Original Paper

Genetic Algorithm with a Changing Grid for Facility Location Problems in a Two-Stage Logistics System with Restricted Inventory Capacity

Hung Wei Cheng ¹¹ and Masahiro ARAKAWA ¹²

Abstract: This paper discusses the construction of a mathematical model and an optimization procedure for facility location problems in a two-stage logistics system consisting of a plant, distribution centers, and shops. The model is constructed to search for the optimal locations of the distribution centers. The goal is to minimize the total distance between plants and distribution centers and between distribution centers and shops considering the demand distribution and restricted inventory capacity at the distribution centers. In addition, the model includes logistic constraints based on the demand distribution. A genetic algorithm is developed to solve the problem. The algorithm is used in a two-step procedure that modifies the grid to search for the optimal locations of the distribution centers within a reasonable computation time. The algorithm is demonstrated on a simple example to investigate its performance and the effect of grid modification.

Key words: facility location problem, two-stage logistics system, genetic algorithm, changing grid, restricted capacity

1 INTRODUCTION

The facility location problem involves finding the optimal locations for facilities on a map. Many researchers have proposed facility location models and procedures for finding the optimal solution. For the facility location problem with logistic constraints, researchers have discussed selecting the locations under the condition that candidate locations are predetermined with transportation cost and/or distance constraints between the facilities. Most of these studies consider a single-stage logistics system [1]-[4]. A multistage logistics system consisting of factories, distribution centers, and shops is a typical model for the transportation of products between production centers and demand locations. Here, distribution center is abbreviated as 'DC'. A typical logistics problem in this context is to decide the quantity of products transported between different locations under transportation capacity and storage capacity constraints. It is clear that the demand at a particular shop depends on the population distribution in the surrounding area. However, the number of shops is usually larger than the number of DCs and can change over time. Therefore, we focus on DC locations given a demand distribution. In addition, we consider constraints on the inventory capacity in DCs and the transportation capacity between different locations. There are studies that treat facility location as a multistage logistics system. Klose and Drexel [5] reviewed studies of facility location models for distribution system design. Sahin and Sural [6], and Teixeira and Antunes [7] reviewed studies of the hierarchical facility location problem. Liu et.al [8] focused on the locations of DCs in a two-stage logistics system, and it is similar to this study. In studies discussed in these reviews and the article, there are predetermined candidate positions, and the optimal locations are selected from the candidates using a mixed integer programming (MIP) model based on a network model. The techniques used to solve the MIP model include branch and bound [8],[9], goal programming
[10], Lagrangian relaxation [11],[12], combination method of Lagrangian relaxation and simulated annealing [13], genetic algorithm [14],[15], and so on. However, placing facilities at arbitrary locations on a map is difficult and was not discussed in these articles. If there are many predetermined candidate positions to place facilities at arbitrary locations on a map, it is difficult to search for the optimal locations in a reasonable computation time. In this study, a genetic algorithm is applied and the following techniques are introduced to reduce the computation time:

(1) Place grids on a map and allocate candidate locations for DCs on the grid,

(2) Reduce grid size and area for optimal locations in the search process.

The problem now involves selecting the optimal location from a set of discrete locations on the grid. We first construct a mathematical model for the facility location problem in a two-stage logistics system considering the demand distribution and restricted inventory capacity of the DCs. A genetic algorithm with controlled grid sizes is proposed. The objective function is the total distance traveled. A simple example is studied to evaluate the effectiveness of the algorithm and reducing grid size, and the area for optimal locations search. In addition, the influence of inventory restrictions is investigated.

2 MODEL

2.1 Model characteristics

Figure 1 shows a network model for a two-stage logistics system. Figure 2 shows sample of facility locations on a two-dimensional map. The logistics system consists of factories, DCs, and shops. The grid points are the candidate DC locations, and the locations of the factories and shops are predetermined. The demand at shops is calculated from a predetermined demand distribution for the surrounding area; a normal distribution is used. DCs are temporarily allocated to specific grid points. The inventory capacity of the DCs affects the quantity of products that can travel between different locations. Therefore, the total quantity of products to be transported influences the number of DCs.

The characteristics of this logistics model are as follows:

(1) This model is developed to resolve two types of problems simultaneously: a problem to decide the amount products to be shipped in a two-stage logistics system, and the problem of deciding locations and the number of DCs.

(2) Priority to select appropriate facilities is adopted to decide routes between different facilities and locations of DCs.

(3) Two types of grid sizes are used to decide locations of DCs in detail. Changing grid size is useful for reducing computation time.

(4) The capable inventory capacity at DCs must be controlled according to the number of small-sized grids included in a large-sized grid.

Introducing grids to seek the optimal facility location is popular in facility location problems. However, different grid sizes are adopted in a two-step process to seek approximate optimal locations and the number of facilities in this study. The two-step process includes characteristics (2) and (3).

2.2 Mathematical model

Constants and variables used in the model are:

Constants
PLANT: Set of factories
DC: Set of distribution centers
SHOP: Set of shops
d_{ij}: distance between factory i and DC j
d_{jk}: distance between DC j and shop k
CP_{ik}: inventory capacity or maximum production

![Schematic diagram of a two-stage logistics system](image-url)
in factory \( i \) at grid point \((l, b)\)

\( CD_{lb}^i \): inventory capacity in DC \( j \) at grid point \((l, b)\)

\( CS_{lb}^k \): predicted demand in shop \( k \) at grid point \((l, b)\)

\( CM_{lb} \): predetermined demand at grid point \((l, b)\)

\( \mu_{lb} \): average demand at grid point \((l, b)\)

\( \sigma_{lb} \): standard deviation of demand at grid point \((l, b)\)

\( S_{lb} \): number of shops at grid point \((l, b)\)

**Variables**

\( R_{ij} \): 0-1 variable; \( R_{ij} \) is 1 when there is transportation from factory \( i \) to DC \( j \), otherwise it is 0

\( R_{jk} \): 0-1 variable; \( R_{jk} \) is 1 when there is transportation from DC \( j \) to shop \( k \), otherwise it is 0

\( X_{ij} \): product quantity in transportation from factory \( i \) to DC \( j \)

\( X_{jk} \): product quantity in transportation from DC \( j \) to shop \( k \)

\( M_{lb} \): 0-1 variable; \( M_{lb} \) is 1 when a DC is located at grid point \((l, b)\), otherwise it is 0

The model is:

**Objective function**

\[
\text{Min} \sum_{i \in \text{PLANT}} \sum_{j \in \text{DC}} R_{ij} d_{ij} + \sum_{j \in \text{DC}} \sum_{k \in \text{SHOP}} R_{jk} d_{jk}
\]

\[
\text{s.t. } \sum_{j \in \text{DC}} X_{ij} \leq CD_{lb}^i \quad \forall i \in \text{PLANT} \quad (2)
\]

\[
\sum_{k \in \text{SHOP}} X_{jk} \leq CD_{lb}^j \quad \forall j \in \text{DC} \quad (3)
\]

\[
\sum_{j \in \text{DC}} X_{jk} = CS_{lb}^k \quad \forall k \in \text{SHOP} \quad (4)
\]

\[
\sum_{i \in \text{PLANT}} \sum_{k \in \text{SHOP}} X_{ij} = \sum_{k \in \text{SHOP}} X_{jk} \quad \forall j \in \text{DC} \quad (5)
\]

\[
X_{ij} \leq CD_{lb}^i R_{ij} \quad \forall i \in \text{PLANT}, \forall j \in \text{DC} \quad (6)
\]

\[
X_{jk} \leq CD_{lb}^j R_{jk} \quad \forall j \in \text{DC}, \forall k \in \text{SHOP} \quad (7)
\]

\[
CM_{lb} = F^{-1}(p| \mu_{lb}, \sigma_{lb}) \quad (8)
\]

\[
p = F(CM_{lb}| \mu_{lb}, \sigma_{lb}) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{CM_{lb}} e^{-\frac{(t-\mu_{lb})^2}{2\sigma^2}} dt \quad (9)
\]

\[
CS_{lb}^k = \sum_{i=1}^{l+1} \sum_{b=1}^{l+b} \frac{CM_{lb}}{S_{lb}} \quad \forall k \in \text{SHOP} \quad (10)
\]

\[
\sum_{i=1}^{l+1} M_{lb} \leq 1 \quad \forall b \quad (11)
\]

\[
\sum_{i=1}^{l+1} M_{lb} \leq 1 \quad \ell = 2, 3, \ldots, L - 1, \forall b \quad (12)
\]

\[
\sum_{\ell=L-1}^{L} M_{lb} \leq 1 \quad \forall b \quad (13)
\]

\[
\sum_{b=1}^{2} M_{lb} \leq 1 \quad \forall \ell \quad (14)
\]

\[
\sum_{b=2}^{b+1} M_{lb} \leq 1 \quad b = 2, 3, \ldots, B - 1, \forall \ell \quad (15)
\]

\[
\sum_{b=B-1}^{B} M_{lb} \leq 1 \quad \forall \ell \quad (16)
\]

---

**Fig. 2** Sample two-dimensional map showing facility locations.
\[ R_{ij} \in \{0,1\}, \quad R_{jk} \in \{0,1\} \quad (17) \]

\[ X_{ij} \geq 0, \quad X_{jk} \geq 0 \quad (18) \]

The objective function in Eq. (1) minimizes the sum of the distances between the factories and the DCs, and between the DCs and the shops. Equation (2) requires the total quantity of the products manufactured to be less than the inventory capacity or the maximum production in the factories. Equation (3) requires the total quantity of products supplied from DCj to all the shops to be less than the inventory capacity of DCj. Equation (4) requires the total quantity of products supplied from all DCs to shop k to be equal to the estimated demand at that shop. Equation (5) requires the total quantity of products supplied from DCj to all shops to be equal to the total quantity of products supplied from all factories to DCj. Equation (6) requires the quantity of products supplied from every factory to every DC to be less than the inventory capacity of the DC. Equation (7) requires the quantity of products supplied from every DC to every shop to be less than the inventory capacity of the DC. Equation (8) specifies the demand calculated from the predetermined demand distribution. Equation (9) specifies the demand at shop k calculated from the predetermined demand distributions of the surrounding area. The quantity of demand at shops is estimated from demand distribution. Equations (10) to (15) require the distance between the locations of two DCs to be greater than or equal to the grid size.

3 SEARCH PROCEDURE

3.1 Genetic algorithm to search for DC locations

In this study, a genetic algorithm (GA) is proposed to find a high-quality solution within a reasonable computation time. Figure 3 shows a representation of the chromosome. The positive numbers in the elements of the chromosome denote factory numbers, DC numbers, and shop numbers. In the left section of the chromosome, DC numbers are calculated from the sum of the original DC number and the number of factories. In the right section of the chromosome, shop numbers are calculated from the sum of the original shop number and the total number of DC candidates.

The numbers in the chromosome indicate the priority of the facilities to which products are transported. If the number in the first element of the chromosome is 1, factory 1 is selected. Then, the route with the lowest transportation cost from factory 1 to the DC is selected. The quantity of products transferred corresponds to the demand at the DC or the availability at the factory. The weight mapping crossover (WMX) procedure is used in the GA [8]. Figure 4 shows a schematic diagram of the WMX procedure. A uniform random number is used to select a cutting point in the chromosome. To prevent the generation of dead chromosomes, numbers in the exchanged part are arranged in ascending order. This process generates a relationship between the numbers in two exchanged parts extracted from parent chromosomes. Then, according to the array of numbers in the exchanged part, numbers are rearranged in the original part. For the mutation, a swapping procedure is adopted in which the numbers in two genes selected randomly are swapped. In the selection procedure for the next generation, the elitist preservation procedure and roulette selection are adopted. Genes in the left and right sections of the chromosome are considered separately in the GA. After the array of facility numbers in a single chromosome is decided,
the right section of the chromosome is decoded to find the quantity of products transported between the DCs and the shops. The left section of the chromosome is then decoded to find the quantity of products transported between the DCs and the factories. A single chromosome is decoded by the following procedure.

Procedure to decode chromosome

**Step 1:** Select facility number from the chromosome.

**Step 2:** Select the nearest facility to that selected in Step 1. The two facilities selected must be a supply-demand pair.

**Step 3:** Investigate the inventory and demand at the facilities selected in Steps 1 and 2 to decide the quantity of products transported.

**Step 4:** If all products have been transported, end the procedure. Otherwise, return to Step 2.

Figure 5 shows a decoding example using a chromosome. The numbers in left side in the chromosome denote a modified number of factories and DCs, and numbers in the right side denote a modified number of DCs and shops. 1", 1', and so on are the modified numbers used in the chromosome.

The order of numbers denotes the priority of facilities selection. From the chromosome, DC1 is selected with regards to logistics between a factory and DCs. In addition, the number of products transported between the factory and DC1 is decided to the maximum number of products capable of being transported or production quantity. Then, if there is inventory at the factory, inventory is assigned as products transported to DC2. On the other hand, Shop2 is selected with regards to logistics between DCs and shops from the right side of the chromosome. Then, the nearest DC (DC1) to Shop2 is selected and the number of products transported from DC1 to Shop2 is decided as the maximum number of products capable of being transported or the inventory of DC1.

### 3.2 Adjusting the grid

It is clear that the solutions obtained depend on the grid used. When the grid is finer, the DCs can be located more precisely. However, fine grids require more computation time to seek the optimal solution. Therefore, we reduce the grid size and area for searching for the optimal locations. In Step 3 of the algorithm, the grid is coarse, and in Step 4, it is finer. In Step 4, the DC locations obtained in Step 3 define the centers of fine grids over a limited area. Step 3 reduces the computation time, and Step 4 seeks high-quality solutions. Figure 6 shows a flowchart of the two-step procedure using the GA. Figure 7 shows the coarse grid of Step 3 and the fine grid of Step 4.

---

**Fig. 4** Schematic diagram of WMX procedure.

**Fig. 5** Example of route selection from a chromosome.
Procedure to seek approximate optimal solutions

**Step 1:** Divide the map into grids of a \( \times \) b.
**Step 2:** Calculate the demand for each grid and for every shop.

**Step 3:**
**Step 3-1:** Combine four grids into a single grid. The centers of the combined grids are treated as candidate DC* locations.
**Step 3-2:** Seek the approximate optimal DC* locations using the GA on the combined grids.

**Step 4:**
**Step 4-1:** Around each DC* location found in Step 3, define a fine grid in a limited area as candidate DC locations.
**Step 4-2:** Search for approximately optimal DC locations on these grids using the genetic algorithm.

### 3.3 Calculation of modified inventory capacity at DCs given in the adjusted grid

In Step 3, since a coarse grid is used, there are fewer candidate locations. Therefore, the remaining candidates should include the quantity of products stored for the removed candidates. Thus, Eq. (3) is relaxed to

\[
CD_{lb}^{j} = \sum_{i=1}^{l+1} \sum_{b=1}^{b+1} CD_{ib} \quad \forall j \in DC
\]

In Eq. (19), \( CD_{lb} \) indicates the inventory capacity for a single DC and is a constant in this study.

### 3.4 Calculation of demand at shops

In this section, a procedure to calculate the demand at shops is described for a simple example consisting of a single factory and six shops. The potential DC locations are on a two-dimensional map. The factory and shop locations (see Fig. 8) are predetermined. The grid points define the candidate DC locations. The number of candidate locations depends on the number of squares in the grid. A Voronoi diagram is drawn to find the area determining the demand at each shop, and Fig. 9 shows this diagram. The area divided by the Voronoi edges is assumed to be the region containing the customers for the associated shop. The regions in a single grid determined by the Voronoi edges are of different sizes. It is assumed that the overall demand assigned to a single grid is distributed equally to

![Fig. 6 Flowchart of two-step procedure to find optimal DC locations.](image)

![Fig. 7 Schematic diagrams of maps denoting candidates for Step 3 and Step 4.](image)
all the different regions. Therefore, the demand at each shop is estimated by Eq. (9), with $\text{Slb}$ set to the number of partial areas. Figure 10 shows the number of partial areas into which each square is divided by the Voronoi edges. Here, an empty square indicates that the number of partial areas is one; there is no Voronoi edge in that square.

4 NUMERICAL EXPERIMENT USING A SIMPLE EXAMPLE

4.1 Characteristics of example

In this section, the GA is evaluated using a simple example. The example consists of a two-dimensional map of $100 \times 100$ points with a single factory and six shops. Table 1 shows the coordinates of the factory and shops. The locations correspond to the points on the map in Fig. 8 and are randomly predetermined. Although the shops are located randomly, the estimated demand at every shop is calculated using the demand distribution. Figure 11 shows the distribution of demand on the map.

4.2 Results and discussion

In Step 4, the number of grid squares around a single DC* affects the solution. The number of squares is denoted by $N$. We set $N$ to 16, which indicates a $4 \times 4$ grid using our method. In this section, our method is compared with the following methods:

Method (1): $N$ is set to 4, indicating a $2 \times 2$ grid.

Method (2): Step 3 of our algorithm.

Method (3): Optimization procedure to solve the MIP given by Eqs. (1) to (15) using our two-step procedure.

Method (4): Optimization procedure to solve the MIP given by Eqs. (1) to (15) using Step 3 of our algorithm.

CPLEX 9.0 (ILOG) is used for Method (3) and (4). In addition, the whole grids shown in Fig. 11 are adopted for Method (2) and (4). The genetic parameters used in our algorithm and Method (2) are: population 50, maximum number of generations 1000, ratio of crossover 0.8, and ratio of mutation 0.2.

Table 2 compares the total distance, the number of DCs, and the computation time of the different methods. Figure 12 shows the DC locations obtained using our method and Method (4). Figure 13 shows the DC locations obtained using our

---

**Fig. 8** Schematic diagram of shops and a factory on a map.

**Fig. 9** Voronoi diagram dividing the map into areas, determining the demand at shops.

**Fig. 10** Number of partial areas into which each square is divided by Voronoi edges.
Table 1 Coordinates of one factory and six shops.

<table>
<thead>
<tr>
<th>Factory</th>
<th>x</th>
<th>y</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>50</td>
<td>6000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shop</th>
<th>x</th>
<th>y</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73</td>
<td>15</td>
<td>1467</td>
</tr>
<tr>
<td>2</td>
<td>87</td>
<td>5</td>
<td>667</td>
</tr>
<tr>
<td>3</td>
<td>39</td>
<td>24</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>93</td>
<td>22</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>72</td>
<td>57</td>
<td>1200</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td>82</td>
<td>450</td>
</tr>
</tbody>
</table>

Table 2 Comparison of total distance, number of DCs, and computational time.

<table>
<thead>
<tr>
<th>Method</th>
<th>Total distance</th>
<th>Number of DCs</th>
<th>Computational time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Method</td>
<td>588.339</td>
<td>13</td>
<td>192</td>
</tr>
<tr>
<td>Method (1)</td>
<td>616.219</td>
<td>12</td>
<td>127</td>
</tr>
<tr>
<td>Method (2)</td>
<td>643.351</td>
<td>12</td>
<td>226</td>
</tr>
<tr>
<td>Method (3)</td>
<td>557.638</td>
<td>12</td>
<td>1570</td>
</tr>
<tr>
<td>Method (4)</td>
<td>557.638</td>
<td>12</td>
<td>73859</td>
</tr>
</tbody>
</table>

method and Method (2). The total distance of the solution generated by our method is only 6Method (2), is inferior to our method with regard to total distance and computational time. This is because the number of candidate DC locations in Method (2) is larger than the number of candidates in the second step of our method. Therefore, in Method (2) the search easily gets trapped at local optimal solutions. A comparison of Method (1) and our method shows that the number of grid squares affects the quality of the solutions in the second step of our method. The model can be adapted to adjust the $4 \times 4$ grids used in the second step, and introducing overlapping areas may be effective.

When the different condition including concentration of demand at center area on a map and four times large area is used in numerical experiment, total distance of the best solution obtained by the proposed method had an error of about 15% from the optimal solution. Furthermore, the following characteristics of the proposed method are obtained from numerical experiments including the different conditions:

1. The number of DCs and distribution of DCs locations obtained by the proposed method are similar to the optimal solution. However, locations of DCs are several grid sizes away from the locations in the optimal solutions.

2. On the condition that demand is concentrated

Fig. 11 Distribution of demand on the map. (Values indicate demand per a single grid)

Fig. 12 Locations of DCs obtained by our method and Method (4).

Fig. 13 Locations of DCs obtained by our method and Method (2).
at center area on a map, it is possible to generate multiple optimal solutions. It tends that DCs are obtained at locations away from the locations in the optimal solutions. The total distance obtained by the proposed method can be similar to the optimal solution.

5 INFLUENCE OF INVENTORY CAPACITY AT DISTRIBUTION CENTERS

In the previous section, the inventory capacity at the DCs is predetermined. It is clear that the capacity affects both the number of DCs and their locations, so different capacities will lead to different solutions. In this section, the impact of the capacity is investigated using our method. Figure 14 shows the number of DCs and the total distance obtained using our method under different capacity conditions. Each mark indicates the average value from 25 trials, and the error bar indicates the standard deviation of the values. This figure shows that increasing the inventory capacity at the DCs reduces the number of DCs and the total distance. When the capacity is set to 6000, the best solution obtained using our method has a minimum total distance of 189.424 under the constraint that the number of DCs is three. Here, the capacity is larger than the total quantity of products transported between different locations. When the capacity is set to 6000 and there is a single DC, the minimum total distance is 229.65 and the DC is located at (70, 30). This distance is larger than that obtained with a capacity of 6000 and three DCs. This result shows that, although increasing the inventory capacity reduces the number of DCs, a single DC may not always give the minimum total distance because of the biased distribution of demand and shops. Figure 15 shows the DC locations of the best solutions obtained using our method under different DC capacities. In addition, it shows the location of a single-DC solution, obtained under the condition that a single DC is allocated. In this example, a large demand is assigned in the upper right area and a small demand is assigned in the upper left area. When the DC capacity is small, many DCs are located in the upper right area. When it is large and few DCs are allocated, the DCs are only partially located in the upper right area. These DC distributions are intuitively reasonable. Furthermore, these results show that our method is useful for solving facility location problems.

6 CONCLUSION

In this study, a mathematical model was constructed and a genetic algorithm was proposed for the facility location problem in a two-stage logistics system considering the demand distribution and logistics. The proposed genetic algorithm is a two-step procedure, and different grids are used in the two steps. The method was evaluated using

![Fig. 14](image1.png) Total distance and number of DCs obtained using our method under different inventory capacity conditions.

![Fig. 15](image2.png) Comparison of DC locations allocated using our method under different inventory capacity conditions.
a simple example with regard to solution quality and computation time. Our algorithm is effective, generating approximately optimal facility locations within a reasonable computation time. In addition, we investigated the influence of inventory capacity at the DCs. In the future, we plan to test the algorithm on an actual problem. Most actual problems are not square; they may be curved or hollow. In addition, there are areas such as ponds and mountains that are difficult to navigate. Therefore, we will need to develop a procedure that enables mapping both complex configuration and simple configurations, and a procedure for generating a variable grid for complex configurations.

REFERENCES


