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Response compensation of temperature sensors: Frequency-domain estimation of thermal time constants

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A technique for estimating in situ thermal time-constant values of temperature sensors in the frequency domain and for compensating the sensor outputs for the response delays is proposed. The scheme developed is complementary to the time-domain response-compensation ones previously reported by Tagawa et al. [Rev. Sci. Instrum. 69, 3370 (1998)]. The scheme provides fast, simple, and flexible data processing based on the fast Fourier transform algorithm, and is potentially applicable to the response compensation of various physical and chemical sensors. © 2003 American Institute of Physics. [DOI: 10.1063/1.1571948]

Most commercially available temperature sensors do not quickly respond to temperature fluctuations, and their cutoff frequencies are usually less than a few Hertz. If even such slow sensors are correctly compensated for the response delay, the response can be improved so as to be 10–100 times faster than the original uncompensated ones. The problem is to accurately obtain the thermal time-constant values. Despite this seemingly simple problem, it is actually very difficult to realize a rational compensation scheme. The time constant of a temperature sensor is a function of many physical quantities such as the physical properties of a measured object (gas or liquid) and its velocity, and this constant can change greatly depending on the circumstances around the sensor. Thus, the time-constant value given in the manufacturer’s specifications is hardly applicable to the response compensation. Meanwhile, dynamic calibration of the sensor response over all conceivable circumstances is virtually impossible. Even if such calibration were possible, we would need extra equipment monitoring the environments around the sensor in order to use the calibration data.

As an approach to resolve the above problem, we proposed a “two-thermocouple probe technique,” in which two temperature sensors of unequal time-constant values were combined so as to estimate in situ the time-constant values in the frequency domain from the sensors’ outputs (raw measurements), and to compensate the sensors for the response delay. In the present study, the two-thermocouple probe technique is extended to estimate time-constant values in the frequency domain and to benefit from the compensation techniques based on the fast Fourier transform (FFT) algorithm. Although the frequency-domain compensation technique may be inapplicable to a thermal field involving stepwise temperature changes owing to the use of the FFT (finite discrete Fourier transform), such a technique will also have the following advantages: (1) it can provide fast data processing; (2) the scheme for the time-constant estimation and the response compensation is simple and flexible; (3) digital noise filtering can be naturally implemented using the inverse FFT (IFFT); and (4) it can deal with sensors of arbitrary dynamic response besides the first-order system. First, the frequency-domain time-constant estimation scheme is developed. Second, the validity of the scheme is tested using experimental results previously reported.

The Fourier transform \( \hat{T}(f) \) [\( f \): frequency] of the sensor output \( T(t) \) (\( t \): time) can be expressed as
\[
\hat{T}(f) = R(f) + jI(f),
\]
where \( R(f) \) and \( I(f) \) are the real and imaginary parts of the Fourier transform. In the following, \( j \) is an imaginary unit and (\( ^* \)) denotes the Fourier transform. In the two-thermocouple probe technique mentioned above, the two temperature sensors of unequal time-constant values are used simultaneously. Then, the Fourier transforms of the temperatures measured by the two sensors, \( T_1(t) \) and \( T_2(t) \), are written as
\[
\hat{T}_1(f) = R_1(f) + jI_1(f),
\]
\[
\hat{T}_2(f) = R_2(f) + jI_2(f).
\]
Usually, the first-order system can provide a good approximation of the dynamic response of most temperature sensors. In the time domain, the first-order system is expressed as
\[
T_g = T + \tau \frac{dT}{dt},
\]
where \( T_g \) is the object temperature to be measured, and \( T \) and \( \tau \) are the sensor output (uncompensated temperature) and the time constant (=1/2\( \pi f_c \)), \( f_c \): cutoff frequency), respectively. In the frequency domain, on the other hand, the dynamic response of the first-order system is expressed by the transfer function \( H(f) = (1 + j2\pi f \tau)^{-1} \), and the relation between \( \hat{T}_g \) (input) and \( \hat{T} \) (output) is given by
\[
\hat{T} = H \hat{T}_g = \hat{T}_g / (1 + j\omega \tau),
\]
where \( \omega \) is an angular frequency (\( =2\pi f \)). Hence, when compensating the two sensor outputs \( T_1 \) and \( T_2 \) with the time constants \( \tau_1 \) and \( \tau_2 \) in the frequency domain, the Fourier transforms of the compensated temperatures, \( \hat{T}_{g1} \) and \( \hat{T}_{g2} \), are obtained from...
\[ \hat{T}_{g1} = (1 + j \omega \tau_1)(R_1 + jI_1), \]
\[ \hat{T}_{g2} = (1 + j \omega \tau_2)(R_2 + jI_2). \]

If the two sensors are detecting an identical temperature \( T_g \), the relation \( \hat{T}_{g1} = \hat{T}_{g2} (= \hat{T}_g) \) should hold. In reality, however, because of the existence of instrumentation noise and the limited spatial resolution of a measurement probe, the relation between \( \hat{T}_{g1} \) and \( \hat{T}_{g2} \) is expressed as \( \hat{T}_{g1} \approx \hat{T}_{g2} \).

Thus, by applying the idea of the two-thermocouple probe technique\(^1\,2\) to Eq. (3), the values of \( \tau_1 \) and \( \tau_2 \) can be estimated. To be more specific, \( \tau_1 \) and \( \tau_2 \) should be determined so as to minimize \( \varepsilon = |\hat{T}_{g1} - \hat{T}_{g2}|^2 \), where \( \hat{\cdot} \) denotes the average in the frequency domain. The concrete steps are given as follows. First, the expansion of Eq. (3) leads to
\[ \hat{T}_{g1} = (R_1 - I_1 \omega \tau_1) + j(I_1 + R_1 \omega \tau_1), \]
\[ \hat{T}_{g2} = (R_2 - I_2 \omega \tau_2) + j(I_2 + R_2 \omega \tau_2). \]

Then, the difference between the two Fourier transforms of the compensated temperatures, \( \hat{T}_{g1} - \hat{T}_{g2} \), is decomposed into the real and imaginary parts, \( e_R \) and \( e_I \), as
\[ e_R = (R_1 - R_2) - I_1 \omega \tau_1 + I_2 \omega \tau_2, \]
\[ e_I = (I_1 - I_2) + R_1 \omega \tau_1 - R_2 \omega \tau_2, \]
and the squared absolute value of the vector \( e_R + je_I \) in the complex plane is averaged in a frequency range \( f_{\text{min}} \leq f \leq f_{\text{max}} \). The average \( \varepsilon \) is written as
\[ \varepsilon = \frac{1}{2} e_R^2 + \frac{1}{2} e_I^2 = \frac{1}{2} \varepsilon_R^2 + \frac{1}{2} \varepsilon_I^2. \]

The lower and upper limits of the frequency range for calculating \( \varepsilon \), \( f_{\text{min}} \), and \( f_{\text{max}} \), are set so as to satisfy the relation \( 0 \leq f_{\text{min}} \leq f_{\text{max}} = \frac{1}{2} f_s \) \(( f_s \) : sampling frequency). The setting of these parameters will be discussed later. Finally, \( \varepsilon \) is minimized using the least squares method to determine the \( \tau_1 \) and \( \tau_2 \) values. Now, the right-hand side of Eq. (6) is expanded as
\[ \varepsilon = P_{11} \tau_1^2 + P_{22} \tau_2^2 - 2P_{12} \tau_1 \tau_2 - 2C_{12} \tau_1 + 2C_{12} \tau_2 \]
\[ + (R_1 - R_2)^2 + (I_1 - I_2)^2, \]
where \( P_{11}, P_{22}, P_{12}, \) and \( C_{12} \) denote
\[ P_{11} = (R_1^2 + I_1^2) \omega^2, \]
\[ P_{22} = (R_2^2 + I_2^2) \omega^2, \]
\[ P_{12} = (R_1 R_2 + I_1 I_2) \omega^2, \]
\[ C_{12} = (R_1 I_2 - R_2 I_1) \omega. \]

Then, the time-constant values that minimize \( \varepsilon \) are obtained by solving simultaneous equations derived from Eq. (7) together with the following conditions:
\[ \frac{\partial \varepsilon}{\partial \tau_1} = 0, \quad \frac{\partial \varepsilon}{\partial \tau_2} = 0. \]

The results are written as follows:

\[ \tau_1 = \frac{C_{12}(P_{22} - P_{12})}{P_{11} P_{22} - (P_{12})^2}, \quad \tau_2 = \frac{C_{12}(P_{12} - P_{11})}{P_{11} P_{22} - (P_{12})^2}. \]

First, two computer-generated first-order responses to a sinusoidal wave with \( \tau_1 \) and \( \tau_2 \) were used as test signals, and it was confirmed that Eq. (10) exactly reproduced the given \( \tau_1 \) and \( \tau_2 \) values. Next, the validity of the proposed scheme was tested using the previous experimental data\(^1\) which were obtained using a temperature probe composed of two \( R \)-type thermocouples (platinum/platinum 87%–rhodium 13%) to measure the fluctuating temperature field formed in a combustion wind tunnel. The experiments were performed under two experimental conditions, cases A and B, and the mean velocities were 1.4 and 5.1 m/s, respectively. A combination of thermocouples 40 and 100 \( \mu \)m in diameter was used in the case A experiment. For case B, thinner thermocouples 25 and 60 \( \mu \)m in diameter, were combined, since the temperature field of case B contained much higher-frequency temperature fluctuations than that of case A.\(^1\) [Of course, if the probe for case B had been used in the case A experiment, the signal to noise (S/N) ratio of the measurements would have been higher than the actual ones. However, the use of the case A probe in the case B experiment would have made the S/N ratio considerably low. Unfortunately, these data are now unavailable, and a detailed examination of the performance of the proposed scheme in the wide range of S/N ratio should be the subject of future investigation.] In the following, the subscripts 1 and 2 denote the thinner and thicker thermocouples of the probe, respectively.

At the beginning of the scheme test, the real and imaginary parts of the discrete Fourier transforms of the uncompensated temperatures (sensor outputs) of case B are shown
in Fig. 1. The sampling frequency $f_s$ and the data length were 4.9 kHz and $2^{12}$ (≈ 4096), respectively. Both ends of the data were made continuous by using a cosine-type data window, which was applied to the 100 data points at each end (Use of the data window had little effect on the present results.) As seen from Fig. 1, the Fourier transforms are considerably damped above 20 Hz (see Fig. 3) because of the response delay of the thermocouples. As shown later (Table I), the cutoff frequencies of the thermocouples 1 and 2 (case B) are about 17 and 4.4 Hz, respectively. Then, the frequency range for calculating $\tau$ of Eq. (6) was set at $0 < f < f_{\text{max}}$, and the change in the time-constant values estimated as a function of $f_{\text{max}}$ was investigated. The results are shown in Fig. 2. As seen clearly in this figure, the time-constant values estimated are kept nearly constant over a wide range except for the low-frequency region of $f_{\text{max}} < 10$ Hz. The present results indicate that the frequency-domain time-constant estimation scheme works successfully. When further increasing $f_{\text{max}}$, the estimated time-constant values gradually decrease since the instrumentation noise and the limited spatial resolution of the probe become increasingly influential.

Table I shows the comparison of the estimated $\tau_1$ and $\tau_2$ values between the present frequency-domain scheme [Eq. (10)] and the previously developed time-domain ones [$e_{\text{min}}^a$ and $R_{\text{max}}^b$ schemes]. The principle of the $e_{\text{min}}$ scheme is to minimize the difference between the two compensated temperatures, and that of the $R_{\text{max}}$ scheme to maximize the correlation coefficient between them. As is obvious from Table I, the time-constant values estimated from the present scheme are consistent with those obtained from the previous time-domain schemes. Finally, the Fourier transforms of compensated temperatures, $\hat{T}_{g1}$ and $\hat{T}_{g2}$, were calculated using Eq. (4) with the estimated time-constant values. The results are shown in Fig. 3. Clearly, the two Fourier transforms agree very well with one another. Figure 4 shows the compensated temperatures reproduced using the IFFT together with a digital finite impulse response low-pass filter with cutoff frequency of 1.2 kHz. As seen from Fig. 4, the two compensated temperatures highly correlate with one another, and the high-frequency temperature fluctuations are reproduced successfully.

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<td>$e_{\text{min}}^a$</td>
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<tr>
<td>$R_{\text{max}}^b$</td>
<td>34.9 ms</td>
<td>9.9 ms</td>
</tr>
<tr>
<td>present</td>
<td>29.3 ms</td>
<td>9.4 ms</td>
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*References*

Fig. 2. Time-constant values estimated using frequency-domain method (case B).

Fig. 3. Discrete Fourier transform (real and imaginary parts) of compensated temperatures (case B).

Fig. 4. Comparison between uncompensated and compensated temperatures (case B).