First of all, the author would like to express his gratitude to the Examining Committee members, Professor Y. Nagano, Professor N. Ohiwa and Professor T. Tsuji for carefully reviewing the manuscript and for making appropriate suggestions.

The author wishes to express his sincere gratitude to his supervisor, Professor Y. Nagano, for his encouragement, guidance, and many helpful suggestions in the progress of this investigation, and for suggesting the subject of this research.

During the author's Ph.D. program at the Nagoya Institute of Technology (NIT), he received much help from the members of the Heat Transfer Laboratory in the Department of Mechanical Engineering. Special thanks are due to Professor T. Tsuji, Dr. Y. Suzuki, Mr. M. Shimada and Mr. H. Hattori for their valuable discussions and helpful comments. The author wishes to thank Dr. M. Tagawa for his valuable suggestions, as well as the graduate students of the Heat Transfer Laboratory for their generous assistance and discussions.

Thanks are also due to Professors S. Murakami and A. Yoshizawa of the Institute of Industrial Science, the University of Tokyo, and to Professor N. Kasagi of the University of Tokyo and Professor H. Kawamura of the Science University of Tokyo for their helpful discussions and incisive comments.

The author also wishes to express his appreciation to Mr. R. Takahashi and Dr. T. Kondoh of the Toyota Central Research and Development Laboratories, Inc. (TCRDL) for providing him with valuable experience and supporting his studies at NIT; to Mr. N. Horinouchi of the mathematical physics group in TCRDL for his help and suggestions in developing the computer programs used in this investigation; and to Mr. K. Suga and Mr. M. Nagaoka in TCRDL for their helpful discussions.

The author is indebted to Dr. H. Sato of Nippondenso Co., Ltd. and Dr. N. Shikazono of Hitachi Ltd. for their valuable and helpful discussions.

Finally, the author would like to express his gratitude and appreciation to his family for their continuous encouragement and support.
Abstract

To calculate complex turbulent fluid flow and heat transfer, four turbulence models are newly proposed.

Two of the models are modified versions of the latest low-Reynolds-number k-ε and two-equation heat-transfer models. The main improvement is achieved by the introduction of the Kolmogorov velocity scale, $u_\varepsilon \equiv (\nu \varepsilon)^{1/4}$, instead of the friction velocity $u_\tau$, to account for the near-wall and low-Reynolds-number effects in both attached and detached flows. After investigating the characteristics of various time scales for the heat-transfer model, a composite time scale which gives weight to a shorter scale among the velocity- and temperature-field time scales is adopted. It is established that the proposed models quite successfully predict turbulent fluid flow and heat transfer in separating and reattaching flows downstream of a backward-facing step. This involves most of the essential physics of complex turbulent heat transfer, under various conditions of the flow Reynolds number and upstream boundary-layer thickness. The flow structure is discussed in detail based on the computational results, which reveal several new mechanistic features of turbulent heat transfer in separating and reattaching flows.

On the other hand, two other models are constructed by introducing the knowledge obtained from the explicit expressions of simplified second-moment closures. One of them is a new nonlinear k-ε model for velocity-field predictions, and the other is a two-equation heat-transfer model incorporating higher-order effects of the velocity and temperature gradients. These models allow application to both wall and free (e.g., homogeneous) turbulent flows, the latter of which has been very difficult to accurately predict with previous two-equation turbulence models. They are reduced to low-Reynolds-number models with the aid of insights obtained from the earlier part of this study. The proposed models quite accurately predict fluid flow and heat transfer in both attached wall and homogeneous turbulent flows. To validate the model performance for complex turbulent flows, the new velocity-field model is applied to channel flows with injection and suction, and to separating and reattaching flows downstream of a backward-facing step. Comparison of the computational results with experimental and direct numerical simulation data indicates that the newly proposed velocity-field model is also effective in calculating complex turbulent flows of engineering interest. Furthermore, the parameters in low-Reynolds-number
model functions are reexamined on the basis of the computational results, yielding some important insights into which factors determine the low-Reynolds-number parameters used in the model functions.
# CONTENTS

Acknowledgments ........................................... ii
Abstract ...................................................... iii
CONTENTS .................................................. v
LIST OF FIGURES ........................................... viii
LIST OF TABLES ............................................. xiv
NOMENCLATURE .............................................. xv

1 INTRODUCTION ............................................ 1
   1.1 Background ........................................... 1
   1.2 Objectives ............................................ 5
   1.3 Organization of Dissertation ......................... 6

2 GOVERNING EQUATIONS .................................... 8
   2.1 Velocity-Field Equations .............................. 8
   2.2 Thermal Field Equations ............................... 9

3 AN IMPROVED $k$-$\varepsilon$ MODEL FOR SEPARATING FLOWS ........................................... 10
   3.1 Turbulence Modeling ................................... 10
      3.1.1 Eddy-viscosity approximation .................... 10
      3.1.2 Original Nagano-Tagawa model ..................... 11
      3.1.3 Modification of model functions .................. 11
   3.2 Model Assessment in Attached Turbulent Flows ....... 14
3.3 Application to Backward-Facing Step Flows ........................................... 15
  3.3.1 Numerical procedure and boundary conditions ............................ 15
  3.3.2 Evaluation of computational accuracy ...................................... 16
  3.3.3 Results and discussion .......................................................... 18

4 AN IMPROVED TWO-EQUATION HEAT-TRANSFER MODEL 39
  4.1 Turbulence Modeling for Thermal Field .......................................... 39
    4.1.1 Modeling eddy diffusivity for heat ..................................... 39
    4.1.2 Model constants and model functions .................................. 42
  4.2 Model Assessment in Attached Flow Heat Transfer ............................ 44
  4.3 Application to Backward-Facing Step Flows .................................... 45
    4.3.1 Model assessment with DNS data on flow field ....................... 45
    4.3.2 Numerical procedure and boundary conditions for thermal field ... 47
    4.3.3 Comparison with experimental data ...................................... 48
    4.3.4 Confusions in previous predictions ..................................... 49
    4.3.5 Thermal field in backward-facing step flow ............................ 52
    4.3.6 Influence of upstream boundary-layer thickness on heat transfer .. 54

5 A NEW TYPE OF NONLINEAR k-ε MODEL 73
  5.1 Concept of Turbulence Modeling ................................................ 73
    5.1.1 Modeling Reynolds stress .................................................. 73
    5.1.2 Extension to a low-Reynolds-number model ................................ 77
  5.2 Relation between Anisotropy Tensors and Velocity Gradients ............... 77
  5.3 Numerical Procedure ............................................................... 79
  5.4 Model Assessment in Fundamental Turbulent Flows ............................. 80
  5.5 Application to Complex Turbulent Flows ....................................... 81
    5.5.1 Channel flow with injection and suction ................................ 81
    5.5.2 Backward-facing step flow .................................................. 83
    5.5.3 Examination of low-Reynolds-number parameters in model functions .. 85

6 A NEW TWO-EQUATION HEAT-TRANSFER MODEL 108
  6.1 Thermal Field Modeling ............................................................ 108
CONTENTS

6.1.1 Modeling turbulent heat flux ...................................... 108
6.1.2 Determination of characteristic time scale ..................... 110
6.1.3 Model constants .................................................. 110
6.1.4 Extension to a low-Reynolds-number model .................... 114
6.2 Validation of Performance of the Proposed Model .............. 114

7 CONCLUSIONS ......................................................... 135

BIBLIOGRAPHY .......................................................... 139
LIST OF FIGURES

3.1 Comparison of channel flow predictions with DNS data ($Re_{r} = 395$): (a) Mean velocity; (b) Eddy viscosity. .................................................. 23

3.2 Friction coefficient of channel flow for various Reynolds numbers: (a) Normalized by bulk velocity; (b) Normalized by centerline velocity. ........ 24

3.3 Friction coefficient of adverse pressure gradient flow. .............................. 25

3.4 Grid systems: (a) Normal resolution; (b) Finer resolution. .............................. 26

3.5 Overview of velocity field in laminar flow ($ER = 1.5$, $Re_H = 150$). ........ 26

3.6 Evaluation of grid and scheme dependence on computational results (Case 3-3): —— 509 × 149; O 255 × 75; □ Another scheme (Kuno et al. 1992); (a) Pressure on walls; (b) Streamwise velocity; (c) Turbulent energy. .......... 27

3.7 Comparison of predicted flow reattachment lengths with experiments. .......... 28

3.8 Streamlines: (a) Case 3-1; (b) Case 3-2; (c) Case 3-3. .............................. 29

3.9 Comparison of flow pattern in recirculating region: Dividing streamline $\psi = 0$ and trace of the point of zero streamwise velocity $\overline{U} = 0$. ........ 30

3.10 Pressure coefficient on walls: (a) Case 3-1; (b) Case 3-2. ........................ 31

3.11 Comparison with experiment of Eaton & Johnston (1980) (Case 3-2): ○ Experiment; ——— Prediction; (a) Streamwise velocity; (b) Turbulent energy; (c) Reynolds shear stress. ........................................... 32

3.12 Comparison with experiment of Kasagi et al. (1993) (Case 3-3, Key as Fig. 3.11): (a) Streamwise velocity; (b) Turbulent energy; (c) Reynolds shear stress. ........................................... 33

3.13 Streamwise variation of maximum values: (a) Streamwise component of turbulent intensity; (b) Reynolds shear stress. .............................. 34
LIST OF FIGURES

3.14 Distributions of eddy viscosity: (a) Comparison with experiment; (b) Streamwise variation of local maximums normalized by free stream velocity and reattachment length. ........................................... 35
3.15 Mean velocity profiles (Case 3-1, opposite wall). ........................................... 36
3.16 Flow features in the recirculating region: ⊙ Case 3-1; □ Case 3-2; △ Case 3-3; ○ Case 3-4; ■ Case 3-5; ● Case 3-6; ● Experiment (Adams & Johnston 1988); ■ Experiment (Eaton & Johnston 1980); (a) Streamwise variation of maximum reverse-flow velocity; (b) Relationship between wall-layer Reynolds number Re_N and skin friction coefficient. .................. 37
3.17 Mean velocity profiles in recirculating region [Computational results correspond to Case 3-2 and experimental data are compiled by Adams & Johnston (1988)]. ........................................... 38
3.18 Mean and fluctuating skin friction coefficients on step wall (Case 3-2). .... 38

4.1 Comparison with experiment in boundary-layer flow (Gibson et al. 1982, \( \bar{U}_c/\nu = 1.41 \times 10^4 \text{ m}^{-1} \)): (a) Mean velocity; (b) Mean temperature. .... 56
4.2 Streamwise development of mean temperature. ........................................... 57
4.3 Streamwise development of turbulent heat flux. ........................................... 58
4.4 Budget of turbulent energy at \( x/H = 4 \) (normalized by \( \bar{U}_o^3/H \)): Convection ○ Model, ----- DNS; Turbulent diffusion □ Model, — — — DNS; Viscous diffusion ◊ Model, - - - - - - DNS; Production △ Model, — — — — DNS; Dissipation ∇ Model, — — — — DNS; (a) Overall view; (b) Close to wall on step side. ........................................... 59
4.5 Grid system (Partial view). ........................................... 60
4.6 Computational results for velocity field (Case 4-1): (a) Streamlines; (b) Mean streamwise velocity; (c) Skin friction coefficient. ........................................... 61
4.7 Comparison with experiment of Vogel & Eaton (1985) (Case 4-1): (a) Stanton number on step side wall; (b) Mean temperature [Key as (a)]. .... 62
4.8 Variation of maximum Stanton number: (a) \( \delta_0/H = 0.7 \); (b) \( \delta_0/H = 1.1 \). .... 63
4.9 Comparison of model results for Stanton number (Case 4-1). ........................................... 64
4.10 Comparison of length scale for turbulent heat transfer (Case 4-1). ........................................... 64
4.11 Budget of turbulent energy close to wall at \( x/H = 4 \) (Key as Fig. 4.4): 
(a) Original LS model; (b) LS+Yap model. 

4.12 Comparison of budget close to wall at reattachment point (Case 4-1, Model A): (a) turbulent energy (normalized by \( \overline{U_0^3}/H \)); (b) temperature variance [normalized by \( \overline{T^2U_0}/H \)]. 

4.13 Variation of turbulent Prandtl number (Model A): (a) Suddenly heated boundary layer; (b) Backward-facing step flow (Case 4-1). 

4.14 Mean temperature profiles normalized by wall parameters (Case 4-1, Model A): (a) Recirculating region; (b) Redeveloping region. 

4.15 Distributions of turbulent heat transfer characteristics (Case 4-1, Model A): (a) Temperature variance; (b) Turbulent heat flux. 

4.16 Effect of upstream boundary-layer thickness on Stanton number (Model A). 

4.17 Effect of upstream boundary-layer thickness on turbulence characteristics \((Re_H = 28000)\): (a) Streamwise variation of local maximum values of eddy viscosity; (b) Cross-streamwise variation of eddy viscosity; (c) Cross-streamwise variation of turbulent energy [Key as (b)]. 

4.18 Comparison of streamwise variation of fluctuating skin friction coefficient and Stanton number on step-side wall \((Re_H = 28000, \text{Model A})\): (a) Fluctuating skin friction coefficient; (b) Stanton number on step-side wall.

5.1 Relation between anisotropy tensors and shear parameter: (a) Comparison with Standard model; (b) Comparison with SH model. 

5.2 Anisotropy tensors versus normal strain rate. 

5.3 Computational results in homogeneous shear flow: (a) Turbulent energy; (b) Dissipation rate. 

5.4 Reynolds stress predictions in homogeneous shear flow: (a) Normal component; (b) Shear component. 

5.5 Anisotropy tensors for homogeneous shear flow. 

5.6 Computational results in channel flow: (a) Mean velocity and turbulent energy; (b) Reynolds shear stress. 

5.7 Budget of turbulent energy in channel flow. 

5.8 Mean velocity profiles at various Reynolds numbers.
LIST OF FIGURES

5.9 Mean velocity and Reynolds shear stress in channel flow with uniform injection and suction: (a) Mean velocity; (b) Reynolds shear stress. ........ 95

5.10 Budget of turbulent energy in channel flow with uniform injection and suction: (a) Injection side; (b) Suction side. .......................... 96

5.11 Mean-velocity profile in channel flow with periodical injection and suction at one side wall: (a) Present model; (b) Yang-Shih (YS1) model; (c) Yang-Shih (YS2) model; (d) Launder-Sharma (LS) model. .................. 97

5.12 Computational results for Case 5 -2: (a) Streamlines; (b) Skin friction coefficient. ......................................................... 98

5.13 Comparison with experiment of Kasagi et al. (1993) (Case 5 -3): O Experiment; ——— Prediction; (a) Streamwise velocity; (b) Turbulent energy; (c) Reynolds shear stress. ........................................... 99

5.14 Budget of turbulent energy in backward-facing step flow at \( x/H = 4 \) (Case 5 -1, Present model): Convection O Model, ——— DNS; Turbulent diffusion □ Model, — — — DNS; Viscous diffusion ◊ Model, - - - - - DNS; Production △ Model, — — — DNS; Dissipation ◀ Model, — — — DNS; (a) Overall view; (b) Close to wall on step side. ... 100

5.15 Budget of turbulent energy in backward-facing step flow at \( x/H = 4 \) (Case 5 -1, YS1 model, Key as Fig. 5.14): (a) Overall view; (b) Close to wall on step side. ..................................................... 101

5.16 Budget of turbulent energy in backward-facing step flow at \( x/H = 4 \) (Case 5 -1, YS2 model, Key as Fig. 5.14): (a) Overall view; (b) Close to wall on step side. ..................................................... 102

5.17 Budget of turbulent energy in backward-facing step flow at \( x/H = 4 \) (Case 5 -1, LS model, Key as Fig. 5.14): (a) Overall view; (b) Close to wall on step side. ..................................................... 103

5.18 Turbulent characteristics in channel flow with periodical injection and suction at one side wall (\( y/\delta = 1/3 \)) : (a) Present model; (b) Yang-Shih (YS1) model; (c) Yang-Shih (YS2) model; (d) Launder-Sharma (LS) model. ... 104
LIST OF FIGURES

5.19 Turbulent characteristics in backward-facing step flow (Case 5-1, $x/X_R = 1$): (a) Present model; (b) Yang-Shih (YS1) model; (c) Yang-Shih (YS2) model; (d) Launder-Sharma (LS) model. ...... 106

6.1 Comparison with experiment of Sirivat & Warhaft (1983) in homogeneous decaying flow (Velocity field): (a) Turbulent energy; (b) Dissipation rate. 119

6.2 Comparison with experiment of Sirivat & Warhaft (1983) in homogeneous decaying flow ($U_1 = 6.3$ m/s): ——— Present model, —— Standard model; (a) Temperature variance; (b) Dissipation rate of $\overline{t^2}/2$; (c) Turbulent heat flux; (d) Ratio of production and dissipation rate. .... 120

6.3 Comparison with experiment of Sirivat & Warhaft (1983) in homogeneous decaying flow ($U_1 = 3.4$ m/s): ——— Present model, —— Standard model; (a) Temperature variance; (b) Dissipation rate of $\overline{t^2}/2$; (c) Turbulent heat flux; (d) Ratio of production and dissipation rate. .... 122

6.4 Comparison with experiment of Tavoularis & Corrsin (1981) in homogeneous shear flow: (a) Temperature variance; (b) Dissipation rate of $\overline{t^2}/2$; (c) Turbulent heat flux; (d) Turbulent Prandtl number and time-scale ratio. 124

6.5 Comparison with experiment in boundary-layer flow (Gibson et al. 1982, $U_o/\nu = 1.41 \times 10^6$ m$^{-1}$): (a) Mean velocity; (b) Mean temperature. .... 126

6.6 Profiles of turbulent quantities in boundary-layer flow (Velocity field): (a) Turbulent energy; (b) Dissipation rate. ......................... 127

6.7 Profiles of turbulent quantities in boundary-layer flow (Thermal field): (a) Temperature variance; (b) Dissipation rate of $\overline{t^2}/2$; (c) Turbulent heat flux; (d) Turbulent Prandtl number and time-scale ratio. ......................... 128

6.8 Comparison of ratio of production and dissipation rate (Velocity field): (a) Homogeneous shear flow; (b) Boundary-layer flow. 130

6.9 Comparison of ratio of production and dissipation rate (Thermal field): (a) Homogeneous shear flow; (b) Boundary-layer flow. 131

6.10 Comparison of shear parameter, $U_{1,2}/k/\varepsilon$: (a) Homogeneous shear flow; (b) Boundary-layer flow. 132
6.11 Comparison of computational results with experiment by Antonia et al. (1977): (a) Mean temperature; (b) Turbulent heat flux; (c) temperature variance. ................................................................. 133

6.12 Variation of turbulent Prandtl number and time-scale ratio: (a) Turbulent Prandtl number; (b) Time-scale ratio. ................................................................. 134
LIST OF TABLES

3.1  Computational conditions for back-step flows ........................................... 17
3.2  Comparison of flow reattachment lengths, $X_R/H$ .................................. 18
4.1  Computational conditions for back-step flows ........................................... 48
4.2  Model functions and model constants ....................................................... 50
5.1  Relation between $b_{12}$ and $\overline{U}_{1,2}k/\varepsilon$ (WTF; Wall-turbulent flow, HSF; Homogeneous shear flow) ...................................... 78
5.2  Model functions and model constants ....................................................... 80
5.3  Skin friction coefficient $C_f$ in the channel flow with injection and suction . 82
5.4  Computational conditions for back-step flows ........................................... 84
5.5  Comparison of flow reattachment lengths, $X_R/H$ .................................. 85
6.1  Relation between estimated value of $C_\lambda$ and shear parameter, $\overline{U}_{1,2}k/\varepsilon$ . 117
NOMENCLATURE

$A_{D1}, A_{D2}, A_{\lambda}, B_{\lambda}$ constants in heat-transfer model functions

$b_{ij}$ anisotropy tensor, $\bar{u}_i\bar{u}_j/2k - \delta_{ij}/3$

$C_D, C_\eta$ constants in nonlinear $k$-$\varepsilon$ model

$C_{D1}, C_{D2}, C_{P1}, C_{P2}$ constants in transport equation for $\varepsilon_t$

$C_f$ mean skin friction coefficient

$C_{f_m}$ mean skin friction coefficient based on bulk velocity, $\tau_w/(\rho U_m^2/2)$

$C'_f$ fluctuating skin friction coefficient

$C_p$ mean static pressure coefficient, $(\bar{P} - \bar{P}_b)/(\rho U_m^2/2)$

$C_s, C_\varepsilon$ constants in GGDH type of turbulent diffusions of $k$ and $\varepsilon$, respectively

$C_{T1}, C_{T2}, C_{T3}$ constants in two-equation heat-transfer model

$C_{t1}, C_{t2}, C_{t3}$ constants in pressure-temperature gradient correlation model

$C_{e1}, C_{e2}$ constants in transport equation for $\varepsilon$

$C_\lambda, C_m$ constants in eddy diffusivity for heat

$C_{\mu}$ constant in eddy viscosity

$c_p$ specific heat at constant pressure

$D$ channel width upstream of step or extra term in Eq. (4.17)

$E$ extra term in Eq. (4.18)

$ER$ channel expansion ratio, $(D + H)/D$

$f_A$ function to account for influence of anisotropy in flow field

$f_B$ function to guarantee the realizability of anisotropy tensor

$f_{D1}, f_{D2}, f_{P1}, f_{P2}$ functions in transport equation for $\varepsilon_t$

$f_R$ function to represent effect of dissimilarity

$f_{t1}, f_{t2}$ functions in GGDH type of turbulent diffusions of $k$ and $\varepsilon$, respectively

$f_\varepsilon$ low-Reynolds-number model function in transport equation for $\varepsilon$

$f_\lambda$ low-Reynolds-number model function in eddy diffusivity for heat
NOMENCLATURE

\( f_\mu \)  
low-Reynolds-number model function in eddy viscosity

\( H \)  
height of backward-facing step

\( h \)  
duct width at exit of wind tunnel

\( k \)  
turbulent kinetic energy, \( \bar{u}_i \bar{u}_i / 2 \)

\( L \)  
length of one period of injection and suction, \( 4\pi \delta \)

\( L_t \)  
turbulent length scale for heat transfer, \( \alpha_t / \sqrt{k} \)

\( M \)  
grid-mesh length in homogeneous decaying flow

\( N_\eta \)  
number of grid points in \( \eta \)-direction

\( N_\xi \)  
number of grid points in \( \xi \)-direction

\( n \)  
local coordinate normal to wall surface

\( \bar{P} \)  
mean static pressure

\( P_{jt} \)  
production term of \( \bar{u}_j \bar{t} \), \(-\bar{u}_j \bar{u}_k \bar{T}_{k,j} - \bar{u}_k \bar{t} \bar{U}_{j,k} \)

\( P_k \)  
production term of \( k \), \(-\bar{u}_i \bar{u}_j \bar{U}_{i,j} \)

\( Pr \)  
molecular Prandtl number

\( Pr_t \)  
turbulent Prandtl number

\( p \)  
fluctuating static pressure

\( q_w \)  
wall heat flux

\( R \)  
time-scale ratio, \( \tau_t / \tau_u \)

\( Re_H \)  
Reynolds number based on step height, \( \bar{U}_0 H / \nu \)

\( Re_m \)  
Reynolds number based on channel bulk velocity, \( 2\bar{U}_m \delta / \nu \)

\( Re_N \)  
Reynolds number based on maximum reverse-flow velocity, \(-\bar{U}_N \bar{y}_N / \nu \)

\( Re_\theta \)  
Reynolds number based on momentum thickness, \( \bar{U}_0 \theta / \nu \)

\( Re_\tau \)  
Reynolds number based on friction velocity, \( u_\tau \delta / \nu \) or \( u^*_\tau \delta / \nu \)

\( Re_0 \)  
Reynolds number based on channel centerline velocity, \( 2\bar{U}_0 \delta / \nu \)

\( R_s \)  
scaling parameter, \( k/\nu S \)

\( R_t \)  
turbulent Reynolds number, \( k^2 / \nu \varepsilon \)

\( S \)  
strain-rate parameter, \( \sqrt{2S_{ij}S_{ij}} \)

\( S_{ij} \)  
strain-rate tensor, \( (\partial \bar{U}_i / \partial x_j + \partial \bar{U}_j / \partial x_i) / 2 \)

\( St \)  
Stanton number, \( q_w / [\rho c_p \bar{U}_0 (\bar{T}_w - \bar{T}_0)] \)

\( \bar{T} \)  
mean temperature

\( \bar{T}^+ \)  
nondimensional mean temperature, \( (\bar{T}_w - \bar{T})/\tau_\tau \)
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{T}_r$</td>
<td>reference temperature, $q_w H / (\rho c_p \alpha)$</td>
</tr>
<tr>
<td>$t$</td>
<td>temperature fluctuation</td>
</tr>
<tr>
<td>$\bar{t}^2$</td>
<td>temperature variance</td>
</tr>
<tr>
<td>$t_r$</td>
<td>friction temperature, $q_w / (\rho c_p u_r)$</td>
</tr>
<tr>
<td>$\bar{U}, \bar{V}, \bar{W}$</td>
<td>mean velocity in $x$, $y$- and $z$-directions, respectively</td>
</tr>
<tr>
<td>$\bar{U}_i$</td>
<td>mean velocity in $i$-direction</td>
</tr>
<tr>
<td>$\bar{U}_m$</td>
<td>bulk velocity of fully-developed channel flow</td>
</tr>
<tr>
<td>$\bar{U}^+$</td>
<td>nondimensional mean velocity, $\bar{U} / u_r$</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>turbulent fluctuation in $x$, $y$- and $z$-directions, respectively</td>
</tr>
<tr>
<td>$u_i$</td>
<td>turbulent fluctuation in $i$-direction</td>
</tr>
<tr>
<td>$u_e$</td>
<td>Kolmogorov velocity scale, $(v \varepsilon)^{1/4}$</td>
</tr>
<tr>
<td>$u_r$</td>
<td>friction velocity, $\sqrt{</td>
</tr>
<tr>
<td>$u_r^*$</td>
<td>averaged friction velocity on both side walls in channel flow with injection and suction</td>
</tr>
<tr>
<td>$X_R$</td>
<td>flow reattachment length</td>
</tr>
<tr>
<td>$x$</td>
<td>Cartesian coordinate in streamwise direction</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Cartesian coordinate in $i$-direction</td>
</tr>
<tr>
<td>$y$</td>
<td>Cartesian coordinate normal to streamwise direction</td>
</tr>
<tr>
<td>$y_k$</td>
<td>nondimensional distance from wall surface, $\sqrt{k_y} / \nu$</td>
</tr>
<tr>
<td>$y^+$</td>
<td>nondimensional distance from wall surface, $u_r y / \nu$</td>
</tr>
<tr>
<td>$y^*$</td>
<td>nondimensional distance from wall surface, $u_r y / \nu$</td>
</tr>
<tr>
<td>$z$</td>
<td>Cartesian coordinate in spanwise direction</td>
</tr>
</tbody>
</table>

**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>molecular diffusivity for heat</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>eddy diffusivity for heat</td>
</tr>
<tr>
<td>$\delta$</td>
<td>half width of channel or 99% boundary layer thickness</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>dissipation rate of turbulent kinetic energy, $\nu (\partial u_i / \partial x_j) (\partial u_i / \partial x_j)$</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>dissipation rate of $\bar{t}^2 / 2$, $\alpha (\partial t / \partial x_j) (\partial t / \partial x_j)$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>generalized coordinate in lower to upper wall direction</td>
</tr>
<tr>
<td>$\theta$</td>
<td>momentum thickness</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Von Kármán's universal constant</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( \nu \)  kinematic viscosity
\( \nu_t \)  eddy viscosity
\( \xi \)  generalized coordinate in inlet-to-outlet direction
\( \rho \)  density
\( \sigma_k, \sigma_\varepsilon \)  constants in isotropic turbulent diffusions of \( k \) and \( \varepsilon \), respectively
\( \sigma_h, \sigma_\phi \)  coefficients of turbulent diffusion terms in \( \overline{t^2} \) and \( \varepsilon \) transport equations, respectively
\( \tau \)  time
\( \tau_c \)  characteristic time scale for turbulent velocity field
\( \tau_m \)  characteristic time scale for turbulent heat transfer
\( \tau_t \)  time scale of temperature field, \( \overline{t^2}/2\varepsilon_t \)
\( \tau_u \)  time scale of velocity field, \( k/\varepsilon \)
\( \tau_w \)  wall shear stress
\( \Phi_{ij} \)  pressure-temperature gradient correlation, \( (p/\rho)\overline{t_{ij}} \)
\( \psi \)  stream function
\( \Omega \)  vorticity parameter, \( \sqrt{2\Omega_{ij}\Omega_{ij}} \)
\( \Omega_{ij} \)  vorticity tensor, \( \left( \frac{\partial \overline{U_j}}{\partial x_i} - \frac{\partial \overline{U_i}}{\partial x_j} \right) / 2 \)

Subscripts

\( e \)  outer edge of boundary layer
\( i, j, k, l, m, n \)  1, 2 and 3 denote \( x-, y- \) and \( z- \)directions, respectively
\( max \)  maximum value on step-side wall
\( max(x) \)  maximum value at a location \( x \)
\( N \)  maximum reverse-flow point at location \( x \)
\( R \)  flow reattachment point
\( w \)  wall surface
\( 0 \)  reference value
\( 1 \)  nearest grid point from wall surface

Special symbols

\( D/D\tau \)  substantial derivative, \( \partial/\partial\tau + \overline{U_j}\partial/\partial x_j \)
\( f, i \)  partial derivative of variable, \( f \), with respect to coordinate, \( x_i \)
\( (\overline{\overline{()}}) \)  ensemble-averaged value
\( (\overline{()})^+ \)  normalization by inner variables
Chapter 1

INTRODUCTION

1.1 Background

In practical fluid machinery, most flows are turbulent and involve complex aspects which play an important role in the performance of devices. For example, in configurations with abruptly expanding walls, flows detach at the corner edge and reattach downstream, which forms a recirculating region. An obstacle settled in a flow duct also causes separations in front and in back of it. These flow detachments and reattachments almost always determine the key structure of the flow field and significantly influence the mechanism of heat transfer. It is well known that flow separations usually increase the drag force which causes an increase of the power supply and a deterioration in the efficiency of fluid machinery. In contrast, turbulent heat transfer is considerably augmented around the reattaching point and the above-mentioned obstacles are often used as an augmentation device in turbulent heat transfer. Thus, a reliable evaluation of turbulent fluid flow and heat transfer is indispensable in optimizing the design of fluid machinery.

On the other hand, the recent remarkable development of high performance computers and numerical schemes provides a powerful tool for predicting turbulent fluid flow and heat transfer. Computational Fluid Dynamics (CFD) is now becoming one of the most useful methods for investigating turbulence, together with the knowledge of turbulence phenomena accumulated so far.

In calculating turbulent fluid flow and heat transfer, turbulence modeling is the most important and difficult problem. So far, mainly due to the computational time and numer-
CHAPTER 1. INTRODUCTION

ical stability, the $k$-$\varepsilon$ model in combination with the assumption of a constant turbulent Prandtl number, $Pr_t$, has been frequently used to predict turbulent heat transfer. Even in complex turbulent flows, e.g., those with separation and reattachment, this methodology has been usually adopted (see Launder 1988). For example, to grasp the essential physics of complex turbulent heat transfer in such flows, several attempts using this approach have been made to simulate the heat transfer downstream of a backward-facing step (Ciofalo & Collins 1989; Djilali et al. 1989; Nešić & Postlethwaite 1992; Dutta & Acharya 1993). Though such an approach is practically useful, the following major problems remain:

1. In calculating turbulent flows with the $k$-$\varepsilon$ model, the wall functions are usually employed as the boundary conditions on solid walls. However, their application to complex phenomena, e.g., a recirculating region and/or a region subjected to a strong pressure gradient, is quite questionable.

2. Performance of the existing $k$-$\varepsilon$ models is insufficient in predicting complex turbulent flows. For example, the calculations with the previous $k$-$\varepsilon$ models usually give a 15-20% underprediction of the flow reattachment length in a backward-facing step flow (Kline et al. 1981; Speziale 1991; Thangam & Speziale 1992), even though it is the most fundamental quantity to be predicted in separating and reattaching flows.

3. In situations where the similarity between the velocity and temperature fields does not hold, as in a boundary-layer or a pipe flow subjected to a sudden change in wall thermal condition (Antonia et al. 1977; Sato et al. 1992), $Pr_t$ is far from constant. This is in contrast to the situation in a simple boundary-layer flow where the velocity and temperature fields develop simultaneously.

To overcome these difficulties, today's $k$-$\varepsilon$ models have been significantly improved so as to work even in the vicinity of the wall; they are called the 'low-Reynolds-number $k$-$\varepsilon$ models'. The principal researchers include Launder & Sharma (1974) [LS], Lam & Bremhorst (1981) [LB], Nagano & Hishida (1987) [NH], Myong & Kasagi (1990) [MK], Nagano & Tagawa (1990) [NT], Zhang & Sousa (1990) [ZS], Kawamura & Hada (1992) [KH], Nagano & Shimada (1993) [NS], Hattori et al. (1993) [HNT] and Yang & Shih
(1993a [YS1], 1993b [YS2]). Some attempts have been made to predict turbulent heat transfer in separating and reattaching flows with low-Reynolds-number $k-\varepsilon$ models. For example, Chieng & Launder (1980) calculated the turbulent heat transfer in separating and reattaching flows downstream of an abrupt expansion pipe with the LS model. The predicted heat transfer coefficients were, however, surprisingly overestimated. Nešić & Postlethwaite (1992) performed the calculation of mass transfer in a separating flow using the LB model at Schmidt numbers higher than $10^3$. The LB model, however, gives the incorrect near-wall limiting behavior of Reynolds stress, and hence the correct prediction of heat transfer cannot be expected in high-Prandtl or Schmidt-number flows. For accurate prediction of heat transfer in high Prandtl number flows, we need to reproduce the near-wall limiting behavior correctly. Among the existing low-Reynolds-number $k-\varepsilon$ models, the NT model (Nagano & Tagawa 1990) is regarded as one of the most reliable. This model can reproduce the near-wall limiting behavior and can provide accurate predictions for the attached turbulent flows such as channel and boundary-layer flows with favorable or adverse pressure gradients.

To exclude the ambiguous assumption of a constant $Pr_t$, great efforts have been made over the last several years in constructing refined two-equation heat-transfer models. This has led to substantial improvements in the accuracy of heat transfer predictions in complex turbulent flows (Nagano & Kim 1988; Nagano et al. 1991; Youssef et al. 1992). The leading researchers in this field include Nagano & Kim (1988) [NK], Nagano et al. (1991) [NTT], Youssef et al. (1992) [YNT], Sommer et al. (1992) [SSL], Hattori et al. (1993) [HNT], Sommer et al. (1994) [SSZ], Sato et al. (1994) [SSN] and Nagano et al. (1994b) [NSY]. The NK model is the original intended two-equation heat-transfer model, while the next four models [NTT – HNT] are extended to satisfy the wall-limiting behavior of turbulence. The last three models [SSZ – NSY] are intended to calculate turbulent heat transfer at different Prandtl numbers. Note that the SSZ model needs a second-order closure model for velocity-field calculations. Among the existing two-equation heat-transfer models, the NTT model (Nagano et al. 1991) is regarded as one of the most reliable. This model can correctly reproduce the near-wall limiting behavior of scalar turbulence quantities for any wall thermal condition (Nagano et al. 1991; Youssef et al. 1992).
CHAPTER 1. INTRODUCTION

As described above, the NT and NTT models, which are representative low-Reynolds-number two-equation turbulence models, are quite useful in accurately predicting fluid flow and heat transfer in attached turbulent flows. However, since the model functions of the NT and NTT models contain the friction velocity $u_r$, they break down around the separating and reattaching points where $u_r = 0$. Thus, contrary to the actual phenomena, the NT and NTT models force the Reynolds stress and turbulent heat flux to vanish at that point. To allow application to complex turbulent flows with separation and reattachment, this problem is urgently in need of a solution.

Another problem in calculating turbulent fluid flow and heat transfer with the existing two-equation turbulence models is that previous models could not predict both wall and free turbulent flows without changing the model constants (Suzuki et al. 1993). This deficiency mainly originates from the eddy-viscosity approximation expressed as follows:

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}, \quad (1.1)$$

$$\alpha_t = C_\lambda k \tau_m. \quad (1.2)$$

In Eqs. (1.1) and (1.2), the model constants $C_\mu$ and $C_\lambda$ are often assigned the values of 0.09 and 0.1, respectively. These values are quite appropriate for the prediction of fluid flow and heat transfer in wall turbulent shear flows, whereas they do not give reasonable predictions in free turbulent (e.g., homogeneous shear) flows. Many remedies have been tried to overcome this difficulty, notably, a zonal approach (Tzuoo et al. 1986), a non-equilibrium eddy-viscosity model (Yoshizawa 1993), modifications of the model constants using the mean-velocity gradient (Shih et al. 1994; Yakhot et al. 1992), and a multiple-scale model (Nagano et al. 1994a). However, these attempts do not always offer reasonable solutions for every flow field, and are now under further investigations.

On the other hand, to develop an improved turbulence model, it is quite useful to refer to the models constructed on the basis of higher-order approximation. Pope (1975) showed an explicit expression of the Reynolds stress $\bar{u}_i \bar{u}_j$ for a two-dimensional algebraic stress model (ASM). Taulbee (1992) developed a new type of explicit ASM expression (EASM) and a nonlinear $k-\varepsilon$ formulation on the basis of Pope's discussion (1975), and applied them to homogeneous shear, plain strain and stagnation flows. However, this model has not been applied to wall-turbulent flows. As for the thermal field model,
Suzuki et al. (1993) proposed a new concept for the expression of turbulent heat flux \( \overline{u_j t} \) using the turbulent heat-flux model of Jones & Musonge (1988). The model of Suzuki et al. (1993), however, does not provide a fully explicit expression for \( \overline{u_j t} \). Moreover, their model is only applicable to free turbulence.

Thus, the development of more sophisticated two-equation turbulence models applicable to predictions of fluid flow and heat transfer in both wall and free turbulent flows is eagerly awaited.

1.2 Objectives

In view of the background described in the previous section, the present study has the following main objectives:

1. To examine what factors determine the low-Reynolds-number parameters used in the model functions;

2. To elucidate what type of hybrid time scale is best suited to the characteristic time scale for turbulent heat transfer under complex thermal conditions;

3. To develop a low-Reynolds-number \( k-\varepsilon \) model applicable to complex turbulent velocity fields with separation and reattachment;

4. To develop a two-equation heat-transfer model applicable to heat transfer problems in complex turbulent flows with separation and reattachment;

5. To show a relation between the separated shear layer and the turbulent phenomena in the near-wall region of a separating and reattaching flow downstream of a backward-facing step;

6. To elucidate an augmentation mechanism of turbulent heat transfer around the reattaching point in a backward-facing step flow;

7. To investigate the theoretical relation between two-equation turbulence models and second-moment closures;

8. To construct a new \( k-\varepsilon \) model applicable to both wall and free turbulent flows;
9. To construct a new two-equation heat-transfer model applicable to heat transfer problems in both wall and free turbulent flows.

1.3 Organization of Dissertation

The subject of this thesis consists of two main areas of inquiry. The first is concerned with the construction of low-Reynolds-number two-equation turbulence models for predicting fluid flow and heat transfer in separating and reattaching flows. The second deals with the construction of new two-equation turbulence models for predicting fluid flow and heat transfer in both wall and free turbulent flows, with the aid of insights into relations between explicit expressions of simplified second-moment closures and conventional two-equation models. Chapters 3 and 4 deal with the first area, and Chapters 5 and 6 with the second.

Chapter 2 presents the ensemble-averaged governing equations for the turbulent velocity and temperature fields used in this study.

In Chapter 3, a modified version of the NT model is proposed (Abe et al. 1992; Abe et al. 1993; Abe et al. 1994a), which allows application to separating and reattaching flows. The principal improvement is the usage of the Kolmogorov velocity scale, \( u_\varepsilon \equiv (\nu \varepsilon)^{1/4} \), instead of the friction velocity \( u_\tau \), to account for the near-wall and low-Reynolds-number effects. In contrast to the friction velocity \( u_\tau \), the velocity scale \( u_\varepsilon \) vanishes at neither the separating nor the reattaching points, so that the proposed model is applicable to complex turbulent flows with separation. Separating and reattaching flows downstream of a backward-facing step are calculated with the proposed model and a detailed investigation of the computational results reveals flow structures in a recirculating region.

Chapter 4 presents a two-equation heat-transfer model for predicting turbulent heat transfer in separating and reattaching flows (Abe et al. 1994b; Abe et al. 1995a), which is a modified version of the NTT model (Nagano et al. 1991; Youssef et al. 1992). The principal improvement is use of the Kolmogorov velocity scale \( u_\varepsilon \) instead of the friction velocity \( u_\tau \), as in the velocity-field model proposed in Chapter 3. Besides this major modification, we examine in detail what type of hybrid time scale is best suited to the characteristic time scale for turbulent heat transfer under complex thermal conditions.
CHAPTER 1. INTRODUCTION

Calculations of turbulent heat transfer in separating and reattaching flows downstream of a backward-facing step are performed. The computational results are compared with the corresponding experimental data to examine the model performance under various conditions of the flow Reynolds number and upstream boundary-layer thickness. Furthermore, the present computational results provide important insights into how the flow-field structures affect the heat-transfer mechanism in separating and reattaching flows.

In Chapter 5, a new $k$-$\varepsilon$ model which incorporates some essential characteristics of second-order closure models is proposed (Abe et al. 1995b; Abe et al. 1995c). The present model is in the category of nonlinear $k$-$\varepsilon$ models, and is extended to a low-Reynolds-number model by employing the Kolmogorov velocity scale $u_\varepsilon$ in the model functions. To verify its basic accuracy, the proposed model is applied to some fundamental wall and free turbulent flows, i.e., a fully developed channel flow and a homogeneous shear flow, respectively. The model is also applied to complex turbulent flows, i.e., channel flows with injection and suction at the wall surfaces, and to separating and reattaching flows downstream of a backward-facing step. In addition, to reveal which factors determine the low-Reynolds-number parameters used in the model functions, the computational results from the present model and representative low-Reynolds-number $k$-$\varepsilon$ models previously proposed are compared and reexamined in detail.

Chapter 6 presents a new two-equation heat-transfer model for predicting heat transfer in both wall and free turbulent flows (Abe et al. 1995d; Abe et al. 1995e). The proposed model incorporates some essential features of the second-order modeling and is reduced to a low-Reynolds-number two-equation model with knowledge obtained from the earlier part of this study. To verify model performance, the present model is tested by application to some significant heat transfer problems in wall and free turbulent flows, i.e., a homogeneous isotropic decaying flow, a homogeneous shear flow, a boundary-layer flow heated from the origin, and a boundary-layer flow subjected to a sudden change in wall thermal conditions.

The important conclusions of this study are described in Chapter 7.
Chapter 2

GOVERNING EQUATIONS

2.1 Velocity-Field Equations

In this study, the following conditions are imposed in calculating a turbulent velocity field:

1. All fluid properties are constant within the flow.

2. Fluid is incompressible and Newtonian.

3. Within the flow, any external body force, e.g., a buoyancy force, does not exist.

Under the above-mentioned conditions, a velocity field is described with the following ensemble-averaged governing equations (Nagano & Hishida 1987; Myong & Kasagi 1990; Nagano & Tagawa 1990):

\[ \frac{\partial \overline{U_i}}{\partial x_i} = 0, \quad (2.1) \]

\[ \frac{D\overline{U_i}}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left\{ \nu \left( \frac{\partial \overline{U_i}}{\partial x_j} + \frac{\partial \overline{U_j}}{\partial x_i} \right) - \overline{u_iu_j} \right\}, \quad (2.2) \]

\[ \frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left\{ \nu \frac{\partial k}{\partial x_j} - \frac{k'}{\rho} \overline{u_j} \right\} - \overline{u_iu_j} \frac{\partial \overline{U_i}}{\partial x_j} - \varepsilon, \quad (2.3) \]

\[ \frac{De}{Dt} = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \varepsilon}{\partial x_j} - \varepsilon' \overline{u_j} - \frac{2 \nu}{\rho} \frac{\partial \overline{u_j}}{\partial x_i} \right) - C_{e1} \frac{e}{k} \frac{\overline{u_iu_j}}{\overline{U_i}} \frac{\partial \overline{U_i}}{\partial x_j} - C_{e2} f_e \frac{\varepsilon^2}{k}, \quad (2.4) \]

where \( k' = u_i u_i / 2 \), \( \varepsilon' = \nu \partial u_i / \partial x_k \partial u_i / \partial x_k \) and \( k = k', \varepsilon = \varepsilon' \), respectively. In Eq. (2.4), \( C_{e1} \) and \( C_{e2} \) are the model constants, and \( f_e \) is the model function to account for the near-wall and low-Reynolds-number effects.
2.2 Thermal Field Equations

In calculating turbulent heat transfer, we consider the following conditions in addition to those imposed on a velocity field as described in Section 2.1:

1. Within the flow, there is no heat generation due to chemical or biological reactions.

2. In wall turbulent flows, internal heat generation originating from the viscous dissipation is negligible small compared with the heat input from wall surfaces.

Under these conditions, we can deal with a thermal field as a forced convection field. Thus, a thermal field can be described with the following ensemble-averaged governing equations (Nagano & Kim 1988; Nagano et al. 1991):

\[
\frac{DT}{Dt} = \frac{\partial}{\partial x_j} \left( \alpha \frac{\partial T}{\partial x_j} - u_j T \right),
\]

\[
\frac{D\epsilon_t}{Dt} = \frac{\partial}{\partial x_j} \left\{ \left( \alpha + \frac{\alpha_t}{\sigma_h} \right) \frac{\partial \epsilon_t}{\partial x_j} \right\} - 2u_j \frac{\partial T}{\partial x_j} - 2\epsilon_t,
\]

\[
\frac{D\epsilon_t}{Dt} = \frac{\partial}{\partial x_j} \left\{ \left( \alpha + \frac{\alpha_t}{\sigma_\phi} \right) \frac{\partial \epsilon_t}{\partial x_j} \right\} - C_{P1} f_{P1} \frac{\epsilon_t}{t^2} u_j \frac{\partial T}{\partial x_j}
\]

\[
- C_{P2} f_{P2} \frac{\epsilon_t}{k} \frac{\partial U_i}{\partial x_j} - C_{D1} f_{D1} \frac{\epsilon_t^2}{t^2} - C_{D2} f_{D2} \frac{\epsilon_t \epsilon_i}{k},
\]

In Eq. (2.7), \(f_{P1}, f_{P2}, f_{D1}\) and \(f_{D2}\) are the model functions to account for the near-wall and low-Reynolds-number effects, and \(C_{P1}, C_{P2}, C_{D1}\) and \(C_{D2}\) are the model constants.

Note that, in this study, the turbulent diffusion terms in Eqs. (2.6) and (2.7) are modeled following the conventional procedure by introducing the eddy diffusivity for heat \(\alpha_t\), and \(\sigma_h\) and \(\sigma_\phi\) are the coefficients of these terms.
Chapter 3

AN IMPROVED $k-\varepsilon$ MODEL FOR SEPARATING FLOWS

3.1 Turbulence Modeling

3.1.1 Eddy-viscosity approximation

In this chapter, a low-Reynolds-number $k-\varepsilon$ model applicable to separating and reattaching flows is proposed. This model is a modified version of the NT model (Nagano & Tagawa 1990), which reproduces the near-wall limiting behavior of turbulence as discussed later.

A turbulent velocity field is described with the governing equations in Section 2.1. On the basis of the isotropic eddy-viscosity approximation, the Reynolds stress and the turbulent diffusion terms in Eqs. (2.3) and (2.4) are expressed as follows:

$$ -\overline{u_i u_j} = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}, \quad (3.1) $$

$$ \left( k' + \frac{p}{\rho} \right) u_j = -\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial x_j}, \quad (3.2) $$

$$ \varepsilon \overline{u_j} + \frac{2\nu}{\rho} \frac{\partial p}{\partial x_i} \frac{\partial u_j}{\partial x_i} = -\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j}, \quad (3.3) $$

with

$$ \nu_t = C_u f_u \frac{k^2}{\varepsilon}, \quad (3.4) $$
where \( \nu_t \) is the eddy viscosity and \( f_\mu \) is the model function to account for the near-wall and low-Reynolds-number effects. In Eqs. (3.2) - (3.4), \( \sigma_k \), \( \sigma_\varepsilon \) and \( C_\mu \) are the model constants.

### 3.1.2 Original Nagano-Tagawa model

In the original NT model, the model functions \( f_\mu \) and \( f_\varepsilon \), which are introduced to reflect the multiple length scales involved in shear flows and to satisfy the requirements for the near-wall limiting behavior of turbulence, are expressed as follows:

\[
f_\mu = \left\{ 1 - \exp \left( -\frac{y^+}{26} \right) \right\}^2 \left( 1 + \frac{4.1}{R_t^1} \right),
\]

\[
f_\varepsilon = \left\{ 1 - \exp \left( -\frac{y^+}{6} \right) \right\}^2 \left[ 1 - 0.3 \exp \left\{ -\left( \frac{R_t}{6.5} \right)^2 \right\} \right],
\]

where \( y^+ = u_+ y / \nu \) and \( R_t = k^2 / \nu \varepsilon \). The model constants in Eq. (2.4) and Eqs. (3.2) - (3.4) are given as follows:

\[
C_\mu = 0.09, \quad \sigma_k = 1.4, \quad \sigma_\varepsilon = 1.3, \quad C_{\varepsilon 1} = 1.45, \quad C_{\varepsilon 2} = 1.9.
\]

This model can reproduce the near-wall asymptotic relations of \( -\overline{uv} \propto y^3 \), \( k \propto y^2 \) and \( \varepsilon \propto y^0 \) correctly, and can predict, with high accuracy, the attached turbulent flows with favorable or adverse pressure gradient (Nagano & Tagawa 1990).

However, Eqs. (3.5) and (3.6) contain the friction velocity \( u_+ \). Thus, if we apply this model to separating flows, it will collapse at a separation point \( u_+ = 0 \) and also at a reattaching location \( u_+ = 0 \).

### 3.1.3 Modification of model functions

A new velocity scale which replaces the friction velocity \( u_+ \) is required so that the NT model can be applied to separating and reattaching flows.

In selecting a new velocity scale, we must satisfy the following essential requirements from the standpoint of turbulence modeling.

1. The velocity scale is composed of characteristic quantities of a turbulent flow.
2. The velocity scale has no singularity at a separating or reattaching point.
3. The new model functions should reproduce the near-wall limiting behavior as correctly as the original NT model. In other words, the selected velocity scale has a finite value at the wall surface.

In what follows, we evaluate the near-wall limiting behavior of velocity. Each component of the velocity can be expanded in terms of $y$ near the wall as follows:

$$
U = A_1 y + A_2 y^2 + A_3 y^3 + \cdots, \quad V = B_2 y^2 + B_3 y^3 + \cdots,
$$

$$
W = C_1 y + C_2 y^2 + C_3 y^3 + \cdots, 
$$

$$
u = a_1 y + a_2 y^2 + a_3 y^3 + \cdots, \quad v = b_2 y^2 + b_3 y^3 + \cdots,
$$

$$
w = c_1 y + c_2 y^2 + c_3 y^3 + \cdots, 
$$

where $y$ is the distance from the wall surface and an overbar ($\bar{\cdot}$) denotes an ensemble-averaged value. The singularity at a separating or reattaching point occurs in the NT model because the velocity scale is determined by the mean velocity component, i.e.,

$$u_r = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\nu \left( \frac{\partial U}{\partial y} \right)_w} = \sqrt{\nu \bar{A}_1},$$

where an expansion coefficient $A_1$ is consistently zero at both separation and reattachment points. It may be expected that, if we determine the velocity scale by the characteristic values of turbulence, the above singularity can be removed.

The obvious velocity scale of turbulence is $\sqrt{k}$, which has been most commonly used in various turbulence models (e.g., Lam & Bremhorst 1981). Recently, Zhang & Sousa (1990) proposed a modification (the ZS model) of the NH model (Nagano & Hishida 1987) with the velocity scale replaced by $\sqrt{k}$.

Though the LB and ZS models can avoid the singularity, their crucial weakness is that the resultant solutions violate the proper near-wall limiting behavior. From Eq. (3.8), the turbulent energy is expressed as follows:

$$k = \frac{1}{2} \left( a_1^2 + c_1^2 \right) y^2 + \cdots. \quad (3.9)$$

The above equation indicates that $\sqrt{k}$ changes linearly with $y$. Thus, the Reynolds stress $-\overline{uv}$ is proportional to $y^4$ in the LB model, and $y^7$ in the ZS model, both of which conflict with the correct behavior $-\overline{uv} \propto y^3$. For the accurate prediction of heat transfer in high Prandtl number fluids where a thermal layer is always very thin, it is extremely important to reproduce the near-wall limiting behavior correctly.

As seen from Eqs. (3.8) and (3.9), $\sqrt{k}$ represents the fluctuating velocity itself, and has no direct relevance to the friction velocity $u_r$. Hence, the choice of $\sqrt{k}$ instead of $u_r$ is not
reasonable. It is more desirable that a new velocity scale couples in some physical manner with the fluctuation of instantaneous friction velocity \( u_r(t) \). The intensity of fluctuation of instantaneous wall shear stress \( \tau_w(t) \) is expressed as follows:

\[
\rho \nu \sqrt{\left( \frac{\partial u}{\partial y} \right)_w^2} = \rho \nu \sqrt{a_1^2}.
\]  

(3.10)

On the other hand, the dissipation rate \( \varepsilon \) near the wall surface is expressed as follows:

\[
\varepsilon = \nu \left\{ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right\} = \nu \left( \frac{a_1^2}{a_1^2 + c_1^2} + \cdots \right).
\]  

(3.11)

From Eqs. (3.10) and (3.11), we can recognize that the Kolmogorov velocity scale \( u_\varepsilon = (\nu \varepsilon)^{1/4} \simeq \left( \nu \sqrt{a_1^2 + c_1^2} \right)^{1/2} \) is closely related to the measure of fluctuating friction velocity \( \left( \nu \sqrt{a_1^2 + c_1^2} \right)^{1/2} \). Furthermore, from Eq. (3.11), the Kolmogorov velocity scale \( u_\varepsilon \) has a finite value \( \left( \nu \sqrt{a_1^2 + c_1^2} \right)^{1/2} \) on the wall surface, thus satisfying the necessary condition to reproduce the near-wall limiting behavior \(-\bar{w}/y^3\).

Accordingly, it is legitimate to introduce the velocity scale \( u_\varepsilon = (\nu \varepsilon)^{1/4} \) instead of the friction velocity \( u_r \). And the model functions may be written as follows:

\[
f_\mu = \left\{ 1 - \exp \left( -\frac{y^*}{1.4} \right) \right\}^2 \left[ 1 + \frac{5}{R^3} \exp \left\{ - \left( \frac{R}{200} \right)^2 \right\} \right],
\]  

(3.12)

\[
f_\varepsilon = \left\{ 1 - \exp \left( -\frac{y^*}{3.1} \right) \right\}^2 \left[ 1 - 0.3 \exp \left\{ - \left( \frac{R}{6.5} \right)^2 \right\} \right],
\]  

(3.13)

where \( y^* = u_\varepsilon y/\nu \).

On the other hand, previous studies with the standard \( k-\varepsilon \) model have suffered from the fact that the reattachment lengths of back-step flows were always underpredicted by 15-20% compared with the experiments. One of the reasons for these underpredictions is that the wall functions have been usually employed even in the separating region. Another reason might be that the values of model constants used are not suitable. Thus, we have reevaluated the model constants of Eq. (2.4) and Eqs. (3.2) - (3.4). First, \( C_\mu \) is set to a standard value of 0.09, because the structure parameter \(-\bar{w}/k\) in separating flows is nearly identical with that in attached flows (Avva et al. 1988a). Next, \( C_{\varepsilon 2} \) is determined to be 1.9 because free turbulent flows are well reproduced by using the typical value of
$C_{e2} = 1.9$ along with the function $[1 - 0.3 \exp\{- (R_d/6.5)^2\}]$ in $f_e$ as shown in the NT model (Nagano & Tagawa 1990).

The other constants to be determined are $C_{e1}, \sigma_k$ and $\sigma_e$. In wall-turbulent flows, $C_{e1}, C_{e2}$ and $\sigma_e$ must satisfy the following relation (Nagano & Tagawa 1990):

$$
C_{e1} = C_{e2} - \frac{k^2}{\sigma_e \sqrt{\nu}}. 
\tag{3.14}
$$

We have investigated the effect of model constants on the computational results, and found that the reattachment length of a backward-facing step flow is sensitive to a value of $C_{e1}$. The reattachment length changes by about 0.2 step height with the $C_{e1}$ variation of 0.01. Based on the reassessment, $C_{e1}$ is determined to be 1.5 as a most appropriate value, and $\sigma_e$ is put at 1.4 so that Eq. (3.14) may be satisfied. Finally, following the lead of Nagano & Tagawa (1990) that the correct profile of the eddy viscosity in internal flows can be obtained by setting $\sigma_k \simeq \sigma_e$, $\sigma_k$ is assigned to 1.4, which is the same value as in the NT model. In sum, the present new model uses the following set of model constants:

$$
C_{\mu} = 0.09, \quad \sigma_k = 1.4, \quad \sigma_e = 1.4, \quad C_{e1} = 1.5, \quad C_{e2} = 1.9. \tag{3.15}
$$

3.2 Model Assessment in Attached Turbulent Flows

In our new proposal, we modified not only the model functions but also some model constants of the original NT model. Thus, to confirm the basic accuracy of the present model, we applied it to the predictions of representative attached turbulent flows, i.e., a fully developed channel flow and a boundary layer flow with an adverse pressure gradient. The latter is known to be difficult to predict accurately with the previous $k-\varepsilon$ models (Nagano & Tagawa 1990).

Distributions of the velocity and the eddy viscosity obtained from the present model are compared with the direct numerical simulation (DNS) data (Kim et al. 1990) in Fig. 3.1. For both the velocity and the eddy viscosity, the present predictions show good agreement with the DNS data. In Fig. 3.2, the computational results for the skin friction coefficient at various Reynolds numbers are shown. It can be seen that the present model predicts accurately the Reynolds-number dependence in a fully turbulent regime, and for the Reynolds number less than $10^4$ the model reflects reasonably the low-Reynolds-number
CHAPTER 3. AN IMPROVED $k$-$\varepsilon$ MODEL FOR SEPARATING FLOWS

effect. The almost perfect agreement in both skin friction coefficients in Figs. 3.2 (a) and 3.2 (b) indicates that the model captures accurately the Reynolds-number dependence of
the wall shear-stress and the ratio of the centerline velocity $U_0$ to the mean bulk velocity
$U_m$. These features pertaining to the Reynolds-number effects may be quite advantageous
in the model application to more complex turbulent flows.

The calculation of an adverse pressure gradient flow was conducted with reference to
the experiment of Samuel & Joubert (1974). The variation of skin friction coefficient with
a flow development in the streamwise direction is shown in Fig. 3.3. It can be seen that
the present results agree well with the experimental data.

The foregoing comparisons demonstrate that the present model can predict quite ac-
curately attached turbulent flows including the effects of both the Reynolds number and
pressure gradient.

3.3 Application to Backward-Facing Step Flows

3.3.1 Numerical procedure and boundary conditions

In calculating the backward-facing step flow, we used the finite-difference method to
discretize the governing equations, employing the third-order upwind difference for the
convection term in Eq. (2.2), the first-order upwind difference for the convection terms
in Eqs. (2.3) and (2.4), and the second-order central difference for the other terms. The
calculations were performed by the MAC method. The generalized coordinate system
was employed and the grid system was non-staggered. Figure 3.4 shows the present
computational grid systems. The finer-resolution grid [Fig. 3.4 (b)] was used to confirm
the grid dependence as will be described later.

The boundary conditions were: $U = V = k = 0$, $\varepsilon_w = 2\nu k_1/n_i^2$ and $\partial P/\partial n = 0$ at
the wall surface, where $n$ is the local coordinate normal to the wall surface, and $n_i$ and
$k_1$ denote the distance and turbulent energy at the nearest grid point from the wall; $U$,
$V$, $k$ and $\varepsilon$ were specified from the experimental conditions together with $\partial^2 P/\partial n^2 = 0$
at the inlet; and $\partial U/\partial x = \partial V/\partial x = \partial k/\partial x = \partial \varepsilon/\partial x = 0$ and $P = 0$ at the outlet.
Some explanations would be appropriate for the boundary conditions of the pressure and
dissipation rate on the wall surface. Strictly, the pressure gradient normal to the wall surface is 
\( \frac{\partial P}{\partial n} = \rho \nu (\partial^2 U_n / \partial n^2) \). The right hand side, however, is known to become almost zero at high Reynolds numbers. As described later, we confirmed the validity of the present boundary condition, \( \frac{\partial P}{\partial n} = 0 \), by comparing the computational results with those obtained with another proper scheme. The strict boundary condition for \( \varepsilon \) at the wall, on the other hand, is 
\( \varepsilon_w = \nu (\partial^2 k / \partial n^2) \). Calculations with this type of boundary condition are, however, unstable because the second-order derivative cannot always be guaranteed to provide a positive value. Thus, the following conditions are usually employed as the boundary condition for the dissipation rate on the wall surface:

\[
\varepsilon_w = 2\nu \left( \frac{\partial \sqrt{k}}{\partial n} \right)^2 \quad \text{or} \quad 2\nu \frac{k_1}{n_1^2}.
\]  

(3.16)  

The validity of the later in Eq. (3.16), which is used here, can be shown with the consideration of the near-wall limiting behavior. From Eqs. (3.9) and (3.11), the following relation is obtained:

\[
2\nu \frac{k}{y^2} = 2\nu \frac{1}{y^2} \left( \frac{a_1^2 + c_1^2}{y^2} \right) + \cdots = \nu \left( \frac{a_1^2 + c_1^2}{y^2} \right) = \varepsilon_w \quad \text{for} \quad y \to 0.
\]  

(3.17)  

In this calculation, the grid system is fine enough to reproduce the near-wall limiting behavior, so that the boundary condition with the values \( k_1 \) and \( n_1 \) at the nearest grid point from the wall as \( k \) and \( y \) in Eq. (3.17) is sufficiently valid.

The calculations were conducted corresponding to six individual experimental cases as shown in Table 3.1.

### 3.3.2 Evaluation of computational accuracy

To ascertain the validity of the present calculation, we performed firstly the calculation of the laminar flow with the grid system shown in Fig. 3.4 (a). Results of the velocity field are presented in Fig. 3.5. The reattachment length obtained is \( X_R/H = 6.43 \pm 0.06 \), which is in good agreement with the result of \( X_R/H \simeq 6.3 \) by Kondoh et al. (1993). In the present calculation, the uncertainty in the reattachment length (±0.06) is approximately equal to the width of grid spacing near the reattachment point. Furthermore, the center point of recirculating region is located at \( x/H \simeq 1.85, y/H \simeq 0.64 \), which is consistent with the result of Kondoh et al. (1993).
Secondly, we conducted the turbulent flow calculations corresponding to Case 3-3 with the two types of grid systems shown in Fig. 3.4 to evaluate the grid dependence of computational results. The comparison is shown in Fig. 3.6, from which we may acknowledge the grid-independent solutions in the calculations.

In addition to the above evaluation, we have carried out the flow calculation corresponding to Case 3-3 with the scheme developed by Kuno et al. (1992), in which only the pressure is located in a staggered manner and the relevant pressure boundary condition is properly taken into account. Comparisons of the computational results with two types of schemes are also included in Fig. 3.6, from which one can see that the results are identical with each other and the present calculation procedure based on the MAC method with the $\partial\overline{P}/\partial n = 0$ pressure boundary condition at the wall is valid.

### Table 3.1: Computational conditions for back-step flows

<table>
<thead>
<tr>
<th>Case</th>
<th>3-1</th>
<th>3-2</th>
<th>3-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ER$</td>
<td>1.5</td>
<td>1.67</td>
<td>1.5</td>
</tr>
<tr>
<td>$Re_H$</td>
<td>46000</td>
<td>38000</td>
<td>5500</td>
</tr>
<tr>
<td>$Re_\theta$</td>
<td>1500</td>
<td>1000</td>
<td>500</td>
</tr>
<tr>
<td>$\theta/H$</td>
<td>0.03</td>
<td>0.03</td>
<td>0.09</td>
</tr>
<tr>
<td>$N_x \times N_\eta$</td>
<td>211 × 101</td>
<td>229 × 101</td>
<td>255 × 75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>3-4</th>
<th>3-5</th>
<th>3-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ER$</td>
<td>1.125</td>
<td>1.25</td>
<td>2.0</td>
</tr>
<tr>
<td>$Re_H$</td>
<td>38000</td>
<td>28000</td>
<td>210000</td>
</tr>
<tr>
<td>$Re_\theta$</td>
<td>6000</td>
<td>4500</td>
<td>8000</td>
</tr>
<tr>
<td>$\theta/H$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>$N_x \times N_\eta$</td>
<td>207 × 125</td>
<td>219 × 125</td>
<td>279 × 125</td>
</tr>
</tbody>
</table>
CHAPTER 3. AN IMPROVED $k$-$\varepsilon$ MODEL FOR SEPARATING FLOWS

Table 3.2: Comparison of flow reattachment lengths, $X_R/H$

<table>
<thead>
<tr>
<th>Case</th>
<th>3-1</th>
<th>3-2</th>
<th>3-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction</td>
<td>7.51 ± 0.03</td>
<td>7.94 ± 0.03</td>
<td>6.52 ± 0.04</td>
</tr>
<tr>
<td>Experiment</td>
<td>7 ± 1</td>
<td>8.0</td>
<td>6.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>3-4</th>
<th>3-5</th>
<th>3-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction</td>
<td>6.18 ± 0.06</td>
<td>6.59 ± 0.04</td>
<td>8.57 ± 0.04</td>
</tr>
<tr>
<td>Experiment</td>
<td>6.2</td>
<td>6.7</td>
<td>8.4</td>
</tr>
</tbody>
</table>

These results assure that the present computations are sufficiently reliable.

3.3.3 Results and discussion

Predictions of the flow reattachment lengths $X_R/H$ for six test cases are compared with the experiments in Fig. 3.7 and Table 3.2. The computational results are seen to be in excellent agreement with the experiments. Note that the experimental value of the reattachment length of Kim et al. (1978, 1980) was originally $(7 ± 1)H$, but Avva et al. (1988b) reported a corrected value of 7.6 $H$ which was derived from the pressure distribution on the wall (see Obi & Peric 1991). As mentioned previously, the reattachment lengths have been underpredicted by 15-20% with the standard $k$-$\varepsilon$ model. With the present model, however, we can predict the reattachment length very accurately. The streamlines in Cases 3-1, 3-2 and 3-3 are depicted in Fig. 3.8. In all cases, the secondary recirculation appears near the step corner as is usually observed experimentally. Furthermore, by comparing the streamlines for Cases 3-1 and 3-3 [Figs. 3.8 (a) and 3.8 (c)], we can find the substantial Reynolds-number dependence of the flow reattachment length with the channel expansion ratio fixed. The present model may be the first to predict the Reynolds-number dependence of the flow reattachment length. Figure 3.7 and Table 3.2 suggest that the reattachment length generally increases with increasing expansion ratio except in the low-Reynolds-number case. This trend was also indicated experimentally by Eaton & Johnston (1981). The separating streamline and the line of zero streamwise velocity for Case 3-6 are shown in Fig. 3.9, compared with the experimental data of Durst.
& Schmitt (1985). Both of them show almost perfect agreement with the experiment, which means that the present computational results capture the overall flow patterns in the recirculating region very well.

The pressure coefficients on the upper and lower walls in Cases 3-1 and 3-2 are presented in Fig. 3.10. Again, the computational results are seen to agree well with the experiments.

Figures 3.11 and 3.12 show the comparisons of the detailed flow field with the experiments of Cases 3-2 and 3-3. Case 3-2 by Eaton & Johnston (1980) is regarded as a representative experiment on a backward-facing step flow. And Case 3-3 by Kasagi et al. (1993) is regarded as having very small measurement uncertainty and thus highly reliable. Concerning the streamwise velocity field, the computational results show excellent agreement with the experimental data in both cases. Distributions of the turbulent energy agree reasonably well with the experiments, though there appears a slight difference in the peak locations and in the near-wall values in the recirculating region. As for the Reynolds stress, the computational results of Case 3-3 conform well to the experiment. The results of Case 3-2, on the other hand, agree very well with the experimental data in the redeveloping region downstream of the reattachment point; but in the recirculating region they predict a little higher maximums.

Streamwise variations of the local maximums of streamwise turbulence intensity and Reynolds shear stress are compared with the experiments in Fig. 3.13. The computed variations of both quantities are qualitatively consistent with the experiments, in which the absolute maximums occur at a location slightly upstream of the reattachment point. Quantitatively speaking, the computational results of the turbulence intensity in Case 3-1 agree very well with the experiments, but in the other two cases the present computations give slightly lower values than the experiments, which might be due to the underlying isotropic assumption of the present model. Concerning the Reynolds stress, the computational results show slightly higher values in all cases except Case 3-4, particularly in the region around and upstream of the reattachment point as seen also in Fig. 3.11 (c).

When predicting turbulent heat transfer with the k-ε model, a vague concept of the turbulent Prandtl number is usually introduced to express the eddy diffusivity for heat
via the eddy viscosity. To avoid this questionable assumption, two-equation heat-transfer models have been developed by Nagano & Kim (1988), Nagano et al. (1991) and so on, in which as a natural consequence the eddy viscosity is used as one of the fundamental parameters in determining the eddy diffusivity for heat. Thus, for the prediction of complex turbulent heat transfer, we need a more detailed knowledge of eddy-viscosity behavior in flows with separation and reattachment. Distributions of the eddy viscosity are shown in Fig. 3.14. As shown in Fig. 3.14 (a), the computed results globally agree with the experiments (see the peak value and its location). From Fig. 3.14 (b), we can grasp the growth mechanism of the eddy viscosity. The computations show almost the same tendency in all cases: (a) The local maximum of eddy viscosity increases linearly with the distance from the separation point; (b) the growth becomes saturated at \( x/X_R \approx 0.7 \) due to the interaction of the detached shear layer with the wall surface and to the subsequent blocking effect of the wall; and (c) the eddy viscosity maintains an almost constant value near the reattachment. The eddy viscosity peaks at a location slightly upstream of the flow reattachment point. The experimental analysis of Vogel & Eaton (1985) has elucidated that the maximum heat transfer is located slightly upstream of the reattachment point, which should be noted for being completely consistent with the tendency of the calculated eddy viscosity. The maximum eddy viscosity at the reattachment point normalized by the free stream velocity and the reattachment length, \( \nu_t/U_0 X_R \), has almost the same value of 0.002 in high Reynolds number flows. It is surprising that, in spite of the calculations under a variety of conditions, such a unique trend of the eddy viscosity is found to exist in all cases of \( Re_H \) larger than \( 10^4 \).

Comparisons of the velocity profiles developing along the opposite wall in Case 3-1 are shown in Fig. 3.15. In the Kim's experiment, the wall shear stresses were obtained by the so-called cross plot technique in which the universal log-law was assumed. However, recently, Nagano et al. (1993) have indicated experimentally that, under the adverse pressure gradient, the universal log-law is no longer valid and the friction velocity is higher than that obtained by the cross plot technique. The most recent DNS data on the boundary-layer flow with an adverse pressure gradient (Spalart & Watmuff 1993) show the same tendency of the friction velocity. Figure 3.15 indicates that the present computational results are in complete agreement with these latest findings. (Note that
CHAPTER 3. AN IMPROVED $k-\varepsilon$ MODEL FOR SEPARATING FLOWS

the back-step flow is subjected to the strong adverse pressure gradient shown in Fig. 3.10.)

The flow features in the reverse-flow region are shown in Fig. 3.16. It can be seen from
Fig. 3.16 (a) that the variations of local maximum of reverse-flow velocity exhibit almost
the same trend in accordance with the experimental data. For the relationship between
the Reynolds number and the skin friction coefficient based on the maximum reverse-flow
velocity, the computational results give the definite dependency as $C_f \propto \frac{Re}{2}$, although
slightly higher than in the experiments. In the calculations, the point of maximum reverse-
flow velocity locates closer to the wall than in the experiments, so the calculated friction
coefficient becomes larger.

The mean velocity profiles in the recirculating region are presented in Fig. 3.17, and the
mean ($C_f$) and fluctuating ($C_f'$) skin friction coefficients along the wall surface are shown
in Fig. 3.18. From Fig. 3.17, one finds the computed velocities are much smaller than
the conventional log-law profile, which qualitatively supports the experiment of Adams
& Johnston (1988). For the skin frictions shown in Fig. 3.18, the computational results
are generally higher than the experimental data of Eaton & Johnston (1980). These
discrepancies in Figs. 3.17 and 3.18 may be due to the same reason as found in the
maximum reverse-flow velocity.

As mentioned previously, the dissipation rate $\varepsilon$ on the wall is closely related to the fluc-
tuating friction coefficient $C_f'$. Considering the near-wall limiting behavior, the following
relation can be derived:

$$\varepsilon_w = \nu \left( a_1^2 + c_1^2 \right) \simeq \nu a_1^2 \left( \frac{2k}{u^2} \right) = \nu \frac{a_1^2}{b_{11} + \frac{1}{3}}, \quad (3.18)$$

where $b_{11}$ is the streamwise component of the anisotropy tensor. From this relation and
Eq. (3.10), $C_f'$ is expressed in terms of $\varepsilon$ and $b_{11}$ as follows:

$$C_f' = \frac{\nu \sqrt{\left( b_{11} + \frac{1}{3} \right) \nu \varepsilon_w}}{\frac{1}{2} \rho U_0^2}. \quad (3.19)$$

Now, as the first estimation, assuming $b_{11} \simeq 0.5$ based on the channel flow data ($\sqrt{a_1^2} \simeq 2\sqrt{c_1^2}$), we can calculate $C_f'$ from the computational results. The present crude estimation
shown in Fig. 3.18 is higher than in the experiments of Eaton & Johnston (1980). However,
as indicated by Kasagi et al. (1993), $b_{11}$ can become smaller than $b_{33}$ near the reattachment
point, so that the calculated $C'_f$ will coincide with the experimental level. Additionally, it is worth noting that the variation of $C'_f$ is very similar to that of the Nusselt (or Stanton) number shown by Vogel & Eaton (1985). A concrete discussion on this correlation will be given in Chapter 4, including a detailed investigation on the budgets of turbulent energy and temperature variance.
Figure 3.1: Comparison of channel flow predictions with DNS data \((Re_{c} = 395)\): (a) Mean velocity; (b) Eddy viscosity.
Figure 3.2: Friction coefficient of channel flow for various Reynolds numbers: (a) Normalized by bulk velocity; (b) Normalized by centerline velocity.
Figure 3.3: Friction coefficient of adverse pressure gradient flow.
CHAPTER 3. AN IMPROVED k-ε MODEL FOR SEPARATING FLOWS

Figure 3.4: Grid systems: (a) Normal resolution; (b) Finer resolution.

Figure 3.5: Overview of velocity field in laminar flow ($ER = 1.5$, $Re_H = 150$).
Figure 3.6: Evaluation of grid and scheme dependence on computational results (Case 3-3): --- 509 x 149; O 255 x 75; □ Another scheme (Kuno et al. 1992); (a) Pressure on walls; (b) Streamwise velocity; (c) Turbulent energy.
Figure 3.7: Comparison of predicted flow reattachment lengths with experiments.
Figure 3.8: Streamlines: (a) Case 3-1; (b) Case 3-2; (c) Case 3-3.
Figure 3.9: Comparison of flow pattern in recirculating region: Dividing streamline $\psi = 0$ and trace of the point of zero streamwise velocity $\bar{U} = 0$. 

$N > 1.0$
Figure 3.10: Pressure coefficient on walls: (a) Case 3-1; (b) Case 3-2.
Figure 3.11: Comparison with experiment of Eaton & Johnston (1980) (Case 3-2): ○ Experiment; ——— Prediction; (a) Streamwise velocity; (b) Turbulent energy; (c) Reynolds shear stress.
Figure 3.12: Comparison with experiment of Kasagi et al. (1993) (Case 3-3, Key as Fig. 3.11): (a) Streamwise velocity; (b) Turbulent energy; (c) Reynolds shear stress.
CHAPTER 3. AN IMPROVED $k-\varepsilon$ MODEL FOR SEPARATING FLOWS

Figure 3.13: Streamwise variation of maximum values: (a) Streamwise component of turbulent intensity; (b) Reynolds shear stress.
Figure 3.14: Distributions of eddy viscosity: (a) Comparison with experiment; (b) Streamwise variation of local maximums normalized by free stream velocity and reattachment length.
Figure 3.15: Mean velocity profiles (Case 3-1, opposite wall).
Figure 3.16: Flow features in the recirculating region: ○ Case 3-1; □ Case 3-2; △ Case 3-3; ○ Case 3-4; □ Case 3-5; ○ Case 3-6; ● Experiment (Adams & Johnston 1988); ■ Experiment (Eaton & Johnston 1980); (a) Streamwise variation of maximum reverse-flow velocity; (b) Relationship between wall-layer Reynolds number $Re_N$ and skin friction coefficient.
CHAPTER 3. AN IMPROVED $k$-$\varepsilon$ MODEL FOR SEPARATING FLOWS

Figure 3.17: Mean velocity profiles in recirculating region [Computational results correspond to Case 3-2 and experimental data are compiled by Adams & Johnston (1988)].

Figure 3.18: Mean and fluctuating skin friction coefficients on step wall (Case 3-2).
Chapter 4

AN IMPROVED TWO-EQUATION HEAT-TRANSFER MODEL

4.1 Turbulence Modeling for Thermal Field

4.1.1 Modeling eddy diffusivity for heat

This chapter deals with a two-equation heat-transfer model for predicting turbulent heat transfer in separating and reattaching flows. To develop a new model, the insights obtained from the discussion in Chapter 3 are taken into account. Furthermore, in this chapter, we discuss in detail what type of hybrid time scale is best suited to the characteristic time scale for turbulent heat transfer under complex thermal conditions. Note that the low-Reynolds-number $k$-$\varepsilon$ model proposed in Chapter 3, which reproduces the correct near-wall limiting behavior and is applicable to separating and reattaching flows, is used as the velocity-field model in the calculations.

A thermal field can be described with the governing equations in Section 2.2. On the basis of the isotropic eddy-viscosity approximation, the turbulent heat flux in Eqs. (2.5) - (2.7) is expressed as follows:

\[-u_j \overline{\tau} = \alpha_t \frac{\partial T}{\partial x_j}, \tag{4.1}\]

where $\alpha_t$ is the eddy diffusivity for heat.
In modeling \( \alpha_t \) in Eq. (4.1), we need to adopt an appropriate turbulent length scale characterizing the turbulent heat transport in both the regions away from and close to the wall, as in the modeling of a velocity field (Myong & Kasagi 1990; Nagano & Tagawa 1990). Furthermore, it is extremely important in modeling \( \alpha_t \) to take into account the relation between the velocity- and temperature-field time scales (Nagano & Kim 1988; Nagano et al. 1991; Youssef et al. 1992).

First, we consider how the time scales should be incorporated in modeling \( \alpha_t \). The eddy diffusivity for heat in the two-equation heat-transfer model can be generally expressed as follows:

\[
\alpha_t \propto k \tau_m = \sqrt{k} \left( \sqrt{\tau_m} \right),
\]

where \( \tau_m \) is a composite (hybrid) time scale characterizing turbulent heat transfer which depends on both the velocity-field time scale, \( \tau_u = k/\varepsilon \), and the temperature-field time scale, \( \tau_t = \bar{T}^2/2 \varepsilon_t \). Equation (4.2) indicates that \( \alpha_t \) consists of the turbulent velocity scale \( \sqrt{k} \) and the turbulent length scale \( \sqrt{\tau_m} \). The composite time scale \( \tau_m \) proposed in the previous studies (e.g., Nagano & Kim 1988; Nagano et al. 1991; Youssef et al. 1992; Sommer et al. 1993, 1994) can be described with the following generalized formula:

\[
\tau_m \propto \left( \tau_u \tau_t^m \right) = \tau_u R^m \quad (l + m = 1),
\]

where \( R = \tau_t/\tau_u \) is the time-scale ratio. Note that this type of hybrid time scale is also used in a turbulent heat-flux model (e.g., see Elghobashi & Launder 1983), and there are several choices in the combination of \( l \) and \( m \). Zeman & Lumley (1976), on the other hand, introduced the following composite time scale in modeling a buoyancy-driven mixed layer:

\[
\tau_m \propto \left( \frac{1}{\tau_u} + \frac{C_m}{\tau_t} \right)^{-1} = \tau_u \frac{R}{C_m + R},
\]

where \( C_m \) is a constant. The composite time scale defined by Eq. (4.4) is the harmonic average of the velocity- and temperature-field time scales. This suggests that the shorter time scale among \( \tau_u \) and \( \tau_t \) is more important for turbulent heat transfer. Note that this type of hybrid time scale has been adopted in a recent turbulent heat-flux model (Shikazono & Kasagi 1993). In the present study, we construct two-equation heat-transfer models based on three kinds of time scales given by Eq. (4.3) with different \( m \) and Eq. (4.4).
On the other hand, we should appropriately introduce a model function in $\alpha_t$ to account for wall-proximity effects. Generalizing the formula for $\alpha_t$ in the NTT model, $\alpha_t$ can be expressed as follows, based on Eq. (4.2), except in immediate proximity to the wall surface:

$$\alpha_t \propto k_{tm} \left\{ 1 - \exp \left( -\frac{y^*}{A_\lambda} \right) \right\} \left\{ 1 - \exp \left( -\frac{y^*}{B_\lambda} \right) \right\}. \quad (4.5)$$

Here, we adopt $y^*$ instead of $y^+$ for the model application to separating and reattaching flows. We determine the constants $A_\lambda$ and $B_\lambda$ with reference to the discussion of Cebeci (1973), i.e., $A_\lambda = 14$, which is the same as in Eq. (3.12), and $B_\lambda = 14/\sqrt{Pr}$.

In close proximity to the wall, the conservation of temperature variance [Eq. (2.6)] is maintained by dissipating almost all $\bar{t}_2$ diffused from the region away from the wall, which is the same mechanism as in the velocity field (Nagano & Tagawa 1990; Kasagi et al. 1992). Therefore, it is appropriate to adopt a high-wavenumber-range scale dominating the dissipation process in modeling $\alpha_t$ close to the wall. Considering that the ratio between the temperature- and velocity-field time scales for dissipative motions is represented by $\sqrt{R/Pr}$ (Shikazono & Kasagi 1993), the eddy diffusivity for heat in proximity to the wall can be expressed as follows:

$$\alpha_t \propto \nu_t \sqrt{\frac{R}{Pr}}. \quad (4.6)$$

The eddy diffusivity for heat given by Eq. (4.6) has exactly the same functional dependency of the time-scale ratio $R$ as in the NTT model, so that this representation for $\alpha_t$ can reproduce the near-wall asymptotic relations correctly in both cases with and without the temperature fluctuations on the wall surface (Nagano et al. 1991; Youssef et al. 1992).

Thus, we propose the following expression for the eddy diffusivity for heat using the characteristic time scale defined by Eq. (4.4) (hereinafter referred to as Model A):

$$\alpha_t = C_\lambda \left[ \frac{k^2}{\varepsilon} \left( \frac{2R}{C_m + R} \right) + 3\sqrt{k} \left( \frac{\nu^3}{\varepsilon} \right)^{\frac{1}{2}} \frac{\sqrt{2R}}{Pr} \exp \left\{ - \left( \frac{R_t}{200} \right)^2 \right\} \right]$$

$$\times \left\{ 1 - \exp \left( -\frac{y^*}{14} \right) \right\} \left\{ 1 - \exp \left( -\frac{\sqrt{Pr} y^*}{14} \right) \right\}. \quad (4.7)$$

Here, we put $C_m = 0.5$ so that the resultant turbulent Prandtl number $Pr_t$ ($= \nu_t/\alpha_t$) may be equal to a standard value of 0.9 when values of $C_\lambda$ and $R$ are set to $C_\lambda = 0.1$ as in the NTT model and $R = 0.5$, as is commonly assumed in an equilibrium boundary-layer heat
transfer. We also examine two formulae for \( \alpha_t \) based on Eq. (4.3) for comparison. The first is the representation with \( m = 1 \) (hereinafter referred to as Model B),

\[
\alpha_t = C_\lambda \left[ \frac{k^2}{\varepsilon} (2R) + 3 \sqrt{k} \left( \frac{\nu^3}{\varepsilon} \right)^{\frac{1}{4}} \frac{\sqrt{2R}}{Pr} \exp \left\{ - \left( \frac{R_t}{200} \right)^2 \right\} \right]
\]

\[
\times \left\{ 1 - \exp \left( - \frac{y^*}{14} \right) \right\} \left\{ 1 - \exp \left( - \frac{\sqrt{Pr} y^*}{14} \right) \right\},
\]

(4.8)

and the second is that with \( m = 1/2 \) (hereinafter referred to as Model C):

\[
\alpha_t = C_\lambda \left[ \frac{k^2}{\varepsilon} \sqrt{2R} + 3 \sqrt{k} \left( \frac{\nu^3}{\varepsilon} \right)^{\frac{1}{4}} \frac{\sqrt{2R}}{Pr} \exp \left\{ - \left( \frac{R_t}{200} \right)^2 \right\} \right]
\]

\[
\times \left\{ 1 - \exp \left( - \frac{y^*}{14} \right) \right\} \left\{ 1 - \exp \left( - \frac{\sqrt{Pr} y^*}{14} \right) \right\}. \quad (4.9)
\]

The prime difference among these three models A - C resides in the representation of the characteristic time scale for turbulent heat transfer in the region away from the wall. Note, however, that Models A - C give almost the same thermal field predictions for an equilibrium flow because the time-scale ratio \( R \) becomes about 0.5 there. Besides, in proximity to the wall surface, Models A - C show exactly the same near-wall limiting behavior, i.e., \( \alpha_t \propto \nu_t \sqrt{R/Pr} \). Thus, all of them can correctly reproduce the near-wall asymptotic relations, regardless of the existence of wall-temperature fluctuations (see Youssef et al. 1992).

### 4.1.2 Model constants and model functions

The model constants and model functions used in Eqs. (2.6) and (2.7), and Eqs. (4.7) – (4.9) are determined in the following way.

First, \( C_\lambda, C_{D_1} \) and \( C_{D_2} \) are set to 0.1, 2.0 and 0.9, respectively, following the lead of the NTT model (Nagano et al. 1991; Youssef et al. 1992). Concerning the turbulent diffusion coefficients \( \sigma_h \) and \( \sigma_\phi \), we assume \( \sigma_h \simeq \sigma_k \) and \( \sigma_\phi \simeq \sigma_\varepsilon \), and set both of them to the same value of 1.6. The constants \( C_{P_1} \) and \( C_{P_2} \) are to be determined with the relation for the “constant-stress and constant-heat-flux layer” (Nagano & Kim 1988; Nagano et al. 1991; Youssef et al. 1992):

\[
C_{D_1} - C_{P_1} = 2R \left\{ C_{P_2} - C_{D_2} + \left( \kappa^2 / Pr_t \right) / \left( \sigma_\phi \sqrt{C_\mu} \right) \right\}. \quad (4.10)
\]
Thus, substituting the standard values of \( C_n = 0.09, R \simeq 0.5, Pr_t \simeq 0.9 \) and \( \kappa \simeq 0.41 \) into 
Eq. (4.10), we obtain the optimum values of \( C_{P1} \) and \( C_{P2} \) after examining the calculated 
results [see Eqs. (4.14) – (4.16)]. The model functions \( f_{D1} \) and \( f_{D2} \) can be expressed 
in the same form as in the NTT model (Nagano et al. 1991) by taking account of the 
wall-limiting behavior:

\[
\begin{align*}
    f_{D1} &= \left\{ 1 - \exp \left( -\frac{y^*}{A_{D1}} \right) \right\}^2, \\
    f_{D2} &= \left( \frac{1}{C_{D2}} \right) (C_{e2} f_2 - 1) \left\{ 1 - \exp \left( -\frac{y^*}{A_{D2}} \right) \right\}^2,
\end{align*}
\]

with \( f_2 = 1 - 0.3 \exp\left\{ -(R_t/6.5)^2 \right\} \), the consequence of Eq. (3.13). Here, \( A_{D1} \) and \( A_{D2} \) are 
determined after numerical optimization [see Eqs. (4.13) – (4.16)]. We put \( f_{P2} \) at unity 
so as to be consistent with the velocity-field model proposed in Chapter 3, as discussed in 
the NTT model (Nagano et al. 1991). In the NTT model, \( f_{P1} \) is also assumed to be unity. 
If we set \( f_{P1} \) to unity, however, we may encounter computational instability in some kinds 
of heat-transfer fields where the similarity between the velocity and temperature fields 
does not exist, as in the experiment of Antonia et al. (1977). On the other hand, the 
NK model (Nagano & Kim 1988) does not suffer from any instability, though it uses the 
model functions of \( f_{P1} = f_{D1} = 1.0 \). From these facts and some computational attempts, 
we have concluded that numerical instability can be avoided by setting \( f_{P1} \simeq f_{D1} \). Thus, 
we set \( f_{P1} = f_{D1} \) in the present model. Note that the present heat-transfer models with 
this modification for \( f_{P1} \) give almost the same results as with the NTT model (Nagano 
et al. 1991) for the fundamental heat transfer problems, as will be shown later.

In sum, the model constants and model functions which are commonly used in Models 
A – C are as follows:

\[
\begin{align*}
    C_\lambda &= 0.1, & C_{D1} &= 2.0, & C_{D2} &= 0.9, & \sigma_h &= 1.6, & \sigma_\phi &= 1.6, \\
    f_{P1} &= f_{D1} = \left\{ 1 - \exp \left( -y^* \right) \right\}^2 \ (i.e., A_{D1} = 1.0), & f_{P2} &= 1.0.
\end{align*}
\]

The model constants optimized for each model are as follows:

\[
\begin{align*}
    C_m &= 0.5, & C_{P1} &= 1.90, & C_{P2} &= 0.60, & A_{D2} &= 5.7 \quad \text{for Model A}, \ (4.14) \\
    C_{P1} &= 1.85, & C_{P2} &= 0.65, & A_{D2} &= 5.5 \quad \text{for Model B}, \ (4.15) \\
    C_{P1} &= 1.95, & C_{P2} &= 0.55, & A_{D2} &= 5.8 \quad \text{for Model C}. \ (4.16)
\end{align*}
\]
Some explanations should be made regarding the dependence of the Prandtl number. The primary subject of the present study is to propose a new heat-transfer model applicable to heat transfer problems in separating and reattaching flows. With this in mind, we have only referred to some established discussions on the Prandtl-number dependence (Cebeci 1973; Shikazono & Kasagi 1993) in constructing the present heat-transfer models, and restricted the model application only to the heat transfer problems in air \((Pr = 0.71)\). To treat the heat transfer at \(Pr \gg 1\) or \(Pr \ll 1\), the dependence of \(Pr\) should be considered in modeling not only the eddy diffusivity for heat but also some model functions. The problem needs more detailed discussions in the future work.

4.2 Model Assessment in Attached Flow Heat Transfer

To confirm the basic accuracy of the present heat-transfer models (Models A, B and C), we applied them to two representative turbulent heat transfer problems in attached flows. The first test case is a boundary-layer heat transfer investigated experimentally by Gibson et al. (1982), and the second one corresponds with the experiment of Antonia et al. (1977). In the first problem (Gibson et al. 1982), the wall surface is heated at a constant wall temperature from the beginning of a boundary-layer development. In the second problem (Antonia et al. 1977), the wall is kept adiabatic in the initial development of a boundary layer, and then suddenly heated with a constant heat flux from a location where the boundary layer has developed to some extent. It should be mentioned that the latter case (Antonia et al. 1977) has been very difficult to simulate accurately with the constant turbulent-Prandtl-number assumption because the analogy between the velocity and temperature fields no longer holds.

The computational procedure used in the present study was the same as that of Hattori et al. (1993), which was based on a finite-volume method developed by Patankar (1980) and Leschziner (1982). The number of grid points across the boundary layer was 201, where the grid points were concentrated in the neighborhood of the wall surface to resolve the viscous sublayer sufficiently and to obtain a grid-independent solution (Hattori et al. 1993). For the technique to specify the boundary conditions, we followed Youssef et
al. (1992). The temperature variation on the wall surface was set to zero to meet the measurement conditions.

The calculated mean-velocity and mean-temperature profiles corresponding to the experiment of Gibson et al. (1982) are shown in Fig. 4.1. From Fig. 4.1 (a), it can be seen that the velocity-field model presented in Chapter 3 predicts the mean-velocity profile as accurately as the NT model (Nagano & Tagawa 1990). We can also acknowledge from Fig. 4.1 (b) that all three models A–C give almost the same results in good agreement with the experimental data of Gibson et al. (1982) as the NTT model (Nagano et al. 1991; Youssef et al. 1992).

The predicted streamwise development of mean-temperature for the second test case (Antonia et al. 1977) is shown in Fig. 4.2, and the relevant development of the turbulent heat flux is presented in Fig. 4.3. The location $x$ in Figs. 4.2 and 4.3 indicates the streamwise distance from the beginning point of heating. From these two figures, it can be seen that all three models A–C show good agreement with the experimental data, whereas the conventional prediction with the constant turbulent Prandtl number, i.e., $Pr_t = 0.9$, shows a substantial underprediction for the mean-temperature variation.

The foregoing comparisons prove that the present models can quite accurately predict both the velocity and temperature fields in attached turbulent flows even with a sudden change of the wall thermal condition.

4.3 Application to Backward-Facing Step Flows

4.3.1 Model assessment with DNS data on flow field

As discussed in Chapter 3, the accurate prediction of heat transfer in separating flows is impossible without the reliable predictions of flow field in the recirculating region. We have shown that the velocity-field model described in Chapter 3 predicts backward-facing step flows quite successfully for various flow conditions.

Recently, Le et al. (1993) performed a DNS of a backward-facing step flow. DNS data are very useful for modeling a flow with separation. Especially, the turbulent-energy budget close to the wall is the most important knowledge on turbulent heat transfer, since
the structure of turbulent heat transfer is almost entirely determined there. Therefore, in order to assess the accuracy of the present model for flow field in further detail, we performed the calculation corresponding to the DNS of Le et al. (1993) and compared the computed results with the DNS data. The computational technique was the same as that in Chapter 3, except that we adopted here the most recent scheme of Kuno et al. (1992) for the pressure calculation, which was used to examine the scheme dependence of the computational results in Chapter 3. The number of grid points used in this calculation was $N_x \times N_y = 389 \times 145$, so the grid spacing was sufficiently fine to obtain a grid-independent solution. The essential validity of the numerical procedure was already discussed in Chapter 3. The Reynolds number was at $Re_H = 5100$, and the channel expansion ratio $ER$ was 1.2. The velocity-field profile of a boundary-layer flow at $Re_{\theta} = 700$ was adopted as the inlet boundary condition, corresponding to the DNS.

The present prediction of flow reattachment length becomes: $X_R/H = 5.92 \pm 0.02$, which agrees well with the DNS value of $X_R/H = 6.0$ (Le et al. 1993). The turbulent-energy budget at $x/H = 4$ in the recirculating region is shown in Fig. 4.4. From Fig. 4.4 (a), one can see that the present results show good agreement with the DNS over the whole region. The production and dissipation terms, which are the leading terms in the budget, are predicted quite accurately, though the convection and turbulent diffusion terms peak around $y/H \approx 1$, being slightly closer to the wall than the DNS data. In the proximity of the wall, the computational results are in good agreement with the DNS data as shown in Fig. 4.4 (b), except that the turbulent diffusion shows an underprediction for the DNS data in the limited region very close to the wall. It should be mentioned, however, that the present result gives the correct near-wall tendency of dissipation rate that a local maximum occurs at the wall. The profile of $\varepsilon$ near the wall indicates that the simplified wall-boundary condition, $\partial \varepsilon / \partial n = 0$, has no validity, although it has been occasionally adopted as a temporary expedient.

As mentioned in Chapter 3, the velocity-field model used in this calculation generally gives somewhat overpredictions of the skin friction coefficient. However, one can see from the above discussions that the turbulent energy budget is successfully predicted. This means that the turbulent length scales in the near-wall region, which are closely related to the turbulent heat transfer, are reasonably calculated. Thus, it can be concluded that
the prediction by the velocity-field model proposed in Chapter 3 is sufficiently reliable in calculating turbulent heat transfer even in the recirculating region of a backward-facing step flow.

4.3.2 Numerical procedure and boundary conditions for thermal field

In calculating the heat transfer in the backward-facing step flow, the finite-difference method was used to discretize the governing Eqs. (2.5) - (2.7). We adopted the third-order upwind difference for the convection term in Eq. (2.5), the first-order upwind difference for the convection terms in Eqs. (2.6) and (2.7), and the second-order central difference for the other terms. Time integration for the thermal field was performed by the Euler-Implicit method. The computational technique and turbulence model for the velocity field were the same as in Section 4.3.1. Figure 4.5 shows the computational grid system. The generalized coordinate system was employed, where only the pressure was located in a staggered position (Kuno et al. 1992). The computational domain was: \(-3.8H \leq x \leq 50H\). And the number of grid points was: \(N_x \times N_y = 287 \times 125\), which was enough to provide grid-independent solutions for the velocity (see Chapter 3). Also, these grid points suffice for temperature-field calculations because the thickness of the conductive sublayer in an air flow \((Pr = 0.71)\) is generally expected to be almost the same as that of the viscous sublayer.

The calculations were conducted corresponding to the experiment of Vogel & Eaton (1985). The channel expansion ratio was \(ER = 1.25\) and the working fluid was air. We computed the thermal field for five variations of the flow conditions, i.e., two cases at different Reynolds numbers \(Re_H\), and three cases with different upstream boundary-layer thicknesses \(\delta_0/H\), as summarized in Table 4.1. The boundary conditions for the thermal field were: \(q_w = \text{constant on the step-side wall at } x > 0 \text{ or 0 on the other wall surfaces}, \quad \bar{t} = 0 \quad \text{and} \quad \varepsilon_{tw} = \alpha(\partial \sqrt{\bar{t}}/\partial n)^2 \text{ at all wall surfaces}; \quad \bar{T} \text{ was uniform at the inlet, and } \bar{t} \text{ and } \varepsilon_{tw} \text{ were set at a sufficiently small value of an order } 10^{-12} \text{ corresponding to experimental perturbations at the inlet; and } \partial^2 T/\partial x^2 = \partial^2 T/\partial x^2 = \partial^2 \varepsilon/\partial x^2 = 0 \text{ at the outlet. It should be noted that the present boundary condition for } \varepsilon_t \text{ at the wall surface is identical
CHAPTER 4. AN IMPROVED TWO-EQUATION HEAT-TRANSFER MODEL

Table 4.1: Computational conditions for back-step flows

<table>
<thead>
<tr>
<th>Case</th>
<th>4-1</th>
<th>4-2</th>
<th>4-3</th>
<th>4-4</th>
<th>4-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_H$</td>
<td>28000</td>
<td>28000</td>
<td>28000</td>
<td>13000</td>
<td>13000</td>
</tr>
<tr>
<td>$\delta_0/H$</td>
<td>1.1</td>
<td>0.7</td>
<td>0.15</td>
<td>1.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>

with the strict one (see Youssef et al. 1992).

4.3.3 Comparison with experimental data

The computational results for the velocity field for Case 4-1 in Table 4.1 are shown in Fig. 4.6. This flow condition is essentially the same as in Case 3-5 in Chapter 3, though the computational scheme and the number of grid points are slightly different. The predicted flow reattachment length is $X_R/H = 6.69 \pm 0.04$, which is almost identical to that obtained in Chapter 3 (see Table 3.2) and is in excellent agreement with the experimental result of Vogel & Eaton (1985), i.e., $X_R/H \approx 6.7$. Overall distributions of the mean velocity and skin friction coefficient are successfully simulated as shown in Figs. 4.6 (b) and 4.6 (c).

The computational results for the thermal field in Case 4-1 are shown in Fig. 4.7. From Fig. 4.7 (a), we can recognize that the predictions of the Stanton number with all three models A – C are in good agreement with the experimental data of Vogel & Eaton (1985), whereas the conventional calculation with $Pr_t = 0.9$ gives substantial overpredictions by as high as 30%. The maximum Stanton numbers predicted by the models A – C are located in the region slightly upstream of the flow reattachment point, which agrees completely with the experimental finding by Vogel & Eaton (1985). Investigating the computational results of three models A – C in detail, we see that Model A shows the best agreement with the experiment, and the position of the maximum Stanton number predicted with Model B is located slightly more upstream than with the other two models.

The predicted mean-temperature profiles, on the other hand, exhibit different tendencies among the models as shown in Fig. 4.7 (b). The variation of mean temperature around a location equal to the step height ($y/H \approx 1$) is very important to examine the
validity of the characteristic time scale used for modeling turbulent heat transfer. From Fig. 4.7 (b), we notice that Model B gives a tendency different from the experimental data of Vogel & Eaton (1985); the others show reasonable variations with a marked kink around the step-height location \( y/H \approx 1 \), and the mean temperature profiles are in good overall agreement with the experiment. The maximum Stanton numbers obtained from the computational results for Cases 4-1, 4-2, 4-4 and 4-5 are summarized in Fig. 4.8, from which we can readily see that the Reynolds-number dependence of the maximum Stanton number differs with models. When the upstream boundary-layer thickness \( \delta_0 \) is fixed, Model B shows the greatest Reynolds-number dependence of the maximum Stanton number. Model A indicates a Reynolds-number dependence greater than do Model C and the constant-\( Pr_t \) calculation. The Reynolds-number dependence predicted by Model C and the constant-\( Pr_t \) calculation are found to be similar to that in a flat-plate flow, which conflicts with the experimental evidence (Vogel & Eaton 1985; Launder 1988). From Fig. 4.8, one can eventually see that Model A gives the best agreement with the experimental data on maximum Stanton numbers.

As a result of these discussions, we have concluded that Model A is the most appropriate model to calculate accurately the turbulent heat transfer in complex flows with separation and reattachment. In what follows, we discuss in more detail the turbulent heat transfer in backward-facing step flows through the computational results obtained from Model A.

### 4.3.4 Confusions in previous predictions

First of all, we compare our computational results with those by previous representative \( k-\varepsilon \) models to confirm the accuracy of our model performance and to investigate the relation between the velocity and temperature fields for the turbulent heat transfer in a separating and reattaching flow. The LS model (Launder & Sharma 1974; Chieng & Launder 1980) and that with the Yap-correction (hereinafter referred to as the LS+Yap model, Launder 1988) are selected for comparison because the former was the first low-Reynolds-number \( k-\varepsilon \) model applied to the calculations of turbulent heat transfer with flow separation and reattachment, while the latter has been most commonly adopted in thermal-field calculations in the near-wall region of separating and reattaching flows.
CHAPTER 4. AN IMPROVED TWO-EQUATION HEAT-TRANSFER MODEL

Table 4.2: Model functions and model constants

<table>
<thead>
<tr>
<th>Model</th>
<th>$f_\mu$</th>
<th>$f_1$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>$\exp\left{-3.4/(1 + R_t/50)^2\right}$</td>
<td>1</td>
<td>$1 - 0.3\exp(-R_t^2)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$C_\mu$</th>
<th>$\sigma_k$</th>
<th>$\sigma_\varepsilon$</th>
<th>$C_{\varepsilon 1}$</th>
<th>$C_{\varepsilon 2}$</th>
<th>$D$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>0.09</td>
<td>1.0</td>
<td>1.3</td>
<td>1.44</td>
<td>1.92</td>
<td>$2\nu\left(\sqrt{k}\right)^2$</td>
<td>$2\nu\nu_t\left(U_{1,j}\right)^2$</td>
</tr>
</tbody>
</table>

(Launder 1988). The model functions and constants in the LS model are summarized in Table 4.2, where the transport equations of $k$ and $\varepsilon$ are expressed as follows:

\[
\frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left\{ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right\} - \overline{u_iu_j} \frac{\partial \overline{U_i}}{\partial x_j} - (\varepsilon + D), \tag{4.17}
\]

\[
\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j} \left\{ \left( \nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right\} - C_{\varepsilon 1}f_1 \frac{\varepsilon}{k} \overline{u_iu_j} \frac{\partial \overline{U_i}}{\partial x_j} - C_{\varepsilon 2}f_2 \varepsilon^2 + E. \tag{4.18}
\]

In Eqs. (4.17) and (4.18), the Reynolds stress $\overline{u_iu_j}$ is given by Eq. (3.1). Note that, in the LS+Yap model, the following term (what is called the 'Yap-correction') is added to the extra term $E$ in Table 4.2 (Launder 1988):

\[
0.83 \left( \frac{k^{3/2}}{2.5\varepsilon y} \right) \left( \frac{k^{3/2}}{2.5\varepsilon y} \right)^2 \frac{\varepsilon^2}{k}. \tag{4.19}
\]

Comparisons of the calculated Stanton numbers for Case 4-1 are shown in Fig. 4.9. Chieng & Launder (1980) and Launder (1988) demonstrated that the heat-transfer coefficients predicted with the LS model were about five times too high in the vicinity of the maximum heat-transfer point, as compared with the experimental data. This was due to the relatively small thickness of viscous (conductive) sublayer and to the calculated too much larger turbulent length scales. Thus, it has been long believed that the low-Reynolds-number form of $k-\varepsilon$ model is not suited to the heat transfer calculations in separating and reattaching flows. Figure 4.9 reconfirms that the LS model gives the same trend as in the previous studies (Chieng & Launder 1980; Launder 1988), in which the Stanton numbers are surprisingly overpredicted. The Yap-correction (Launder 1988)
given by Eq. (4.19) is regarded as a powerful measure to reduce the near-wall turbulent length scale in a separating flow, in particular near the flow reattachment point around which the maximum heat transfer occurs. From Fig. 4.9, we recognize that while the Yap-correction offers remarkable improvements over the original LS model, it still suffers from overpredictions. On the other hand, the prediction with Model A definitely gives the best agreement with the experimental data.

Comparisons of the turbulent length scales for heat transfer at the flow reattachment point, \( L_t = \alpha_t/\sqrt{k} \), obtained from the computational results are shown in Fig. 4.10. Note that the difference in length scale between Model A and the constant-\( Pr_t \) calculation in Fig. 4.10 is due to the difference of the characteristic time scale, since they use completely the same velocity-field data. From Fig. 4.10, we can see that the proposed model gives the proper turbulent length scale near the wall and the order of magnitude of calculated Stanton numbers in Fig. 4.9 is proportional to that of the length scales in the near-wall region of \( 0.02 \leq y/H \leq 0.04 \), where the eddy diffusivity for heat dominates over the molecular diffusivity.

As discussed before, in order to extend the applicability of a heat-transfer model to complex flows with separation and reattachment, the accurate prediction of the turbulent velocity field in the near-wall region is indispensable. Thus, to examine the accuracy of turbulence models in the near-wall region of separating flows, calculations with the LS and LS+Yap models, corresponding to the DNS of Le et al. (1993), were performed. Comparisons of the turbulent-energy budget with the DNS data are shown in Fig. 4.11. Two points should be mentioned with regard to this figure. First, in the near-wall region \( y/H \leq 0.05 \), most of the predicted terms with the LS model show an unacceptable overestimation in comparison with the DNS data as shown in Fig. 4.11 (a). In particular, the model predictions of turbulent diffusion, viscous diffusion and total dissipation rate are more than twice as large as the DNS data. Note that, in these models, the total dissipation rate is expressed as follows:

\[
\varepsilon_{\text{total}} = \varepsilon + D = \varepsilon + 2\nu \left( \frac{\partial \sqrt{k}}{\partial y} \right)^2.
\]  

Figure 4.11 (a) also shows that the spiky behavior of the production term appears in the immediate vicinity of the wall but not in the DNS data. These enormous overpredic-
tions and unexpected variations are thought to be the main reason why the original LS model overpredicts the heat-transfer coefficient in the recirculating region. Secondly, it is surprising that the LS+Yap model does not accurately reproduce the turbulent-energy budget of the DNS data as shown in Fig. 4.11 (b). The predicted distribution of the total dissipation rate is worse than by the original one in the region $y/H \geq 0.05$. Moreover, in close proximity to the wall, the viscous diffusion and total dissipation rate terms have the respective local peak values around $y/H \approx 0.02$. This conflicts with the DNS data, according to which each local peak appears just at the wall. Again, it should be emphasized that the dissipation rate predicted by the present velocity-field model shows the correct behavior in the proximity of the wall and agrees very well with the DNS data in the region $y/H \geq 0.05$, without any additional amendment term (see Fig. 4.4).

4.3.5 Thermal field in backward-facing step flow

Comparisons of the budgets of both turbulent energy $k$ and temperature variance $\overline{\theta^2}$ close to the wall at the reattachment point in Case 4-1 are shown in Fig. 4.12. As shown in Fig. 4.12 (a), the production term of the turbulent-energy equation is negligibly small near the reattachment point because the mean shear of the velocity $\partial U/\partial y$ almost vanishes there. As a result, in this region, the turbulent diffusion term dominates the gain side of the budget. Such a turbulent-energy balance is never seen in an ordinary attached shear flow. On the other hand, the budget of temperature variance essentially remains similar to that in a flat-plate or channel flow (Kasagi et al. 1992) because the temperature field with heat input from the wall has the substantial mean temperature gradient $\partial \overline{T}/\partial y$ even in the recirculating-flow region. Therefore, in the thermal field, the main term on the gain side is the production term. As a consequence, no similarity exists between the velocity and temperature fields in this type of heat-transfer situation.

Here, a question arises: Can one accurately predict the turbulent heat transfer in separating flows with the constant-$Pr_t$ model, if accurate velocity-field data are given? As shown previously, the accurate prediction for a suddenly-heated flow is impossible with the constant-$Pr_t$ model. In that case, there also exists dissimilarity between the velocity and temperature fields. On the other hand, reasonable predictions can be achieved with two-equation heat-transfer models as shown in Figs. 4.2 and 4.3. Figure 4.13 shows
the turbulent-Prandtl-number distributions in the suddenly-heated flow corresponding to the experiment of Antonia et al. (1977) and in the backward-facing step flow for Case 4-1, both of which are computed using Model A. The calculated turbulent Prandtl numbers in both cases are much higher than the conventional value of $Pr_t = 0.9$ and, of course, not constant. The present results for the former case agree with the experimental fact indicated by Sato et al. (1992). From Fig. 4.13 (b), the Yap-correction mentioned before seems to be too effective in reducing the turbulent length scale by substantially overestimating the total dissipation rate, though it leads to a reasonable heat-transfer coefficient with $Pr_t = 0.9$. The internal inconsistency, however, cannot be avoided, and the significant departure from the DNS data on the energy balance shown in Fig. 4.11 (b) is a salient example.

The mean temperature profiles normalized by the wall parameters for Case 4-1 are shown in Fig. 4.14, and the distributions of temperature variance and turbulent heat flux in Fig. 4.15. Corresponding profiles for the "Boundary Layer," obtained with Model A for the experiment of Gibson et al. (1982), are shown in these figures for comparison. From Fig. 4.14, it can be seen that the computed temperature profiles lie considerably lower than in the boundary layer, in agreement with the experimental data\(^1\) (Vogel & Eaton 1985). This tendency of the temperature profile appears to be essentially similar to that of the velocity field as discussed in Chapter 3. Figure 4.15 indicates the characteristics of turbulent heat transfer in separating flows. Obviously, the near-wall profiles of scalar turbulence are completely different from the familiar shapes in a boundary layer. From these results, we may conclude that the conventional log-law is inapplicable not only to the velocity but also to the temperature in a separating and reattaching flow.

\(^1\)The friction temperature is expressed as $t_r = (q_w / \rho c_p u_r) = (q_w / \rho c_p \bar{U}_0) / (u_r / \bar{U}_0)$. In calculating $t_r$ from the experimental data, a value of $q_w / \rho c_p \bar{U}_0$ is obtained from the wall temperature (Vogel & Eaton 1985, Fig. 9) and the Stanton number (Vogel & Eaton 1985, Fig. 12). On the other hand, a value of $u_r / \bar{U}_0$ is obtained from the distribution of mean skin friction coefficient (Vogel & Eaton 1985, Fig. 10).
4.3.6 Influence of upstream boundary-layer thickness on heat transfer

The effect of the upstream boundary-layer thickness on the Stanton number distribution is shown in Fig. 4.16, from which we can ascertain that the present model reproduces the characteristics of the Stanton number, according to which a peak value increases as the upstream boundary-layer thicknesses decrease. It should be emphasized that this quantitative agreement has never been achieved with any previous turbulence model to date. The streamwise variations of local maximums of eddy viscosity $\nu_t$, and cross-streamwise variations of eddy viscosity $\nu_t$ and turbulent energy $k$ at the reattachment point for Cases 4-1, 4-2 and 4-3, are shown in Fig. 4.17. Streamwise variations of the local $\nu_t$ maximums show a common tendency: the local maximum increases linearly with the distance from the separation point and peaks at a location slightly upstream of the reattachment point. The maximum values range from 0.002 to 0.0025 as shown in Fig. 4.17 (a), consistent with the results in Chapter 3. Moreover, the present investigation has elucidated that a slight difference in the growth rate of eddy viscosity greatly influences the heat-transfer rate. As pointed out by Vogel & Eaton (1985) and Adams & Eaton (1988), the shear-layer turbulence increases with a decrease of the upstream boundary-layer thickness. Accordingly, the eddy viscosity in the shear-layer can be expected to increase with the decrease in upstream boundary-layer thicknesses, as seen in Fig. 4.17 (a). Further, we notice that the effect of the initial boundary-layer thickness on the turbulence quantities in the shear-layer region is maintained well into the region very close to the wall surface, as shown in Figs. 4.17 (b) and 4.17 (c). Thus, the streamwise variations of turbulence quantities (e.g., $k$ and $\nu_t$) in the near-wall region are greatly affected by the corresponding variations in the separated shear layer.

On the other hand, as suggested by Vogel & Eaton (1985), the streamwise variation of the fluctuating skin friction coefficient $C'_f$ is very similar to that of the heat-transfer coefficient. As discussed in Chapter 3, $C'_f$ defined by Eq. (3.19) is closely related to the dissipation rate on the wall $\varepsilon_w$. Needless to say, the dissipation rate in the vicinity of the wall is closely related to the turbulent energy there, so the streamwise variation of $C'_f$ is expected to follow the turbulent energy (or eddy viscosity) variation shown in Fig. 4.17.
The fluctuating skin friction and heat-transfer coefficients are shown in Fig. 4.18, where \( C'_f \) is obtained from Eq. (3.19) with the assumption that \( b_{11} \simeq 0.5 \) (see Chapter 3). The predicted distributions of the fluctuating skin friction coefficient \( C'_f \) show a reasonable coincidence with the experimental data of Vogel & Eaton (1985) as shown in Fig. 4.18 (a), though the calculations follow curves a little higher than do the experiments. However, as mentioned in Chapter 3, Kasagi et al. (1993) indicated experimentally that \( b_{11} \) can become smaller than \( b_{33} \) near the reattachment point, so the actual level of the calculated \( C'_f \) can thus be lower than that in Fig. 4.18 (a), leading to better agreement with the experimental data. It is also seen from Fig. 4.18 that the present computations support the experimental conclusion by Vogel & Eaton (1985) that the variation of the fluctuating skin friction coefficient \( C'_f \) is very similar to that of the heat-transfer coefficient [Fig. 4.18 (b)].

From these discussions, it can be concluded that the heat-transfer coefficient strongly depends on the near-wall turbulence intensity, which is essentially dominated by the variation of turbulent energy (or eddy viscosity) in the separated shear layer near the reattachment point. This causal sequence, one must conclude, is the main reason why the maximum heat-transfer coefficient increases with the decrease in upstream boundary-layer thicknesses.
Figure 4.1: Comparison with experiment in boundary-layer flow (Gibson et al. 1982, \( \overline{U_e}/\nu = 1.41 \times 10^6 \text{ m}^{-1} \)): (a) Mean velocity; (b) Mean temperature.
Figure 4.2: Streamwise development of mean temperature.
Figure 4.3: Streamwise development of turbulent heat flux.
Figure 4.4: Budget of turbulent energy at $x/H = 4$ (normalized by $U_0^3/H$): Convection $\bigcirc$ Model, $\cdots\cdots$ DNS; Turbulent diffusion $\square$ Model, $\cdots\cdots$ DNS; Viscous diffusion $\diamondsuit$ Model, $\cdots\cdots\cdots$ DNS; Production $\triangle$ Model, $\cdots\cdots\cdots$ DNS; Dissipation $\nabla$ Model, $\cdots\cdots\cdots$ DNS; (a) Overall view; (b) Close to wall on step side.
Figure 4.5: Grid system (Partial view).
Figure 4.6: Computational results for velocity field (Case 1-1): (a) Streamlines; (b) Mean streamwise velocity; (c) Skin friction coefficient.
Figure 4.7: Comparison with experiment of Vogel & Eaton (1985) (Case 4-1): (a) Stanton number on step side wall; (b) Mean temperature [Key as (a)].
Figure 4.8: Variation of maximum Stanton number: (a) $\delta_0/H = 0.7$; (b) $\delta_0/H = 1.1$. 
Figure 4.9: Comparison of model results for Stanton number (Case 4-1).

Figure 4.10: Comparison of length scale for turbulent heat transfer (Case 4-1).
Figure 4.11: Budget of turbulent energy close to wall at $x/H = 4$ (Key as Fig. 4.4): (a) Original LS model; (b) LS+Yap model.
Figure 4.12: Comparison of budget close to wall at reattachment point (Case 4-1, Model A): (a) turbulent energy (normalized by $\bar{U}_0^3/H$); (b) temperature variance [normalized by $\bar{T}_r^2 \bar{U}_0/H$, Key as (a)].
Figure 4.13: Variation of turbulent Prandtl number (Model A): (a) Suddenly heated boundary layer; (b) Backward-facing step flow (Case 4-1).
Figure 4.14: Mean temperature profiles normalized by wall parameters (Case 4-1, Model A): (a) Recirculating region; (b) Redeveloping region.
Figure 4.15: Distributions of turbulent heat transfer characteristics (Case 4-1, Model A):
(a) Temperature variance; (b) Turbulent heat flux.
Figure 4.16: Effect of upstream boundary-layer thickness on Stanton number (Model A).
Figure 4.17: Effect of upstream boundary-layer thickness on turbulence characteristics ($Re_H = 28000$): (a) Streamwise variation of local maximum values of eddy viscosity; (b) Cross-streamwise variation of eddy viscosity; (c) Cross-streamwise variation of turbulent energy [Key as (b)].
Figure 4.18: Comparison of streamwise variation of fluctuating skin friction coefficient and Stanton number on step-side wall ($Re_H = 28000$, Model A): (a) Fluctuating skin friction coefficient; (b) Stanton number on step-side wall.
Chapter 5

A NEW TYPE OF NONLINEAR $k$-$\epsilon$ MODEL

5.1 Concept of Turbulence Modeling

5.1.1 Modeling Reynolds stress

In this chapter, a new type of $k$-$\epsilon$ model which incorporates some essential characteristics of Reynolds stress and algebraic stress models (ASM) is proposed.

As mentioned in Chapter 1, Pope (1975) showed the explicit expression of the Reynolds stress, $\overline{u_i u_j}$, for a two-dimensional ASM. Gatski & Speziale (1993) recently extended Pope’s idea to a three-dimensional formula, and discussed the relation between the explicit ASM (EASM) expression and the nonlinear $k$-$\epsilon$ formulation proposed by Speziale (1987). The knowledge obtained from this discussion is worth noting for the development of the coming generation of two-equation turbulence models.

In this study, we construct a new type of expression of the Reynolds stress $\overline{u_i u_j}$, whose essential feature lies in the introduction of the EASM concept (Pope 1975; Taulbee 1992; Gatski & Speziale 1993) to the nonlinear $k$-$\epsilon$ formulation proposed by Speziale (1987).

A turbulent velocity field is described with the governing equations in Section 2.1. The nonlinear $k$-$\epsilon$ model by Speziale (1987) is expressed as follows:
\[
\overline{u_iu_j} = \frac{2}{3} k \delta_{ij} - 2 \nu \epsilon_{ij} - 4 C_D \frac{\nu_k^2}{k} \left( S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj} \right) \\
+ 4 C_D \frac{\nu_k^2}{k} \left( S_{ik} S_{kj} - \frac{1}{3} S_{mn} S_{mn} \delta_{ij} \right). \tag{5.1}
\]

In Eq. (5.1), \( S_{ij} \) and \( \Omega_{ij} \) are the strain-rate and the vorticity tensors, respectively. Note that the Oldroyd derivative in the original formulation by Speziale (1987) is omitted here. The eddy viscosity \( \nu_t \) in Eq. (5.1) is expressed by Eq. (3.4), where \( f_\mu \) is the model function [not included in the original model (Speziale 1987)] to account for the near-wall and low-Reynolds-number effects as described in Chapter 3. By normalizing the anisotropy tensor \( b_{ij} \), the strain-rate tensor \( S_{ij} \) and the vorticity tensor \( \Omega_{ij} \) as

\[
b^*_{ij} = C_D b_{ij} \quad , \quad S^*_{ij} = C_D \tau_c S_{ij} \quad , \quad \Omega^*_{ij} = 2 C_D \tau_c \Omega_{ij}, \tag{5.2}
\]

we can rewrite Eq. (5.1) as follows (hereinafter referred to as the Standard model):

\[
b^*_{ij} = -S^*_{ij} - \left( S^*_{ik} \Omega^*_{kj} - \Omega^*_{ik} S^*_{kj} \right) + 2 \left( S^*_{ik} S^*_{kj} - \frac{1}{3} S^*_{mn} S^*_{mn} \delta_{ij} \right). \tag{5.3}
\]

In Eq. (5.2), \( C_D \) is the model constant and \( \tau_c \) is a characteristic time scale of turbulence given by

\[
\tau_c = \frac{\nu_t}{k}. \tag{5.4}
\]

On the other hand, the EASM by Pope (1975) (hereinafter referred to as the Original model) is expressed as

\[
b^*_{ij} = \frac{3}{3 + 6 \Omega^* - 2 S^*} \left\{ -S^*_{ij} - \left( S^*_{ik} \Omega^*_{kj} - \Omega^*_{ik} S^*_{kj} \right) + 2 \left( S^*_{ik} S^*_{kj} - \frac{1}{3} S^*_{mn} S^*_{mn} \delta_{ij} \right) \right\}, \tag{5.5}
\]

where \( S^* = S^*_{ij} S^*_{ij} \) and \( \Omega^* = \Omega^*_{ij} \Omega^*_{ij} \). This model was derived from the transport equation of Reynolds stress \( \overline{u_iu_j} \) in an equilibrium field, and included the essential features of second-moment closure models (Pope 1975; Gatski & Speziale 1993). Comparing Eqs. (5.3) and (5.5), it can easily be understood that the contents of \{ \} in Eq. (5.5) completely coincide with Eq. (5.3). This suggests that a more sophisticated \( k-\epsilon \) model can be constructed by introducing the important aspects of the coefficient in front of \{ \} in Eq. (5.5). Thus, in this study, we propose the expression given by Eq. (5.5) with the variables defined by Eq. (5.2) as the basic representation for the Reynolds stress tensor.

The Reynolds stress expression given by Eq. (5.5) has, however, a significant problem, i.e., mathematical (phenomenological) inaccuracies may occur with the increase of
$S^2$, because the denominator of the coefficient can be zero or negative. As a solution, Gatski & Speziale (1993) proposed a modification of the coefficient using the Padé type approximation (hereinafter referred to as the Padé model) expressed as follows:

\[ \frac{3 \left(1 + S^2\right)}{3 + S^2 + 6\Omega^2 S^2 + 6\Omega^2} . \] (5.6)

The modification by Eq. (5.6) is, however, inadequate to satisfy the realizability of $b_{ij}$ even in the two-dimensional flow field, though it guarantees the non-negative value of the coefficient. Therefore, in the present study, Eq. (5.5) is reexamed and modified to satisfy the realizability of the Reynolds stress, as discussed below. First, we rewrite the coefficient of Eq. (5.5) as follows:

\[ \frac{3}{3 + 6\Omega^2 - 2S^2} = \frac{1}{1 + \frac{22}{3} \left(\frac{\Omega^2}{4}\right) + \frac{2}{3} \left(\frac{\Omega^2}{4} - S^2\right)} . \] (5.7)

The parameter $(\Omega^2/4 - S^2)$ [i.e., $(CDt_c)^2(\Omega^2 - S^2)$, where $S^2 = S_{ij}S_{ij}$ and $\Omega^2 = \Omega_{ij}\Omega_{ij}$] in Eq. (5.7) is one of the most important measures in turbulence, since it indicates how the flow field deviates from the condition of pure shear flow. As recognized from Eq. (5.7), the foregoing mathematical inaccuracies do not occur in pure shear flows, where only $S_{12}$, $S_{21}(= S_{12})$, $\Omega_{12}(= S_{12})$ and $\Omega_{21}(= -S_{12})$ exist so that $(\Omega^2/4 - S^2) = 0$. On the contrary, improper behavior may result when the flow field greatly deviates from pure shear flow, since the normal strain rate is much larger than the shear strain rate, i.e., $S^2 \gg \Omega^2/4$.

On that basis, Eq. (5.7) is modified as follows, in which a model function $f_B$ is introduced to guarantee the realizability under the condition of $S^2 \gg \Omega^2/4$:

\[ \frac{1}{1 + \frac{22}{3} \left(\frac{\Omega^2}{4}\right) + \frac{2}{3} \left(\frac{\Omega^2}{4} - S^2\right)} f_B . \] (5.8)

In Eq. (5.8), it is desirable to model $f_B$ so that its effect disappears in pure shear flows such as a fully developed channel flow and a homogeneous shear flow. Thus, the following formulation is adopted which satisfies these requirements and seems to be the simplest:

\[ f_B = 1 + C_n \left(\frac{\Omega^2}{4} - S^2\right) , \] (5.9)

where $C_n$ is the model constant.

Thus, the final expression of the Reynolds stress in the present model is
\[ b_{ij}^* = \frac{1}{1 + \frac{22}{3} \left( \frac{\Omega^2}{4} + \frac{3}{2} \left( \frac{\Omega^2}{4} - S^2 \right) \right) f_B \left\{ -S_{ij}^* \right. \]
\[ \left. - \left( S_{ij}^* \Omega_{kj}^* - \Omega_{ik}^* S_{kj}^* \right) + 2 \left( S_{ik}^* S_{kj}^* - \frac{1}{3} S_{mn}^* S_{mn}^* \delta_{ij} \right) \right\}. \] \] (5.10)

Note that the present model (5.10) can be rewritten as follows with the conventional form as in Eq. (5.1):
\[ \frac{u_i u_j}{\mu} = \frac{2}{3} k \delta_{ij} + \frac{1}{1 + \left( \frac{C_D \nu_t}{k} \right)^2 \left\{ \frac{22}{3} \Omega^2 + \frac{3}{2} \left( \Omega^2 - S^2 \right) \right\} f_B \left\{ -2 \nu_t S_{ij} \right. \]
\[ \left. - 4 C_D \frac{\nu_t^2}{k} \left( S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj} \right) + 4 C_D \frac{\nu_t^2}{k} \left( S_{ik} S_{kj} - \frac{1}{3} S_{mn} S_{mn} \delta_{ij} \right) \right\}, \] (5.11)

where \( f_B = 1 + C_n (C_D \nu_t/k)^2 (\Omega^2 - S^2) \).

Turbulent diffusion terms in Eqs. (2.3) and (2.4) are modeled using the generalized gradient diffusion hypothesis (GGDH) by Daly & Harlow (1970) as follows:
\[ \left( \frac{k'}{\rho} \right) u_j = -C_s f_{t1} \tau_c u_j \frac{\partial k}{\partial x_i}, \] (5.12)
\[ \varepsilon u_j + \frac{2 \nu}{\rho} \frac{\partial p}{\partial x_i} \frac{\partial u_j}{\partial x_i} = -C_s f_{t2} \tau_c u_j \frac{\partial \varepsilon}{\partial x_i}, \] (5.13)
where \( C_s \) and \( C_{\varepsilon} \) are the model constants, and \( f_{t1} \) and \( f_{t2} \) are the model functions activating in the vicinity of the wall.

The model constants are determined as follows. First, \( C\mu \) and \( C_D \) are set to 0.12 and 0.8, respectively, in order to properly reproduce equilibrium states in both wall-turbulent \( (U_{1,2} k/\varepsilon \simeq 3.3, b_{12} \simeq -0.15) \) and homogeneous shear \( (U_{1,2} k/\varepsilon \simeq 6, b_{12} \simeq -0.15, \) Tavoularis & Corrsin 1981) flows with the guarantee of realizability under the condition of \( U_{1,2} k/\varepsilon \gg 1 \). In Eq. (5.9), \( C_n \) is set to 5.0, so that realizability can be satisfied for any value of \( U_{1,1} k/\varepsilon \) \( (= -U_{2,2} k/\varepsilon) \) in the two-dimensional flow field, and so that the influence of the model function \( f_B \) should be as small as possible. The model constant \( C_{\varepsilon} \) for the turbulent diffusion is set to 1.4 based on the relation for "constant stress layer" (Nagano & Tagawa 1990) and \( C_s \) is assigned the same value as \( C_{\varepsilon} \), i.e., \( C_s = 1.4 \). In Eq. (2.4), \( C_{e1} \) and \( C_{e2} \) are set to the generally accepted values, i.e., \( C_{e1} = 1.45 \) and \( C_{e2} = 1.9 \) (Nagano & Tagawa 1990).

In sum, the present model uses the following set of model constants:
\[ C\mu = 0.12, \quad C_D = 0.8, \quad C_n = 5.0, \quad C_s = 1.4, \quad C_{\varepsilon} = 1.4, \quad C_{e1} = 1.45, \quad C_{e2} = 1.9. \] (5.14)
5.1.2 Extension to a low-Reynolds-number model

The near-wall and low-Reynolds-number effects are modeled as follows. Concerning $f_\mu$ in Eq. (3.4) and $f_\epsilon$ in Eq. (2.4), we basically follow the concept described in Chapter 3, also taking account of some recent findings suggested by Nagano & Shimada (1993). As a result, the present model functions become

$$f_\mu = \left[1 + \frac{35}{3} \exp\left\{-\left(\frac{R_t}{30}\right)^{\frac{3}{4}}\right\}\right] \left\{1 - f_w(26)\right\}, \quad (5.15)$$

$$f_\epsilon = \left[1 - 0.3 \exp\left\{-\left(\frac{R_t}{6.5}\right)^2\right\}\right] \left\{1 - f_w(3.7)\right\}, \quad (5.16)$$

where

$$f_w(A) = \exp\left\{-\left(\frac{y^*}{A}\right)^2\right\}, \quad R_t = \frac{k^2}{\nu \epsilon}, \quad y^* = \frac{u_* y}{\nu}, \quad u_* = (\nu \epsilon)^{\frac{1}{4}}. \quad (5.17)$$

The model functions used in Eqs. (5.12) and (5.13) are given by

$$f_{t1} = 1 + 5.0 f_w(5), \quad f_{t2} = 1 + 4.0 f_w(5). \quad (5.18)$$

These model functions, $f_{t1}$ and $f_{t2}$, increase the turbulent diffusion in the vicinity of the wall. As a result, the turbulent energy budget (including the profile of the dissipation rate) can successfully be predicted (see Nagano & Shimada 1993).

The most important feature of the present model functions is the introduction of the Kolmogorov velocity scale $u_*$ instead of the friction velocity $u_\tau$, to account for the near-wall and low-Reynolds-number effects in both attached and detached flows (see Chapter 3). This model can reproduce the correct near-wall asymptotic relations of turbulence as well as the model proposed in Chapter 3, i.e., $k \propto y^3$, $\epsilon \propto y^6$, $\nu_t \propto y^3$ and $-\overline{u v} \propto y^3$ for $y \to 0$.

5.2 Relation between Anisotropy Tensors and Velocity Gradients

The capability of a turbulence model to accurately predict both wall-turbulent and homogeneous shear flows can be assessed by examining the relation between the anisotropy tensor $b_{ij}$ and the shear component of the velocity gradient $\overline{U_{12}}$. The variations of $b_{ij}$ with the shear parameter $\overline{U_{12}}k/\epsilon$ are shown in Fig. 5.1. The relations between $b_{12}$ and $\overline{U_{12}}k/\epsilon$
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Table 5.1: Relation between $b_{12}$ and $\mathoverline{U}_{1,2}k/\varepsilon$ (WTF; Wall-turbulent flow, HSF; Homogeneous shear flow)

<table>
<thead>
<tr>
<th></th>
<th>WTF</th>
<th>HSF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_{1,2}k/\varepsilon$</td>
<td>$b_{12}$</td>
</tr>
<tr>
<td>DNS(WTF), Experiment(HSF)</td>
<td>3.3</td>
<td>-0.15</td>
</tr>
<tr>
<td>Present Model</td>
<td>3.4</td>
<td>-0.15</td>
</tr>
<tr>
<td>Standard Model</td>
<td>3.3</td>
<td>-0.15</td>
</tr>
<tr>
<td>SH Model</td>
<td>3.7</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

in typical wall-turbulent and homogeneous shear flows are summarized in Table 5.1. In these figure and table, the results by the Standard model (5.3) and the model proposed by Shih et al. (1994) (hereinafter referred to as the SH model) are also included for comparison. The Reynolds stress of the SH model is expressed as follows:

$$\overline{u_iu_j} = \frac{2}{3}k\delta_{ij} - \frac{2/3}{5.5 + \left(\frac{k^2}{\varepsilon}\right)} \left(\frac{\varepsilon}{\varepsilon}\right) \left(\overline{U_{i,j}} + \overline{U_{j,i}}\right)$$

$$+ \frac{1}{1000 + \left(\frac{k^2}{\varepsilon}\right)^3 + \left(\frac{k^2}{\varepsilon}\right)^3} \left(\frac{\varepsilon}{\varepsilon}\right)^3 \left\{ -4 \left(\overline{U_{i,k}}\overline{U_{k,j}} + \overline{U_{j,k}}\overline{U_{k,i}} - \frac{2}{3}\overline{U_{m,n}}\overline{U_{n,m}}\delta_{ij}\right) 
\right\} + 13 \left(\overline{U_{i,k}}\overline{U_{j,k}} - \frac{1}{3}\overline{U_{m,n}}\overline{U_{m,n}}\delta_{ij}\right) - 2 \left(\overline{U_{k,i}}\overline{U_{k,j}} - \frac{1}{3}\overline{U_{m,n}}\overline{U_{m,n}}\delta_{ij}\right) \right\}, \quad (5.19)$$

where $S$ and $\Omega$ are the strain-rate and vorticity parameters defined by $S = \sqrt{2S_{ij}S_{ij}}$ and $\Omega = \sqrt{2\Omega_{ij}\Omega_{ij}}$, respectively. Note that, in Table 5.1, the terms WTF and HSF denote, respectively, wall-turbulent [$P_k/\varepsilon = 1$] and homogeneous shear [$P_k/\varepsilon = (C_{r2} - 1)/(C_{r1} - 1)$] flows, where $P_k$ is the production term of turbulent energy, i.e., $P_k = -\overline{u_iu_j}$. It can be seen that the present model (5.10) successfully reproduces both wall-turbulent and homogeneous shear flows compared with the Standard model (5.3) as shown in Table 5.1, where the WTF data correspond to the DNS of Kim et al. (1990) and the experimental data on HSF to Tavoularis & Corrsin (1981). Since the relationship between the anisotropy tensor $b_{12}$ and the shear parameter $U_{1,2}k/\varepsilon$ in the Standard model (5.3) is linear as shown in Fig. 5.1 (a), it cannot essentially predict wall-turbulent and homogeneous shear flows simultaneously, where $b_{12}$ in both situations are almost equal.
(\approx -0.15) for the different values of \(\bar{U}_{1,2} k/\varepsilon\) (see Table 5.1). We can also understand from Table 5.1 that the accuracy of the present model is better than that of the SH model, since the latter shows some discrepancies in the WTF region. These discrepancies usually cannot be overlooked in this type of two-equation model.

As already mentioned, the breakdown of realizability occurs in the condition of \(S_{\Omega}^2 \gg \Omega_{\Omega}^2/4\), in which the normal component of the velocity gradient \(U_{1,1}\) (i.e., elongation or contraction) is dominant. Figure 5.2 shows a comparison among the variations of \(b_{ij}\) with \(U_{1,1} k/\varepsilon\) obtained by the Original model (5.5), the Padé model (5.6), the SH model (5.19) and the present model (5.10). It can easily be understood that the realizability constraint is not satisfied in the Original and Padé models, though the Padé type modification proposed by Gatski & Speziale (1993) considers the positiveness of the coefficient in Eq. (5.5) as previously described. On the other hand, the present and SH models satisfy realizability. Moreover, in marked contrast to the SH model, the behavior of the present model is quite similar to that of the Original model in the region of \(U_{1,1} k/\varepsilon < 3\). As seen from Figs. 5.1 (b) and 5.2, the anisotropy predicted by the SH model is much weaker than that by the present model, which results in a difference of accuracy in predicting homogeneous shear flow, as will be shown.

5.3 Numerical Procedure

All the following calculations were performed using the finite difference method described in Chapters 3 and 4, except for homogeneous shear flow which was calculated using a simple algorithm by the Runge-Kutta method. In the finite difference calculations, the primitive variables, \(U_i\), \(k\) and \(\varepsilon\), were colocated, whereas the pressure was located in a staggered position (Kuno et al. 1992).

Recently, many types of low-Reynolds-number \(k-\varepsilon\) models have been proposed in which various types of low-Reynolds-number parameters are adopted, e.g., \(\gamma = u_{\kappa} y/\nu\), \(R_{\kappa} = k^2/\nu\varepsilon\), \(y_k = \sqrt{k y/\nu}\), \(R_s = k/\nu S\) and so on. However, it is not clear which factors determine the low-Reynolds-number parameters used in the model functions, and which parameter is best suited for scaling the near-wall region.

Thus, in this study, three of the other low-Reynolds-number \(k-\varepsilon\) models were selected
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Table 5.2: Model functions and model constants

<table>
<thead>
<tr>
<th>Model</th>
<th>$f_\mu$</th>
<th>$f_1$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YS1</td>
<td>$1 + \left( \frac{1}{\sqrt{R_1}} \right) \left{ 1 - \exp \left( -a_1 y_k - a_3 y_k^2 - a_5 y_k^3 \right) \right}^{1/2}$</td>
<td>$\sqrt{R_t} / (\sqrt{R_t} + 1)$</td>
<td>$\sqrt{R_t} / (\sqrt{R_t} + 1)$</td>
</tr>
<tr>
<td>YS2</td>
<td>$1 + \left( \frac{1}{\sqrt{R_1}} \right) \left{ 1 - \exp \left( -b_1 R_s - b_2 R_s^2 - b_3 R_s^3 \right) \right}^{1/2}$</td>
<td>$\sqrt{R_t} / (\sqrt{R_t} + 1)$</td>
<td>$\sqrt{R_t} / (\sqrt{R_t} + 1)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$C_\mu$</th>
<th>$\sigma_k$</th>
<th>$\sigma_\varepsilon$</th>
<th>$C_{\varepsilon 1}$</th>
<th>$C_{\varepsilon 2}$</th>
<th>$D$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YS1</td>
<td>0.09</td>
<td>1.0</td>
<td>1.3</td>
<td>1.44</td>
<td>1.92</td>
<td>0</td>
<td>$\nu \nu_t \left( \overline{U_{i,j,k}} \right)^2$</td>
</tr>
<tr>
<td>YS2</td>
<td>0.09</td>
<td>1.0</td>
<td>1.3</td>
<td>1.44</td>
<td>1.92</td>
<td>0</td>
<td>$\nu \nu_t \left( \overline{U_{i,j,k}} \right)^2$</td>
</tr>
</tbody>
</table>

\[
y_k = \sqrt{k} y / \nu, \quad R_s = k / \nu S, \quad R_t = k^2 / \nu \varepsilon, \quad (a_1, a_3, a_5) = (1.5 \times 10^{-4}, 5 \times 10^{-7}, 1 \times 10^{-10}), \quad (b_1, b_2, b_3) = (3 \times 10^{-4}, 6 \times 10^{-5}, 2 \times 10^{-6})
\]

for comparison, i.e., the YS1 (Yang & Shih 1993a), YS2 (Yang & Shih 1993b) and LS (Launder & Sharma 1974) models. The YS1 and YS2 models are summarized in Table 5.2, where the transport equations of $k$ and $\varepsilon$ are given by Eqs. (4.17) and (4.18), and the Reynolds stress $\overline{u_i u_j}$ is expressed by Eq. (3.1) (concerning the LS model, see Table 4.2). Note that $R_s = k / \nu S$ used in the YS2 model is a recently proposed parameter with no reference to the distance from the wall (Yang & Shih 1993b).

5.4 Model Assessment in Fundamental Turbulent Flows

To confirm the basic accuracy of the present model, we applied it to two representative turbulent flows, i.e., homogeneous shear flow corresponding to the experiment by Tavoularis & Corrsin (1981) and channel flow corresponding to the DNS by Kim et al. (1990).

Computational results in the homogeneous shear flow are shown in Figs. 5.3 and 5.4,
where $h$ is the duct width at the exit of the wind tunnel. Results by the Standard model (5.3) and the SH model (5.19) are also included. From these figures, it is clear that accuracy in calculating homogeneous shear flow is dramatically improved by the present model. The comparison of the anisotropy is shown in Fig. 5.5. The computational results of the present model agree well with the experimental data of Tavoularis & Corrsin (1981), whereas the SH model shows considerably weak anisotropy. The SH model does not take into account compatibility with higher order closure models, although it considers realizability. This seems to explain the difference in accuracy between the present and SH models.

Figures 5.6–5.8 show the results of the channel flow with the present model calculated under the DNS condition of Kim et al. (1990). In Fig. 5.6, two types of grid points are tested to examine grid dependence, i.e., 151 and 301. The Reynolds number based on the friction velocity $u_\tau$ and the half width of the channel $\delta$ was $Re_\tau = u_\tau \delta / \nu = 395$. From Fig. 5.6 (a), one can see that almost perfect grid-independent solutions are obtained in the calculations. Also, from Figs. 5.6 and 5.7, it can be seen that mean velocity, turbulent energy, Reynolds shear stress and dissipation rate are quite successfully predicted, and that all terms of the turbulent energy budget are in good agreement with the DNS data (Fig. 5.7). The accuracy of the mean-velocity results is validated in a wide range of the Reynolds numbers as shown in Fig. 5.8.

5.5 Application to Complex Turbulent Flows

5.5.1 Channel flow with injection and suction

The introduction of flow injection and suction is one of the most important strategies to control turbulent flows and heat transfer. It is well known that even a small amount of mass injection and suction (only about 0.3 % of the bulk-flow rate) greatly influences the turbulent structures (Sumitani & Kasagi 1993). Thus, the present model was applied to this type of turbulent flow to assess the accuracy in the flows with complex boundary conditions. Two types of flow conditions were considered. One was uniform injection at one wall and uniform suction at the same rate at the other wall corresponding to the DNS
CHAPTER 5. A NEW TYPE OF NONLINEAR \( k-\varepsilon \) MODEL

Table 5.3: Skin friction coefficient \( C_f \) in the channel flow with injection and suction

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Injection side</th>
<th>Suction side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction</td>
<td>( 8.85 \times 10^{-3} )</td>
<td>( 6.40 \times 10^{-3} )</td>
<td>( 1.31 \times 10^{-2} )</td>
</tr>
<tr>
<td>DNS</td>
<td>( 8.64 \times 10^{-3} )</td>
<td>( 6.29 \times 10^{-3} )</td>
<td>( 1.27 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

by Sumitani & Kasagi (1993). The other was periodic injection and suction at one side wall corresponding to the DNS by Miyake et al. (1994). The latter case seems particularly difficult to accurately predict, since a periodic change in pressure gradient is induced by flow injection and suction, so that the flow field may show characteristics of both favorable and adverse pressure gradient flows (Miyake et al. 1994).

Concerning the former test case (Sumitani & Kasagi 1993), the Reynolds number based on the averaged friction velocity on both-side walls, \( u_r^{**} \), and the half width of the channel \( \delta \) was \( Re_r = u_r^{**} \delta / \nu = 150 \), and the injection and suction velocity was set to \( \overline{V}/u_r^{**} = 0.05 \). The calculation was conducted with the grid points of 101. Profiles of the mean velocity and Reynolds shear stress are shown in Fig. 5.9, and skin friction coefficients \( C_f \) are shown in Table 5.3, in which \( (\;)^+ \) denotes the values normalized by friction velocity on each side wall, and \( (\;)^{**} \) are those normalized by the averaged friction velocity \( u_r^{**} \). The results for channel flow without any injection and suction corresponding to the DNS of Kuroda et al. (1993) are also included for comparison. These data make it clear that the present model has the capability to accurately predict the mean velocity, Reynolds shear stress and friction coefficient. Ohashi et al. (1993) concluded from their computational results that the non-dimensional distance, \( y^* \), with \( u_r \) was not suitable for predicting this type of flow field. As demonstrated by the present results, however, the use of \( y^* \) is not always harmful to accuracy. This confusion seems to occur because the Reynolds number in this test case is extremely low, i.e., as low as the critical Reynolds number of the laminarization for most low-Reynolds-number models. Therefore, some models may give laminarized solutions which may lead to a major deviation from the DNS data. The budgets of turbulent energy are shown in Fig. 5.10. It can be seen that all the terms of the budget on the injection side are accurately predicted, whereas those on the suction side show some discrepancies with the DNS data of Sumitani & Kasagi.
CHAPTER 5. A NEW TYPE OF NONLINEAR k-\( \varepsilon \) MODEL

(1993). This issue needs more detailed study in the future.

As for the latter test case with a periodic change in pressure gradient (Miyake et al. 1994), the Reynolds number was \( \text{Re}_x = u_x^* \delta / \nu = 150 \), and the injection and suction velocity was set to \( \overline{V}/u_x^* = -0.5 \sin(2\pi x/L) \) at one side wall, where \( L(=4\pi \delta) \) was the length of one period of injection and suction. The number of grid points was \( 185 \times 101 \). The variations in mean velocity obtained from the present model and three low-Reynolds-number models, i.e., YS1, YS2 and LS, are shown in Fig. 5.11 (the computational results are those on the side without flow injection and suction). In this type of flow field, the mean-velocity profile shifts up and down across the line of the log law according to the mean-pressure gradient. The mean-velocity profiles of the present and YS1 models show the correct tendency for both the phase and amplitude of this shift across the log-law line. In contrast, the YS2 and LS models show considerable overpredictions at \( x = 5L/8 \) and \( 6L/8 \). It seems that the low-Reynolds-number parameters adopted in the model functions account for the difference in computational accuracy. This issue will be discussed in detail in the later section.

5.5.2 Backward-facing step flow

To validate the accuracy in complex separating and reattaching turbulent flows, we applied the present new model to the flows downstream of a backward-facing step. The calculations were conducted corresponding to five representative cases as shown in Table 5.4.

The reattachment lengths obtained from the computational results are shown in Table 5.5 with the corresponding experimental and DNS data. From Table 5.5, we can understand that the present model can successfully predict the reattachment lengths over a wide range of the expansion ratio and Reynolds number as well as the model proposed in Chapter 3.

The streamlines and the skin friction coefficient in Case 5-2 obtained by the present model are shown in Fig. 5.12. The accuracy of the skin friction coefficient in the recirculating region is considerably improved compared with the linear \( k-\varepsilon \) model proposed in Chapter 3. A comparison of the computational results with the experiment of Kasagi et al. (1993) (Case 5-3) is shown in Fig. 5.13, in which predictions by the present model
### Table 5.4: Computational conditions for back-step flows

<table>
<thead>
<tr>
<th>Case</th>
<th>5-1</th>
<th>5-2</th>
<th>5-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ER$</td>
<td>1.2</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td>$Re_H$</td>
<td>5100</td>
<td>28000</td>
<td>5500</td>
</tr>
<tr>
<td>$Re_\theta$</td>
<td>700</td>
<td>4500</td>
<td>500</td>
</tr>
<tr>
<td>$\theta/H$</td>
<td>0.14</td>
<td>0.16</td>
<td>0.09</td>
</tr>
<tr>
<td>$N_\xi \times N_\eta$</td>
<td>$389 \times 145$</td>
<td>$325 \times 125$</td>
<td>$303 \times 125$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>5-4</th>
<th>5-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ER$</td>
<td>1.67</td>
<td>2.0</td>
</tr>
<tr>
<td>$Re_H$</td>
<td>38000</td>
<td>100000</td>
</tr>
<tr>
<td>$Re_\theta$</td>
<td>1000</td>
<td>4000</td>
</tr>
<tr>
<td>$\theta/H$</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>$N_\xi \times N_\eta$</td>
<td>$341 \times 111$</td>
<td>$359 \times 125$</td>
</tr>
</tbody>
</table>

are in good agreement with the experimental data. The turbulent energy budgets at $z/H = 4$ in Case 5-1 corresponding to the DNS of Le et al. (1993), obtained by the present model and three low-Reynolds-number models, i.e., YS1, YS2 and LS models, are shown in Figs. 5.14–5.17. Note that, in these models, the total dissipation rate is estimated as $\varepsilon_{\text{total}} = \varepsilon + D$. We can definitely declare from these figures that the present model gives the best predictions compared with the other three models, all of which considerably overpredict the molecular diffusion, turbulent diffusion and dissipation rate in the near-wall region. Furthermore, predictions by the YS2 and LS models show the spiky behavior of the production term in the immediate vicinity of the wall, which is in conflict with the DNS data of Le et al. (1993). It is also suggested that the low-Reynolds-number parameters adopted in the model functions greatly influence the computational results.
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Table 5.5: Comparison of flow reattachment lengths, $X_R/H$

<table>
<thead>
<tr>
<th>Case</th>
<th>5-1</th>
<th>5-2</th>
<th>5-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction</td>
<td>$5.92 \pm 0.03$</td>
<td>$6.41 \pm 0.06$</td>
<td>$6.60 \pm 0.04$</td>
</tr>
<tr>
<td>Experiment</td>
<td>6.0</td>
<td>6.7</td>
<td>6.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>5-4</th>
<th>5-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction</td>
<td>$8.12 \pm 0.04$</td>
<td>$8.67 \pm 0.05$</td>
</tr>
<tr>
<td>Experiment</td>
<td>8.0</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Details are to be discussed in the next section.

5.5.3 Examination of low-Reynolds-number parameters in model functions

As mentioned before, many types of low-Reynolds-number $k$-$\varepsilon$ models have been proposed in which various types of low-Reynolds-number parameters are adopted. One of the most important issues, on which recent research focuses, is the problem of whether or not an accurate low-Reynolds-number model can be constructed without the distance from the wall, $y$. It is usually said that the low-Reynolds-number parameters without $y$ are desirable for a more general-purpose low-Reynolds-number $k$-$\varepsilon$ model (Yang & Shih 1993b). Most of the proposed models without $y$ have, however, been applied only to relatively simple flow fields for the assessment of accuracy, whereas these models were constructed for application to complex turbulent flows as represented by flows with injection, suction and separation. Furthermore, as revealed by the above discussions, the low-Reynolds-number parameters adopted in the model functions show considerable difference in the accuracy of flow-field prediction. Therefore, we investigate the characteristics of the low-Reynolds-number parameters in complex turbulent flows based on the computational results obtained before.

First, based on computational results shown in Fig. 5.11 and Figs. 5.14-5.17, the examined models can be divided by whether or not their predictions reasonably agree with the
experimental and DNS data. One group consists of the present and YS1 models, and the other of the YS2 and LS models. It is the former group that gives reasonable predictions for both the channel flow with periodic injection and suction, and the backward-facing step flow. It should be noted that this grouping coincides with whether or not $y$ is included in the model functions, as shown in Eq. (5.15), Table 4.2 and Table 5.2.

Streamwise variations in turbulent characteristics at the location $y/\delta = 1/3$ in the channel flow with periodic injection and suction are shown in Fig. 5.18, in which all variables are normalized by the value averaged in the streamwise direction at that $y$ location. It can be seen that the behavior of $\sqrt{k}$ and $u_\varepsilon$ is quite similar, as shown in Figs. 5.18 (a) and 5.18 (b), whereas $R_t$ shows a different variation whose phase shifts by about $\pi$ from $\sqrt{k}$ and $u_\varepsilon$. This is a novel feature since $R_t$ consists of $\sqrt{k}$ and $u_\varepsilon$, i.e., $R_t = k^2/\nu \varepsilon = (\sqrt{k}/u_\varepsilon)^4$. A slight difference in the variations between $\sqrt{k}$ and $u_\varepsilon$ may be amplified in $R_t$ by the fourth power relation which can cause anomalous behavior. This phase shift can also be observed in Figs. 5.18 (c) and 5.18 (d), in which a more definite discrepancy appears between the above variables. Furthermore, it is worth noting that, among all variables shown in Fig. 5.18, only $R_s$ shows a different behavior in the two groups mentioned above. The prediction of $R_s$ by the YS2 model [Fig. 5.18 (c)] shows a tendency similar to that of $R_t$ by the LS model [Fig. 5.18 (d)], which may lead to similar predictions of the mean- and friction-velocity profiles in these two models. From these discussions, it can be more readily understood that the low-Reynolds-number parameters constructed by more than two of the turbulence characteristics, i.e., $k$, $\varepsilon$ and $S$, may result in unexpected behavior due to an amplification of the difference between the variations in turbulent characteristics. Thus, care must be taken when using such parameters.

The cross streamwise variations in turbulent characteristics in the backward-facing step flow of Case 5-1 at the reattachment point, $x/X_R = 1$, are shown in Fig. 5.19, in which all variables are normalized by the maximum value in the section. It is clear that $R_s$ and $R_t$ very rapidly increase in the immediate vicinity of the wall and decrease before the step height location of $y/H = 1$. This kind of behavior is undesirable as a scaling parameter in the near-wall region. The rapid increase in low-Reynolds-number parameters results in a thinning of the viscous sublayer. Launder (1988) indicates that the LS model gives an immoderate thinning of the viscous sublayer and an excessively large
turbulent length scale near the reattachment point in separating flows, which leads to an unacceptable overprediction of the heat transfer coefficient. Therefore, care is needed in the introduction of $R_t$ for scaling the near-wall region. On the other hand, as discussed in Chapters 3 and 4, parameters such as $u_s$ and $R_s$ constructed by the mean-velocity gradient pose a problem at the separating and reattaching points, where the near-wall limiting behavior of Reynolds shear stress given by the models using these parameters changes along with changes in that of the mean velocity, i.e., $\bar{U} \propto y^2$ for $y \to 0$. In addition, parameters with the mean-velocity gradient do not seem to be essential for flow regions where the production term of turbulent energy is relatively small compared with the other terms, e.g., the shear-free side in Couette-Poiseuille flows and the recirculating region in separating flows. In such flow fields, turbulent diffusion is usually the dominant term on the gain side of the turbulent energy budget (Le et al. 1993). Thus, the inclusion of $R_s$ in the model functions is not always reasonable where complex turbulent flows are concerned.

In sum, the following knowledge emerged from the present investigation: (1) The turbulent Reynolds number, $R_t = k^2/\nu \varepsilon$, and the recently proposed parameter, $R_s = k/\nu S$, neither of which include the distance from the wall, should be carefully introduced in application to complex turbulent flows; (2) The essential difficulty in constructing the low-Reynolds-number parameter without $y$ seems to lie in the necessity of more than two of the turbulence characteristics; (3) The combination of two turbulence characteristics, e.g., $R_t = k^2/\nu \varepsilon$, could result in characteristics quite different from the original ones, even if the profiles of both show a similar tendency in the flow field.
Figure 5.1: Relation between anisotropy tensors and shear parameter: (a) Comparison with Standard model; (b) Comparison with SH model.
Figure 5.2: Anisotropy tensors versus normal strain rate.
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Figure 5.3: Computational results in homogeneous shear flow: (a) Turbulent energy; (b) Dissipation rate.
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Figure 5.4: Reynolds stress predictions in homogeneous shear flow: (a) Normal component; (b) Shear component.
Figure 5.5: Anisotropy tensors for homogeneous shear flow.
Figure 5.6: Computational results in channel flow: (a) Mean velocity and turbulent energy; (b) Reynolds shear stress.
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Figure 5.7: Budget of turbulent energy in channel flow.

Figure 5.8: Mean velocity profiles at various Reynolds numbers.
Figure 5.9: Mean velocity and Reynolds shear stress in channel flow with uniform injection and suction: (a) Mean velocity; (b) Reynolds shear stress.
Figure 5.10: Budget of turbulent energy in channel flow with uniform injection and suction: (a) Injection side; (b) Suction side.
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Figure 5.11: Mean-velocity profile in channel flow with periodical injection and suction at one side wall: (a) Present model; (b) Yang-Shih (YS1) model; (c) Yang-Shih (YS2) model; (d) Launder-Sharma (LS) model.
Figure 5.12: Computational results for Case 5-2: (a) Streamlines; (b) Skin friction coefficient.
Figure 5.13: Comparison with experiment of Kasagi et al. (1993) (Case 5-3): ○ Experiment; ——— Prediction; (a) Streamwise velocity; (b) Turbulent energy; (c) Reynolds shear stress.
Figure 5.14: Budget of turbulent energy in backward-facing step flow at \( x/H = 4 \) (Case 5-1. Present model): Convection \( C \) Model, \( \cdots \) DNS; Turbulent diffusion \( D \) Model, \( \cdots \) DNS; Viscous diffusion \( \Diamond \) Model, \( \cdots \cdots \cdots \cdots \) DNS; Production \( \Delta \) Model, \( \cdots \cdots \cdots \cdots \) DNS; Dissipation \( \nabla \) Model, \( \cdots \cdots \cdots \cdots \) DNS; (a) Overall view; (b) Close to wall on step side.
Figure 5.15: Budget of turbulent energy in backward-facing step flow at $x/H = 4$ (Case 5-1, YS1 model, Key as Fig. 5.14): (a) Overall view; (b) Close to wall on step side.
Figure 5.16: Budget of turbulent energy in backward-facing step flow at $x/H = 4$ (Case 5-1, YS2 model, Key as Fig. 5.14): (a) Overall view; (b) Close to wall on step side.
Figure 5.17: Budget of turbulent energy in backward-facing step flow at $x/H = 4$ (Case 5-1, LS model, Key as Fig. 5.14): (a) Overall view; (b) Close to wall on step side.
Figure 5.18: Turbulent characteristics in channel flow with periodical injection and suction at one side wall ($y/\delta = 1/3$): (a) Present model; (b) Yang-Shih (YS1) model; (c) Yang-Shih (YS2) model; (d) Launder-Sharma (LS) model.
Figure 5.18: (continued)
CHAPTER 5. A NEW TYPE OF NONLINEAR $k$-$\varepsilon$ MODEL

Figure 5.19: Turbulent characteristics in backward-facing step flow (Case 5-1, $x/X_R = 1$): (a) Present model; (b) Yang-Shih (YS1) model; (c) Yang-Shih (YS2) model; (d) Lauder-Sharma (LS) model.
Figure 5.19: (continued)
Chapter 6

A NEW TWO-EQUATION HEAT-TRANSFER MODEL

6.1 Thermal Field Modeling

6.1.1 Modeling turbulent heat flux

This chapter deals with a new type of two-equation heat-transfer model for predicting heat transfer in both wall and free turbulent flows. The essential concept in developing a new model follows the discussion in Chapter 5.

A thermal field can be described with the governing equations in Section 2.2. In Eqs. (2.6) and (2.7), the eddy diffusivity for heat $\alpha_t$ in the turbulent diffusion terms has the following form

$$\alpha_t = C_\lambda k \tau_m , \quad (6.1)$$

which is essentially identical to Eq. (4.2). In Eq. (6.1), $\tau_m$ is the characteristic time scale dominating turbulent heat transfer as described in Chapter 4.

In this chapter, we construct a new type of expression for the turbulent heat flux $\overline{u_j t}$ based on its transport equation in an equilibrium field, i.e., the convective and diffusive effects being neglected. On this equilibrium hypothesis, the transport equation for $\overline{u_j t}$ can be written as follows:

$$0 = P_{jt} + \Phi_{jt} , \quad (6.2)$$

where $P_{jt}$ is the production term and $\Phi_{jt}$ is the pressure-temperature gradient correlation.
As for $\Phi_{jt}$, we adopt the Quasi-Isotropic model (Launder 1976) given by

$$\Phi_{jt} = -C_{t1}\frac{u_j t}{\tau_m} + C_{t2}u_k u_k \overline{U}_{j,k} + C_{t3}u_k u_k \overline{U}_{j,k}.$$  

(6.3)

Substituting Eq. (6.3) into Eq. (6.2) yields the following relation:

$$\overline{u_j t} = -C_{T1}\tau_m u_j u_k \overline{T}_{j,k} - C_{T2}\tau_m u_k u_k \overline{U}_{j,k} + C_{T3}\tau_m u_k u_k \overline{U}_{j,k},$$

(6.4)

where $C_{T1} = 1/C_{t1}$, $C_{T2} = (1 - C_{t2})/C_{t1}$ and $C_{T3} = C_{t3}/C_{t1}$. By normalizing Eq. (6.4) by $(\sqrt{k} \sqrt{t^2})$, we obtain

$$\overline{u_j t} = -(\delta_{jk} + 3b_{jk}) \left\{ \frac{2}{3} C_{T1} \left( \frac{\sqrt{k} \tau_m}{\sqrt{t^2}} \right) \overline{T}_{j,k} \right\}$$

$$- \left\{ \left( C_{T2} - C_{T3} \right) \tau_m S_{j,k} + \left( C_{T2} + C_{T3} \right) \tau_m \Omega_{j,k} \right\} \frac{u_k l}{\sqrt{k} \sqrt{t^2}}.$$  

(6.5)

If we introduce the nondimensional, rescaled variables

$$a_{j*} = \frac{u_j t}{\sqrt{k} \sqrt{t^2}}, \quad b_{jk*} = 3b_{jk}, \quad T_{k*} = \frac{2}{3} C_{T1} \left( \frac{\sqrt{k} \tau_m}{\sqrt{t^2}} \right) \overline{T}_{j,k},$$

$$S_{j,k*} = (C_{T2} - C_{T3}) \tau_m S_{j,k}, \quad \Omega_{j,k*} = (C_{T2} + C_{T3}) \tau_m \Omega_{j,k},$$

(6.6)

then Eq. (6.5) reduces to the simpler form

$$a_{j*} = -\left( \delta_{jk} + b_{jk*} \right) T_{k*} - \left( S_{j,k*} + \Omega_{j,k*} \right) a_{k*}.$$  

(6.7)

In the two-dimensional space, by solving Eq. (6.7), we can obtain the following explicit expression for the non-dimensional turbulent heat flux $a_{j*}$:

$$a_{j*} = \frac{1}{1 + \frac{1}{2} (\Omega_{*2} - S_{*2})} \left\{ -\delta_{jk} - b_{jk*} + \left( S_{j,k*} + \Omega_{j,k*} \right) + \left( S_{j,l*} + \Omega_{j,l*} \right) b_{lk*} \right\} T_{k*},$$

(6.8)

where $S_{*2} = S_{j,k*} S_{j,k*}$ and $\Omega_{*2} = \Omega_{j,k*} \Omega_{j,k*}$. When we consider only the first term in Eq. (6.8), the conventional eddy-viscosity form can be obtained. Retaining the first and second terms, on the other hand, leads to the expression known as the generalized gradient diffusion hypothesis (GGDH) (Daly & Harlow 1970). This suggests that a more sophisticated explicit expression for the turbulent heat flux in the two-equation heat-transfer model may be constructed by introducing the important aspects involved in Eq. (6.8). Thus, in this study, we propose the expression (6.8) with the variables defined by Eq. (6.6) as the basic representation for the turbulent heat flux. Note that the present model is still in the category of two-equation heat-transfer modeling, and the computing time and stability remain almost the same as in the previous two-equation models.
6.1.2 Determination of characteristic time scale

The characteristic time scale $\tau_m$ is generally expressed as follows:

$$\tau_m = \tau_u f_R f_A f_\lambda = \left( \frac{k}{\varepsilon} \right) f_R f_A f_\lambda ,$$  \hspace{1cm} (6.9)

where $f_R$ is the model function to represent the effect of dissimilarity between the velocity and temperature fields, and generally written as a function of the time-scale ratio $R$. The model function $f_A$ accounts for the influence of anisotropy in the flow field, and is expressed with the invariants of anisotropy tensor. On the other hand, $f_\lambda$ is the model function to account for the near-wall and low-Reynolds-number effects.

As for $f_R$, we adopt the following formula, in which the composite time scale defined by the harmonic average of $\tau_u$ and $\tau_i$ is incorporated:

$$f_R = \frac{2R}{0.5 + R} .$$  \hspace{1cm} (6.10)

It has already been revealed in Chapter 4 that this type of hybrid time scale is best suited to the characteristic time scale for turbulent heat transfer under complex thermal conditions. Concerning $f_A$, we adopt the simple expression

$$f_A = \frac{2}{1 + 3.5b^2} ,$$  \hspace{1cm} (6.11)

where $b^2$ is defined by $b^2 = b_i b_j$. The conception of $f_A$ expressed by Eq. (6.11) is consistent with the generally accepted understanding that the characteristic time scale for the slow term, i.e., the first term in Eq. (6.3), becomes smaller with an increase of anisotropy in the flow field. The constant, 3.5, in the denominator of Eq. (6.11) is determined to predict the turbulent heat transfer in both homogeneous isotropic decaying and homogeneous shear flows, which is validated in the later section. Note that, to conform to the conventional modeling procedure, the characteristic time scale expressed by Eq. (6.9) is also used in modeling the eddy diffusivity for heat given by Eq. (6.1). The model function $f_\lambda$ to account for the near-wall and low-Reynolds-number effects will be discussed in the later section.

6.1.3 Model constants

We determine the model constants $C_{T1}$, $C_{T2}$ and $C_{T3}$ in Eq. (6.6) in the following way.
CHAPTER 6. A NEW TWO-EQUATION HEAT-TRANSFER MODEL

First, the model constants $C_{T2}$ and $C_{T3}$ are determined by examining the heat-flux ratio $\overline{uT}/\overline{vT}$ obtained from Eq. (6.8). We consider a pure shear flow, where only $U_{1,2}$ is not null in the velocity gradient tensor. In this case, Eq. (6.8) can be rewritten as

$$a_j^{**} = \frac{1}{1 + \frac{2C_{T2}C_{T3}}{(C_{T2} + C_{T3})^2} \left( \frac{\delta_{jk}}{\Omega_{**}} + \left( S_{jk}^{**} + \Omega_{jk}^{**} \right) \frac{b_{ik}^{**}}{w_{ik}^{**}} \right)} \overline{T_k^{**}}. \quad (6.12)$$

When the mean-temperature gradient exists only in the $x$- or $y$-direction, the turbulent heat fluxes, $\overline{uT}$ and $\overline{vT}$, are expressed from Eq. (6.12) as follows:

- For $\overline{T_{1,1}} \neq 0$:

$$\overline{uT} = \frac{1}{1 + C_{T2}C_{T3}(\tau_m U_{1,2})^2} \left\{ -\left( \frac{1}{3} + b_{11} \right) + b_{12} \left( C_{T2}\tau_m U_{1,2} \right) \right\} 2C_{T1} (k\tau_m) \overline{T_{1,1}},$$

$$\overline{vT} = \frac{1}{1 + C_{T2}C_{T3}(\tau_m U_{1,2})^2} \left\{ -b_{12} + \left( \frac{1}{3} + b_{11} \right) \left( -C_{T3}\tau_m U_{1,2} \right) \right\} 2C_{T1} (k\tau_m) \overline{T_{1,1}}. \quad (6.13)$$

- For $\overline{T_{2,2}} \neq 0$:

$$\overline{uT} = \frac{1}{1 + C_{T2}C_{T3}(\tau_m U_{1,2})^2} \left\{ -b_{12} + \left( \frac{1}{3} + b_{22} \right) \left( C_{T2}\tau_m U_{1,2} \right) \right\} 2C_{T1} (k\tau_m) \overline{T_{2,2}},$$

$$\overline{vT} = \frac{1}{1 + C_{T2}C_{T3}(\tau_m U_{1,2})^2} \left\{ -\left( \frac{1}{3} + b_{22} \right) + b_{12} \left( -C_{T3}\tau_m U_{1,2} \right) \right\} 2C_{T1} (k\tau_m) \overline{T_{2,2}}. \quad (6.14)$$

From Eqs. (6.13) and (6.14), the heat flux ratio, which is one of the most important values to be predicted, becomes:

- For $\overline{T_{1,1}} \neq 0$:

$$\frac{\overline{uT}}{\overline{vT}} = -\left( \frac{1}{3} + b_{11} \right) + b_{12} \left( C_{T2}\tau_m U_{1,2} \right) -b_{12} + \left( \frac{1}{3} + b_{11} \right) \left( -C_{T3}\tau_m U_{1,2} \right) \cdot \quad (6.15)$$

- For $\overline{T_{2,2}} \neq 0$:

$$\frac{\overline{uT}}{\overline{vT}} = -b_{12} + \left( \frac{1}{3} + b_{22} \right) \left( C_{T2}\tau_m U_{1,2} \right) -\left( \frac{1}{3} + b_{22} \right) + b_{12} \left( -C_{T3}\tau_m U_{1,2} \right) \cdot \quad (6.16)$$

Now, we apply these expressions to the equilibrium-state solutions for homogeneous shear flows corresponding to the experiment of Tavoularis & Corrsin (1981), and determine the model constants $C_{T2}$ and $C_{T3}$. The experiment of Tavoularis & Corrsin (1981) gives the
following equilibrium values: $U_{1,2}k/\varepsilon \approx 6.2$, $b_{11} + 1/3 \approx 0.53$, $b_{22} + 1/3 \approx 0.18$ and $b_{12} \approx -0.15$. Using these values and $R \approx 0.35$ (Tavoularis & Corssin 1981), we can estimate from Eqs. (6.10) and (6.11) that $f_{RfA} \approx 0.75$. Then, $\tau_mU_{1,2}$ can be calculated as follows:

$$\tau_mU_{1,2} = f_{RfA} \left\{ \frac{(k/\varepsilon)U_{1,2}}{f_{RfA}} \right\} \approx 0.75 \times 6.2 \approx 4.6.$$  \hspace{1cm} (6.17)

In this study, the model constants $C_{T2}$ and $C_{T3}$ are assigned the values 0.18 and 0.02, respectively. Hence, Eqs. (6.15) and (6.16) give the heat-flux ratio $\overline{u_t}/\overline{v_t} \approx -6.5$ in the case of $\overline{T}_1 \neq 0$, and $\overline{u_t}/\overline{v_t} \approx -1.8$ in the case of $\overline{T}_2 \neq 0$. These values are reasonably identical with those obtained by the experiment (Tavoularis & Corssin 1981) and the DNS (Rogers et al. 1986) as well.

Next, the model constant $C_{T1}$ may be determined by examining the turbulent Prandtl number $Pr_t$ in homogeneous shear flows. We consider the case where only $U_{1,2}$ and $T_{1,2}$ are not zero. Comparing the expression (6.14) with that of the conventional type (i.e., $-\overline{v_t} = \alpha_t\overline{T}_2$), we can see that the coefficient $C_\lambda$ in Eq. (6.1) corresponds to the following in Eq. (6.14):

$$C_\lambda = \frac{-2C_{T1}}{1 + C_{T2}C_{T3} \left( \tau_mU_{1,2} \right)^2} \left\{ -\left( \frac{1}{3} + b_{22} \right) + b_{12} \left( -C_{T3}\tau_mU_{1,2} \right) \right\}. \hspace{1cm} (6.18)$$

The experiment of Tavoularis & Corssin (1981) shows that the model constant $C_\mu$ in the eddy-viscosity expression, $u_t = C_\mu k^2/\varepsilon$, should be around 0.045 and the turbulent Prandtl number is $Pr_t \approx 1.1$. Thus, $C_\lambda$ is estimated as follows with the relation $\alpha_t = \nu_t/Pr_t$:

$$C_\lambda f_{RfA} = C_\mu/Pr_t \approx 0.045/1.1 = 0.041. \hspace{1cm} (6.19)$$

The estimate of $C_\lambda f_{RfA}$ from Eq. (6.18) with $f_{RfA} \approx 0.75$ mentioned previously and the model constant $C_{T1} = 0.18$ becomes $C_\lambda f_{RfA} \approx 0.042$, which is in almost perfect agreement with Eq. (6.19). It should be mentioned here that the foregoing values for the model constants $C_{T1}$, $C_{T2}$ and $C_{T3}$ are also pertinent to the prediction of heat transfer in wall turbulent shear flows.

The model constants in $\varepsilon_t$ equation (2.7) are determined by examining the equilibrium solution for a homogeneous shear flow. The transport equations for $\overline{t^2}$ and $\varepsilon_t$ in a homogeneous shear flow are expressed as follows:

$$\frac{D\overline{t^2}}{Dt} = P_t - 2\varepsilon_t, \hspace{1cm} (6.20)$$
\[ \frac{D \varepsilon_t}{D \tau} = C_{P_1} \frac{\varepsilon_t}{\varepsilon_t} \frac{P_t}{2} + C_{P_2} \frac{\varepsilon_t}{k} P_k - C_{D_1} \frac{\varepsilon_t^2}{\varepsilon_t} - C_{D_2} \frac{\varepsilon \varepsilon_t}{k}. \] (6.21)

In this situation, \( \tau_t \) has a constant value and the following relation holds:
\[ \frac{D \tau_t}{D \tau} = \frac{D}{D \tau} \left( \frac{\tau_t^2}{2 \varepsilon_t} \right) = \frac{1}{2 \varepsilon_t} \frac{D \varepsilon_t}{D \tau} - \frac{\tau_t^2}{2 \varepsilon_t^2} \frac{D \varepsilon_t}{D \tau} = 0. \] (6.22)

The substitution of Eqs. (6.20) and (6.21) into Eq. (6.22) yields the following relation:
\[ \frac{P_t}{2 \varepsilon_t} = \frac{1}{1 - \frac{C_{P_1}}{2}} \left[ \left( 1 - \frac{C_{D_1}}{2} \right) + \left\{ C_{P_2} \left( \frac{P_k}{\varepsilon} \right) - C_{D_2} \right\} R \right]. \] (6.23)

In Eq. (6.23), we put \( C_{D_1} = 2.0 \) and \( C_{D_2} = 0.9 \) in conformity with previous studies (Nagano et al. 1991; Youssef et al. 1992), so that Eq. (6.23) becomes
\[ \left( \frac{P_t}{2 \varepsilon_t} \right) \left( \frac{1}{R} \right) = \frac{C_{P_2} \left( \frac{C_{\varepsilon_2}-1}{C_{\varepsilon_1}-1} \right) - 0.9}{1 - \frac{C_{P_1}}{2}}. \] (6.24)

Note that, in Eq. (6.24), we have used the equilibrium solution for the velocity field, i.e., \( P_k/\varepsilon = (C_{\varepsilon_2} - 1)/(C_{\varepsilon_1} - 1) \), where \( P_k \) is the production term of \( k \), and \( C_{\varepsilon_1} \) and \( C_{\varepsilon_2} \) are the model constants in the transport equation for \( \varepsilon \) (see Chapter 5). The experimental data of Tavoularis & Corrsin (1981) give that \( P_t/(2 \varepsilon_t) \approx 1.25 \) and \( R \approx 0.35 \). Hence, we can estimate the left-hand side of Eq. (6.24) as follows:
\[ \left( \frac{P_t}{2 \varepsilon_t} \right) \left( \frac{1}{R} \right) \approx 3.6. \] (6.25)

The model constants \( C_{P_1} \) and \( C_{P_2} \) should satisfy the constraint for the constant stress/heat-flux layer in wall turbulence (Nagano et al. 1991; Youssef et al. 1992) as well as the relation (6.24). Also, the standard values of \( C_{\varepsilon_1} \) and \( C_{\varepsilon_2} \) are 1.45 and 1.9, respectively. Taking into account these requirements along with a slight numerical optimization, we adopt the values of \( C_{P_1} = 1.9 \) and \( C_{P_2} = 0.55 \). These model constants are essentially identical to those of the model proposed in Chapter 4, i.e., \( C_{P_1} = 1.9, C_{P_2} = 0.6 \). With these model constants \( (C_{P_1} = 1.9, C_{P_2} = 0.55, C_{\varepsilon_1} = 1.45 \text{ and } C_{\varepsilon_2} = 1.9) \), the right-hand side of Eq. (6.24) consequently gives \((P_t/(2 \varepsilon_t))(1/R) = 4\) [cf., Eq. (6.25)].

The turbulent diffusion coefficients \( \sigma_h \) and \( \sigma_\phi \) in Eqs. (2.6) and (2.7) are determined with reference to the recent studies (Nagano & Shimada 1993; Nagano et al. 1994b):
\[ \sigma_h = \frac{1.8}{f_{t1}}, \quad \sigma_\phi = \frac{1.4}{f_{t2}}. \] (6.26)
where \( f_{t1} \) and \( f_{t2} \) defined by Eq. (5.18) are effective only in the vicinity of the wall as described in Chapter 5. Concerning the model constant \( C_{\lambda} \) in Eq. (6.1), we adopt a generally accepted value of \( C_{\lambda} = 0.1 \).

In sum, the model constants used in the present model are as follows:

\[
\begin{align*}
C_{\lambda} & = 0.1, \quad C_{T1} = 0.18, \quad C_{T2} = 0.18, \quad C_{T3} = 0.02, \\
C_{P1} & = 1.9, \quad C_{P2} = 0.55, \quad C_{D1} = 2.0, \quad C_{D2} = 0.9. 
\end{align*}
\] (6.27)

### 6.1.4 Extension to a low-Reynolds-number model

To apply the model to the near-wall region, the low-Reynolds-number and near-wall effects should be properly considered. Thus, we extend the foregoing model to a low-Reynolds-number type, following the discussion in Chapter 4 with some minor modifications based on the recent knowledge (Nagano & Shimada 1993; Nagano et al. 1994b).

First, concerning the model function \( f_{\lambda} \) in Eq. (6.9), we adopt the following form:

\[
f_{\lambda} = \left[ 1 + \frac{24}{R_t^3 f_{\lambda} f_{A}} \frac{2}{r_{Pr}} \exp \left\{ -\left( \frac{R_t}{30} \right)^{1.4} \right\} \right] \left\{ 1 - f_{w} (26) \right\}, \tag{6.28}\]

where \( f_{w}(A) \) is defined by Eq. (5.17). In the proximity to the wall, \( f_{A} \) given by Eq. (6.11) should be modified not to give an illegitimate influence to the field. Thus, in this study, the following modification is introduced:

\[
f_{A} = \left( \frac{2}{1 + 3.5\sqrt{b^2}} \right) \left\{ 1 + \left( \frac{1 + 3.5\sqrt{b^2}}{2} - 1 \right) f_{w} (26) \right\}. \tag{6.29}\]

The above expression for \( f_{A} \) is completely identical to Eq. (6.11) in the region far from the wall, and its effect vanishes (i.e., \( f_{A} \to 1 \)) in the vicinity of the wall. As for other model functions, the following formulae are adopted based on the discussion in Chapter 4.

\[
\begin{align*}
&f_{P1} = 1 - f_{w} (3), \quad f_{P2} = 1, \quad f_{D1} = 1 - f_{w} (3), \quad f_{D2} = \left( \frac{1}{C_{D2}} \right) (C_{e2} f_2 - 1) \{ 1 - f_{w} (7) \}, \\
&f_{2} = 1 - 0.3 \exp \{ - (R_t / 6.5)^2 \} \tag{Nagano et al. 1991}. \tag{6.30}
\end{align*}
\]

### 6.2 Validation of Performance of the Proposed Model

To validate the prediction accuracy of the proposed model, we applied it to some fundamental turbulent heat transfer fields, i.e., a homogeneous isotropic decaying flow...
CHAPTER 6. A NEW TWO-EQUATION HEAT-TRANSFER MODEL

(Sirivat & Warhaft 1983), a homogeneous shear flow (Tavoularis & Corrsin 1981), a flat-plate boundary-layer flow heated from the origin (Gibson et al. 1982), and a boundary-layer flow subjected to a sudden change of the wall thermal condition (Antonia et al. 1977). Calculations of the first two cases were performed with the Runge-Kutta method. The computational procedure used for other cases was completely the same as that used in Chapter 4. As for the velocity-field model in these calculations, we used the low-Reynolds-number $k$-$\varepsilon$ model proposed in Chapter 5, which is applicable to both wall and free turbulent flows. In calculating the heat transfer in a homogeneous isotropic decaying and a homogeneous shear flows, computational results with the Standard model [i.e., $C_u = 0.09$, $C_{t1} = 1.45$ and $C_{t2} = 1.9$ (Nagano & Tagawa 1990) for the velocity field, and $C_{t} = 0.1$, $C_{P1} = 1.8$, $C_{P2} = 0.77$, $C_{D1} = 2.0$, $C_{D2} = 0.9$ (Nagano et al. 1994b) and $\tau_m = k/\varepsilon$ in Eq. (6.1) for the thermal field] are also included for comparison.

The variations of $k$ and $\varepsilon$ in the homogeneous isotropic decaying flow corresponding to the experiment of Sirivat & Warhaft (1983) are shown in Fig. 6.1, where the computational results are in excellent agreement with the experimental data. Comparisons of the thermal field predictions with the experimental data are shown in Figs. 6.2 and 6.3. The computational results shown in Figs. 6.1 – 6.3 are normalized by the initial values in the calculations. Note that the initial values of $\varepsilon_i$ in the calculations with the proposed model are estimated using $k$, $\varepsilon$, $\overline{t^2}$ and $\overline{u^2}$ obtained from the experiment of Sirivat & Warhaft (1983). In the calculations with the Standard model, the same values of $k$, $\varepsilon$, $\overline{t^2}$ and $\varepsilon_i$ as estimated above are used as the initial values. From these figures, it can be seen that the prediction accuracy is considerably improved with the proposed model. The turbulent heat flux with the Standard model is surprisingly underpredicted, which may lead to underpredictions of the temperature variance $\overline{t^2}$ and dissipation rate $\varepsilon_i$. Here, if we accurately predict the heat transfer in this type of flow fields with the conventional heat-transfer models, the model constant $C_{t}$ in Eq. (6.1) need to be set around 0.3, which is considerably larger compared with the generally accepted value, i.e., $C_{t} = 0.1$.

Figure 6.4 shows the comparison of model predictions in the homogeneous shear flow corresponding to the experiment of Tavoularis & Corrsin (1981). In this flow field, the performance of the velocity-field model is already validated in Chapter 5 (see Figs. 5.3 – 5.5). We can readily understand from Fig. 6.4 that the proposed model dramatically
improves the prediction accuracy compared with the Standard model. In particular, the
heat flux in streamwise direction is successfully predicted with the new model, whereas
the Standard model cannot essentially calculate it. As described before, if we accurately
predict the heat transfer in homogeneous shear flows with the conventional heat-transfer
models, the model constant $C_\lambda$ in Eq. (6.1) should be around 0.04, which is considerably
smaller than the generally accepted value of $C_\lambda = 0.1$.

The computational results in the boundary-layer flow corresponding to the experiment
of Gibson et al. (1982) are shown in Figs. 6.5 - 6.7. As is clear from Fig. 6.5, the new
models for velocity- and temperature-fields give quite reasonable predictions as well as
do the models proposed in Chapters 3 and 4 (see Fig. 4.1). The peak values of turbulent
energy and temperature variance are around 5 as shown in Figs. 6.6 (a) and 6.7 (a), which
support the generally accepted knowledge. It can also be seen from Fig. 6.6 (b) that the
profile of dissipation rate gives the peak value just at the wall surface, which is consistent
with the recent insight obtained from the DNS (Kim et al. 1990; Kuroda et al. 1993).
Furthermore, the heat flux in streamwise direction is successfully predicted except for the
limited region in the proximity to the wall as shown in Fig. 6.7 (c). The predicted heat-
flux ratios $-\overline{u_t}/\overline{v_t}$ are around 1.4, which reasonably agree with the experimental finding
in wall-turbulent flows, i.e., $-\overline{u_t}/\overline{v_t} = 1.5-1.7$ (Antonia et al. 1977; Gibson et al. 1982;
Hishida et al. 1986). We can see from Fig. 6.7 (d) that the profiles of turbulent Prandtl
number $Pr_t$ and time-scale ratio $R$ in the near-wall region are in good agreement with the
DNS data of Kasagi et al. (1992). As discussed in Chapter 4, the generally accepted value
of $C_\lambda = 0.1$ is quite reasonable in predicting the heat transfer in wall-turbulent flows.

The turbulent characteristics in the homogeneous shear and boundary-layer flows ob-
tained from the computational results are compared in Figs. 6.8 - 6.10. From Figs. 6.8
and 6.9, we can see that, in the homogeneous shear flow, the ratios of the production
to destruction terms for velocity and temperature fields show considerably different as-
psects, whereas those in a boundary-layer flow show almost the same tendency. This may
be one of the reasons which cause a difficulty in calculating the turbulent heat transfer
in both wall and homogeneous shear flows with the previous two-equation models. The
variations of the shear parameter $\overline{U_{1,2}}k/\varepsilon$ shown in Fig. 6.10 give the generally accepted
tendency, i.e., $\overline{U_{1,2}}k/\varepsilon \simeq 6.2$ in the homogeneous shear flow and $\overline{U_{1,2}}k/\varepsilon \simeq 3.3$ in the
Table 6.1: Relation between estimated value of $C_\lambda$ and shear parameter, $\overline{U_{1,2} k}/\varepsilon$

<table>
<thead>
<tr>
<th></th>
<th>HIDF</th>
<th>WTF</th>
<th>HSF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\lambda$</td>
<td>0.3</td>
<td>0.1</td>
<td>0.04</td>
</tr>
<tr>
<td>$\overline{U_{1,2} k}/\varepsilon$</td>
<td>0</td>
<td>3.3</td>
<td>6.2</td>
</tr>
</tbody>
</table>

local-equilibrium region of the boundary-layer flow.

From these discussions, it is understood that the conventional two-equation heat-transfer models have an essential difficulty in predicting the heat transfer in both wall and free turbulent flows. It is of great importance that the estimated value of $C_\lambda$ in Eq. (6.1) becomes smaller with an increase of the shear parameter $\overline{U_{1,2} k}/\varepsilon$, as shown in Table 6.1. Note that, in Table 6.1, the terms HIDF, WTF and HSF denote homogeneous isotropic decaying, wall-turbulent and homogeneous shear flows, respectively. This indicates that the characteristic time scale dominating turbulent heat transfer becomes smaller with an increase of the shear rate (or anisotropy), including both wall and free turbulent flow fields. The insight obtained here is consistent with the generally accepted knowledge as described in Eq. (6.11).

The predicted streamwise developments of mean-temperature, turbulent heat flux and temperature variance corresponding to the experiment of Antonia et al. (1977) are shown in Fig. 6.11, from which it can be seen that the new model gives reasonable predictions as well as do the models proposed in Chapter 4 (see Figs. 4.2 and 4.3). Note that the results of the constant-$Pr_t$ calculation in Fig. 6.11 (a) are identical to those in Fig. 4.2. Figure 6.12 shows the streamwise developments of turbulent Prandtl number and time-scale ratio. As indicated in Chapter 4, the calculated turbulent Prandtl numbers are much higher than the conventional value of $Pr_t = 0.9$ and, of course, not constant. On the other hand, the values of time-scale ratio are lower than the conventional value of $R \simeq 0.5$. It should be repeated that the present results of the turbulent Prandtl number agree with the experimental fact indicated by Sato et al. (1992).

Finally, it should be emphasized that accurate predictions of the heat transfer in both wall and free turbulent flows cannot be obtained without a reasonable representation for the characteristic time scale $\tau_m$ taking into account the dissimilarity between the velocity
and temperature fields, anisotropy in the flow field, magnitude of the shear rate, and near-wall and low-Reynolds-number effects.
Figure 6.1: Comparison with experiment of Sirivat & Warhaft (1983) in homogeneous decaying flow (Velocity field): (a) Turbulent energy; (b) Dissipation rate.
Figure 6.2: Comparison with experiment of Sirivat & Warhaft (1983) in homogeneous decaying flow ($\bar{U}_1 = 6.3$ m/s): Present model, Standard model; (a) Temperature variance; (b) Dissipation rate of $\overline{\epsilon^2}/2$; (c) Turbulent heat flux; (d) Ratio of production and dissipation rate.
CHAPTER 6. A NEW TWO-EQUATION HEAT-TRANSFER MODEL

Figure 6.2: (continued)
Figure 6.3: Comparison with experiment of Sirivat & Warhaft (1983) in homogeneous decaying flow ($\overline{U}_1 = 3.4$ m/s): ——— Present model, —— Standard model; (a) Temperature variance; (b) Dissipation rate of $\overline{t^2}/2$; (c) Turbulent heat flux; (d) Ratio of production and dissipation rate.
Figure 6.3: (continued)
Figure 6.4: Comparison with experiment of Tavoularis & Corrsin (1981) in homogeneous shear flow: (a) Temperature variance; (b) Dissipation rate of \( \overline{\varepsilon}^2 / 2 \); (c) Turbulent heat flux; (d) Turbulent Prandtl number and time-scale ratio.
Figure 6.4: (continued)
Figure 6.5: Comparison with experiment in boundary-layer flow (Gibson et al. 1982, $\overline{U}_e/\nu = 1.41 \times 10^6 \text{ m}^{-1}$): (a) Mean velocity; (b) Mean temperature.
Figure 6.6: Profiles of turbulent quantities in boundary-layer flow (Velocity field): (a) Turbulent energy; (b) Dissipation rate.
Figure 6.7: Profiles of turbulent quantities in boundary-layer flow (Thermal field): (a) Temperature variance; (b) Dissipation rate of $\overline{\nu^2}/2$; (c) Turbulent heat flux; (d) Turbulent Prandtl number and time-scale ratio.
CHAPTER 6. A NEW TWO-EQUATION HEAT-TRANSFER MODEL

Figure 6.7: (continued)
Figure 6.8: Comparison of ratio of production and dissipation rate (Velocity field): (a) Homogeneous shear flow; (b) Boundary-layer flow.
CHAPTER 6. A NEW TWO-EQUATION HEAT-TRANSFER MODEL

Figure 6.9: Comparison of ratio of production and dissipation rate (Thermal field): (a) Homogeneous shear flow; (b) Boundary-layer flow.
Figure 6.10: Comparison of shear parameter, $\bar{U}_{1,2}k/\varepsilon$: (a) Homogeneous shear flow; (b) Boundary-layer flow.
Figure 6.11: Comparison of computational results with experiment by Antonia et al. (1977): (a) Mean temperature; (b) Turbulent heat flux; (c) temperature variance.
Figure 6.12: Variation of turbulent Prandtl number and time-scale ratio: (a) Turbulent Prandtl number; (b) Time-scale ratio.
Chapter 7

CONCLUSIONS

To calculate complex turbulent fluid flow and heat transfer, four turbulence models have been developed. The conclusions obtained from the present study are summarized as follows.

In Chapter 3, a modified version of the NT model has been proposed which allows application to separating and reattaching flows. The main results obtained from Chapter 3 are as follows:

1. To allow for the application of low-Reynolds-number turbulence models to complex turbulent flows with separation, it is legitimate to substitute the Kolmogorov velocity scale $u_\varepsilon = (\nu \varepsilon)^{1/4}$ for the friction velocity $u_\tau$ because the velocity scale $u_\varepsilon$ vanishes at neither the separating nor reattaching points, in contrast to the friction velocity $u_\tau$.

2. The proposed model using the velocity scale $u_\varepsilon$ in the model functions is applicable to complex turbulent flows with separation and reattachment, holding the correct near-wall asymptotic relations of turbulence.

3. The mean flows and turbulent quantities of backward-facing step flows are predicted quite successfully by the proposed model. In particular, the calculated flow reattachment lengths show excellent agreement with measurements under a variety of experimental conditions.

4. The streamwise variation of eddy viscosity normalized by the free stream velocity and reattachment length shows almost the same tendency at high Reynolds
numbers.

5. The velocity profiles along the opposite wall deviate from the standard log-law in the reattachment region, a phenomenon which is mainly due to the presence of an adverse pressure gradient in a separating and reattaching flow. This tendency is completely consistent with the latest experimental findings.

6. With the information from an anisotropy tensor, it is possible to estimate the fluctuating friction coefficient on the wall, which is regarded as one of the most important turbulent characteristics closely related to the heat transfer in separating and reattaching flows.

In Chapter 4, to predict turbulent heat transfer in separating and reattaching flows, a two-equation heat-transfer model, which is a modified version of the NTT model, has been developed. The following are the major conclusions obtained from Chapter 4:

1. The introduction of the Kolmogorov velocity scale $u_*$ in the model functions is also effective in developing a heat-transfer model applicable to complex turbulent heat transfer problems in separating and reattaching flows.

2. The accurate prediction of heat transfer in separating flows is impossible without reliable predictions of the flow field in the vicinity of the wall in the recirculating region. Thus, it is indispensable to use a low-Reynolds-number turbulence model such as the model described in Chapter 3, which can fully resolve the near-wall region.

3. The composite time scale, which gives weight to a shorter scale among the velocity- and temperature-field time scales, is the most appropriate for the characteristic time scale for turbulent heat transfer.

4. The proposed model quite successfully predicts complex turbulent heat transfer in an attached boundary-layer flow subjected to a sudden change in wall thermal conditions, and in a separating and reattaching flow downstream of a backward-facing step. Especially, the calculated Stanton numbers in backward-facing step flows are in excellent agreement with the experimental data under various conditions of the Reynolds number and upstream boundary-layer thickness.

5. The turbulent-energy balance in the flow reattachment zone is totally different
from that in a boundary-layer flow. The budget of temperature variance, on the other hand, remains similar to that in a boundary-layer flow, at least qualitatively.

6. In the recirculating region, the turbulent Prandtl number has a value substantially higher than the standard one of $Pr_t = 0.9$. Such a phenomenon can also be seen in some classes of attached flows subjected to a sudden change in wall thermal conditions, and thus should be regarded as a natural consequence of dissimilarity between the velocity and temperature fields.

7. The conventional log-laws not only for the velocity but also for the temperature cannot be applied to complex turbulent flows with separation.

8. The heat-transfer coefficient strongly depends on the near-wall turbulence intensity, which is essentially dominated by the variation in turbulent energy (or eddy viscosity) in the separated shear layer near the reattachment point.

In Chapter 5, a new type of $k$-$\varepsilon$ model which incorporates some essential characteristics of second-moment closures has been proposed. The key results obtained from Chapter 5 are as follows:

1. By considering the relation between the explicit expressions of simplified second-order closure models and the conventional type of $k$-$\varepsilon$ models, a more sophisticated $k$-$\varepsilon$ model can be constructed. This new model maintains the advantage of the existing conventional $k$-$\varepsilon$ models, e.g., computing time, memory capacity and stability.

2. In the present model, the realizability of the anisotropy tensor is sufficiently guaranteed in two-dimensional flow field, with only a minor modification for the Reynolds-stress expression.

3. The calculations show that the present model is capable of predicting both wall-turbulent and homogeneous shear flows, the latter of which has been very difficult to accurately predict with previous two-equation turbulence models.

4. In channel flows with injection and suction at the wall surfaces, and in separating and reattaching flows downstream of a backward-facing step, a comparison of computational results with experimental and DNS data indicates that the present model is also effective in calculating complex turbulent flows of engineering interest.
5. Computational results show that the Kolmogorov velocity scale $u_\varepsilon$ in the model functions is useful in predicting turbulent flows with complex boundary conditions, i.e., flow injection and suction at the wall surfaces.

6. Some valuable insights emerged from a detailed investigation of the parameters in the model functions based on a comparison of computational results by the present model with those of several existing models. In particular, the use of low-Reynolds-number parameters constructed by more than two of the turbulence characteristics poses great difficulty in application to complex turbulent flows. For example, the low-Reynolds-number models using $R_t = k^2/\nu \varepsilon$ and $R_s = k/\nu S$ show considerable discrepancies with experimental and DNS data, although these parameters are generally regarded as effective because they do not include the distance from the wall.

In Chapter 6, a new type of two-equation heat-transfer model for predicting heat transfer in both wall and free turbulent flows has been presented. The following are the important conclusions obtained from Chapter 6:

1. By taking into account the relation between the explicit expressions of simplified second-order modeling and the conventional type of two-equation heat-transfer models, a more sophisticated two-equation heat-transfer model can be developed; one which maintains the advantages of the existing conventional models.

2. Application to some fundamental turbulent flows, i.e., a homogeneous isotropic decaying flow, a homogeneous shear flow, a boundary-layer flow heated from the origin and a boundary-layer flow subjected to a sudden change in wall thermal conditions, shows that the proposed model can successfully predict the turbulent heat transfer in both wall and homogeneous turbulent flows, though such a prediction had been close to impossible with existing two-equation heat-transfer models.

In summary, the turbulence models newly proposed in this study are considered highly effective for analyzing complex fluid flow and heat transfer involving flow separation and reattachment.
BIBLIOGRAPHY


Avva, R. K., Kline, S. J. & Ferziger, J. H. 1988b Computation of the turbulent flow over a backward-facing step using the zonal modeling approach. TF-33, Thermosciences Division, Department of Mechanical Engineering, Stanford University.


Kim, J. et al. 1990 The collaborative testing of turbulence models (Organized by P. Bradshaw et al.). Data disk No. 4.


BIBLIOGRAPHY


Leschziner, M. A. 1982 An introduction and guide to the computer code PASSABLE. UMIST.


