Multi-Objective Genetic Algorithm to Design Manufacturing Process Line Including Feasible and Infeasible Solutions in Neighborhood

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Abstract: This paper treats multi-objective problem for manufacturing process design. A purpose of the process design is to decide combinations of work elements assigned to different work centers. Multiple work elements are ordinarily assigned to each center. Here, infeasible solutions are easily generated by precedence relationship of work elements in process design. The number of infeasible solutions generated is ordinarily larger than that of feasible solutions generated in the process. Therefore, feasible and infeasible solutions are located in any neighborhood in solution space. It is difficult to seek high quality Pareto solutions in this problem by using conventional multi-objective evolutional algorithms. We consider that the problem includes difficulty to seek high quality solutions by the following characteristics: (1) Since infeasible solutions are resemble to good feasible solutions, many infeasible solutions which have good values of objective functions are easily sought in the search process, (2) Infeasible solutions are useful to select new variable conditions generating good feasible solutions in search process. In this study, a multi-objective genetic algorithm including local search is proposed using these characteristics. Maximum value of average operation times and maximum value of dispersion of operation time in all work centers are used as objective functions to promote productivity. The optimal weighted coefficient is introduced to control the ratio of feasible solutions to all solutions selected in crossover and selection process in the algorithm. This paper shows the effectiveness of the proposed algorithm on simple model.

Keywords: Process design, process line, feasible and infeasible solution, multi-objective genetic algorithm, mix production, simulation

1. Introduction

This paper treats multi-objective problem for process design. A purpose of process design is to decide combinations of work elements assigned to all work centers. Here, the work elements denote works to produce a single product and multiple work elements are ordinarily assigned to every center. There is possibility that infeasible solutions are generated by precedence relationship of work elements in process design. In the process of process design, the number of infeasible solutions sought is ordinarily larger than that of feasible solutions. However, there are the cases that infeasible solutions are superior to feasible solutions in terms of objective functions.

This problem is treated as multi-objective problem in order to design process line for high productivity. Especially mix production appeared in a large number of process lines is focused. The following functions are introduced as multi-objective functions to evaluate the productivity of process line for mix production: the maximum value of average operation times in all work centers and the maximum value of dispersion of operation times in all work centers.

However, this problem is including feasible and infeasible solutions in any neighborhood in solution
space. Therefore, it is difficult to seek high quality feasible solutions by using evolutionary algorithms.

In past studies related to process design, many researchers proposed heuristic methods and rule-based procedures [1-6]. Here, one of the most conventional and popular procedures for process design is the ranked positional weight technique [6]. This is a heuristic procedure that has been used to develop many designs. However, process design is focused on a single-objective problem for line balance in these studies. In addition, infeasible solutions are neglected or infeasible solutions are modified to feasible solutions in the search process. In the case of conventional genetic algorithm, lethal chromosomes are modified to normal chromosomes to prevent from generating infeasible solutions.

We consider that a problem of process design includes the following characteristic causing difficulty to seek high quality solutions:

(1) Many infeasible solutions which have good values of objective functions are easily sought in the search process, since infeasible solutions are resemble to good feasible solutions,

(2) Infeasible solutions are useful to select new variable conditions generating good feasible solutions in search process.

When new conditions are generated from existing solutions in evolutilional algorithm under the condition that feasible and infeasible solutions have same possibility to be selected, Characteristic (1) causes generation of a large number of infeasible solutions in the search process.

On the other hand, a genetic operation using Characteristic (2) is expected to promote seeking high quality feasible solutions since various patterns of chromosomes are generated. Therefore, multi-objective genetic algorithm (MOGA) including a search procedure using Characteristic (2) is proposed in this paper. Both chromosomes which generate feasible and infeasible solutions are used to select as parent chromosomes in crossover process and as chromosomes preserved for next generation in the proposed algorithm. Probability to select chromosomes generating infeasible solutions is controlled in crossover and selection for next generation in the proposed algorithm. Then, high quality feasible solutions are expected to be sought.

In this paper, the characteristics of the proposed algorithm are explained and the performance of the proposed algorithm is evaluated on simple model for process design for mix production.

2. Process Design

2.1 Characteristics of Process Design and Objective Functions

Fig.2 shows schematic diagram of assignment process for process design. Process design denotes that all work elements required to manufacture products are assigned to work centers. A precedence relationship is constructed among different work elements and this relationship is affected as constraints to assign the work elements to the work centers. Therefore, when a procedure relationship is not satisfied between different work elements assigned to works, infeasible solutions are generated. Ordinarily, since multiple work elements are assigned to each single work center, infeasible solutions which take good resultant values of objective functions are easily generated.

![Fig. 1 Schematic diagram of search process for process design.](image-url)
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Minimum of work centers is calculated from production plan. In conventional process design, target cycle time is calculated to satisfy production plan. Here, target cycle time denotes time interval to manufacture required products from a production line in predetermined period. The cycle time is defined as the maximum operation time of work elements assigned among work centers.

The minimum number of work centers ‘\(N_{\text{min}}\)’ and target cycle time ‘\(T_c\)’ are calculated from Eqs. (1) and (2) from production plan.

\[
N_{\text{min}} = \left\lceil \frac{T_{\text{all}}}{T_c} \right\rceil \quad (1)
\]

\[
T_c = \frac{S}{Q} \quad (2)
\]

Here, \(T_{\text{all}}\) is total operation time to complete a product. \(S\) is planning period where products are able to be produced. \(Q\) is total amount of products required to produce. \(\lceil \cdot \rceil\) denotes floor function.

A purpose of conventional process design is to assign work elements to work centers under the condition that maximum operation time among work centers is equal to or smaller than the target cycle time. Process line calculated under the condition easily causes large difference of operation times among work centers. Since maximum operation time among work centers is practical cycle time, minimizing the practical cycle time is required to promote productivity. Large variation of operation time causes idle time at continuous work centers. Therefore, minimizing variation of operation time at all work centers is effective not to decrease productivity.

From this discussion, maximum of operation time in work centers ‘\(\text{Max}(\mu_m)\)’ and dispersion of operation time in work centers ‘\(\text{Max}(s_m)\)’ are introduced as objective functions to evaluate the quality of process design in this study. These functions need to minimize simultaneously to increase productivity. Eqs. (3) and (4) denote the maximum value of average operation time and the maximum value of dispersion of operation time in all work center. These are objective functions. ‘\(m\)’ denotes work center.

\[
\text{Max } (\mu_m) \quad (3)
\]

\[
\text{Max } (s_m) \quad (4)
\]

\[
\mu_m = \sum_i \mu_i \delta_{im} \quad \forall m \quad (5)
\]

\[
s_m = \sum_i s_i \delta_{im} \quad \forall m \quad (6)
\]

Here, \(\mu_i\) and \(s_i\) denote average operation time and dispersion of operation time of work element ‘\(i\)’ respectively. \(\delta_{im}\) is 0-1 variable which takes 1 when work element ‘\(i\)’ is assigned to work center ‘\(m\)’. Otherwise, the variable takes 0.

In Fig. 2, arrows show direction of operation time at work centers to improve productivity in the process of assignment of work elements. When maximum of average operation time in all work centers is decreased by change of work elements among work centers, practical cycle time is reduced and productivity is increased. In addition, when maximum of dispersion of operation time in all work centers is decreased, standard deviation of operation time is decreased and idle time caused by variation of operation time among different work centers is reduced.

The mathematical model of this problem is written as following:

\[
\text{Min } C_{\text{max}} \quad (7)
\]

\[
\text{Min } D_{\text{max}} \quad (8)
\]

\[
\text{Subject to}
\]

\[
C_{\text{max}} \geq \sum_{i=1}^{l} \delta_{ik} p_i \quad \forall k \quad (9)
\]

\[
D_{\text{max}} \geq \sum_{i=1}^{l} \delta_{ik} s_i \quad \forall k \quad (10)
\]
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\[ \sum_{k=1}^{K} \delta_{ik} = 1 \quad \forall i \quad (11) \]
\[ k + M (\delta_{ik} - 1) \leq u_i \quad \forall i, \forall k \quad (12)\]
\[ u_i \leq K (1 - \delta_{ik}) + k \quad \forall i, \forall k \quad (13)\]
\[ u_i \leq u_j \quad (14)\]

Here, Eqs. (7) and (8) denote objective functions. \(C_{\text{max}}\) and \(D_{\text{max}}\) denote maximum value of average operation time and maximum value of dispersion of operation time among work centers respectively. \(k\) and \(i\) denote work center's number and work element's number respectively. In this model, both \(k\) and \(i\) take integer numbers from one. \(K\) and \(I\) denote the maximum number of work centers and the maximum number of work elements. In addition, \(u_i\) denotes a variable which presents work center's number to which work element \(i\) is assigned. \(M\) is a large number and it takes \(2K\) in this problem. When this mathematical model is resolved, the optimal solutions are obtained and the solutions are used to compare with the proposed algorithm.

2.2 Bi-Objective Genetic Algorithm Including Evaluation of Infeasible Solutions

New type of multi-objective genetic algorithm is proposed to resolve bi-objective problem for process design in this study. In a problem of process design, it is known that many infeasible solutions are ordinarily located nearby feasible solutions in solution space from numerical experiment in past. Therefore, it is not easy to seek approximate optimal Pareto solutions by using conventional multi-objective genetic algorithm. Many infeasible solutions are ordinarily superior to feasible solutions in terms of resultant values of objective functions. The proposed algorithm uses infeasible solutions in order to effectively seek approximate optimal Pareto solutions. Both feasible and infeasible solutions are adopted as chromosomes to select for crossover process and to preserve chromosomes for next generation. Values of criteria of infeasible solutions are modified by Eq. (15) in order to control probability of selection of infeasible solutions.

\[ \text{Fitness}(i) = KX \cdot \text{value}(i) \quad (15)\]

Here, \(\text{Fitness}(i)\) and \(\text{value}(i)\) denote fitness of chromosome \('i'\) used for selection and resultant value of criterion of chromosome \('i'\) respectively. \(KX\) denotes weighted coefficient to modify value of criterion of infeasible solutions. \(KX\) takes 1 or a number more than 1. A simple MOGA starts after \(KX\) is determined in the proposed algorithm. By introducing the coefficient, we consider that an excess of infeasible solutions is suppressed and solutions which have smaller values of objective functions are effectively sought. The process of the proposed MOGA is described in the following.

(1) Weighted coefficients located at left side and right side, ‘KL0’ and ‘KR0’, are determined to be initial values. Here, KL0 is smaller than KR0. Parameter to control weighted coefficient number ‘\(\alpha\)’ is determined to be constant number and the maximum number of iteration to change the weighted coefficient ‘MAXITER’ is determined to be a constant number.

(2) The number of iteration is determined to be 0. The coefficient located at left side ‘KL’ and the coefficient located at right side ‘KR’ are calculated from Eqs.(16) and (17).

\[ KL = \alpha (KR0-KL0)+KL0 \quad (16) \]
\[ KR = KR0-\alpha (KR0-KL0) \quad (17) \]

(3) The ranked positional weight technique is executed to generated an initial solution.

(4) Initial population is generated by using the solution generate in Step (3).

(5) The coefficient located at left side ‘KL’ is assigned to \(KX\). Then, a single MOGA is executed using the coefficient determined to be \(KX\).

(6) The coefficient located at right side ‘KR’ is assigned to \(KX\). Then, a single MOGA is executed using the coefficient determined to be \(KX\). Move to Step (8).

(7) A single MOGA is executed using the
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coefficient determined to be KX.

(8) Pareto solutions obtained using the coefficient located at left side ‘KL’ is compared with that obtained using the coefficient located at right side ‘KR.’

(9) Move to Step (10) when feasible solutions obtained by using the coefficient located at left side ‘KL’ are superior to feasible solutions obtained by using the coefficient located at right side ‘KR.’ Move to Step (11) when feasible solutions obtained by using the coefficient located at right side ‘KR’ are superior to solutions at the other side. Otherwise, move to Step (12).

(10) Each weighted coefficient is changed by using the following equations:

\[ KR_0 = KR, \]
\[ KR = KL, \]
\[ KL = KL_0 + \alpha (KR_0 - KL_0). \]

Then, the coefficient located at left side ‘KL’ is assigned to KX. Move to Step (12).

(11) Each weighted coefficient is changed by using the following equations:

\[ KL_0 = KL, \]
\[ KL = KR, \]
\[ KR = KR_0 - \alpha (KR_0 - KL_0). \]

Then, the coefficient located at right side ‘KR’ is assigned to KX.

(12) The number of iteration is added one.

(13) Complete the algorithm when the number of iterations is equal to MAXITER. Otherwise, go back to Step (7).

In this study, \( \alpha \) takes 0.3820. Here, this search process to control one dimensional variable using this constant number is similar to ‘Golden search method.’ Pareto and feasible solutions obtained by using different coefficients are compared with each other in Steps (8) and (9). When the number of the Pareto and feasible solutions obtained by using the weighted coefficient KX is larger than that of the Pareto and feasible solutions obtained by using the other coefficient, the coefficient KX regards as superior condition to the other coefficient.

The process of a single MOGA in Steps (5), (6), and (7) is described in the following.

(S1) Population ‘POP’ and maximum generation ‘MaxGen’ are predetermined.

(S2) Current generation takes 0.

(S3) The weighted coefficient ‘KX’ is predetermined.

(S4) Crossover and mutation:

(S4-1) Select parent chromosomes by roulette selection.

(S4-2) Generate new chromosomes by crossover.

(S4-3) Investigate whether new chromosomes are identical to chromosomes included in the population or not. If a new chromosome is NOT identical to any chromosome, go to Step (S5) after the chromosome is put into the population. When there is a chromosome which is identical to a new chromosome, go to Step (S4-4) in terms of the new chromosome.

(S4-4) Execute mutation process for the new chromosome and go back to Step (S4-3).

(S5) Encode the new chromosomes and calculate values of objective functions of solutions obtained from the chromosomes.

(S6) Selection of chromosomes for new generation:

(S6-1) Select Pareto solutions;

(S6-2) Parallel selection; and

(S6-3) Roulette selection.

(S7) Judge completion of algorithm.

Complete the algorithm when the current generation is equal to the maximum number of generations ‘MAXGen.’ Otherwise, go back to Step (S4) after one is added to the current generation.

Population is continuously used under the condition of different weighted coefficients. Therefore, solutions are continuously improved in population while weighted coefficient is changed in the proposed algorithm.

In roulette selection in Step (S6-3), values of objective functions of a solution obtained from a chromosome are modified to one dimensional
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criterion by using Eqs.(20) and (21). Furthermore, inverse of the one dimensional criterion is used as probability to select chromosomes in roulette selection because the objective functions are minimized in this problem.

\[ f = \sqrt{f_1^2 + f_2^2} \quad (20) \]

\[ \tilde{f} = \frac{f_i - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}} \quad (21) \]

Fig. 3 shows a sample of representation of a chromosome. The chromosome is constructed of an array of work element numbers and an array of work center numbers. Numbers located in the same element number in both arrays denote that work element represented in the element is assigned to work center represented in the other element. In an array of work center number, numbers are arranged in the ascending order. In order to operate elements in an array of work center's numbers for crossover and mutation, new type of array is generated by the following modification: 1 is assigned to the last element in each partial array of same work center number and 0 is assigned to other elements in the new array. Fig. 4 denotes an example of the new array. The number of '1' included in the array is same as the number of work centers. Therefore, work centers to which work elements are assigned can be controlled by operating locations of '1' in the array.

In Step (S4-1), several types of criterion are used in roulette selection: inverse of every objective function and inverse of the one dimensional criterion defined by Eqs.(20) and (21). New chromosomes equally are generated in crossover which uses different criterion in roulette selection. When bi-objective functions in the problem are \( f_1 \) and \( f_2 \), \( 1/f_1 \), \( 1/f_2 \), and \( 1/f \) calculated by Eqs. (20) and (21) are used as criterion in the roulette selection.

In the crossover in Step (S4-2), two-point crossover is adopted to operate elements in an array of work element numbers and an array of work center numbers independently. Two elements are arbitrarily selected in the array of two chromosomes and parts of the array between the selected elements are exchanged with each other. Lethal genes are prevented by the following process: (a) in an array of work element numbers, when numbers in the original partial array are included in the exchanged partial array, elements of the numbers in the exchanged partial array are used. The other numbers in the original partial array are rearranged in the exchanged partial array according to the order of the numbers in the original partial array. (b) in an array of work center numbers, '1' is deleted at any elements in the exchanged partial array when the number of '1' in the exchanged partial array is larger than that in the original partial array. On the other hand, '1' is added at any element in the exchanged partial array when the number of '1' in the exchanged partial array is smaller than that in the original partial array.

As for mutation process in Step (S4-4), swap is performed between any two elements by iteration of random number. Here, the random number is determined between 1 and 10 by uniform random number.

Solutions which have different chromosome and same values of objective functions are stored in population. Population is continuously used and updated in all process of the proposed algorithm. Therefore, local search is used to seek good feasible solution by changing widely the coefficient.

![Fig. 3 Representation of chromosomes (Upper: A series of work element numbers, Lower: A series of work center numbers).](image1)

![Fig. 4 Representation of a series of work center numbers changed for crossover and mutation (Upper: An original representation, Lower: A representation changed for crossover and mutation).](image2)
3. Numerical Experiment

3.1 Numerical conditions

In order to evaluate the performance of the proposed algorithm, process design is performed on simple model by using the proposed algorithm. In this paper, process design for mix production is discussed. The model is constructed based on mix production including three types of products named as Product A, B, and C. Here, Product A is core product. Product B is a product based on core product and several parts are exchanged. Product C is a product constructed by including additional parts in the core product. Product A, B, and C are manufactured from 11, 11, and 14 work elements respectively. Each work element is determined as assembly of a part to semi-finished product. Table 1 shows operation times of work elements. ‘Av’ and ‘#D’ denote average of operation time and standard deviation of operation time respectively. Arrow to going to the left denotes that data in the column is the same as data in the left column.

Fig. 5 shows procedure diagram of work elements of all products. The numbers in circles denote work element numbers in this figure. Character ‘A’, ‘B’, and ‘C’ denotes Product A, B, and C. When work elements are similar to each other in mix production, the elements are combined to construct a single work element. Groups of work elements surrounded by polygons are regard as single work elements in the diagram.

The amounts of products A, B, and C to produce in production plan are 100, 220, and 100 respectively. The period for production is 8400 min. (20 days×7 hours). Table 2 shows operation times in terms of groups of work elements regarded as single work elements, and these times are calculated from Eqs. (22) and (23).

When multiple work elements are assigned to work center $r$, average operation time $\mu_r$ and dispersion of operation time $s_r$ are calculated at the work center from Eqs. (22) and (23) considering the number of products.

\[
\mu_r = \frac{\sum_i P_i \sum_k \mu_{ik} \delta_{rik}}{\sum_i P_i} \tag{22}
\]

\[
s_r = \frac{\sum_i P_i \sum_k s_{ik} \delta_{rik}}{\sum_i P_i} \tag{23}
\]

$P_i$ denotes the required amount of Product $i$. $\mu_{ik}$ and $s_{ik}$ denote an average operation time and dispersion of operation time of work element $k$ to produce Product $i$.

<table>
<thead>
<tr>
<th>Product A</th>
<th>Time</th>
<th>No.</th>
<th>Product B</th>
<th>Time</th>
<th>No.</th>
<th>Product C</th>
<th>Time</th>
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<tbody>
<tr>
<td></td>
<td>Av</td>
<td>#D</td>
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Table 1 Operation time of work elements of products (No.: work element number, Av: average time, #D: standard deviation)
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Fig. 5 Precedence diagram of work elements of all products

Table 2 Operation times of work elements combined by similarity of works (No.: work element number, Av: average time, #D: standard deviation)

<table>
<thead>
<tr>
<th>No.</th>
<th>Av</th>
<th>#D</th>
<th>Work element number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1'</td>
<td>6.57</td>
<td>0.82</td>
<td>1(Product A,C), 12(Product B)</td>
</tr>
<tr>
<td>2'</td>
<td>6.57</td>
<td>1.25</td>
<td>2(Product A,C), 13(Product B)</td>
</tr>
<tr>
<td>5'</td>
<td>5.57</td>
<td>0.58</td>
<td>5(Product A,C), 14(Product B)</td>
</tr>
<tr>
<td>7'</td>
<td>7.57</td>
<td>2.15</td>
<td>7(Product A,C), 15(Product B)</td>
</tr>
</tbody>
</table>

\( \delta_{rik} \) is 0—1 variable. When work element \( k \) to produce Product \( i \) is assigned to work center \( r \), \( \delta_{rik} \) takes 1. Otherwise, \( \delta_{rik} \) takes 0.

The minimum number of work centers is calculated from production plan by using Eqs. (1) and (2), and the number of work centers is determined to be five. Therefore, five work centers are assumed to be located in process line in this problem. As for genetic operator used in the proposed algorithm, both population and the maximum generations are determined to be 100.

3.2 Evaluation of the Performance of the Proposed Algorithm

Production lines designed by the proposed algorithm are compared with the lines designed by the different methods. Different methods and characteristics of designed lines are shown as follows:

(D1) A production line designed by using the proposed algorithm. Here, in terms of weighted coefficients to control values of criteria of infeasible solutions, the initial value of the weighted coefficient on left side, KL0, and the initial value of the coefficient on right side, KR0, are determined as 0.0 and 20.0 respectively. The number of iteration to change weighted coefficient takes 15.

(D2) A production line designed by the ranked positional weight technique for process design.

(D3) A production line designed by resolving mathematical model for single objective functions presented in Section 2.1. The result is calculated by using free software `glpk.'

The method D2 is used to design process for production line aimed at satisfying production plan. A purpose of this method is that work elements are assigned to work centers below target cycle time. Since this method tends to generate large difference of operation times among work centers, productivity of the line is ordinarily inferior to that of the line designed by optimal methods. In the process design obtained by Method D2, the maximum value of average operation time takes 19.42 and the maximum value of dispersion of operation time takes 23.77 among work centers.

On the other hand, the different lines designed by Method D3 give 15.99 as the maximum value of average operation time among work centers and 19.36 as the maximum value of dispersion of operation time among work centers respectively.

Fig. 6 shows solutions obtained by Method D1. The distribution obtained by Method D1 is one of popular and common results obtained by five trials. The figure shows distribution of all solutions in the final population and the solutions obtained by Method D2 and Method D3. Filled circles and opened squares denote feasible solutions and infeasible solutions respectively. The solutions obtained by the proposed algorithm include the optimal solutions obtained for single objective functions by using Method D3. In the figure, several infeasible solutions are superior to feasible solutions and feasible solutions are generated at middle area between both axes. The proposed algorithm generates superior feasible solutions to the method of Method D2.

In order to evaluate productivity of the lines, event-driven simulation is executed by using data of
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Fig.6 Distribution of solutions obtained by different methods.

Fig.7 Comparison of completion time of products in production plannt methods

work elements assigned at each work center by different methods, because different products flow in line for mix production and difference of operation times between successive work centers tends to be large. Fig. 7 shows comparison of productivity of the lines designed by different methods. Productivity of the line is evaluated by time to complete 420 products determined in production plan. It iterates the simulation 100 times. Solution 'A' and Solution 'B' generated by the proposed algorithm are evaluated using simulation in Fig.6. Fig.7 shows that the proposed algorithm is clearly superior to Method D2 in the terms of productivity. These results show the proposed algorithm can generate Pareto and feasible solutions to provide high productivity for the problem. Since solution ‘A’ is smaller than solution ‘B’ in terms of maximum value of average operation times, average completion time of products obtained by solution ‘A’ is smaller than that obtained by the other solution. On the other hand, since solution ‘B’ is smaller than solution ‘A’ in terms of maximum value of dispersion of operation times, standard deviation of computational times obtained by solution ‘B’ is barely smaller than that obtained by the other solution.

3.3 The Characteristics of the Proposed Algorithm

In order to investigate the characteristics of the proposed algorithm in terms of the weighted coefficient, the proposed algorithm is compared with the single MOGA in the proposed algorithm under the conditions that the coefficient takes several static values between 1.0 and 50.0. Maximum generation takes 200 in order to take adequate time to seek Pareto solutions. Five trials are performed on each coefficient.

When the weighted coefficient takes 1.0, the condition denotes that both chromosomes obtaining feasible solutions and infeasible solutions have the same probability to be selected in the process of crossover and selection for next generation in the single MOGA. When the weighted coefficient takes large number such as 20.0, 30.0, and 50.0, the condition denotes that chromosomes generating feasible solutions have higher probability to be selected for crossover and selection for the next generation than chromosomes generating infeasible solutions.

Fig. 8 shows the ratio of feasible solutions to all solutions obtained by the single MOGA utilizing constant coefficients at the maximum generation. Each bar denotes average of the resultant ratios calculated from five trials and each error bar denotes standard deviation of the ratios. This figure shows that it is difficult to seek feasible solutions when the weighted coefficient takes less than 10.0. Therefore, it is difficult to seek high quality feasible and Pareto solutions under these conditions. When the weighted coefficient takes 1.0, no feasible solution is sought.
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The ratio of feasible solutions to all solutions obtained by the single MOGA.

On the other hand, when the weighted coefficient takes greater than 10.0, the ratio takes larger numbers. This result denotes that solutions are easily selected from bad feasible solutions when the coefficient takes large number. Therefore, we consider that local search to widely change the coefficient is useful to prevent from selecting solutions from bad feasible solutions.

Fig. 9 shows distributions of all feasible solutions obtained by the single MOGA at the maximum generation when the weighted coefficient takes different numbers.

Fig. 9 Distributions of all feasible solutions obtained by the single MOGA at the maximum generation when the weighted coefficient takes different numbers.

The weighted coefficient which generates adequate quality feasible solutions is not clear ordinarily. Therefore, control of the coefficient included in the proposed method is effective to seek feasible and Pareto solutions for problem generating many infeasible solutions such as process design.

4. Conclusions

This study treats multi-objective problem for process design for mix production. Infeasible solutions are easily generated more than feasible solutions in the problem because precedence relationship of work elements does not easily satisfy by changing assignment of the elements slightly.

New type of a multi-objective genetic algorithm is proposed and the proposed MOGA includes local search including control of the probability to select feasible solutions while both feasible solutions and infeasible solutions are stored. The proposed MOGA is performed on simple model of process design for mix production to evaluate the algorithm. Numerical experiments show that the proposed algorithm is effective to design production line which provides high productivity as well as high quality feasible and Pareto solutions. In addition, the results show the effectiveness of control to select infeasible solutions in the proposed algorithm.

In future study, the proposed algorithm is applied to large scale and complex problem of process design. Furthermore, search process in the single MOGA is
developed as PSO and MOGA including specific local search to effectively generate high quality feasible.

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References