Shoitsuru Kato, Yutaka Tada, Yasuhiro Takeda, Yasuhiro Hirai, Yuichiro Nagatsu

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Correlation of Power Consumption for Propeller and Pfaudler Type Impellers

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Power consumption in unbaffled and baffled agitated vessels with propeller and pfaudler type impellers is measured over a wide range of Reynolds numbers from laminar to turbulent flow regions. The power correlation for the propeller and pfaudler type impellers is derived from modification of the previously proposed power correlation for a paddle impeller. The calculated correlation values agree well with experimental ones, and the same correlation can be applied to both propeller and pfaudler type impellers.

Introduction
Mixing and agitating vessels are widely used in chemical, biochemical, food and other industries. Power consumption is the most important parameter to estimate mixing performance. To estimate power consumption, the correlation of Nagata et al. (1956) has traditionally been used. However, this correlation was developed for two-blade paddle impellers, which do not always have the same numerical values of power consumption as those for multi-blade impellers. Kamei et al. (1995, 1996) and Hiraoka et al. (1997) developed the new correlation of power consumption shown in Table 1, and this correlation is more accurate than Nagata’s. However, the new correlation also cannot calculate the power consumption for other types of impellers, such as propeller and pfaudler type impellers used as axial flow impellers.

The propeller and pfaudler type impellers are used for low-viscosity liquid and solid—liquid suspensions, and the propeller type has been widely used in vessels ranging from portable type to large tanks. There are no correlations for these impellers. Therefore, we have developed a new correlation of power consumption for propeller and pfaudler type impellers, based on the correlations of Kamei and Hiraoka.

1. Experimental
A schematic diagram of a mixing vessel is shown in Figure 1. The vessel for the measurement of power consumption is a flat-bottom cylindrical vessel of inner diameter \( D = 200 \text{ mm} \). Three kinds of baffled conditions were used: unbaffled, four baffles of \( B_w = D/10 \) (i.e. the standard baffled condition), and fully baffled. The baffles were plate type. Also used were two kinds of impellers, a propeller and a pfaudler type, as shown in Figure 2. These propellers are not the marine type that have a constant pitch ratio; instead, they are variable pitch impellers, which are similar to pitched paddle impellers. The diameter, blade width and blade angle are shown in Table 2. The propeller impeller was symmetrically set up at one-half the level of the liquid depth (\( C/H = 0.5 \)) and at one-fourth (\( C/H = 0.25 \)) to obtain down-flow. The pfaudler type impeller was set up at the same level as the propeller (\( C/H = 0.25, 0.5 \)) as well as slightly above the bottom (bottom clearance of 1 mm). For measurement of the power consumption, the liquids used were desalted water and varying starch-syrup solutions (\( \mu = 0.003 - 13 \text{ Pa \cdot s} \)). The liquid was filled to the height equal to the vessel diameter (\( H = D \)). The power consumption \( P = 2\pi nT \) was measured with the shaft torque \( T \) and rotational speed \( n \) by using a torque meter (ST-3000, Satake Chemical Equipment Mfg., Ltd.). The range of rotational speed was from 60 to 540 rpm to avoid a large vortex at the liquid-free surface of the vessel center.

Fig. 1 Schematic diagram of experimental apparatus
Table 1 Correlation of power number for paddle impeller

<table>
<thead>
<tr>
<th>Unbaffled condition</th>
<th>Baffled condition</th>
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<tbody>
<tr>
<td>$N_{p0} = {1.2 \pi^{0.6} }^{[8 \theta/(D^2 H)]}f$</td>
<td>$N_{p0} = {1.2 \pi^{0.6} }^{[8 \theta/(D^2 H)]}f$</td>
</tr>
<tr>
<td>$f = C_f/Re_G + C_g [(C_p/Re_G) + Re_G]^{-1} \left( f_{cr}/\gamma \right)^{1/3}$</td>
<td>$f = C_f/Re_G + C_g [(C_p/Re_G) + Re_G]^{-1} \left( f_{cr}/\gamma \right)^{1/3}$</td>
</tr>
<tr>
<td>$Re_G = {2 \pi n_p / \theta H }$</td>
<td>$Re_G = {2 \pi n_p / \theta H }$</td>
</tr>
<tr>
<td>$C_L = 0.215 n_p / \theta H [1-(d/D)^2] + 1.83 \beta \sin \theta \theta / \theta_0 (n_p / 2 \sin \theta)^{1.3}$</td>
<td>$C_L = 0.215 n_p / \theta H [1-(d/D)^2] + 1.83 \beta \sin \theta \theta / \theta_0 (n_p / 2 \sin \theta)^{1.3}$</td>
</tr>
<tr>
<td>$C_t = [1.96 X^{1.3} + (0.25) X^{1.3} + (0.333)]^{-1}/0.2 \pi \cos \theta \theta / \theta_0$</td>
<td>$C_t = [1.96 X^{1.3} + (0.25) X^{1.3} + (0.333)]^{-1}/0.2 \pi \cos \theta \theta / \theta_0$</td>
</tr>
<tr>
<td>$m = [(0.71 X^{0.373})^{0.7} + (0.333)]^{-1}/0.2 \pi \cos \theta \theta / \theta_0$</td>
<td>$m = [(0.71 X^{0.373})^{0.7} + (0.333)]^{-1}/0.2 \pi \cos \theta \theta / \theta_0$</td>
</tr>
<tr>
<td>$X = n_p / \theta \sin \theta$</td>
<td>$X = n_p / \theta \sin \theta$</td>
</tr>
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</table>

Table 2 Geometry of impellers used

<table>
<thead>
<tr>
<th>Propeller 1</th>
<th>Propeller 2</th>
<th>Pfaunder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d/D = 0.365$</td>
<td>$d/D = 0.345$</td>
<td>$d/D = 0.475$</td>
</tr>
<tr>
<td>$b/d = 0.253$</td>
<td>$b/d = 0.326$</td>
<td>$b/d = 0.126$</td>
</tr>
<tr>
<td>$n_p = 3$</td>
<td>$n_p = 3$</td>
<td>$n_p = 3$</td>
</tr>
<tr>
<td>$\theta = \pi/4$</td>
<td>$\theta = \pi/6$</td>
<td>$\theta = \pi/2$</td>
</tr>
</tbody>
</table>

2.1 Unbaffled condition

When the correlations in Table 1 were used for the propeller and pfaunder type impellers, the estimated values agreed with the measured ones in the laminar region, but the estimated ones were approximately 1.5—2 times as large as the measured ones in the transition and turbulent regions, as shown in Figure 3.

The correlations of Table 1 were developed for paddle and pitched paddle impellers. The blades of the propeller and pfaunder impellers do not have sharp edges. The laminar term $C_L$ in Eq. (T.1) in Table 1 can be used without modification. Because the deviation from correlations to measured values in the turbulent region was large, the turbulent terms $C_t$ and $m$ in Table 1 were modified to reproduce the measured values, as follows.

$$C_t = [(3.8 X^{3.3})^{0.7} + (0.25) X^{1.3} + (0.333)]^{-1}/0.2 \pi \cos \theta \theta / \theta_0$$

$$m = [(0.8 X^{0.373})^{0.7} + (0.333)]^{-1}/0.2 \pi \cos \theta \theta / \theta_0$$

2.2 Fully baffled condition

As shown in Table 1, three kinds of equations must be used according to the value of the impeller similarity parameter ($n_p / \theta b / d$) for a paddle impeller; however, only one equation is needed for the propeller and pfaunder impellers, as follows.

$$N_{p0} = [(1+x)^{-1/3}]^{N_{p0}}$$

$$x = 4.5(B_w / D) n_b^{0.6}/N_{p0}^{0.2} + N_{p0}/N_{pmax}$$

In the present experimental conditions, the values of $N_{pmax}$ for the propeller and pfaunder impellers were almost the same as $N_p$ of the standard baffled condition, shown in Figure 3.

2.3 Baffled condition

The exponent of the blade angle term only was modified, as follows.

$$N_{p0} = [(1+x)^{-1/3}]^{N_{p0}}$$

$$x = 4.5(B_w / D) n_b^{0.6}/(2 \theta \pi^{0.72} N_{p0}^{0.2}) + N_{p0}/N_{pmax}$$
2.4 Correlations of power number

The new correlations for the propeller and the Pfaudler type impellers are shown in Table 3, Figures 4, 5 and 6 show the values estimated by Table 3 and the measured ones. The same correlations can be used for the propeller and the Pfaudler type impellers, regardless of the clearance between the vessel bottom and impeller.

![Fig. 4](image1.png)

**Fig. 4** Power diagram of propeller type impeller (1)

![Fig. 5](image2.png)

**Fig. 5** Power diagram of propeller type impeller (2)

![Fig. 6](image3.png)

**Fig. 6** Power diagram of Pfaudler type impeller

### Table 3 New correlations of power number for propeller and Pfaudler

**Unbaffled condition**

\[ N_{\text{P0}} = \left(1.2 \pi \beta f \right)[10^7 (d/D)^2 H] \]

\[ f = \frac{C_t}{R_{\text{e}}} + \frac{C_m}{[(C_m/R_{\text{e}}) + R_{\text{e}}]^2} + \left(1 - \frac{C_m}{R_{\text{e}}} \right)^{1.5} \]

\[ R_{\text{e}} = \frac{\eta \pi n_d (d/D)^2 (4d/B_d) R_{\text{e}}}{[0.8(0.373)^{2.8} + (0.333)^{2.8}]^{1.7/8}} \]

\[ C_t = 3 \left(0.25 \right)^{7.8} + (0.25)^{7.8} \]

\[ m = \left(0.8 \pi \right)^{0.373} (3.8)^{2.8} \]

\[ C_u = 23.8(d/D)^{2.24} (sin H/H) \]

\[ f_c = 0.0151(d/D)^{0.3088} \]

\[ X = n_p^{0.7} b \sin 1.6 \theta / H \]

\[ \beta = 2 \ln (D/d) [(d/D) - (d/D)] \]

\[ \gamma = \left[ \pi n_d (d/D)^2 (\beta d/d)^{1.7/3} \right] \]

\[ n_p = 0.71(0.157 + [n_p, \ln (D/d)]^{0.611}) / \left[ n_p^{0.52} \left(1 - (d/D)^2 \right) \right] \]

**Baffled condition**

\[ N_{\text{P0}} = \left(1 + x^3 \right)^{1.5} N_{\text{Pmax}} \]

\[ x = 4.5(B_B/D)^{0.5} \left(2 \theta \pi + 0.92 \right) \]

\[ N_{\text{Pmax}} = 6.5 \left( n_p^{0.8} \sin 1.8 \theta / d \right)^{1.7} \]

### Conclusions

A new correlation of power consumption, based on the correlation of Kamei and Hiraoka, was developed for propeller and Pfaudler type impellers, and it was shown that the estimated values of the power number agree very closely with the measured ones. In future work, this correlation will be expanded to other impellers.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Value</th>
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<tbody>
<tr>
<td>( b )</td>
<td>height of impeller blade [m]</td>
</tr>
<tr>
<td>( C )</td>
<td>clearance between bottom and impeller [m]</td>
</tr>
<tr>
<td>( D )</td>
<td>vessel diameter [m]</td>
</tr>
<tr>
<td>( d )</td>
<td>impeller diameter [m]</td>
</tr>
<tr>
<td>( f )</td>
<td>friction factor [-]</td>
</tr>
<tr>
<td>( H )</td>
<td>liquid depth [m]</td>
</tr>
<tr>
<td>( N_p )</td>
<td>power number (=( P/\rho \omega d^3 )) [-]</td>
</tr>
<tr>
<td>( N_{\text{P0}} )</td>
<td>power number in unbaffled condition [-]</td>
</tr>
<tr>
<td>( N_{\text{Pmax}} )</td>
<td>power number in fully baffled condition [-]</td>
</tr>
<tr>
<td>( n )</td>
<td>impeller rotational speed [-]</td>
</tr>
<tr>
<td>( n_b )</td>
<td>number of baffle plates [-]</td>
</tr>
<tr>
<td>( n_p )</td>
<td>number of impeller blades [-]</td>
</tr>
<tr>
<td>( P )</td>
<td>power consumption [W]</td>
</tr>
<tr>
<td>( R_{\text{e}} )</td>
<td>impeller Reynolds number (=( d^2 \rho / \mu )) [-]</td>
</tr>
<tr>
<td>( R_{\text{e}0} )</td>
<td>modified Reynolds number [-]</td>
</tr>
<tr>
<td>( T )</td>
<td>shaft torque [N \cdot m]</td>
</tr>
</tbody>
</table>
\[ \theta = \text{angle of impeller blade} \quad [-] \]
\[ \mu = \text{liquid viscosity} \quad [\text{Pa} \cdot \text{s}] \]
\[ \rho = \text{liquid density} \quad [\text{kg/m}^3] \]

**Literature Cited**


