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Response compensation of thermistors: Frequency response and identification of thermal time constant

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The purpose of this study is to improve the accuracy of general-purpose temperature sensors (commercially available thermistors) in fluctuating temperature measurement and to extend their applicability. First, heat transfer models reflecting the internal structures of plate-type and spherical thermistors were proposed to obtain theoretical frequency response to temperature fluctuations. Second, parameters dominating the response characteristics of the thermistor were extracted, and it was shown that the thermistor response can be approximated by a first-order system. Finally, a flexible response compensation technique for the thermistor was constructed, the development of which has been very difficult because the thermal time constant varies greatly depending on the flow velocity and physical properties of the fluid surrounding the thermistor. The technique utilized a two-thermocouple probe method—rational identification of thermal time constants and digital compensation for response delay—proposed by Tagawa et al. [Rev. Sci. Instrum. 69, 3370 (1998)], and greatly improved the accuracy of fluctuating temperature measurement by the thermistor. Although the level of the improvement depends on the fluid type measured and the signal-to-noise ratio of temperature signals, the present response compensation technique worked successfully. As a result, the compensated thermistor became 5 to 50 times as fast as the uncompensated (original) one. © 2003 American Institute of Physics. [DOI: 10.1063/1.1542668]

I. INTRODUCTION

Most general-purpose temperature sensors are designed with particular priority on physical and chemical durability, and therefore their response to temperature fluctuation is fairly slow. The highest measurable frequency is at most 0.01–0.1 Hz. Although much faster temperature sensors are available, such sensors generally are fragile and have no practical endurance and stability. Hence, improvements in the response of general-purpose temperature sensors through the compensation for response delay would significantly improve their performance and applicability.

The response characteristics of a temperature sensor can usually be expressed by the first-order system:

$$T_n = T + \frac{\tau}{\pi} \frac{d\pi}{dt},$$  \hspace{1cm} (1)

where $T_n$ is the fluid temperature to be measured; $T$ is the temperature corresponding to the sensor output; $\tau$ is the thermal time constant (henceforth termed simply a time constant); and $i$ is time. The time constant in Eq. (1) is defined as $\tau = \rho c D/nh$ ($\rho$: density of sensor material; $c$: specific heat; $h$: heat transfer coefficient; $D$: representative length of sensor; and $n$ denotes the parameter as determined by the geometrical configuration of the sensor). The length $D$ is the thickness of a plate-type sensor or the diameter of a cylindrical or spherical one, and the value of $n$ is 2, 4, and 6 for plate-type, cylindrical, and spherical sensors, respectively.

Various response-compensation techniques based on Eq. (1) can now be used to improve the accuracy of fluctuating temperature measurement. In reality, however, the time-constant value is often unknown or unavailable even when given in the sensor manufacturer’s specifications. This is because the time constant can change largely depending on the flow velocity and/or physical properties of the fluid surrounding the sensor. These characteristics have complicated the response compensation techniques that appear simple.

There is much research on the frequency response of various temperature sensors and the compensation for the delay of the response. Thanks to recent advances in computer performance, computer-intensive compensation techniques have been widely developed. For example, as cited in previous papers, 1,2 various methods for compensating the response of a fine-wire thermocouple have been proposed so far. For a fine-wire thermocouple with a diameter no greater than 0.1 mm, there is no need to consider the internal temperature distribution in the radial direction of the wire. If a bead at the junction is sufficiently small (for example, less than 2 to 3 times the wire diameter) and the length-to-diameter ratio is larger than approximately 200, the frequency response can be accurately modeled by the first-order system. Most thermistors, on the other hand, have a multi-tiered structure and their temperature-sensitive bodies are often large, and therefore the responses show rather complicated characteristics compared to those of fine-wire thermocouples. Thus, details of the response characteristics of thermistors are not yet fully understood, and identification of the parameters dominating the thermistor dynamic re-

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response is still necessary. In particular, to accurately compensate for the delay of the response, appropriate mathematical expressions of the response characteristics are needed.

Several theoretical and experimental investigations on the thermistor response have been reported. Lueck et al. theoretically analyzed a coated-thermistor response in water, and showed that the theoretical results correlated well with the observed frequency response. In their analysis, a multilayered thermal conduction model was employed with the aid of a theory introduced by Fabula, and the surrounding water was treated as a thermally diffusive layer with the thickness of the diffusive layer expressed as a function of the water flow velocity. Hill reported that the dynamic response of a particular fast response thermistor could be scaled by the square root of the water flow velocity over the range of 0.5–2 m/s. Gregg and Meagher experimentally investigated the square root of the water flow velocity over the range of a particular fast response thermistor could be scaled by the square root of the water flow velocity. Hill reported that the dynamic response of a particular fast response thermistor could be scaled by the square root of the water flow velocity over the range of 0.5–2 m/s. Gregg and Meagher experimentally investigated the response characteristics of glass rod thermistors 0.4 mm in diameter, and showed that the frequency response to temperature frequencies less than 25 Hz was accurately described by a double-pole filter, although a single-pole form (first-order system) was an equally good representation for less than 10 Hz. Fuehrer et al. theoretically investigated the effects of thermistor lead wires on frequency response. They indicated that the lead wires acted as heat transfer fins and improved the frequency response over that of an isolated bead (temperature-sensitive body), although temperature distribution inside the bead was not considered in the analysis. Balko et al. simulated the step response of a hermetically sealed thermistor immersed in water using a finite element simulation technique to obtain temperature distributions inside a multilayered structure consisting of a spherical thermistor (a temperature-sensitive body coated with glass and a layer of Parylene-C) and a water layer. Recently, Storck modeled a commercially available thermistor using precise computational grids, and simulated the step response using a finite element method for solving heat conduction equations under convective boundary conditions.

In the present study, a “two-thermocouple probe technique” previously developed is applied to thermistor measurement to greatly improve the accuracy and reliability in fluctuating temperature measurement. The two-thermocouple probe technique consists of a rational scheme for identifying the thermal time constants of temperature sensors and a reliable compensation technique based on digital signal processing. First, the thermistor response to fluid-temperature fluctuation was theoretically investigated using a heat-transfer model reflecting the internal structure of a thermistor. Second, the applicability of the two-thermocouple probe technique was experimentally validated using thermistor measurements.

II. THEORETICAL ANALYSIS OF FREQUENCY RESPONSE OF THERMISTOR

A. Heat transfer model of thermistor

For the theoretical analysis of frequency response of a thermistor, an accurate heat-transfer model reflecting the internal structure of the thermistor is needed. Figure 1 shows the heat transfer models of plate-type and spherical thermistors together with the coordinate systems for the analysis. The heat-transfer model of the plate-type thermistor shown in Fig. 1 (a) can be expressed mathematically by the following set of equations.

Temperature-sensitive body (0 \(< x < x_i\)):

\[
\rho_1 c_1 \frac{\partial T_1}{\partial t} = \lambda_1 \frac{\partial^2 T_1}{\partial x^2}.
\]  

(2)

Film/coating (\(x_i < x < x_s\)):

\[
\rho_2 c_2 \frac{\partial T_2}{\partial t} = \lambda_2 \frac{\partial^2 T_2}{\partial x^2}.
\]  

(3)

Boundary conditions:

(i) \(x = 0\) : \(\frac{\partial T_1}{\partial x} = 0\),

(ii) \(x = x_i\) : \(T_1 = T_2\),

(iii) \(x = x_i\) : \(\lambda_1 \frac{\partial T_1}{\partial x} = \lambda_2 \frac{\partial T_2}{\partial x}\),

(iv) \(x = x_s\) : \(\frac{\partial T_2}{\partial x} = h(T_s - T_2)\),

where \(T\), \(\rho\), \(c\), \(\lambda\), and \(h\) are temperature, density, specific heat, heat conduction, and heat transfer coefficient, respectively. The subscripts 1 and 2 denote the temperature sensitive body and the film/coating, respectively. The subscripts 1 and 2 denote the temperature sensitive body and the film/coating, respectively. The subscripts 1 and 2 denote the temperature sensitive body and the film/coating, respectively. The subscripts 1 and 2 denote the temperature sensitive body and the film/coating, respectively. The subscripts 1 and 2 denote the temperature sensitive body and the film/coating, respectively. The subscripts 1 and 2 denote the temperature sensitive body and the film/coating, respectively.

The Fourier integral is applied to the theoretical analysis of the frequency response. Then, \(T_1\), \(T_2\), and \(T_g\) can be expressed as

\[
T_1(t,x) = \int_{-\infty}^{\infty} e^{j\omega t} \hat{T}_1(\omega,x) d\omega,
\]

\[
T_2(t,x) = \int_{-\infty}^{\infty} e^{j\omega t} \hat{T}_2(\omega,x) d\omega,
\]

\[
T_g(t) = \int_{-\infty}^{\infty} e^{j\omega t} \hat{T}_g(\omega) d\omega,
\]

where \(j\) is an imaginary unit, and \(\omega\) is an angular frequency \((= 2\pi f)\). The substitution of Eq. (5) into Eqs. (2)–(4) yields

\[
T_1(t,x) = \int_{-\infty}^{\infty} e^{j\omega t} \hat{T}_1(\omega,x) d\omega,
\]

\[
T_2(t,x) = \int_{-\infty}^{\infty} e^{j\omega t} \hat{T}_2(\omega,x) d\omega,
\]

\[
T_g(t) = \int_{-\infty}^{\infty} e^{j\omega t} \hat{T}_g(\omega) d\omega,
\]
and

\[ H_P(\omega, x) = \frac{\cosh(\Omega_1 x)}{D_P}, \]

\[ D_P = \cosh(\Omega_1 x) \cosh(\Omega_2 d) \left[ 1 + \frac{\Omega_2 x_s}{\text{Bi}} \tanh(\Omega_2 d) \right] + \sinh(\Omega_1 x_s) \sinh(\Omega_2 d) \]

\[ \times \left[ 1 + \frac{\Omega_2 x_s}{\text{Bi}} \coth(\Omega_2 d) \right] \frac{\lambda_1 \Omega_1}{\lambda_2 \Omega_2}. \]

The symbols \( \Omega_1 \) and \( \Omega_2 \) in Eq. (6) are defined, respectively, by

\[ \Omega_1 = \sqrt{\frac{j}{\omega \rho a}} = (1 + j) \sqrt{\frac{\omega}{(2a)}}, \]

\[ \Omega_2 = \sqrt{\frac{j}{\omega \rho a}} = (1 + j) \sqrt{\frac{\omega}{(2a)}}, \]

where \( a = (\lambda/\rho c) \), \( d = (x_s - x_i) \), and \( \text{Bi} = (h x_s/\lambda_2) \) are thermal diffusivity, film/coating thickness, and the Bio number, respectively. Hyperbolic sine and cosine functions for complex numbers used in Eq. (6) are given by

\[ \sinh(a + j\beta) = \sinh a \cos \beta + j \cosh a \sin \beta, \]

\[ \cosh(a + j\beta) = \cosh a \cos \beta + j \sinh a \sin \beta, \]

where \( a \) and \( \beta \) are real numbers.

Similarly, the frequency response of the spherical thermistor shown in Fig. 1(b), \( H_S(\omega, r) \), can be calculated to obtain the following:

\[ H_S(\omega, r) = \left[ \sinh(\Omega_1 r)/(\Omega_1 r) \right]/D_S, \]

\[ D_S = \cosh(\Omega_1 r_s) \cosh(\Omega_2 d) \left[ \frac{\lambda_1}{\lambda_2} \right] \]

\[ \times \left[ 1 - \frac{(1/\text{Bi})}{\Omega_1 r_s} \tanh(\Omega_1 r_s) \right] \times \left[ 1 - \frac{(1/\text{Bi})}{\Omega_2 r_s} \tanh(\Omega_2 d) \right] \]

\[ + \sinh(\Omega_1 r_s) \sinh(\Omega_2 d) \]

\[ \times \left[ 1 - \frac{(1/\text{Bi})}{\Omega_1 r_s} \tanh(\Omega_1 r_s) \right]. \]

where \( \text{Bi} = h r_s/\lambda_2 \). It should be noted here that Eqs. (6) and (9) are not rationalized, since most computer programs can handle complex numbers directly.

As mentioned in the Introduction, Lueck et al. derived an analytical frequency response of coated thermistors. Their theory is based on a three-layer heat transfer model in which the water around a thermistor is treated as a thermally diffusive layer. The present theory, on the other hand, uses the heat transfer coefficient \( h \) for describing the thermal boundary condition at the thermistor surface as shown in Eq. (4). Thus the heat transfer at the thermistor surface is treated differently.

### B. Verification of theoretical results

Most thermistors have spherical configurations. Hence, the detailed investigation of the frequency response of a spherical thermistor using Eq. (9) revealed factors dominating the response characteristics.

The temperature-sensitive body of a thermistor is composed of metal oxides. However, the materials and physical properties are often unknown. In the following, we investigate the response characteristics of a spherical thermistor simulating Fenwal 112-104 KAJ-B01 [the physical properties are given in Table I (row A)], for which Storck et al. analyzed the internal temperature field using a finite element method and simulated the step response. In the present analysis, the heat transfer coefficient at the thermistor surface, \( h \), is necessary for calculating the frequency response using Eq. (9), and is obtained from the following correlation by Whitaker:

\[ \text{Nu} = 2 + (0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{1/3}) \text{Pr}^{1/4} \left( \frac{\mu_b}{\mu_s} \right)^{1/4}, \]

where the Nusselt, Reynolds, and Prandtl numbers (\( \text{Nu} \), Re, and \( \text{Pr} \)) are defined as \( \text{Nu} = h D_s/\lambda_s \), \( \text{Re} = U D_s/\nu_g \), and \( \text{Pr} = \nu_g/\alpha_g \), respectively [\( U \): flow velocity; \( D_s \) is the outer diameter of the thermistor (= \( D + 2d \)). The working fluid is air or water. Their physical properties are evaluated at 20 °C, and the viscosity at the bulk temperature, \( \mu_b \), is assumed to be identical to that at a surface temperature of the thermistor, \( \mu_s \).
Prior to analysis of the frequency response of the spherical thermistor, the validity of Eq. (9) was examined using the following three test cases.

First, the time-constant value predicted by Eq. (9) was compared with that obtained by Storck et al. They simulated the step response of the thermistor in an airflow velocity of 8 m/s. Using the result, we can estimate the time-constant value from the time the step response reaches 0.632 (= 1 − e^−1), and obtain τ = 1.36 s. From the frequency response of Eq. (9), on the other hand, we obtain τ = 1.27 s under the same conditions as the simulation. [physical properties of the temperature-sensitive body and coating are given in Table I (row A)]. [The time-constant value was determined from the relation τ = 1/(2πf_c), where f_c is the cutoff frequency at which the gain becomes √(1/2)(−30 dB).] The present theoretical result is consistent with the simulation by Storck et al.

Second, the present theoretical result was compared to the experimental frequency response of a spherical thermistor, which was observed by Lueck et al. The physical properties of the thermistor are given in Table I (row B) for the temperature-sensitive body (D = 0.15 mm) and the coating (d = 0.015 mm). When the temperature and velocity of water flow were 8 °C and 1.25 m/s, respectively, the time-constant value observed was 1.06 × 10^{-2} s. The present analysis, Eq. (9), gives τ = 1.18 × 10^{-2} s under the same condition, which agrees well with the observed time-constant value.

Third, the time-constant value of a noncoated thermistor was predicted and compared to the experimental result. A dummy thermistor (D = 1.0 mm, d = 0 mm) was made by solidifying a dollop of adhesive liquid epoxy resin on a junction of the type-K thermocouple (25 μm in wire diameter). The time-constant value measured in still air was 3.8 s, while Eq. (9) gives 4.1 s [the physical properties are given in Table I (row C)]. The two results agree within 8%.

C. Parametric analysis of thermistor response

To identify dominant parameters of the thermistor response to temperature fluctuation, the frequency response of a spherical thermistor simulating Fenwal 112-104 KAJ-B01 [D = 0.5 mm; d = 0.25 mm; physical properties are listed in Table I (row A)] was calculated under various computational conditions.

Equation (9) provides the frequency response for a given position of the temperature-sensitive body of a thermistor. The following uses the response at the center of the temperature-sensitive body (r = 0 mm), H_f (ω, 0), to represent the frequency response of a thermistor. The adequacy of this treatment is discussed later in Fig. 6.

Figure 2 shows the bode diagram for the thermistor response placed in still air, airflow of 8 m/s, and still water. The upper and lower figures of Fig. 2 show the gain and phase of the response, respectively. As seen in Fig. 2, the frequency response is greatly improved as the airflow velocity is increased from 0 to 8 m/s, and even when the fluids are equally still, the response in water is approximately ten times as fast as that in air. Such characteristics are well recognized.

It is noteworthy that the frequency responses deviate from the first-order system at the high-frequency region. In the first-order system given by Eq. (1), the gain at the high-frequency region is attenuated by −20 dB/dec, and the phase lag approaches −π/2. As shown in Fig. 2, the frequency response in still air exhibits such characteristics. However, the linear region of −20 dB/dec in the gain diagram narrows with increasing airflow velocity, while the response becomes much faster. Thus, to compensate for the thermistor response delay adequately, it is necessary to make a compensation scheme to respond to such a significant change in the frequency response.

When using a thermistor to measure fluctuating temperature without response compensation, the highest measurable frequency is 0.01 Hz in still air and 0.1 Hz in airflow of 8 m/s. When compensating the thermistor response, assuming a first-order system [Eq. (1)], temperature fluctuations of a much higher frequency can be reproduced. The upper limit of reproducible frequencies may depend on the noise level of the thermistor output signal (signal-to-noise ratio) and on the performance of the measurement instruments used. For example, if it is possible to reproduce a temperature-fluctuation frequency for which the gain (Fig. 2) is larger than 0.01 (usually this is not difficult), the measurable frequency reaches 2 Hz for still air and 5 to 6 Hz for airflow of 8 m/s. The frequency response in water shows that the gain is damped faster than that of the first-order system in the high-frequency region. Hence, the response compensation based on Eq. (1) leads to some undercompensation for high-frequency temperature fluctuations whose gain becomes lower than, for example, 0.1 as in Fig. 2. To improve the
accuracy of the compensation for the high-frequency temperature fluctuations, we need to use frequency-domain compensation techniques\(^2\) with the aid of the fast Fourier transform.

To know what parameters dominate the frequency response of a spherical thermistor, the parameter values of Eq. (9) were systematically changed. The results are shown in Figs. 3–6. In these figures, the response in still air (Fig. 2) is treated as a reference. When the value of one parameter is changed, the other parameter values are kept identical to the reference values.

Figure 3 shows the effects of the sizes of the temperature-sensitive body and the coating on the frequency response of a thermistor. In Fig. 3(a), the diameter of the temperature-sensitive body \((D)\) is changed while the thickness of the coating \((d)\) is fixed at 0.25 mm. In Fig. 3(b), \(d\) is changed with \(D\) fixed at 0.5 mm. Naturally, the thermistor response is improved by decreasing \(D\) and/or \(d\), and the linear region of \(-20\) dB/dec in the gain diagram extends. As a result, the response characteristics can be accurately approximated by the first-order system. However, the thermistor response cannot drastically improve unless both \(D\) and \(d\) are reduced simultaneously.

Next, the effects of the thermal conductivities of a temperature-sensitive body and a coating \((\lambda_1\) and \(\lambda_2\)) on the frequency response were investigated. The effects of \(\lambda_1\) and \(\lambda_2\) are shown in Figs. 4(a) and 4(b), respectively, where \(\lambda_1\) and \(\lambda_2\) are changed within the range of \(0.1–20\) W/(m K). As seen from Fig. 4, the frequency response in the low-frequency region below \(0.1\) Hz hardly changes, even as the thermal conductivities of the thermistor materials are changed. Storck et al.\(^9\) has also noted this feature.

In still water, however, a change in the thermal conductivity of the coating \((\lambda_2)\) can strongly affect the frequency response, as shown in Fig. 5. This is because the thermistor response in water is approximately ten times as fast as that in air (see Fig. 2), and the effect of \(\lambda_2\) on the frequency response becomes pronounced in the high-frequency region.

FIG. 3. Effect of sensor structure \((D\) and \(d)\).

FIG. 4. Effect of \(\lambda_1\) or \(\lambda_2\) \((\text{in still air})\).

FIG. 5. Effect of \(\lambda_2\) \((\text{in still water})\).
III. IDENTIFICATION OF TIME CONSTANT AND RESPONSE COMPENSATION

If the time constant of a temperature sensor is accurately identified and correct compensation for the response delay is made, the reliability of fluctuating temperature measurement will be greatly improved. As a result, the applicability of general-purpose (commercially available) temperature sensors will be extended further.

In the present study, a two-thermocouple probe technique developed previously has been used to identify the time constants of thermistors and to compensate for the response delay. The technique is based on digital signal processing and can respond to changes in environmental conditions around the temperature sensors, e.g., velocity and/or physical properties of fluid. In this technique, two temperature sensors of unequal time constants are combined into a probe. Although there is no strict requirement for the selection of the two sensors to be combined, it seems appropriate to select two sensors whose time-constant ratio is 2 to 3. In the present experiment, two thermistors, Ishizuka 103JT-025A (JT) and 103AT-2 (AT), were combined. The JT thermistor is made of a rectangular temperature-sensitive body (chip) sandwiched between plastic film, and the AT thermistor is made of a spherical body coated with epoxy-resin.

From the above theoretical investigations, it is known that the response characteristics of thermistors can usually be approximated by the first-order system. Although there appears to be some deviation from this for water flow, the first-order system provides an acceptable approximation. Thus, Eq. (1) was used to represent the response characteristics of the above two thermistors. In the following, subscripts A and B denote those two thermistors. Then, fluid temperatures measured by the thermistors A and B, \( T_{gA} \) and \( T_{gB} \), can be expressed by the following equation:

\[
T_{gA} = T_A + \tau_A G_A, \\
T_{gB} = T_B + \tau_B G_B, \tag{11}
\]

where \( T_A \) and \( T_B \) are the output temperatures of the thermistors A and B, respectively, and \( \tau_A \) and \( \tau_B \) denote the time constants that should be unequal (\( \tau_A \neq \tau_B \)). The time derivative of the thermistor output, \( G \), is defined by \( G = dT/dt \). As for two adjacent thermistors, the relation \( T_{gA} = T_{gB} \) holds. Thus, we can determine the values of \( \tau_A \) and \( \tau_B \) by minimizing the mean square value of the difference between \( T_{gA} \) and \( T_{gB} \),

\[
e = (T_{gB} - T_{gA})^2.
\]

The result is written as follows:

\[
\tau_A = \frac{G_A^2 G_A \Delta T - G_A G_B G_B \Delta T}{G_A^2 G_B - (G_A G_B)^2}, \tag{12}
\]

\[
\tau_B = \frac{G_A G_B G_A \Delta T - G_A^2 G_B \Delta T}{G_A^2 G_B - (G_A G_B)^2},
\]

where \( \Delta T = T_B - T_A \).

In the present experiment, the fluid temperatures of air and water flows were measured using a probe composed of the above JT and AT thermistors. A 12-bit analog-to-digital converter was used to digitize the thermistor outputs together with a personal computer for data collection and signal processing. The Savitzky–Golay scheme was used to compute \( G \) in Eq. (12) to shorten the processing time. Details of the data processing are given in previous studies.
Figure 7 shows the step responses of the thermistors when the probe held in air of 30 °C was plunged instantaneously into still water of 83 °C. The time-constant values of the JT and AT thermistors determined by Eq. (12) were 0.17 and 0.52 s, respectively. As shown in Fig. 7, the temperatures compensated with those time-constant values successfully reproduced the stepwise temperature changes that we expected.

In an airflow experiment, a fluctuating temperature field was generated by oscillating the probe above a cylindrical electric heater set at the exit of an upright wind tunnel. Further details of the experimental apparatus are given in previous papers.\textsuperscript{2,14} To examine the overall accuracy of the time-constant identification and the response compensation, a type-K thermocouple 100 μm in wire diameter was employed, which provided a reference temperature (in the present experiment, compensation for the thermocouple response delay was unnecessary).

Figures 8(a) and 8(b) show the compensated temperatures in airflows of 0 and 8.7 m/s, respectively. The time-constant values of the JT and AT thermistors for still air were 4.6 and 11.7 s, respectively, and were consistent with those given in the manufacturer’s specifications for the thermistors (JT: 5 s; AT: 15 s). (The time-constant values obtained were somewhat smaller than those in the specifications. Probably, these differences are due to the existence of a weak flow relative to the temperature probe, since the probe was oscillating above the heater and, as a result, the weak flow was induced even in still air. In addition, to be more accurate, the variation in time-constant values caused by the velocity fluctuation needs to be considered. However, since the velocity fluctuation of the present flow field was weak, the time constants may be regarded as invariable.) As seen from Fig. 8, the raw temperature signals from JT and AT do not trace the thermocouple output (fluid temperature) at all, and the present fluctuating temperature field cannot be accurately measured with these thermistors. When the thermistor responses were compensated using Eq. (11) together with the time-constant values estimated from Eq. (12), the compensated temperature signals were closely correlated with the thermocouple output. This means that the present response compensation technique enables fairly accurate measurement of the fluid temperature fluctuations using commercially available thermistors. To measure fluid temperature fluctuation accurately, a sufficiently fast temperature sensor is needed so that the power-spectrum-density (psd) distribution of the temperature fluctuation can peak at the lower side of the frequency band being measured. Thus, unless we use a slow sensor that makes the psd peak in the high-frequency region where the sensor response deviates largely from the first-order system, the present compensation technique can reproduce fluid temperature fluctuations accurately, as shown in Fig. 8.
Finally, the time-constant values in airflows of various velocities were measured. The results are shown in Fig. 9. Concurrently, the time-constant value of the AT thermistor was predicted by regarding it as a spherical thermistor. The result is included in Fig. 9. [The JT thermistor is made of a rectangular temperature-sensitive body (chip) sandwiched between polyester film. Although the time constant of the JT thermistor may be identified by treating it as a spherical thermistor, such treatment did not seem well-grounded. Thus, the time-constant value of the JT thermistor was not predicted.] Since the internal structure and the physical properties of the AT thermistor were not fully stated, the following data were used in reference to the specifications. The size parameters \( D \) and \( d \) were set as \( D = 1.6 \) mm and \( d = 0.2 \) mm, respectively. The temperature-sensitive body and the coating were assumed to be made from metal oxides (row A in Table I) and epoxy resin (row C), respectively. By calculating Eq. (9) under such conditions, the predicted time-constant values are obtained as shown in Fig. 9 (dotted line). As seen from Fig. 9, the prediction represents the measurement behavior successfully. Hence, if the internal structure of the thermistor and its physical properties are fully specified, the thermistor response can be predicted using Eq. (9).

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**APPENDIX: DERIVATION OF FREQUENCY RESPONSE EQUATIONS**

Using the Fourier integral of Eq. (5), each term of Eqs. (2) and (3) can be expressed as

\[
\frac{\partial T_1}{\partial t} = \int_{-\infty}^{\infty} j \omega e^{j \omega t} \hat{T}_1 d\omega, \\
\frac{\partial T_2}{\partial t} = \int_{-\infty}^{\infty} j \omega e^{j \omega t} \hat{T}_2 d\omega, \\
\frac{\partial^2 T_1}{\partial x^2} = \int_{-\infty}^{\infty} e^{j \omega t} \frac{\partial^2 \hat{T}_1}{\partial x^2} d\omega, \\
\frac{\partial^2 T_2}{\partial x^2} = \int_{-\infty}^{\infty} e^{j \omega t} \frac{\partial^2 \hat{T}_2}{\partial x^2} d\omega. \\
\]

(A1)

From Eq. (A1), Eqs. (2) and (3) are transformed into the following equations:

\[
\frac{\partial^2 [\hat{T}_1(\omega,x)]}{\partial x^2} - \frac{j \omega}{a_1} \hat{T}_1(\omega,x) = 0, \\
\frac{\partial^2 [\hat{T}_2(\omega,x)]}{\partial x^2} - \frac{j \omega}{a_2} \hat{T}_2(\omega,x) = 0. \\
\]

(A2)

Equation (A2) is composed of two ordinary differential equations, and the solution is given by

\[
\hat{T}_1(\omega,x) = C_1 e^{\Omega_1 x} + C_2 e^{-\Omega_1 x}, \\
\hat{T}_2(\omega,x) = C_3 e^{\Omega_2 x} + C_4 e^{-\Omega_2 x},
\]

where \( \Omega_1 \) and \( \Omega_2 \) are defined by Eq. (7). Similarly, the boundary conditions (i)–(iv) of Eq. (4) are transformed into

\[
\left. \frac{\partial \hat{T}_1}{\partial x} \right|_{x=0} = 0, \\
\hat{T}_1|_{x=s_i} = \hat{T}_2|_{x=s_i}, \\
\lambda_1 \left. \frac{\partial \hat{T}_1}{\partial x} \right|_{x=s_i} = \lambda_2 \left. \frac{\partial \hat{T}_2}{\partial x} \right|_{x=s_i}, \\
\lambda_2 \left. \frac{\partial \hat{T}_2}{\partial x} \right|_{x=s_i} = h(\hat{T}_g - \hat{T}_2|_{x=s_i}).
\]

(A4)

The coefficients \( C_1–C_4 \) in Eq. (A3) need to be determined using Eq. (A4) to obtain the solution of Eq. (A2). Then, the frequency response \( H_p(\omega,x) \) can be expressed as follows:

\[
H_p(\omega,x) = \frac{\hat{T}_1(\omega,x)}{\hat{T}_g(\omega)}. \\
\]

(A5)

After substituting the solution Eq. (A3) into the right-hand side of Eq. (A5), the calculation leads to Eq. (6).

In the same way, the frequency response of a spherical thermistor [Eq. (9)] can be derived. The heat-conduction equations for a temperature-sensitive body (0 \(< r \leq r_s \)) and a coating (\( r_i \leq r \leq r_s \)) are given by

\[
\frac{\partial T_1}{\partial t} = a_1 \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{2}{r} \frac{\partial T_1}{\partial r} \right), \\
\frac{\partial T_2}{\partial t} = a_2 \left( \frac{\partial^2 T_2}{\partial r^2} + \frac{2}{r} \frac{\partial T_2}{\partial r} \right). \\
\]

(A6)

Now, Eq. (A6) is transformed into a simple form by introducing a new variable \( u \) defined by \( u(t,r) = rT(t,r) \). Thus, rewriting Eq. (A6) using \( u_1 = rT_1 \) and \( u_2 = rT_2 \), the following is obtained:

\[
\frac{\partial u_1}{\partial t} = a_1 \frac{\partial^2 u_1}{\partial r^2}, \\
\frac{\partial u_2}{\partial t} = a_2 \frac{\partial^2 u_2}{\partial r^2}. \\
\]

(A7)

Now, Eq. (A7) has the same form as Eqs. (2) and (3). Similarly, by transforming the boundary conditions (it should be noted that \( u_1 = 0 \) at \( r = 0 \)), we obtain Eq. (9) for the frequency response of the spherical thermistor.

As for a cylindrical sensor, since the above transformation cannot be used, the analytical treatment becomes fairly complicated. In such a case, a numerical method will facilitate the procedures to obtain the frequency response of the sensor. Actually, when numerically solving Eq. (A2) under the boundary condition of Eq. (A4) using a finite volume method, the same result as that of Eq. (6) was obtained. Furthermore, it was confirmed that the numerical results for a spherical thermistor also agreed completely with those of Eq. (9).
12 S. Whitaker, AIChE J. 18, 361 (1972).