QUASI-COHERENT STRUCTURES IN TURBULENT BOUNDARY LAYER SUBJECTED TO ADVERSE PRESSURE GRADIENT

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QUASI-COHESENT STRUCTURES IN
TURBULENT BOUNDARY LAYER SUBJECT TO ADVERSE PRESSURE GRADIENT

by

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Abstract

A turbulent boundary layer subjected to a sustained adverse pressure gradient is experimentally investigated. Waveforms of fluctuating velocity components in the boundary layer, especially in the near-wall region, are remarkably elongated in time compared with those in zero-pressure-gradient flows, and thus time scales increase with an increasing pressure gradient parameter $P^+$. The increase in the time scales is not in proportion to the corresponding increase in the conventional viscous time scale $\nu/u^2$. It is found that the Taylor time scale is the most appropriate to describe the essential characteristics of non-equilibrium adverse pressure gradient flows. Even the near wall-limiting behavior of streamwise velocity fluctuations for different $P^+$ is well correlated in the coordinates based on the Taylor time scale.

However, a change in the coherent structures may not be identified solely with a change in the time scale. To identify any scale-irrelevant structures hidden in the flow, we next investigate the dynamical features of adverse-pressure-gradient flows. As the pressure gradient parameter $P^+$ increases, the turbulent energy and shear stress transport, $\overline{\nu u^2}$ and $\overline{\nu u\upsilon}$, become significant in the direction toward the wall from the regions away from the wall, in contrast to those in zero-pressure-gradient cases. The quadrant splitting and trajectory analyses reveal that obvious changes do occur in large-amplitude sweep motions ($Q_4$) and outward interactions ($Q_1$). On the other hand, the contributions from other coherent motions, especially the ejection motions ($Q_2$), significantly decrease and grow longer in duration, i.e., these motions are dull and less active. Moreover, multi-point simultaneous measurements with five X-probes are made to depict the kinematic pictures of the effects of the adverse pressure gradient on the eddy structures.
Acknowledgments

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Finally, the author would like to express his gratitude and appreciation to his family for their continuous encouragement and support.
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Nomenclature

$C_p$  
wall static pressure coefficient, $C_p = (P - P_0)/(\rho U_0^2/2)$

$C_{pq}$  
expansion coefficient, Eq. (4.6)

$E_{11}$  
wave number spectrum of velocity fluctuation $u$, Eq. (3.13)

$E_u, E_v$  
spectrum functions of velocity fluctuations $u$ and $v$, Eqs. (3.3) and (3.4)

$E_{\dot{u}}$  
dissipation spectrum of velocity fluctuations $u$, Eq. (3.5)

$f$  
frequency

$f'$  
dimensionless frequency, $f' = f \nu/U_0^2$

$f''$  
dimensionless frequency, $f'' = f \tau_E$

$H$  
threshold, $H = |\hat{u}| \hat{v}|$; shape factor, $H = \delta^*/\theta$

$h$  
threshold, $h = |\hat{u}|$ and/or $|\hat{v}|$

$H_n$  
Hermite polynomial, Eq. (4.7)

$I_i$  
indicator function, Eq. (4.2)

$k$  
turbulent kinetic energy, $k = (\bar{u}^2 + \bar{v}^2 + \bar{w}^2)/2$

$k'$  
instantaneous turbulent kinetic energy, $k' = (u^2 + v^2 + w^2)/2$

$k_1$  
wave number, $k_1 = 2\pi f/U$

$k_{pq}$  
cumulant

$N_0$  
average number of zeros of $u$-fluctuation per unit time

$P$  
joint p.d.f. for velocity fluctuations $u$ and $v$

$\mathcal{P}$  
mean pressure

$P^+$  
dimensionless pressure gradient parameter, $P^+ = \nu(d\mathcal{P}/dx)/\rho u_r^3$
\( P_0 \)  
reference inlet pressure

\( R_{ij} \)  
two-point autocorrelation function

\( R_u \)  
autocorrelation coefficient of \( u \)

\( \tilde{R}_u \)  
short-time averaged autocorrelation coefficient of \( u \), Eq. (3.18)

\( R_\theta \)  
Reynolds number based on momentum thickness, \( R_\theta = \bar{U} e \theta / \nu \)

\( \bar{T}_B \)  
mean burst period

\( T_{DP} \)  
data-processing time

\( T_E \)  
Eulerian integral time scale, \( T_E = \int_0^\infty R_u(\Delta t) d(\Delta t) \)

\( \bar{T}_i, \Delta T_i \)  
mean period and mean duration of the \( i \)th-quadrant motion

\( t, t_o \)  
time and reference time

\( \Delta t \)  
lag time; sampling interval

\( U \)  
instantaneous velocity in \( x \) direction

\( \bar{U}, \bar{V} \)  
mean velocities in \( x \) and \( y \) directions

\( \bar{U}_e \)  
free-stream velocity

\( \bar{U}_m \)  
apparent velocity measured by a hot-wire near wall

\( \bar{U}_0 \)  
reference inlet velocity

\( \bar{U} \)  
short time averaged velocity in \( x \) direction, Eq. (3.19)

\( u, v, w \)  
fluctuating velocity components in \( x, y \) and \( z \) directions

\( u_\tau \)  
friction velocity, \( u_\tau = \sqrt{\tau_w / \rho} \)

\( W_m \)  
weighted p.d.f. of moment \( m \), Eq. (4.10)

\( x, y, z \)  
streamwise, wall-normal and spanwise coordinates

\( \bar{Y} \)  
average position of interface between turbulent and non-turbulent fluid

\( y^+ \)  
dimensionless distance from wall, \( y^+ = u_\tau y / \nu \)

**Greek Symbols**

\( \alpha \)  
universal constant, Eq. (3.13)

\( \beta \)  
Clauser pressure gradient parameter, \( \beta = (\delta^* / \tau_w) d\bar{P} / dx \)

\( \gamma \)  
intermittency factor
boundary layer thickness defined at location where mean velocity is equal to 99% of free-stream velocity

displacement and momentum thicknesses

dissipation rate of $k$

eigenvalue

dynamic viscosity

kinematic viscosity, $\nu = \mu/\rho$

density

r.m.s. value of burst period; r.m.s. value of difference between instantaneous and average position of interface between turbulent and non-turbulent fluid

sign functions, Eq. (4.4)

Taylor time scale or Eulerian dissipation time scale, $\tau_E = \sqrt{2u^2/(\partial u/\partial t)^2}$

Taylor time scale in linear (viscous) sublayer

time scale corresponding to mean shear rate, $\tau_m = (\partial \overline{U}/\partial y)^{-1}$

time scale for energy-containing eddies, $\tau_u = k/\varepsilon$

wall shear stress

Kolmogorov time scale, $\tau_\eta = \sqrt{\nu/\varepsilon}$

eigenfunction

random variable

outer edge of boundary layer

reference inlet point

normalization by r.m.s. value

time mean value

conditional average in the $i$th-quadrant of the $(u, v)$-plane

short time averaged value

normalization by inner variables $(u_\tau, \nu)$
ensemble-averaged value

differentiation by time

**Abbreviation**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>APG</td>
<td>adverse-pressure-gradient</td>
</tr>
<tr>
<td>DNS</td>
<td>direct numerical simulation</td>
</tr>
<tr>
<td>p.d.f.</td>
<td>probability density function</td>
</tr>
<tr>
<td>r.m.s.</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>ZPG</td>
<td>zero-pressure-gradient</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background

In theory as well as in practice, it is of fundamental importance to investigate the effects of pressure gradients on the structure of turbulent boundary layers. From the viewpoint of engineering, the efficiency of fluid machinery, such as a diffuser and turbine blades, is often restricted by the occurrence of separation due to a pressure rise in the flow direction. Therefore, it is very important to elucidate the effects of an adverse-pressure-gradient (APG) and establish a flow control method to avoid flow separation. In addition, the study of turbulent boundary layers subjected to a pressure gradient promises the further benefit of a deeper understanding of wall turbulence, which may not appear under an equilibrium zero-pressure-gradient (ZPG) condition.

For more than forty years, turbulent boundary layers with a sustained APG have been studied by many investigators. Among these previous studies, the experimental investigations conducted by Clauser (1954) and Bradshaw (1967) and the theoretical one by Rotta (1962) are famous with regard to the equilibrium boundary layer (with the pressure gradient parameter kept constant), which preserves the self-similar characteristics in the flow direction. The equilibrium
APG boundary layer has produced some interest in the statistical features of the self-similarity, and is thus now the subject of experiments (Skåre and Krogstad 1994) and direct numerical simulations (DNS) (Stoke et al. 1998).

In real situations, however, the equilibrium condition is not necessarily satisfied, and the various characteristics of turbulent flows may emerge from more general conditions. For example, the development of a boundary layer after a sudden change in external conditions (Townsend 1976, see Fig. 1.1) is typical. Thus, many researchers have been investigating the boundary layers under non-equilibrium pressure-gradient conditions. Research on non-equilibrium APG boundary layers has been conducted both experimentally (Samuel and Joubert 1974) and numerically (Spalart and Watmuff 1993, Na and Moin 1996). However, the lack of accurate information and superficial discussions on the effects of APGs leave much room for further investigation of non-equilibrium boundary layers.

From a previous experiment on an APG turbulent boundary layer with the same experimental apparatus as that used in the present study (Nagano et al. 1992), it was found that: (1) the standard log-law velocity profile for a ZPG boundary layer does not hold in APG turbulent boundary layers; (2) near-wall distributions of r.m.s. velocity fluctuations cannot scale with the wall parameters \( u_\tau \) and \( \nu \); and (3) the response time of turbulence to the imposed APG, which relates closely to the redistribution process of turbulent kinetic energy, differs among the streamwise, wall-normal and spanwise velocity components. Although the viscous wall unit is a standard parameter for scaling the equilibrium turbulent boundary layers (Skåre and Krogstad 1994), the above imply that another characteristic time or length scale must be introduced in order to scale the non-equilibrium APG flows.

Recently, it has been confirmed that near-wall quasi-coherent structures in a ZPG flow play a key role in the turbulent transport mechanism (Robinson 1991), and there has been an increasing number of studies dealing with so-called boundary-layer-control in such a flow. However, there is only scanty information on the quasi-coherent structures indispensable for the control of
turbulence in APG flow.

1.2 Objectives

Given the background described in the previous section, the present study has the following six main objectives:

1) to reveal the in-depth turbulent structure inherent in APG flows;

2) to find an appropriate time scale for the universal scaling of the near-wall turbulent statistics in APG flows;

3) to understand the physical significance and roles of the time scale in characterizing the APG flows;

4) to find the structures, if any, which cannot be described in terms of the time scale;

5) to investigate the effects of APGs on the quasi-coherent structures using conditional sampling techniques;

6) to gain a deeper insight into the effects of APGs on eddy structures by multi-point simultaneous measurement.

1.3 Organization of Dissertation

The present thesis consists of two main parts: one concerned with the statistical characteristics and the scaling law for a non-equilibrium APG boundary layer; and the other concerned with the dynamical characteristics of the quasi-coherent structures in the APG boundary layer.

Chapter 2 describes the experimental arrangement and the measurement techniques employed in this study.
In Chapter 3, the statistical characteristics and the changes in the temporal features of the APG boundary layer are presented, and the scaling law for this layer is revealed (Nagano et al. 1997, 1998).

In Chapter 4, the dynamical characteristics of the quasi-coherent structures in the APG boundary layer are investigated (Houra et al. 1999, 2000) using conditional sampling techniques, i.e., quadrant splitting (Lu and Willmarth 1973, Nagano and Tagawa 1988, 1990) and trajectory analyses (Nagano and Tagawa 1995), and through observation of the instantaneous flow fields interpolated with the Karhunen-Loève expansion (Holmes et al. 1996).

The major conclusions of the study are summarized in Chapter 5.

The interpolation method with the Karhunen-Loève expansion is described in Appendix A.
Chapter 1. Introduction

Normal development
Region of rapid change

Inner equilibrium layer
Total head varying along streamlines
Reynolds stress constant along streamlines
Critical surface
Total head constant along streamlines

Figure 1.1 Development of a boundary layer after a sudden change in external conditions (from Townsend 1976; Fig. 7.12).
Chapter 2

Experimental Apparatus

The experimental apparatus used for this study is the same as that described by Nagano et al. (1992). The test section is shown in Fig. 2.1. The working fluid (air) flows successively through the settling chamber, two-dimensional contraction, test section, plenum chamber and blower. The test section (Fig. 2.1) is composed of a flat-plate on which a turbulent boundary layer develops, and a roof-plate to adjust the pressure gradients. The aspect ratio at the inlet to the test section is 13.8 (50.7 mm high × 700 mm wide). Under the present measurement conditions, the free-stream turbulence level is below 0.15% and velocity non-uniformities in the $y$-(normal to the wall) and $z$-(spanwise) directions are within 0.17% ($2.2 \, \text{mm} \leq y \leq 48.5 \, \text{mm}$) and 0.63% ($-200 \, \text{mm} \leq z \leq 200 \, \text{mm}$), respectively. Therefore, a nearly ideal, two-dimensional uniform inflow is obtained. To generate a stable turbulent boundary layer on the flat-plate, a row of equilateral triangle plates (length of one side: 10 mm; thickness: 1 mm) is located at the inlet to the test section as a tripping device. It was confirmed that even at the end of the test section the laminar boundary layer on the pressure-adjusting roof-plate is separated by the uniform free-stream from the objective turbulent boundary layer developing on the flat-plate. Thus, there are no interactions between the two (see Nagano et al. 1992).

Wall static pressures were measured with a Göttingen-type manometer equipped with a
microscope (measurement accuracy: $\pm 0.01$ mm). Pressure taps with a 0.5 mm diameter are located at 20 mm apart from the centerline of the flat-plate and at 50 mm intervals from the inlet.

Velocity measurements were made with hot-wire probes, i.e., a handmade subminiature (Ligrani and Bradshaw 1987) normal hot-wire (diameter: 3.1 $\mu$m; length: 0.6 mm), and two types of specially devised X-probes (diameter: 3.1 $\mu$m; length: 0.6 mm $\simeq 7.5 \nu/u_\tau$; wire spacing: 0.30 mm $\simeq 3.8 \nu/u_\tau$ for measurement of $u$ and $v$, and 0.23 mm $\simeq 2.9 \nu/u_\tau$ for $u$ and $w$). An array of five X-probes aligned in the wall-normal direction is employed for simultaneous measurement in a flow field (see Fig. 2.2). The shape of this array is designed to measure the flow field simultaneously, where the effects of APG become significant [i.e., the buffer region: $y^+ \simeq 20$, the log region: $y^+ \simeq 100$, the outer edge of the log region: $y^+ \simeq 200$, and two locations in the outer layer: $y^+ \simeq 300$ and $y^+ \simeq 400$ ($y/\delta_{99} \simeq 0.7$)].

To convert the hot-wire outputs into velocity components, we used the well-established look-up-table method (Lueptow et al. 1988). In addition, the bias error, which is ascribed to the finite separation of the wires when using an X-probe, was removed in accordance with the procedure described by Tagawa et al. (1992). As a result, the measured velocity fluctuations near the wall in the ZPG flow show good quantitative agreement with the DNS data (Spalart 1988) for the ZPG flow (see Nagano et al. 1992). As a traversing mechanism, we used a finely adjustable positioning device equipped with a micrometer having a positioning accuracy of $\pm 0.01$ mm. The absolute distance from the wall is determined by measuring, with a telescope, the distance between the real wire and its reflected image on the flat-plate.

The friction velocities in the APG flows are determined with the method of Nagano et al. (1992). In the vicinity of the wall, the apparent velocities $\overline{U}_m^+$ measured with hot wires deviate systematically from the linear profiles, $\overline{U}^+ = y^+$, as the wall is approached. This can be ascribed to the wall proximity effect of hot-wire outputs. Previous extensive studies investigating this wall proximity effect (for example, Oka and Kostić 1972, Hebbar and Melnik 1978,
Bhatia et al. 1982, Janke 1987, Chew et al. 1995) have confirmed experimentally and numerically that once the material of the wall, the geometrical factors of the hot-wire, and the operating overheat ratio are given, the amount of this deviation can be determined universally, independent of values of wall shear stress, so that \( \overline{U}_m^+ = f(y^+) \). As seen in Fig. 2.3, the apparent velocities \( \overline{U}_m^+ \) in the present experiments collapse very well onto a unique curve \( f(y^+) \), and are consistent with the previous findings. We utilize this relationship to determine the friction velocities \( u_\tau \) as follows. First, we determine the friction velocities by using the established Clauser method (Clauser 1954) in the ZPG flows, where the existence of the log-law has been definitely confirmed, and determine the near-wall relationship \( \overline{U}_m^+ = f(y^+) \). Then, in APG flows, we use this relationship in reverse as a calibration curve to determine the friction velocities \( u_\tau \).

In this study, we have conducted the measurements in a turbulent boundary layer under both APG and ZPG flow conditions, as shown in Fig. 2.4. The important flow parameters of the present measurements are listed in Tables 2.1 (for the investigation of the statistical features in Chapter 3) and 2.2 (for the conditional analysis in Chapter 4). In the APG flow, the pressure gradient \( dC_p/dx \) keeps a nearly constant value of 0.6 m\(^{-1}\) over the region \( 65 \text{ mm} \leq x \leq 700 \text{ mm} \), and then decreases slowly \( (x) \) is the streamwise distance from a tripping point). On the other hand, the pressure gradient parameter normalized by inner variables \( P^+ \) and the Clauser parameter \( \beta \) increase monotonously, thus yielding moderate to strong adverse pressure gradients (Huffman and Bradshaw 1972).
Chapter 2. Experimental Apparatus

Table 2.1  Flow parameters ($U_0 = 10.8$ m/s).

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>$U_e$ (m/s)</th>
<th>$u_r$ (m/s)</th>
<th>$\delta_{99}$ (mm)</th>
<th>$\delta^*$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$H$ (mm)</th>
<th>$R_{\theta}$ (mm)</th>
<th>$P^+$ (mm)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZPG 525</td>
<td>10.8</td>
<td>0.481</td>
<td>13.3</td>
<td>2.27</td>
<td>1.56</td>
<td>1.45</td>
<td>1070</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>925</td>
<td>10.8</td>
<td>0.465</td>
<td>19.9</td>
<td>3.39</td>
<td>2.38</td>
<td>1.43</td>
<td>1620</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>APG 523</td>
<td>9.08</td>
<td>0.390</td>
<td>16.2</td>
<td>3.31</td>
<td>2.19</td>
<td>1.52</td>
<td>1290</td>
<td>$9.12 \times 10^{-3}$</td>
<td>0.77</td>
</tr>
<tr>
<td>723</td>
<td>8.18</td>
<td>0.307</td>
<td>24.6</td>
<td>5.83</td>
<td>3.62</td>
<td>1.61</td>
<td>1880</td>
<td>$1.93 \times 10^{-2}$</td>
<td>2.19</td>
</tr>
<tr>
<td>925</td>
<td>7.54</td>
<td>0.251</td>
<td>34.2</td>
<td>9.53</td>
<td>5.47</td>
<td>1.74</td>
<td>2660</td>
<td>$2.56 \times 10^{-2}$</td>
<td>3.95</td>
</tr>
<tr>
<td>1121</td>
<td>6.68</td>
<td>0.197</td>
<td>46.1</td>
<td>14.49</td>
<td>7.72</td>
<td>1.88</td>
<td>3350</td>
<td>$2.87 \times 10^{-2}$</td>
<td>5.32</td>
</tr>
</tbody>
</table>

Table 2.2  Flow parameters ($U_0 = 11.0$ m/s).

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>$U_e$ (m/s)</th>
<th>$u_r$ (m/s)</th>
<th>$\delta_{99}$ (mm)</th>
<th>$\delta^*$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$H$ (mm)</th>
<th>$R_{\theta}$ (mm)</th>
<th>$P^+$ (mm)</th>
<th>$\beta$</th>
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<td>17.5</td>
<td>3.85</td>
<td>2.48</td>
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<td>1.81</td>
<td>2950</td>
<td>$3.08 \times 10^{-2}$</td>
<td>5.18</td>
</tr>
</tbody>
</table>

Figure 2.1  Test section. (All dimensions in millimeters)
Chapter 2. Experimental Apparatus

Figure 2.2 Array of five X-probes for multi-point simultaneous measurement.

(All dimensions in millimeters)
Figure 2.3 Near-wall distributions of apparent velocities $U_m^+$ in various zero-pressure-gradient flows.

Figure 2.4 Development of the pressure coefficient.
Chapter 3

Statistical Characteristics in Adverse-Pressure-Gradient Flow

3.1 Mean Velocity

Figure 3.1 shows the development of the two kinds of turbulent boundary layer. One is an objective turbulent boundary layer subjected to adverse pressure gradients, and the other is a turbulent boundary layer without the pressure gradient for comparison (i.e., as a control). The pressure distribution shown in Fig. 2.4 does not preserve the equilibrium condition (i.e., $\beta = \text{const.}$); thus, the mean velocity profile in an adverse-pressure-gradient flow develops in such a way as to increase the thickness of the layer and the shape factor $H(=\delta^*/\theta)$.

Figure 3.2 shows the mean velocity profiles normalized by the friction velocity $u_\tau$. The friction velocities are obtained using the least-squares curve fitting method (to the data in the range $1.1 \leq y^+ \leq 5$), in which corrections were made for the mean velocity $\bar{U}^+$ near the wall ($y^+ < 3.6$) by subtracting the quantity $\Delta U_m^+ = f(y^+) - \bar{U}^+$ shown in Fig. 2.3 from the apparent measurement $\bar{U}_m^+$ in APG flows.

As is clearly seen from this figure, the velocity profiles in APG flows lie below the following
Chapter 3. Statistical Characteristics in APG Flow

‘standard’ log-law profile for ZPG flows:

\[ U^+ = 2.44 \ln y^+ + 5.0. \] (3.1)

This important characteristic of the APG flows conforms to our previous result (Nagano et al. 1992), and is also confirmed by the direct numerical simulation (DNS) of Spalart and Watmuff (1993) and by the recent measurement of Debisschop and Nieuwstadt (1996). Moreover, this finding is corroborated by the recent DNS for an APG recovery region of a backward-facing step flow (Le et al. 1997).

The mean velocity profiles in the various outer coordinates shown in Figs. 3.3 (a), (b) and (c) represent other characteristics of the APG boundary layer. Figure 3.3 (a) shows the defect profiles written as:

\[ \frac{(U - U_e)}{u_\tau} = g(y/\delta_{99}, \beta), \] (3.2)

where \( \beta \) is the pressure gradient parameter, which represents the effects of APG. With increasing \( \beta \) (\( P^+ \) increases in the same manner as \( \beta \); thus, in what follows, \( P^+ \) is used to describe the effects of APG), the defect in the mean velocity \( U \) from the free-stream velocity \( U_e \) becomes larger in the near-wall region in APG flows than in ZPG flows. Figure 3.3 (b) shows the mean velocity profiles normalized by the free-stream velocity \( U_e \). Note that on the basis of the normalization by \( U_e \), the mean velocity gradient in the outer region of the APG flow becomes larger than that in the ZPG flow. Thus, this may account for the significant production of the turbulent kinetic energy in the outer region (e.g., Bradshaw 1967). However, as shown in Fig. 3.3 (c), the mean velocity profiles normalized by the ‘constant’ reference inlet velocity \( U_0 \) show little change in the velocity gradient in the outer region of the APG flows (Bradshaw 1994).
3.2 Turbulence Intensities

Figure 3.4 shows the distributions of the turbulence intensities in APG flows, normalized by the friction velocity \( u_\tau \). As \( P^+ \) increases, all of the velocity fluctuation components \( (\sqrt{u'^2}/u_\tau, \sqrt{v'^2}/u_\tau, \sqrt{w'^2}/u_\tau) \) become remarkably large in the outer region of the APG boundary layers. The intensity profiles of the fluctuating velocity components \( u, v \) and \( w \), normalized by the free-stream velocity \( U_0 \) at the inlet to the test section are presented in Figs. 3.5 and 3.6. The abscissa is the distance from the wall normalized with the boundary layer thickness \( \delta_{99} \). With an increasing APG effect, the reduction in turbulence intensities can be seen in the wall region (see Fig. 3.5, \( y/\delta_{99} < 0.4 \)), whereas all the profiles in the outer layer are kept unchanged (see Fig. 3.6). Thus, turbulence intensities are considered to be unchanged along streamlines of the mean flow lying outside the wall region, since the streamlines and the lines of constant \( y/\delta_{99} \) are approximately the same. The APG changes the intensities of velocity fluctuations near the wall in the order of streamwise \( (u) \), spanwise \( (w) \), and wall-normal \( (v) \) components. These profiles cannot be correlated in conventional wall coordinates even in the near-wall region. As shown in Fig. 3.7, the distributions of \( \sqrt{u'^2}/u_\tau \) near the wall follow each \( P^+ \)-dependent straight line that coincides with the origin. The same tendency is also confirmed from the DNS (Spalart and Watmuff 1993). This means that the viscous wall unit cannot be used to describe the unique features of the present and DNS’s APG flows.

3.3 Reynolds Shear Stress

Figure 3.8 (a) shows the profiles of Reynolds shear stress in the wall coordinates, \( -\overline{uv}/u_\tau^2 \) in APG flows. With increasing \( P^+ \), \( -\overline{uv}/u_\tau^2 \) increases in the outer region. Thus, the constant-stress-layer relationship \( -\overline{uv}/u_\tau^2 \sim 1 \) observed in ZPG flows is no longer valid. This, too, may account for the nonexistence of the universal law of the wall in APG boundary layers. The Reynolds shear stress profiles, \( -\overline{uv}/U_0^2 \), in the same outer coordinate as the intensity profiles in
Figs. 3.5 and 3.6 are shown in Figs. 3.8 (b) and (c), respectively. With an increasing APG effect, a reduction is seen in the Reynolds shear stress in the wall region [see Fig. 3.8 (b), \(y/\delta_9 < 0.4\)], whereas the profiles in the outer layer remain unchanged [see Fig. 3.8 (c)].

### 3.4 Instantaneous Signal Traces

To understand the basic mechanism of the above feature of APG flows, we have investigated the characteristics of instantaneous signal traces of the fluctuating velocity components \(u\) and \(v\) together with the Reynolds shear stress \(uv\). The results in the near-wall region and those at the outer edge of the log-law region are shown in Figs. 3.9 (a) and (b), respectively, for comparison with the ZPG flow. A circumflex “ˆ” denotes the normalization by the respective r.m.s. value. It is quite clear from Fig. 3.9 (a) that, despite having nearly the same \(R_\theta\) value, the time scales of the velocity fluctuations in the wall region of the APG flow are extremely elongated and become different from those in the ZPG flow; that is, turbulent motions of the APG flow become gentler and less active, which may correspond to the observed low production of turbulence energy (Nagano et al. 1992). Corresponding to this retardation, the viscous time scale, \(\nu/u_\tau^2\), described in Fig. 3.9 (a) is also extremely elongated in the APG flow. In the outer region [Fig. 3.9 (b)], on the other hand, there is a small (but not negligible) difference in the instantaneous signal traces between the ZPG and APG flows, and the time scale in the outer layer \(\delta_9/U_\infty\) is also elongated.

### 3.5 Spectra

The power spectra of the velocity fluctuations \(u\) and \(v\) in the log region \((y^+ \approx 50)\) are presented in Figs. 3.10 (a) and (b), and Figs. 3.11 (a) and (b), against the dimensionless frequency \(f' = f\nu/U_0^2\). In Figs. 3.10 (a) and (b), lines indicating the traditional \(-1, -5/3\) and \(-7\) power-law spectra are also included for comparison. The definitions of the spectra are as
follows:

\[
\overline{u^2} = \int_0^\infty E_u(f') \, df' = \int_0^\infty f' E_u(f') \, d(\ln f'),
\]

(3.3)

\[
\overline{v^2} = \int_0^\infty E_v(f') \, df' = \int_0^\infty f' E_v(f') \, d(\ln f'),
\]

(3.4)

As expected from the waveforms in Fig. 3.9, the frequencies of energy-containing eddies in both spectra gradually shift toward the lower frequency with increasing \(P^+\). To clarify the APG effect on velocity fluctuations at high frequencies, we have examined the spectra of \(\partial u/\partial t\), defined as

\[
\left(\frac{\partial u}{\partial t}\right)^2 = \int_0^\infty E_u(f') \, df' = \int_0^\infty f' E_u(f') \, d(\ln f'),
\]

(3.5)

which may be considered an approximation of the dissipation, and present them in Fig. 3.11 (c). The spectra of \(\partial u/\partial t\) also shift toward the lower frequency as \(P^+\) increases. Such changes in power and dissipation spectra are observed in both the near-wall and outer regions.

### 3.6 Scaling Law

The above facts indicate that an adverse pressure gradient has a strong influence on turbulence statistics selectively in the near-wall region, and that it is the time scale that represents the essential characteristics of APG turbulent boundary layers. Thus, we proceed to investigate the flow structures from the viewpoint of the temporal behavior of turbulence quantities so as to obtain an appropriate time scale in order to provide a universal scaling law for the near-wall turbulence statistics of APG flows.

In the present study, we have examined the turbulence structures of the APG flow using the following six distinct characteristic time scales (see Nomenclature for definitions):
• viscous time scale: $\nu/u_r^2$
• Kolmogorov time scale: $\tau_\eta$
• Taylor time scale: $\tau_E$
• integral time scale: $T_E$
• time scale for energy-containing eddies: $\tau_u$
• time scale corresponding to mean shear rate: $\tau_m$

where $k$ and $\varepsilon$ are the turbulent kinetic energy and its dissipation rate, respectively. Note that the viscous time scale $\nu/u_r^2$ is uniquely determined at a given $x$ location (the value of the time scale corresponding to the mean shear rate $\tau_m$ at the wall reduces to the viscous time scale: $\nu/u_r^2 = \tau_m|_{y=0} = (\partial U/\partial y)^{-1}|_{y=0}$), while the other parameters vary locally in the $y$ direction.

We have employed the following methods in order to estimate the Taylor time scale $\tau_E$:

(i) direct calculation of the time derivative of $u$ with finite differences of various orders, e.g.,
$$\frac{\partial u}{\partial t} \bigg|_i = \frac{u[(i+1)\Delta t] - u[(i-1)\Delta t]}{2 \Delta t},$$
where $\Delta t$ is the sampling interval;

(ii) integration of the dissipation spectrum, written by the $u$-spectrum $E_u(f)$,
$$\left(\frac{\partial u}{\partial t}\right)^2 = (2\pi)^2 \int_0^\infty f^2 E_u(f) \, df;$$

(iii) the zero-crossing method proposed by Laufer and Liepmann (see Hinze 1975),
$$\tau_E = \frac{\sqrt{2}}{\pi N_0},$$
where $N_0$ is the average number of zeros of the $u$-fluctuations per unit time;

(iv) well-known definition of the autocorrelation coefficient $R_u(\Delta t)$,
$$R_u(\Delta t) \simeq 1 - \left(\frac{\Delta t}{\tau_E}\right)^2, \quad \frac{1}{\tau_E^2} = -\frac{1}{2} \frac{\partial^2 R_u}{\partial (\Delta t)^2} \bigg|_{\Delta t=0},$$
where $\Delta t$ is the lag time.
The estimated results of the Taylor time scale $\tau_E$ using these methods (except for the zero-crossing method) correspond well with each other, if the signal is clearly not suffering from the effects of the noise at high frequencies.

The integral time scale $T_E$ can be estimated by the following two methods:

(i) integration of the autocorrelation coefficient $R_u(\Delta t)$,

$$T_E = \int_0^\infty R_u(\Delta t) \, d(\Delta t); \quad (3.10)$$

(ii) limiting the value to the zero frequency of the $u$-spectrum $E_u(f)$

$$T_E = \lim_{f \to 0} \frac{E_u(f)}{4u^2}. \quad (3.11)$$

The above methods produce nearly the same value of $T_E$.

There are three common practices in evaluating the dissipation rate $\varepsilon$ of turbulent kinetic energy $k$:

(i) with the use of the isotropic relation and Taylor’s hypothesis, one gets

$$\varepsilon = 15\nu \left( \frac{\partial u}{\partial x} \right)^2 = 15\nu \frac{1}{U^2} \left( \frac{\partial u}{\partial t} \right)^2, \quad (3.12)$$

where the time derivative of $u$ is estimated in the same way as in the Taylor time scale $\tau_E$;

(ii) fitting the inertial subrange of the $u$-spectrum to the Kolmogorov law (Bradshaw, see Lawn 1971) gives

$$E_{11}(k_1) = \alpha \varepsilon \frac{4}{3} k_1^{-\frac{5}{3}}, \quad \overline{u^2} = \int_0^\infty E_{11}(k_1) \, dk_1, \quad (3.13)$$

where $k_1 = 2\pi f / U$ is the wave number, and $\alpha (= 0.51$ for inner layer, $0.55$ for outer layer) is the universal constant;

(iii) using as the closing term in the budget of the turbulent kinetic energy $k$:

$$\varepsilon = -\left( \overline{u'^2} - \overline{v'^2} \right) \frac{\partial U}{\partial x} - \overline{u'v'} \frac{\partial U}{\partial y} - U \frac{\partial k}{\partial x} - V \frac{\partial k}{\partial y} - \frac{\partial}{\partial y} (\overline{vk'}) + \nu \frac{\partial^2 k}{\partial y^2} \quad (3.14)$$
where $k' = (u^2 + v^2 + w^2)/2$, and the unmeasured part of the turbulent diffusion term (the fifth term in the right hand side) is estimated by $\overline{vw^2} = \frac{1}{2}(\overline{vu^2} + \overline{v^3})$.

For the outer region of the boundary layer, these three methods are found to provide nearly the same value of $\varepsilon$. With decreasing $y^+$, however, Eq. (3.14) offers more probable values as determined by comparison with the results of the DNS data (Spalart 1988) in a ZPG flow (see Fig. 3.12), and thus is adopted for the estimation of the dissipation rate $\varepsilon$.

The distributions of the time scales are shown in Figs. 3.13 ~ 3.17. Using the above six time scales, we have analyzed the temporal turbulent structures of the APG flows. As a result, the Taylor time scale $\tau_E$ was found to be the most appropriate for representing the temporal behavior of turbulence quantities and for universally scaling the turbulence statistics. Generally, in high-Reynolds number flows, the Taylor time scale $\tau_E$ and the viscous dissipation $\varepsilon$ are related to each other through the expression $\varepsilon = 30\nu u^2/(U^2\tau_E^2)$.

The distributions of the measured Taylor time scale $\tau_E$ in the $y$ direction are shown in Fig. 3.14. One may find that $\tau_E$ increases with increasing pressure gradient parameters $P^+$. In proximity to the wall, however, $\tau_E$ becomes almost constant for a given $P^+$. In the following, we apply the characteristic time scale $\tau_E$ to scaling various turbulence statistics.

### 3.6.1 Waveforms and Spectra

The sample results of scaling raw waveforms in Fig. 3.9 with $\tau_E$ are presented in Fig. 3.18. The corresponding various spectra in the log region ($y^+ \simeq 50$) arranged with a new dimensionless frequency, $f'' = f\tau_E$, are presented in Figs. 3.19 and 3.20. Both the waveforms and the spectra are well correlated irrespective of the pressure gradients. Based on these figures, we conclude that the Taylor time scale is the best scaling parameter for both ZPG and APG flows.
3.6.2 Wall-Limiting Behavior

Next, we present the scaling of the wall-limiting behavior of streamwise turbulence intensity $\sqrt{\overline{u'^2}}$ in Fig. 3.21. Here, $\tau_{Es}$ defined by a value of $\tau_E$ at the outer edge of the viscous sublayer, i.e., $y^+ = 3$, is adopted, and the coordinate $y$ is normalized with the characteristic length scale $u_\tau \tau_{Es}$. If the viscous wall unit is used as a length scale, remarkable differences appear in the wall-limiting behavior between the ZPG and APG flows, with a systematic deviation from the ZPG case, as depicted in Fig. 3.7. On the other hand, the use of the time scale $\tau_{Es}$ makes all the profiles collapse with the following linear relation:

$$\frac{\sqrt{\overline{u'^2}}}{u_\tau} = 3.34 \frac{y}{u_\tau \tau_{Es}}, \quad (3.15)$$

irrespective of the values of $P^+$ as shown in Fig. 3.21.

We should notice that the above relation does not need the friction velocity $u_\tau$. Thus, Eq. (3.15) only indicates the relation:

$$\overline{u'^2} = 3.34 y \sqrt{\frac{\left(\frac{\partial \bar{u}}{\partial t}\right)^2}{\frac{\partial \tau_{w}'}{\partial t} u_\tau^2/\nu}}, \quad (3.16)$$

Bradshaw (1997) pointed out that for small $y$, $u = \tau_{w}'/y/\mu$, and Eq. (3.16) shows the relation:

$$\overline{\tau_{w}'^2} / \tau_{w}^2 = 3.34 \sqrt{\frac{\left(\frac{\partial \tau_{w}'}{\partial t} u_\tau^2/\nu\right)^2}{\frac{\partial \tau_{w}'}{\partial t} u_\tau^2/\nu}}, \quad (3.17)$$

where $\tau_{w}'$ is the fluctuating component of wall shear stress $\tau_{w}$.

3.6.3 Mean Burst Period

We now investigate the relation between $\tau_E$ and the characteristic time scale pertaining to the bursting phenomena, $T_B$, obtained from the short-time averaged autocorrelation function method (Kim et al. 1971, Hishida and Nagano 1979). The short-time averaged autocorrelation
coefficient $\tilde{R}_u(\Delta t)$ is defined as:

$$
\tilde{R}_u(\Delta t, t_o; T_{DP}) = \frac{1}{T_{DP}} \int_{t_o}^{t_o+T_{DP}} [U(t) - \tilde{U}] [U(t + \Delta t) - \tilde{U}] \, dt \times \frac{1}{T_{DP}} \int_{t_o}^{t_o+T_{DP}} [U(t + \Delta t) - \tilde{U}]^2 \, dt
$$

where $\Delta t$ is the lag time, $t_o$ is an arbitrary reference time, $U(t)$ is instantaneous streamwise velocity, $\tilde{U}$ is the short-time averaged velocity defined as:

$$
\tilde{U} = \frac{1}{T_{DP}} \int_{t_o}^{t_o+T_{DP}} U(t) \, dt,
$$

and $T_{DP}$ is a data-processing time, which must be determined. We obtained the mean burst period $T_B$ as the ensemble-averaged value of the delay time when the autocorrelation coefficient $\tilde{R}_u(\Delta t)$ takes the first maximum (see Fig. 3.22). When we adopt a short time range, say from 3 to 7 $T_B$, for the data processing time $T_{DP}$, the mean burst period is independent of the data processing time $T_{DP}$, as shown in Fig. 3.23. The p.d.f. of the burst period may be represented by a log-normal distribution, as shown in Fig. 3.24. For the present study, this form of p.d.f. is given by

$$
P(T_B) = \frac{1}{\sqrt{2\pi} B T_B} \exp \left[ -\frac{1}{2} \left( \frac{\ln T_B - c}{B} \right)^2 \right], \quad 0 < T_B < \infty,
$$

and

$$
B^2 = \ln \left( 1 + \frac{\sigma^2}{T_B^2} \right),
$$

$$
c = \ln T_B - \frac{1}{2} B^2,
$$

where $T_B$ is the mean burst period and $\sigma$ is the r.m.s. value of the burst period.

There are many arguments about the scaling of mean burst period in APG flows (e.g., White and Tiederman 1990, Tillman and Kistler 1996). Bandyopadhyay (1982) claimed that a universal value of the nondimensional period between bursts does not exist, although this might be an extreme view. It becomes obvious from Fig. 3.25 (a) that the mean burst period $T_B$ changes
significantly with \( P^+ \). This increase in the mean burst period cannot be correlated to the conventional viscous time scale \( \nu/u^2 \), as shown in Fig. 3.25 (b). However, as seen in Fig. 3.25 (c), the normalized period \( T_B/\tau_{Es} \) tends to collapse for any pressure gradient level. This means that \( \tau_E \) is closely related to the dynamical coherent structure. It should be noted that the Taylor time scale \( \tau_E \) is also suitable for scaling the mean burst period in a favorable pressure gradient flow (Misu et al. 1992), and the event durations of spanwise vorticity fluctuation in ZPG flows (Klewicki and Falco 1996).

### 3.6.4 Intermittency Factor

Next, we obtained the intermittency factors, which represent the fraction of time for which the flow at the selected position is turbulent in the outer region of the boundary layer. To identify the turbulent and non-turbulent regions, we used the method of Hedley and Keffer (1974), which is based on the time derivatives of the fluctuating velocities \( u \) and \( v \). Compared with the ZPG flows, the characteristic time scales for \( u \) and \( v \) in the APG flows are further elongated in the near-wall region as well as in the outer region (although here only slightly). If we therefore use the same threshold level in both ZPG and APG flows without stretching the raw waveforms, the intermittency factors in the inner region of the APG flows show the existence of a non-turbulent zone (i.e., less than one). Therefore, in order to take into account the change in the time scale in APG flows, we have stretched the signals in accordance with the relevant Taylor time scale. By doing so, we can obtain the rational distributions of intermittency factors by keeping an identical threshold level for both ZPG and APG flows. As shown in Fig. 3.26, the distributions of intermittency factors in APG flows become identical to those in the ZPG flows. The solid line in Fig. 3.26 is the Gaussian error function expressed by the following equation:

\[
\gamma = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left[-\frac{(y - \bar{Y})^2}{2\sigma^2}\right] dy, \quad (3.23)
\]
where the average position of the interface between turbulent and non-turbulent fluid is $Y = 0.9 \delta_{99}$, and the r.m.s. value of the difference between the instantaneous and average positions of the interface is $\sigma = 0.14 \delta_{99}$.

These results suggest that if we choose the proper scaling parameter based on the knowledge of turbulence structures, we may describe the features of adverse pressure gradient flows uniquely, even under non-equilibrium conditions.

### 3.7 Concluding Remarks

In this chapter, the statistical characteristics of turbulence quantities in an adverse-pressure-gradient (APG) turbulent boundary layer were investigated. We also attempted to find a time scale which would provide a universal scaling law for the near-wall turbulence statistics of APG flows. As a result, the following may be concluded:

1. In the APG flow, the characteristic time scale of the flow is elongated exceedingly in the near-wall region, compared with a ZPG flow at nearly the same $R_\theta$. This difference should be closely related to the progressive decrease in turbulence intensities in the near-wall region, and can be ascribed to the retardation of turbulence production.

2. In the outer region, there is a slight difference in the instantaneous velocity signals and the distributions of turbulence intensities between the ZPG and APG flows.

3. The Taylor time scale $\tau_E$ is the most appropriate for describing the essential characteristics of the near-wall structure of non-equilibrium APG flows.

4. The conventional scaling law using the viscous time scale $\nu/u_Z^2$ cannot be applied to the scaling of the near-wall statistics for non-equilibrium APG flows. Instead of $\nu/u_Z^2$, the Taylor time scale $\tau_E$ in the near-wall region, combined with $u_+$, may provide the best scaling law.
Figure 3.1 Development of the mean velocity profiles in adverse-pressure-gradient flows.
Figure 3.2 Mean velocity profiles in wall coordinates in adverse-pressure-gradient flows.
Figure 3.3 Mean velocity profiles in outer coordinates in adverse-pressure-gradient flows.
Chapter 3. Statistical Characteristics in APG Flow

Figure 3.4   Turbulence intensities of fluctuating velocity components in wall coordinates.
Chapter 3. Statistical Characteristics in APG Flow

Figure 3.5 Turbulence intensities of fluctuating velocity components.
Chapter 3. Statistical Characteristics in APG Flow

Figure 3.6 Turbulence intensities of fluctuating velocity components.
Figure 3.7 Wall-limiting behavior of $\sqrt{\frac{u'^2}{u_T^2}}$.

Figure 3.8 (a) Reynolds shear stress profiles in adverse-pressure-gradient flows.
Figure 3.8 Reynolds shear stress profiles in adverse-pressure-gradient flows.
Chapter 3. Statistical Characteristics in APG Flow

\[ P^+ = 0, \quad R_\theta = 1780 \]

\[ \hat{u}, \hat{v}, \hat{u}\hat{v} \]

\[ P^+ = 2.04 \times 10^{-2}, \quad R_\theta = 2110 \]

\[ \hat{u}, \hat{v}, \hat{u}\hat{v} \]

(a) Inner layer \((y^+ \simeq 18, \ y/\delta_{99} \simeq 0.03)\)

\[ P^+ = 0, \quad R_\theta = 1780 \]

\[ \hat{u}, \hat{v}, \hat{u}\hat{v} \]

\[ P^+ = 2.04 \times 10^{-2}, \quad R_\theta = 2110 \]

\[ \hat{u}, \hat{v}, \hat{u}\hat{v} \]

(b) Outer layer \((y^+ \simeq 260, \ y/\delta_{99} \simeq 0.5)\)

Figure 3.9 Signal traces of \(\hat{u}, \hat{v}\) and \(\hat{u}\hat{v}\).
Figure 3.10  Power spectra of velocity fluctuation in the log region ($y^+ \simeq 50, y/\delta_99 \simeq 0.1$).
Figure 3.11 Power spectra of velocity fluctuation in the log region ($y^+ \simeq 50, y/\delta_{99} \simeq 0.1$).
Figure 3.12  Dissipation rate $\varepsilon$ in zero-pressure-gradient flow.

Figure 3.13  Distributions of Kolmogorov time scale $\tau_\eta$. 
Chapter 3. Statistical Characteristics in APG Flow

Figure 3.14 Distributions of Taylor time scale $\tau_E$.

Figure 3.15 Distributions of integral time scale $T_E$. 
Chapter 3. Statistical Characteristics in APG Flow

Figure 3.16  Distributions of time scale for energy-containing eddies $\tau_u$.

Figure 3.17  Distributions of time scale corresponding to mean shear rate $\tau_m$. 
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\[ P^+ = 0, \quad R_\theta = 1780 \]

\[ P^+ = 2.04 \times 10^2, \quad R_\theta = 2110 \]

(a) Inner layer \((y^+ \simeq 18, \, y/\delta_{99} \simeq 0.03)\)

(b) Outer layer \((y^+ \simeq 260, \, y/\delta_{99} \simeq 0.5)\)

Figure 3.18 Signal traces of \(\hat{u}, \hat{v} \) and \(\hat{u}\hat{v}\), time being normalized by Taylor time scale \(\tau_E\).
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Figure 3.19  Power spectra of velocity fluctuation arranged with dimensionless frequency $f''$ in the log region ($y^+ \simeq 50, y/\delta_{99} \simeq 0.1$).
Figure 3.20 Power spectra of velocity fluctuation arranged with dimensionless frequency $f''$ in the log region ($y^+ \approx 50$, $y/\delta_99 \approx 0.1$).
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Figure 3.21 Scaling of wall-limiting behavior of streamwise intensity $\sqrt{u'^2}$ with Taylor time scale $\tau_{Es}$. 

\[ \frac{\sqrt{u'^2}}{u_\tau} = 3.34 \frac{y}{u_\tau \tau_{Es}} \]
\[ \tilde{R}_u(\Delta t) \]

\[ y^+ = 18.9 \]
\[ T_{DP}/\bar{T}_B \approx 5 \]

Figure 3.22 Short-time averaged autocorrelation coefficient $\tilde{R}_u(\Delta t)$.

\[ T_{DP} \]

\[ \bar{T}_B \]

\[ y^+ = 18.9 \]

Figure 3.23 Dependence of data-processing time $T_{DP}$ on $\bar{T}_B$. 
Figure 3.24 P.d.f. of bursting periods in zero-pressure-gradient flow.

Figure 3.25 (a) Mean period of intermittent bursts.

\[ y^+ = 18.9 \]
\[ \overline{T_B}/\tau_{Es} = 6.94 \]
\[ \sigma/\tau_{Es} = 3.31 \]
Figure 3.25 Mean period of intermittent bursts.
Figure 3.26  Intermittency factors $\gamma$ in adverse-pressure-gradient flows.
Chapter 4

Quasi-Coherent Structures in Adverse-Pressure-Gradient Flow

In this chapter, we investigate the dynamical features of the quasi-coherent structures in adverse-pressure-gradient (APG) flows in order to identify any scale-irrelevant structures hidden in such flows.

4.1 Turbulent Transport

The structural differences in quasi-coherent motions reflect on higher-order turbulence statistics, especially third-order moments (Nagano and Tagawa 1988, 1990). Figures 4.1 (a) and (b) show the time-averaged turbulent transport, i.e., third-order moments, of the turbulent energy component \( u^2 \) and Reynolds shear stress \( uv \) normalized by the friction velocity \( u_\tau \), respectively. The definite effects of the APG are clearly seen on both third-order moments. As shown in Fig. 4.1 (a), the positive region of \( vu^2 \) in the ZPG flows for \( y^+ > 15 \) disappears partly as \( P^+ \) increases. Since third-order moments are predominately sensitive to the change in coherent structures such as ejections and sweeps (Nagano and Tagawa 1988), this result indicates that internal structural changes do occur in the APG boundary layers. Negative values of \( vu^2 \) in
the near-wall to outer regions demonstrate the existence of turbulent energy transport toward
the wall from the regions away from the wall. This important characteristic of the APG flows
conforms to our previous results (Nagano et al. 1992), and is also consistent with the results of
Bradshaw (1967), Cutler and Johnston (1989), and Skåre and Krogstad (1994). From Fig. 4.1
(b), it can be seen that a similar inward transfer takes place in the turbulent transport of the
Reynolds shear stress. It should be noted that, as $P^+$ increases, turbulent transport in the APG
boundary layer occurs in the direction completely opposite to that in the ZPG boundary layer.

4.2 Instantaneous Characteristics of Turbulence Quantities

To understand the above features of the APG flows in more detail, we investigated the in-
stantaneous characteristics of the coherent motions. Figures 4.2 (a) and (b) show instantaneous
signals of $uv$, $vu^2$ and $vuv$ together with $u$- and $v$-fluctuations at $y^+ \approx 30$, where the time-
averaged values of triple products $vu^2$ and $vuv$ become nearly maximum in the ZPG flow [see
Figs. 4.1 (a) and (b)]. A circumflex “$\hat{\cdot}$” denotes the normalization by the respective r.m.s.
value. The time on the abscissa is normalized by the Taylor time scale $\tau_E$, which is the most
appropriate for scaling the period of the coherent motions, irrespective of pressure gradients
(Nagano et al. 1997, 1998, see section 3.6). The mean burst periods have nearly the same value
of $10\tau_E$ in both ZPG and APG flows at this location.

In the ZPG flow, large-amplitude fluctuations of the triple products are generally associated
with the fluid motions categorized as the second- and fourth-quadrant events in the $(u, v)$-plane,
i.e., the ejection ($Q2$) and sweep ($Q4$) motions (for the classification see Fig. 4.3). In Fig. 4.2 (a),
very large-amplitude fluctuations of $\hat{v}\hat{u}^2$ and $\hat{v}\hat{u}\hat{v}$ are skewed toward the positive and negative
sides, respectively, thus indicating that the ejection ($Q2$) is the principal contributor here. On
the other hand, as shown in Fig. 4.2 (b), in the APG flow the sweep motions ($Q4$) occur much
more frequently than the ejections ($Q2$), besides which the large-amplitude outward interaction
(Q1) is no longer negligible. Thus, the fluctuations of $\hat{v}\hat{u}^2$ and $\hat{v}\hat{u}\hat{v}$ are respectively skewed to the negative and positive sides, which is quite opposite to what occurs in ZPG flows.

4.3 Mean Frequency and Mean Duration of Events

Next, we investigate the temporal differences between the ZPG and APG flows. The definitions of a period $T_i$ and duration $\Delta T_i$ of events are shown in Fig. 4.4 (for ejection: $i = 2$). Figures 4.5 (a) and (b) show the dimensionless mean frequencies $(\bar{T}_i/\tau_E)^{-1}$ of each event normalized by the Taylor time scale $\tau_E$ in the log region ($y^+ \simeq 50$), as a function of a threshold level $H (= |\hat{u}\hat{v}|)$. At $H = 0$, the frequencies of each motion are essentially equivalent (Nagano and Hishida 1990), irrespective of pressure gradients. This result also validates the scaling with the Taylor time scale $\tau_E$. As the threshold level $H$ becomes larger (i.e., restricted to the large amplitude motions), the frequencies of the $Q2$- (ejection) and $Q4$- (sweep) motions exceed those of the interactions ($Q1$, $Q3$) in the ZPG flow, as shown in Fig. 4.5 (a). In the APG flow, however, the sweep motion ($Q4$) is dominant at any threshold level and the $Q1$-motion (outward interaction) occurs more frequently than the $Q3$-motion (wallward interaction), as shown in Fig. 4.5 (b).

Figures 4.6 (a) and (b) show the mean durations of each event $\bar{\Delta T}_i$ normalized by the Taylor time scale $\tau_E$ in the log region ($y^+ \simeq 50$). In the ZPG flow [Fig. 4.6 (a)], the mean durations of the ejections ($Q2$) and the sweeps ($Q4$) are much longer than those of the interactions ($Q1$, $Q3$). In the APG flow, however, as shown in Fig. 4.6 (b), the mean duration of the sweeps ($Q4$) decreases relatively to that of the ejections ($Q2$), and the mean duration of the wallward interactions ($Q3$) increases relatively to that of the outward interactions ($Q1$). This corresponds to the fact that the large-amplitude $Q4$- and $Q1$-motions of shorter duration occur frequently in the APG flow, as will be discussed later.
4.4 Fractional Contribution to Third-Order Moments

The contributions to $u^l v^m$ from a particular quadrant $Q_i$ ($i = 1, 2, 3, 4$) may be written as:

$$
(u^l v^m)_i = \lim_{T \to \infty} \frac{1}{T} \int_0^T u^l(t) v^m(t) I_i(t) \, dt,
$$

where $I_i(t)$ is the indicator function defined as:

$$
I_i(t) = \begin{cases} 
1: \text{when the point } (u, v) \text{ in the } (u, v)-\text{plane is the } i\text{-th-quadrant}, \\
0: \text{otherwise.}
\end{cases}
$$

The fractional contributions of different quadrant motions to $\overline{v^2}$ and $\overline{uv}$, normalized by the friction velocity $u_\tau$, are shown in Figs. 4.7 and 4.8, respectively. In the ZPG flow, as is apparent from Figs. 4.7 (a) and 4.8 (a), the turbulent transport of $u^2$ and $uv$ in the wall-normal direction is dominated mainly by the $Q_2$- and $Q_4$-motions. Since the net values of the triple products are determined by the disparity in contributions between these two types of motions (Nagano and Tagawa 1988), $\overline{v^2}$ and $\overline{uv}$ result in positive and negative values, respectively, in the log region ($y^+ \simeq 50$) of the ZPG flow where the ejections ($Q_2$) become larger than the sweeps ($Q_4$). However, in the APG flow [Figs. 4.7 (b) and 4.8 (b)], the contributions from the coherent motions, especially the ejections ($Q_2$), significantly decrease. Thus, the triple products $\overline{v^2}$ and $\overline{uv}$ in the log region of the APG flow have net values different from those in the ZPG flow, i.e., $\overline{v^2}$ and $\overline{uv}$ become negative and positive, respectively [see Figs. 4.1 (a) and (b)].

The lines in Figs. 4.7 and 4.8 represent the theoretical predictions from the following equation, based on a sophisticated cumulant-discard method (Nagano and Tagawa 1988).

$$
(\hat{u}^l \hat{v}^m)_i = \sigma_{u,i}^l \sigma_{v,i}^m \int_0^\infty \left[ \int_0^\infty \hat{u}^l \hat{v}^m P(\sigma_{u,i} \hat{u}, \sigma_{v,i} \hat{v}) \, d\hat{v} \right] d\hat{u},
$$

where $P(\hat{u}, \hat{v})$ is the joint probability density function (p.d.f.) for $u$- and $v$-fluctuations, and $\sigma_{u,i}$ and $\sigma_{v,i}$ are sign functions which represent the signs of $u$ and $v$ of the $i$th-quadrant in the $(u, v)$-plane:

$$
\sigma_{u,i} = (1, -1, -1, 1), \quad \sigma_{v,i} = (1, 1, -1, -1).
$$
Using cumulants and Hermite polynomials, the joint p.d.f. $P(\hat{u}, \hat{v})$ can be written (Nagano and Tagawa 1988) as

$$P(\hat{u}, \hat{v}) = \frac{1}{2\pi} \exp \left[ -\frac{1}{2}(\hat{u}^2 + \hat{v}^2) \right] \sum_{p,q=0}^{p+q \leq 4} C_{pq} H_p(\hat{u}) H_q(\hat{v}),$$  \hspace{1cm} (4.5)

where the expansion coefficient $C_{pq}$ represented by cumulants $k_{pq}$, and an Hermite polynomial $H_n(\chi)$ are given by

$$C_{00} = 1, \quad C_{10} = C_{01} = 0, \quad C_{20} = C_{02} = 0, \quad C_{11} = k_{11} = \overline{uv},$$

$$C_{30} = \frac{1}{6} k_{30} = \frac{1}{6} \overline{u^3}, \quad C_{03} = \frac{1}{6} k_{03} = \frac{1}{6} \overline{v^3},$$

$$C_{21} = \frac{1}{2} k_{21} = \frac{1}{2} \overline{u^2v}, \quad C_{12} = \frac{1}{2} k_{12} = \frac{1}{2} \overline{uv^2},$$

$$C_{40} = \frac{1}{24} k_{40} = \frac{1}{24} (\overline{u^4} - 3), \quad C_{04} = \frac{1}{24} k_{04} = \frac{1}{24} (\overline{v^4} - 3),$$

$$C_{31} = \frac{1}{6} k_{31} = \frac{1}{6} (\overline{u^3v} - 3\overline{uv}), \quad C_{13} = \frac{1}{6} k_{13} = \frac{1}{6} (\overline{u^3v} - 3\overline{uv}),$$

$$C_{22} = \frac{1}{4} (k_{22} + 2k_{11}^2) = \frac{1}{4} (\overline{u^2v^2} - 1),$$  \hspace{1cm} (4.6)

and

$$H_n(\chi) = (-1)^n \exp \left( \frac{\chi^2}{2} \right) \frac{d^n}{d\chi^n} \exp \left( -\frac{\chi^2}{2} \right).$$  \hspace{1cm} (4.7)

Thus, substitution of Eqs. (4.4) ~ (4.7) into Eq. (4.3) yields

$$\overline{(\hat{u}^l \hat{v}^m)}_i = \frac{1}{2\pi} \sum_{p,q=0}^{p+q \leq 4} \sigma_{\nu,i}^{l+p} \sigma_{\nu,i}^{m+q} C_{pq} B_{l,p} B_{m,q},$$  \hspace{1cm} (4.8)

where $B_{j,k}$ is defined as

$$B_{j,k} = \int_0^\infty \chi^j H_k(\chi) \exp(-\chi^2/2) d\chi.$$  \hspace{1cm} (4.9)

Equation (4.8) with $(l, m) = (2, 1)$ and $(l, m) = (1, 2)$ produces the fractional contributions $\overline{(vu^2)}_i$ and $\overline{(vuv)}_i$, respectively. As seen in Figs. 4.7 and 4.8, the theoretical predictions correspond well with the experimental data in both ZPG and APG flows.

### 4.5 Weighted P.D.F.s of Third-Order Moments

An analysis of fractional contributions in the $(u, v)$-plane alone is not sufficient to ascertain the detailed changes in quasi-coherent structures in the APG flow. Hence, we investigated the
weighted p.d.f.s of triple products $\overline{vu^2}$ and $\overline{uvu}$ in the $(u, v)$-plane (Nagano and Tagawa 1988, 1990):

$$W_m(\hat{u}, \hat{v}) = m P(\hat{u}, \hat{v}), \quad m = \hat{v}\hat{u}^2 \text{ and } \hat{u}\hat{v},$$

(4.10)

where $P(\hat{u}, \hat{v})$ is the joint p.d.f. for $u$- and $v$-fluctuations. The integrated value of $W_m(\hat{u}, \hat{v})$ in each quadrant becomes the fractional contribution $(m_i)$, and integration over the whole $(u, v)$-plane reduces to the conventional time-averaged value $\overline{m}$.

The typical distributions of $W_m$ for $m = \hat{v}\hat{u}^2$ and $\hat{u}\hat{v}$ in the log region ($y^+ \simeq 50$) are shown in Figs. 4.9 and 4.10, respectively. The theoretical predictions from the joint p.d.f. $P(\hat{u}, \hat{v})$ given in Eq. (4.5) are also shown in comparison with the measured results. In these figures, the shaded regions represent $|W_m| \geq 0.01$, and the light and dark regions show positive and negative values, respectively. The contour lines demonstrate that the value of $|W_m|$ is 0.01, 0.02, 0.04, 0.08.

In the APG flow [Figs. 4.9 (b) and 4.10 (b)], the amplitude of the sweep motions ($Q4$) becomes much larger than that of the ejections ($Q2$). Furthermore, the extent of the p.d.f.s in the outward interaction ($Q1$) also becomes larger in the APG flow. The theoretical predictions [Figs. 4.9 (d) and 4.10 (d)] reproduce precisely the change in the extent of the distributions in the ejection ($Q2$) and sweep ($Q4$) motions. The remarkable extent of the changes in the outward interaction ($Q1$) is also reproduced.

In the outer region, however, these striking differences are not seen between the ZPG and APG flows.

### 4.6 Trajectory Analysis

To clarify the changes in coherent motions in APG flows, we conducted the trajectory analysis developed by Nagano and Tagawa (1995) and extracted the key patterns and flow modules contributing to the momentum transport, which consist of three successive quadrants in the $(u,$
The trajectory analysis is composed of the following three steps (Nagano and Tagawa 1995):

(i) Using the fluctuating velocity components $u$ and $v$, fluid motions are classified into the four types, $Qi$ ($i = 1, 2, 3, 4$), where $i$ denotes the $i$th-quadrant of the $(u, v)$-plane.

(ii) Obtain a time-series of quadrant sequences, $Qi(1), Qi(2), \ldots, Qi(j), \ldots$, where $i$ changes with time according to the classification and $j$ is an integer index which increases by one whenever a trajectory crosses a boundary of a quadrant of the $(u, v)$-plane. However, if the condition $|\hat{u}(j)| \leq h$ and/or $|\hat{v}(j)| \leq h$ is satisfied, $Qi(j)$ is set to equal $Qi(j - 1)$ to reject a meaningless quadrant change in the trajectory.

(iii) The resultant time series of quadrants is decomposed and classified into a total 36 patterns which are the permutations and combinations of three quadrants, $Qi_1-Qi_2-Qi_3$. In this processing, if the conditions $|\hat{u}\hat{v}(j - 1)| \leq H, |\hat{u}\hat{v}(j)| \leq H$ and $|\hat{u}\hat{v}(j + 1)| \leq H$ are satisfied simultaneously, the pattern is discarded as an insignificant event unrelated to the coherent motions.

The thresholds $(h, H)$ are set to $(0.25, 1.07)$ following the results of Nagano and Tagawa (1995). For example, the trajectory of the turbulent motion shown in Fig. 4.11 is judged to be $Q4-Q3-Q2-Q3-Q2$ due to the existence of very small-scale, negligible fluctuations. However, by introducing the threshold $h$, this scheme allows the appropriate recognition of the trajectory as $Q4-Q3-Q2$ (for more details, see Nagano and Tagawa 1995).

The results of fractional contributions to $\overline{vu^2}$ and $\overline{vuv}$ in the log region ($y^+ \simeq 50$) of the ZPG and APG flows are summarized in Table 4.1 together with the number of detected events. It is seen from the table that the $Q4-Q1-Q4$ and the $Q2-Q3-Q2$ patterns occur most frequently, and that they make very large contributions to the third-order moments, irrespective of pressure gradients. These trajectories are called the sub-patterns (Nagano and Tagawa 1995). The $Q1-Q4-Q1$ and $Q3-Q2-Q3$ patterns also occur frequently, and they make large contributions to the
third-order moments. These patterns often follow the corresponding sub-patterns as $Q_1$-$Q_4$-$Q_1$-$Q_4 \cdots$, as will be discussed later. On the other hand, the $Q_2$-$Q_1$-$Q_4$, $Q_2$-$Q_3$-$Q_4$, $Q_4$-$Q_1$-$Q_2$ and $Q_4$-$Q_3$-$Q_2$ patterns are considered to be the important trajectories, since they have an interaction ($Q_1$ or $Q_3$) between the two coherent motions (i.e., $Q_2$ and $Q_4$), which appear at the interface between the ejection ($Q_2$) and sweep ($Q_4$) motions and correspond well to the VITA events ($Q_2$-$Q_1$-$Q_4$ and $Q_2$-$Q_3$-$Q_4$ patterns: the positive events $\partial u/\partial t > 0$; $Q_4$-$Q_1$-$Q_2$ and $Q_4$-$Q_3$-$Q_2$ patterns: the negative events $\partial u/\partial t < 0$). Thus, these trajectories represent the essential features of near-wall turbulence, and they are called the key patterns (Nagano and Tagawa 1995).

In order to obtain further details of the characteristics of the trajectories, we performed an ensemble averaging of them. The procedure consists of the following steps (Nagano and Tagawa 1995):

(i) to extract fluid motions which belong to a specified pattern;

(ii) to calculate the mean duration for each quadrant of the pattern;

(iii) to perform ensemble averaging by adjusting the temporal duration of each event to the mean duration of the respective $Q_i$ motions while keeping the phase relations between $u$ and $v$.

We can formulate this procedure as follows:

$$\langle \chi(t) \rangle \text{ for } Q_{i_1}-Q_{i_2}-Q_{i_3} = \begin{cases} 
\frac{1}{N} \sum_{n=1}^{N} \chi(r_{1,n} t + t_{o1,n}) & \text{for } Q_{i_1} \text{- motion,} \\
\frac{1}{N} \sum_{n=1}^{N} \chi(r_{2,n} t + t_{o2,n}) & \text{for } Q_{i_2} \text{- motion,} \\
\frac{1}{N} \sum_{n=1}^{N} \chi(r_{3,n} t + t_{o3,n}) & \text{for } Q_{i_3} \text{- motion,}
\end{cases} \quad (4.11)$$

where $\chi$ denotes a fluctuating component, $t$ is time, $N$ is the number of occurrences of the pattern $Q_{i_1}$-$Q_{i_2}$-$Q_{i_3}$, $n$ is a label number which increases by one when the trajectory analysis
detects a fluid motion belonging to the specified pattern, \( r_{i, n} \) is an adjusting factor for the ensemble averaging defined as \( r_{i, n} = \Delta T_{i, n} / \Delta T_i \). \( \Delta T_{i, n} \) denotes the individual duration of the \( Q_i \) motions of the pattern labeled \( n \), and \( \Delta T_i \) is the mean duration of the \( Q_i \) motions of the pertinent pattern. Thus, \( \Delta T_i = \frac{1}{N} \sum_{n=1}^{N} \Delta T_{i, n} \), and \( t_{o1, n}, t_{o2, n} \) and \( t_{o3, n} \) are the reference points in time when \( Q_{i1}, Q_{i2} \)- and \( Q_{i3} \)-motions of the \( Q_{i1} \)-\( Q_{i2} \)-\( Q_{i3} \) pattern begin, respectively.

Figures 4.12, 4.13 and 4.14 show the ensemble-averaged velocity fluctuations \( \langle \hat{u} \rangle, \langle \hat{v} \rangle, \langle \hat{u} \hat{v} \rangle, \langle \hat{v} \hat{u} \rangle \) and \( \langle \hat{v} \hat{u} \rangle \) of the identified key patterns (\( Q_2-Q_1-Q_4 \) and \( Q_2-Q_3-Q_4 \), \( Q_4-Q_1-Q_2 \) and \( Q_4-Q_3-Q_2 \)) and the sub-patterns (\( Q_4-Q_1-Q_4 \) and \( Q_2-Q_3-Q_2 \)) in the log region \( (y^+ \approx 50) \) of the ZPG and APG flows.

In the ZPG flow, as shown in Fig. 4.12 (a) and Fig. 4.13 (a), the amplitude of the \( Q_2 \)-motions in the \( Q_2-Q_1-Q_4 \), \( Q_2-Q_3-Q_4 \), \( Q_4-Q_1-Q_1 \) and \( Q_4-Q_3-Q_2 \) patterns is relatively larger than that of the \( Q_4 \)-motions. In addition, as shown in Fig. 4.14 (a), both the \( Q_4-Q_1-Q_4 \) and the \( Q_2-Q_3-Q_2 \) patterns are very active flow patterns, though the amplitude of the \( Q_2-Q_3-Q_2 \) pattern is slightly larger than that of the \( Q_4-Q_1-Q_4 \) pattern. On the other hand, in the APG flow, as shown in Figs. 4.12 (b), 4.13 (b) and 4.14 (b), the amplitude of the ensemble-averaged fluctuations decreases in the \( Q_2 \)-motions and increases in the \( Q_4 \)- and the \( Q_1 \)-motions of each pattern. Moreover, the durations of the \( Q_4-Q_1-Q_4 \) pattern and the \( Q_2-Q_3-Q_2 \) pattern become shorter and longer in the APG flow, respectively [see Fig. 4.14 (b)].

Figures 4.15 ~ 4.20 show the ensemble-averaged vector maps in the ZPG and APG flows, which consist of the velocity components \( \langle u \rangle \) and \( \langle v \rangle \) in Figs. 4.12, 4.13 and 4.14.

### 4.7 Multi-Point Simultaneous Measurement

There is also little knowledge of what effects the APG produces on the interaction between the large-scale structures in the outer layer and the small-scale coherent structures near the wall. Therefore, we conducted simultaneous measurement with five X-probes (Fig. 2.2) to obtain
multi-point information in the wall-normal direction. Figures 4.21, 4.22 and 4.23 display the examples of instantaneous velocity vectors around a time when the flow in the log region \( (y^+ \simeq 50) \) is identified as having various patterns. These vectors are obtained by using the Karhunen-Loève expansion based on the eigenfunctions up to the fifth eigenmode (see Appendix A), and depicted with respect to an observer’s coordinate moving at a speed of about \( 0.6 \overline{U}_e \), which is equal to the mean streamwise velocity at \( y^+ \simeq 50 \) in both ZPG and APG flows. The shaded regions in the figures represent \( -uv/u_t^2 \geq 1 \), and the light and dark regions show the ejection \((Q2)\) and sweep \((Q4)\), respectively. The contour lines demonstrate that the value of \( -uv/u_t^2 \) is 1, 2, 4, 8.

Figures 4.23 (a) and (b) display the velocity vectors around a time when the flow is identified as having the \( Q2-Q3-Q2 \) pattern in the ZPG flow and the governing \( Q4-Q1-Q4 \) pattern in the APG flow, respectively. In the ZPG flow, as shown in Fig. 4.23 (a), the ejection \((Q2)\) that lifts up in the near wall region (around the time: \( t/\tau_E \simeq 1 \)) and the sweep \((Q4)\) that intrudes into the inner layer strongly interact with each other at \( y^+ \simeq 100 \). On the other hand, in the APG flow, as shown in Fig. 4.23 (b), large-scale sweep motions from the outer to the inner region predominate. The \( Q4 \)-motions then reflect near the wall without significant loss of the momentum. Moreover, the \( Q4-Q1-Q4 \) patterns seem to continue as \( Q4-Q1-Q4-Q1-Q4 \cdots \). Thus, the \( Q1 \)-motion observed in the \( Q4-Q1-Q4 \) pattern is not a noise-like small motion, but a subsequent reflection of the sweep \((Q4)\) near the wall.

4.8 Concluding Remarks

In this chapter, we investigated the dynamical characteristics of the quasi-coherent structures in an adverse-pressure-gradient (APG) flow using conditional sampling techniques (quadrant splitting and trajectory analyses), and through observations of the instantaneous flow fields. As a result, the following may be concluded:
(1) In the near-wall region of the APG flow, large-amplitude sweep motions \((Q4)\) and outward interactions \((Q1)\) occur frequently.

(2) The contributions from the ejection \((Q2)\) significantly decrease. Thus, the turbulent transport quantities, \(\overline{w^2}\) and \(\overline{uv}\), in the log region of the APG flow become negative and positive, respectively.

(3) In the APG flow, marked transfer of high momentum fluid toward the wall \((Q4)\), and the following reflection near the wall without significant loss of the momentum \((Q4-Q1-Q4\) pattern), occur more frequently with a short duration time. On the other hand, the duration of the \(Q2-Q3-Q2\) pattern in the APG flow becomes longer, and there are less active motions, than in the ZPG flow.

(4) From observations of the instantaneous velocity vector field interpolated with the Karhunen-Loève expansion, it is confirmed that fluid motions from the outer to inner layers predominate in the APG flow.
### Table 4.1 Summary of frequency of occurrence and contribution to third-order moments for all 36 possible trajectories in the log region ($y^+ \approx 50$).

<table>
<thead>
<tr>
<th>Trajectories</th>
<th>$P^+ = 0$</th>
<th></th>
<th>$P^+ = 3.08 \times 10^{-2}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of events</td>
<td>Contribution to $\langle vu^2 \rangle / u_x^3$</td>
<td>Number of events</td>
<td>Contribution to $\langle vu^2 \rangle / u_x^3$</td>
</tr>
<tr>
<td>Q1-Q2-Q1</td>
<td>102</td>
<td>0.12</td>
<td>0.04</td>
<td>39</td>
</tr>
<tr>
<td>Q1-Q2-Q3</td>
<td>122</td>
<td>0.25</td>
<td>-0.13</td>
<td>64</td>
</tr>
<tr>
<td>Q1-Q2-Q4</td>
<td>46</td>
<td>0.03</td>
<td>0.02</td>
<td>13</td>
</tr>
<tr>
<td>Q1-Q3-Q1</td>
<td>4</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Q1-Q3-Q2</td>
<td>74</td>
<td>0.09</td>
<td>-0.05</td>
<td>21</td>
</tr>
<tr>
<td>Q1-Q3-Q4</td>
<td>16</td>
<td>-0.01</td>
<td>0.00</td>
<td>10</td>
</tr>
<tr>
<td>Q1-Q4-Q1</td>
<td>719</td>
<td>-0.39</td>
<td>0.41</td>
<td>325</td>
</tr>
<tr>
<td>Q1-Q4-Q2</td>
<td>187</td>
<td>0.11</td>
<td>-0.03</td>
<td>41</td>
</tr>
<tr>
<td>Q1-Q4-Q3</td>
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<td>-0.15</td>
<td>0.10</td>
<td>70</td>
</tr>
<tr>
<td>Q2-Q1-Q2</td>
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<td>0.27</td>
<td>-0.12</td>
<td>32</td>
</tr>
<tr>
<td>Q2-Q1-Q3</td>
<td>43</td>
<td>0.04</td>
<td>-0.03</td>
<td>10</td>
</tr>
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<td>Q2-Q1-Q4</td>
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<td>-0.10</td>
<td>0.04</td>
<td>21</td>
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</tbody>
</table>

Total       | 6276      | 2.43     | -1.14                      | 2207     | -2.61    | 2.78 |
Figure 4.1 Distributions of turbulent transport (third-order moments) in adverse-pressure-gradient flows.
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Figure 4.2 Simultaneous signal traces of third-order moments $\hat{v}\hat{u}^2$ and $\hat{v}\hat{u}\hat{v}$, time being normalized by Taylor time scale $\tau_E (y^+ \approx 30)$. 

(a) $P^+ = 0$

(b) $P^+ = 3.08 \times 10^{-2}$
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Figure 4.3 Classification of fluid motions in the $(u, v)$-plane.

Figure 4.4 Period $T_i$ and duration $\Delta T_i$ of events ($i = 2$: ejection).
Figure 4.5 Mean frequencies of events normalized by $\tau_E$ in the log region ($y^+ \simeq 50$).

Figure 4.6 Mean durations of events normalized by $\tau_E$ in the log region ($y^+ \simeq 50$).
Figure 4.7  Fractional contributions of different quadrant motions to the third-order moment \( \overline{vu^2} \).
Figure 4.8 Fractional contributions of different quadrant motions to the third-order moment $\overline{uvw}$.
Figure 4.9  Weighted p.d.f. of $\hat{v}^2$ in the log region ($y^+ \simeq 50$). The shaded regions represent $|W_{\hat{v}^2}| \geq 0.01$, and the light and dark regions show positive and negative values, respectively. The contour lines demonstrate that the value of $|W_{\hat{v}^2}|$ is 0.01, 0.02, 0.04, 0.08.
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Figure 4.10 Weighted p.d.f. of $\hat{v} \hat{u} \hat{v}$ in the log region ($y^+ \approx 50$). The shaded regions represent $|W_{\hat{v}\hat{u}\hat{v}}| \geq 0.01$, and the light and dark regions show positive and negative values, respectively. The contour lines demonstrate that the value of $|W_{\hat{v}\hat{u}\hat{v}}|$ is 0.01, 0.02, 0.04, 0.08.
Introducing \( h = 0.25 \) meaningless change

Figure 4.11 Trajectory analysis based on the quadrant-sequences on the \((u, v)\)-plane (from Nagano and Tagawa 1995; Fig. 3).
Figure 4.12 Ensemble-averaged characteristics of velocity fluctuations ($Q_2-Q_1-Q_4$ and $Q_2-Q_3-Q_4$ patterns) in the log region ($y^+ \approx 50$).
Figure 4.13 Ensemble-averaged characteristics of velocity fluctuations (Q4-Q1-Q2 and Q4-Q3-Q2 patterns) in the log region ($y^+ \approx 50$).
Figure 4.14 Ensemble-averaged characteristics of velocity fluctuations (Q4-Q1-Q4 and Q2-Q3-Q2 patterns) in the log region ($y^+ \approx 50$).
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Figure 4.15 Vector maps of the ensemble-averaged flow patterns in the log region \((y^+ \simeq 50)\) of zero-pressure-gradient flow.

Figure 4.16 Vector maps of the ensemble-averaged flow patterns in the log region \((y^+ \simeq 50)\) of adverse-pressure-gradient flow \((P^+ = 3.08 \times 10^{-2})\).
Figure 4.17  Vector maps of the ensemble-averaged flow patterns in the log region ($y^+ \simeq 50$) of zero-pressure-gradient flow.

Figure 4.18  Vector maps of the ensemble-averaged flow patterns in the log region ($y^+ \simeq 50$) of adverse-pressure-gradient flow ($P^+ = 3.08 \times 10^{-2}$).
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Figure 4.19   Vector maps of the ensemble-averaged flow patterns in the log region \((y^+ \simeq 50)\) of zero-pressure-gradient flow.

Figure 4.20   Vector maps of the ensemble-averaged flow patterns in the log region \((y^+ \simeq 50)\) of adverse-pressure-gradient flow \((P^+ = 3.08 \times 10^{-2})\).
Figure 4.21 Instantaneous velocity vectors. The shaded regions represent $-uv/u_x^2 \geq 1$, and the light and dark regions show the ejection ($Q^2$) and sweep ($Q^4$), respectively. The contour lines demonstrate that the value of $-uv/u_x^2$ is 1, 2, 4, 8.
Figure 4.22 Instantaneous velocity vectors. The shaded regions represent $-uv/u_x^2 \geq 1$, and the light and dark regions show the ejection ($Q_2$) and sweep ($Q_4$), respectively. The contour lines demonstrate that the value of $-uv/u_x^2$ is 1, 2, 4, 8.
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Figure 4.23 Instantaneous velocity vectors. The shaded regions represent $-\frac{uv}{u^2} \geq 1$, and the light and dark regions show the ejection ($Q^2$) and sweep ($Q^4$), respectively. The contour lines demonstrate that the value of $-\frac{uv}{u^2}$ is 1, 2, 4, 8.
Chapter 5

Conclusions

Non-equilibrium turbulent boundary layers subjected to an adverse pressure gradient (APG) were investigated experimentally. The conclusions obtained in this study are summarized in the following.

In Chapter 3, the statistical characteristics of turbulence quantities in the APG flow are investigated. We also attempt to find a time scale which would provide a universal scaling law for the near-wall turbulence statistics of APG flows. The conclusions are as follows:

(1) In the APG flow, the characteristic time scale of the flow is elongated exceedingly in the near-wall region, compared with a ZPG flow at nearly the same $R_\theta$. This difference should be closely related to the progressive decrease in turbulence intensities in the near-wall region, and can be ascribed to the retardation of turbulence production.

(2) In the outer region, there is a slight difference in the instantaneous velocity signals and the distributions of turbulence intensities between the ZPG and APG flows.

(3) The Taylor time scale $\tau_E$ is the most appropriate for describing the essential characteristics of the near-wall structure of non-equilibrium APG flows.
(4) The conventional scaling law using the viscous time scale $\nu/u^2$ cannot be applied to the scaling of the near-wall statistics for non-equilibrium APG flows. Instead of $\nu/u^2$, the Taylor time scale $\tau_E$ in the near-wall region, combined with $u_\tau$, may provide the best scaling law.

In Chapter 4, we investigate the dynamical characteristics of the quasi-coherent structures in the APG flow using the conditional sampling techniques (quadrant splitting and trajectory analyses), and through observations of the instantaneous flow fields. The conclusions are as follows:

1. In the near-wall region of the APG flow, large-amplitude sweep motions ($Q4$) and outward interactions ($Q1$) occur frequently.

2. The contributions from the ejection ($Q2$) significantly decrease. Thus, the turbulent transport quantities, $\bar{vu}$ and $\bar{uv}v$, in the log region of the APG flow become negative and positive, respectively.

3. In the APG flow, marked transfer of high momentum fluid toward the wall ($Q4$), and the following reflection near the wall without significant loss of the momentum ($Q4-Q1-Q4$ pattern), occur more frequently with a short duration time. On the other hand, the duration of the $Q2-Q3-Q2$ pattern in the APG flow becomes longer, and there are less active motions, than in the ZPG flow.

4. From observations of the instantaneous velocity vector field interpolated with the Karhunen-Loève expansion, it is confirmed that fluid motions from the outer to inner layers predominate in the APG flow.
Appendix A

Interpolation Method with Karhunen-Loève Expansion

To display the instantaneous velocity vector field on the basis of the multi-point simultaneous data from the array of five X-probes, the data at different $y$ values from the measurement points were interpolated by utilizing the Karhunen-Loève expansion (Holmes et al. 1996), as described below (the flowchart is shown in Fig. A.1):

(i) First, we measure the two-point autocorrelation function at each measurement location using the multi-point simultaneous data.

(ii) With the measured correlation functions, we make a reasonable estimate of $R_{ij}(y_i, y_j) = \overline{u(y_i)u(y_j)}$ of the $(M + N) \times (M + N)$ matrix, which is composed of the correlations at the measured points $M (= 5)$ and the points to be interpolated $N$ (see Fig. A.2). Then, we solve the following matrix eigenvalue problem, and obtain the eigenvalues $\lambda_n$ and the corresponding normalized eigenfunctions $\varphi_n(y_i)$, which are orthogonal to each other.

\[ R_{ij}(y_i, y_j) \varphi_n(y_j) = \lambda_n \varphi_n(y_i), \quad i, j = 1, 2, \ldots, (M + N) \quad (A1) \]

(iii) The Karhunen-Loève expansion can be used to reconstruct a random stochastic variable
from the least numbers of the orthogonal bases. The number of data points is limited to five (in the present measurement); thus, we reconstruct the interpolated signals using the eigenfunctions up to the fifth eigenmode.

\[ u(y_i, t) \simeq \sum_{n=1}^{5} a_n(t) \varphi_n(y_i) \]  

(A2)

(iv) The time-dependent coefficients \( a_n(t) (n = 1, 2, \ldots, 5) \) can be obtained from Eq. (A2) with the measured known data \( u(y_i, t) \) and the eigenfunctions \( \varphi_n(y_i) \) obtained from Eq. (A1).

(v) Instantaneous velocities at the interpolated positions can be estimated from Eq. (A2).

(vi) We recalculate the autocorrelation function \( R_{ij}(y_i, y_j) = \overline{u(y_i)u(y_j)} \) using Eq. (A2).

We repeat the foregoing process until a good convergence is obtained for \( R_{ij}(y_i, y_j) = \overline{u(y_i)u(y_j)} \).

This interpolating procedure for the \( v \)-component is similar to that for \( u \).

To verify the interpolation procedure, we utilized the DNS database of a turbulent channel flow (Iida et al. 1997, Reynolds number based on channel half width and friction velocity is \( Re_\tau = 100 \)). The viscous sublayer is excluded from the domain of this interpolation, because its characteristics are different from those of other regions and hence difficult to interpolate with the limited number of eigenmodes. Note that the Karhunen-Loève expansion can be formulated for any subdomain, which is advantageous to the convergence of the expansion (Moin and Moser 1989). The convergence then reaches almost 98% for both \( \overline{u'^2} \) and \( \overline{v'^2} \) with up to the fifth eigenmode in the domain \( 14 \leq y^+ \leq 100 \) (\( M = 5, N = 16 \)). The interpolated results of \( u \)- and \( v \)-fluctuations are quite good for both the statistics [Fig. A.3 (a)] and instantaneous behavior [Fig. A.3 (b)].
Appendix A. Interpolation Method with K-L Expansion

Estimation of correlation matrix.
Calculation of eigenvalues $\lambda_n$ and eigenfunctions $\varphi_n$.
Calculation of time-dependent coefficients $a_n(t)$.
Calculation of instantaneous velocities.

Convergence?

Start

Measuring two-point autocorrelation.

Estimation of correlation matrix.

Calculation of eigenvalues $\lambda_n$ and eigenfunctions $\varphi_n$.
Calculation of time-dependent coefficients $a_n(t)$.
Calculation of instantaneous velocities.

Figure A.1 Flowchart of the interpolation method with Karhunen-Loève Expansion.

Two-point autocorrelation function:

$$ R_{ij}(y_i, y_j) = u(y_i)u(y_j) $$

Reasonably estimated values

- Measured positions, $M$
- Interpolated positions, $N$

Figure A.2 Two-point autocorrelation function $R_{ij}(u_i, u_j)$. 
Appendix A. Interpolation Method with K-L Expansion

Figure A.3 Interpolated results for DNS database of turbulent channel flow ($Re_\tau = 100$).
Bibliography


Bradshaw, P. 1997 Private communication.


Bibliography


