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A Diversity Order Design of Linearly Precoded MU-MIMO Downlink System

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Abstract—Recently, Multi-User MIMO (MU-MIMO) downlink system which uses multiple antennas at Base Station (BS) and accommodates multiple users with multiple receive antennas attracts much attention. It can achieve a high channel capacity when the channel state information is known both at BS and each User Equipment (UE). There exist linear and non-linear MU-MIMO downlink schemes, but the linear scheme is considered more easily to be implemented and adjusted. Linear Block Diagonalization (BD) has been proposed in order to cancel the Inter-User Interference (IUI), and combined with the Eigenbeam-Space Division Multiplexing (E-SDM). In this paper, by noting that the block size of each user channel matrix in BD becomes (number of receive antennas of each UE) × (nullity of the matrix constituted by other users), we design the BD + E-SDM scheme based on the downlink quality of each UE, i.e., the diversity order. We made simulations for some examples and show the effectiveness of design method based on the diversity order of each UE.

Keywords—MU-MIMO, block diagonalization, E-SDM, nullity, diversity order

I. INTRODUCTION

Recently, spatially multiplexed MU-MIMO downlink system in which a Base Station (BS) can transmit multiple streams to each User Equipment (UE) without IUI attracts much attention [1]-[7]. In MU-MIMO downlink communication, the Channel State Information (CSI) between BS and each UE has to be shared at both of them, but by increasing the number of transmit antennas at BS, the frequency efficiency and the channel capacity of downlink can be improved. Transmit signals from BS to each UE located at different position are spatially multiplexed and the IUI received at each UE is pre-excluded by the precoding at BS. As representative methods for excluding the IUI at BS, there exist BD [2] and Channel Inversion (CI) [3] both categorized as linear schemes, and Dirty Paper Coding (DPC) [4], Tomlinson-Harashima Precoding (THP) [5] and Vector Perturbation (VP) [6],[7] as nonlinear schemes. Although nonlinear schemes attain high downlink channel capacity, the complexity is high and the design method is difficult. The linear CI method has a problem on increasing transmit power and the restriction of sum rate characteristics. The linear BD method consumes much degree of freedom on making nulls, but it eliminates IUI completely and it matches the E-SDM [8] very well. Hence the BD + E-SDM seems practical solution for the MU-MIMO downlink.

In this paper, we investigate the MU-MIMO downlink system in which both BS and UE’s have multiple antennas. The BS eliminates the IUI using BD and transmits multiple streams to each UE using E-SDM for the block matrix in the block diagonalized channel matrix. We note that the size of the block matrix for UE in the block diagonalized matrix becomes (number of receive antennas of UE) × (nullity of the channel matrix in which channel matrix of UE is excluded from the entire channel matrix). When the channel is i.i.d. flat Rayleigh fading, as the diversity order of the first eigen mode in E-SDM for the block channel matrix of UE is also given by the above product, we propose a design method in MU-MIMO downlink in which the diversity order of the first eigen mode is designated. We made simulations for some design examples and show the effectiveness of the proposed design method.

The paper is organized as follows. In Section II, the MU-MIMO downlink model is introduced. In Section III, we propose the design method of MU-MIMO downlink based on the receive quality of UE, i.e., the diversity order of 1st eigen mode channel of UE. In Section IV, we show some examples of design method and clarify the BER characteristics. The paper is concluded with Section V with the most important results and future work. In appendix, we derive the theoretical BER in BD+E-SDM when the number of transmit stream is one.

II. MU-MIMO DOWNLINK MODEL

We show the transmission model of MU-MIMO downlink in Fig.1. We denote the number of transmit antennas at BS and the number of UE’s as $N_t$ and $N_r$ respectively. When the number of receive antennas of UE $k$ $(k=1,\cdots,N_r)$ is $n_{rk}$, the total number of receive antennas of all UE’s are given by $N_r=\sum_{k=1}^{N_r}n_{rk}$. Denoting the channel matrix for UE $k$ as $H_k$ $(n_{rk} \times N_t)$, the entire channel matrix from BS to UE’s is expressed as

$$H=[H_1^T \ H_2^T \ \cdots \ H_{N_r}^T]$$ (1)

We denote the matrix $\tilde{H}_k$ in which the channel matrix $H_k$ of UE $k$ is excluded from the entire channel matrix $H$ as

$$\tilde{H}_k=[H_1^T \ \cdots \ H_{k-1}^T \ H_{k+1}^T \ \cdots \ H_{N_r}^T]$$ (2)

with the size of $(N_r-n_{rk}) \times N_t$. By making the Singular Value Decomposition (SVD) on $\tilde{H}_k$, we get

$$\tilde{H}_k=\tilde{U}_k \Sigma_k \tilde{V}_k^H$$ (3)
where \( \tilde{V}_k^o \) is the null space of \( \tilde{H}_k \) and it orthogonalizes \( H_k (k' \neq k) \), i.e., \( H_k \tilde{V}_k^o = 0 \). Using the null space \( \tilde{V}_k^o \) for each UE, the entire channel matrix \( H \) is block diagonalized as

\[
B = \begin{bmatrix}
H_1 \\
\vdots \\
H_k \\
\vdots \\
H_N
\end{bmatrix}
\begin{bmatrix}
\tilde{V}_1^o & \cdots & \tilde{V}_k^o & \cdots & \tilde{V}_N^o
\end{bmatrix} =
\begin{bmatrix}
D_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & D_N
\end{bmatrix}
\]

(4)

The number of column vectors of \( \tilde{V}_k^o \) becomes the nullity \( n^l_k \) of \( \tilde{H}_k \). The nullity of a matrix is given by the difference between the number of columns and the rank of a matrix. The nullity \( n^l_k \) of matrix \( \tilde{H}_k \) is expressed as

\[
n^l_k = \text{nullity}(\tilde{H}_k) = N_k - \text{rank}(\tilde{H}_k)
\]

(5)

As the rank of a matrix is given by the minimum between the number of rows and columns, the rank of \( \tilde{H}_k \) is expressed as

\[
\text{rank}(\tilde{H}_k) = \min(N_k - n_{mk}, N_k)
\]

(6)

If the nullity \( n^l_k \geq 1 \), then the null space \( \tilde{V}_k^o \) exists and the BD becomes available. Accordingly, the condition for realizing BD is expressed as

\[
n^l_k = N_k - \text{rank}(\tilde{H}_k) = N_k - N_k + n_{mk} > 0
\]

(7)

where we set \( \text{rank}(\tilde{H}_k) = \min(N_k - n_{mk}, N_k) = N_k - n_{mk} \) in (6). In some literature, there is a description that the condition for realizing BD is \( N_k \geq N_{mk} \), but correctly the condition of \( n^l_k = N_k - N_k + n_{mk} > 0 \) must be satisfied. Therefore even in the case of \( N_k < N_{mk} \), the BD is realizable. For example, for the parameters of \( N_k = 4 \), \( N_k = 2 \), \( n_{mk} = 3 \), \( N_k = 6 \), the nullity of \( n^l_k = 4 \) is \( 4 - 6 + 3 = 1 > 0 \) and the BD is realizable for \( N_k = 4 < N_{mk} = 6 \) [2]. Finally we know that the size of block channel matrix \( D_k \) in (4) is given by \( n_{mk} \times n^l_k \).

III. DESIGN METHOD OF MU-MIMO DOWN LINK BASED ON RECEIVE QUALITY OF UE

We assume that the channel matrix \( H \) represents the quasi-static flat Rayleigh fading where each element of \( H \) follows the i.i.d. complex Gaussian random variable. From the section II, in the block diagonalized matrix \( B \) in (4), the size \( n_{mk} \times n^l_k \) of the block matrix \( D_k \) for the UE is determined by the number of receive antennas \( n_{mk} \) of UE and the nullity \( n^l_k \) of the matrix \( \tilde{H}_k \). The diversity order of the 1st eigen mode channel having the channel gain of \( \lambda_{max} \) is given by the product of number of rows and columns of block channel matrix \( D_k \) [9]. Hence, the diversity order \( n_{mk} \) of the 1st eigen mode channel of the block channel matrix \( D_k \) for the UE is determined as \( n_{mk} = n_{mk} 	imes n^l_k \). It is also known that the diversity order of the 2nd eigen mode channel is given by \( ((n_{mk} - 1) \times (n^l_k - 1)) \) and the diversity order of the \( i \)-th eigen mode channel decreases as \( ((n_{mk} - (i - 1)) \times (n^l_k - (i - 1))) \). Using those findings, we propose the design method of MU-MIMO down link based on the diversity order of the stream of UE indicating the receive quality.

A. Design Method of MU-MIMO Down Link 1

We consider the case where the number of transmit antennas of BS \( N_k \) is fixed, the number of active UE’s is determined and known at BS and also the number \( n_{mk} \) of receive antennas at each UE is known at BS. In this case, the total number of receive antennas of all UE’s is \( N_k = \sum n_{mk} \). For the block channel matrix \( D_k \) with the size of \( n_{mk} \times n_{mk} \), from (7) the nullity \( n^l_k \) is given by

\[
n^l_k = N_k - N_k + n_{mk}
\]

(8)

Accordingly, the diversity order \( n_{mk} \) of the 1st eigen mode channel for UE is expressed as

\[
n_{mk} = n_{mk} \times n^l_k
\]

(9)

When there are some inactive UE’s, the number of total receive antennas of all UE’s \( N_k \) decreases and the nullity for UE \( n^l_k \) in (8) increases.

B. Design Method of MU-MIMO Down Link 2

We consider the case where the number \( N_k \) of all UE’s is determined and the UE \( i \) has the minimum number of receive antennas \( n_{mk} \) among UE’s. When the desired diversity order of the 1st eigen mode channel of UE \( i \) is denoted as \( n_{min} \), it holds \( n^l_k = n_{mk} \times n^l_k \) and \( n_{mk} = n_{min} / n_{max} \). If \( n_{min} \) is designated, the number of transmit antennas at BS is determined from (7) as

\[
N_k = n_{min} \times N_k - n_{max}
\]

(10)

where the size of block channel matrix of \( D_k \) of UE \( i \) having the minimum number of receive antennas \( n_{max} \) is given by \( n_{max} \times n_{max} \). On the other hand, the nullity of UE \( k \) \( n^l_k = \text{nullity}(\tilde{H}_k) \) other than the UE \( i \) is expressed as \( n^l_k = N_k - N_k + n_{mk} > n_{max} \), which means \( n^l_k > n_{max} \). The size \( D_k \) of block channel matrix of UE \( i \) is given as \( n_{min} \times n^l_k \). In this way, by designating the minimum diversity order \( n_{min} \) of the 1st eigen mode channel of UE \( i \), the diversity orders of other UE’s other than UE \( i \) can have larger ones. This is the design method in which the number of transmit antennas \( N_k \) at BS is finally determined based on the requirement of UE \( i \) with the worst receive quality.

C. Design Method of MU-MIMO Down Link 3

There exists the case where the number of transmit antennas \( N_k \) at BS is smaller than the number of total receive antennas \( N_k \) of all UE’s, i.e., \( N_k < N_k \), under the positive nullity condition \( n^l_k = N_k - N_k + n_{mk} > 0 \) which enables the BD as stated in II. If the following condition

\[
N_k = n_{min} + N_k - n_{max}, \quad n^l_k = N_k - N_k + n_{mk} > 0, \quad N_k < N_k
\]

(11)

is satisfied, then the BD+E-SDM transmission is available even though \( N_k < N_k \).

IV. EXAMPLES OF DESIGN METHOD OF MU-MIMO DOWN LINK

We show some examples for evaluating the design methods A–C. The BER characteristics are simulated by MATLAB to verify the diversity order of each UE in Fig.1. The abscissa of BER
characteristic is taken as the transmit SNR [10]. The transmit SNR is defined as the ratio of the total transmit power $P$ to the receive noise power $\sigma_n^2$ per a receive antenna of each UE and is given by

$$\frac{S}{N}_{\text{eaves}} = \frac{P}{\sigma_n^2}$$  \hspace{1cm} (12)

where the variance of complex Gaussian random variable $h_i$ in the channel matrix $H$ in (1) is set to $E[|h_i|^2] = 1$.

A. Example of Design Method of MU-MIMO Down Link 1

When the number of transmit antennas at BS $N_t = 6$, the number of UE’s $N_u = 3$ and number of receive antennas of UE is all equal with $n_{u1} = n_{u2} = n_{u3} = 1$, the number of total receive antennas becomes $N_r = n_{u1} + n_{u2} + n_{u3} = 3$. We call it BD+E-SDM 6$x(1,1,1)$ model. In this case, the channel matrix $H$ is $3 \times 6$ and $H_i$ becomes $2 \times 6$. For the UE’s $k = 1 \sim 3$, the nullity of $H_k$ is calculated as $n_{\ell_k} = 6 - 2 \times 4$, $k = 1, 2, 3$. The size of block channel matrix $D_k$ is given by $n_{a_k} \times n_{\ell_k} = 1 \times 4$, $k = 1 \sim 3$ and the diversity order $D_k$ with 1 stream transmission to UE’s $k = 1 \sim 3$ becomes equal and is given by $n_{a_k} = n_{a_1} \times n_{\ell_1} = 1 \times 4 = 4$, $k = 1 \sim 3$.

In Fig.2, the BER characteristic of BD+E-SDM 6$x(1,1,1)$ model with QPSK (2bps/Hz) modulation and 1 stream transmission is shown as the orange circle. From Fig.2, the diversity order of 4 is observed for UE 1~3, because the BER falls by $10^{-3}$ between 15~20 dB of transmit SNR. Likewise, the BER results of BD+E-SDM 3$x(1,1,1)$ and 6$x(1,1,1,1)$ are also shown in Fig.2.

For BD+E-SDM 3$x(1,1,1)$, the nullity becomes $n_{\ell_1} = 1$ and the size of block matrix $D_1$ is given by $n_{a_1} \times n_{\ell_1} = 1 \times 1$. Thus the diversity order is calculated as $n_{\ell_1} = n_{a_1} \times n_{\ell_1} = 1$ and we can confirm it from the slope in Fig.2. When comparing 6$x(1,1,1)$ with 3$x(1,1,1)$, by using more antennas at BS, 6$x(1,1,1)$ can achieve the higher nullity of 4 rather than 1 leading to the communication quality improvement of UE.

Next, BD+E-SDM 6$x(1,1,1,1)$ model is also shown in Fig.2. In this case, the number of UE’s $N_u = 4$, the nullity becomes $n_{\ell_1} = N_r - N_u = 6 - 4 = 2$, $k = 1 \sim 4$ and the size of $D_k$ is given by $1 \times 3$. When comparing 6$x(1,1,1,1)$ with 6$x(1,1,1)$, the increase of UE’s causes the decrease of nullity from 4 to 3 and we can find the BER degradation in Fig.2.

Also in Fig.2, we can confirm that the simulated BER coincides exactly with the theoretical BER. The theoretical BER is derived in appendix.

B. Example of Design Method of MU-MIMO Down Link 2

When $N_u = 3$ and $n_{u1} = 3$, $n_{u2} = 2$, $n_{u3} = 1$, it becomes $N_r = n_{u1} + n_{u2} + n_{u3} = 6$. In this case, the number of receive antenna $n_{u1} = 1$ of UE 3 is minimum, thus we can say $n_{\ell_{u1}} = n_{u2} = n_{u3} = 1$. If the desired diversity order of the 1st eigen mode channel of UE3 is $n_{\ell_{min}} = 1$, from the relation of $n_{\ell_{u1}} = n_{u2} \times n_{\ell_{u2}}$, it holds $n_{\ell_{min}} = n_{u1} / n_{\ell_{u1}} = n_{u2}$. Hence, $N_r$ is calculated as $N_r = n_{\ell_{min}} + N_u = n_{\ell_{min}} + 3 = 6 - 1 = 5$ and the size of $D_1$ for UE 3 becomes $1 \times 3$. The nullity of UE 2 is given by $n_{\ell_2} = \text{nullity}(H_2) = N_r - N_u = 5 - 3 = 2$, and the size of $D_2$ becomes $3 \times 4$. The diversity order of the 1st eigen mode channel of UE 2 is calculated as $n_{\ell_2} = n_{u2} \times n_{\ell_2} = 2 \times 4 = 8$. Likewise, the nullity of UE 1 is given by $n_{\ell_1} = \text{nullity}(H_1) = N_r - N_u = 5 - 1 = 4$, and the size of $D_1$ becomes $3 \times 4$. The diversity order of the 1st eigen mode channel of UE 1 is calculated as $n_{\ell_1} = n_{u1} \times n_{\ell_1} = 3 \times 4 = 12$.

Fig. 2. BER characteristics of BD+E-SDM (3$x(1,1,1)$, 6$x(1,1,1,1)$, 6$x(1,1,1)$, Modulation:QPSK)

Fig. 3. BER characteristics of BD+E-SDM (8$x(3,2,1)$, Fixed transmission rate of 6(bps/Hz), Modulation formats are optimally selected from 64QAM, 16QAM and QPSK.)

Fig. 4. BER characteristics of BD+E-SDM (6$x(3,2,1)$, Fixed transmission rate of 6(bps/Hz), Modulation formats are optimally selected from 64QAM, 16QAM and QPSK.)

Fig. 5. BER characteristics of BD+E-SDM (Modulation:QPSK)
$D_i$ becomes $3 \times 5$. The diversity order of the 1st eigen mode channel of UE 1 is calculated as $n_d = n_s \times n_t = 3 \times 5 = 15$.

Fig.3 shows the BER characteristics of UE’s 1–3 in BD+E-SDM $8 \times (3,2,1)$ model. The BER of UE 3 marked by blue * transmits only 1 stream using the 1st eigen mode and is equivalent to BD+MRT (Maximum Ratio Transmission) [11]. The diversity order of 3 is observed because the BER falls by about $10^{-3}$ during $25\text{dB} \sim 35\text{dB}$ of transmit SNR. On the other hand, the BER of UE 2 and UE 1 marked by red circle and green triangle respectively are the average BER of all streams where UE 2 and UE 1 transmit 2 and 3 streams using 2 and 3 eigen values respectively. As stated before, when multiple streams are transmitted in parallel using E-SDM, the optimum selection of modulation formats and the optimum power assignment to each stream are done to make the average BER of all streams small as possible [8]. For UE 3, UE 2 and UE 1, under the constraint of constant bit rate of 6(bps/Hz), the entire transmit power is assigned to one stream of UE 64QAM. Also for selected modulation formats to multiple streams, the optimum transmit power assignment to each stream has been done. But for UE 3, there is no selection on modulation formats and the entire transmit power is assigned to the one stream of 64QAM. In addition, UE 1, UE 2 and UE 3 share the total transmit power from BS equally, i.e., $1/3$ of total transmit power.

In Fig.3, if the desired diversity order of the 1st eigen mode channel of UE 3 is changed from $n_d = 3$ to 1, then it becomes $N_r = 6$. The BER characteristics are shown in Fig.4. We can confirm that the diversity order of UE 3 is actually 1 from Fig.4.

### C. Example of Design Method of MU-MIMO Down Link

We consider the case where $N_r = 2$, $n_s = 3$, $n_g = 3$ and $N_g = 6$. In this case, from the relation $N_r = n_{g,m} + N_g - n_{g,m}$, when $n_{g,m} = 1$, it holds $N_r = 1 + 6 - 3 = 4$ and $N_g = 4 < N_g = 6$. In this $4 \times (3,3)$ model, the size of block channel matrix of $D_i$ of UE $k$ becomes $n_{g,m} \times n_{t,k} = 3 \times 1$ and 1 stream transmission becomes available. The diversity order becomes $n_d = n_{g,m} = 3 \times 1 = 3$. In Fig.5, the BER characteristics of $4 \times (3,3)$ model are shown. For comparison, the BER characteristics of $4 \times (3,1)$ and $4 \times (2,2)$ are also shown in Fig.5. The nullity of UE in $4 \times (1,1)$ model is calculated as $n_{t,k} = N_r - N_g + n_{u} = 4 - 2 + 1 = 3$ and the size of block channel matrix of $D_i$ becomes $1 \times 3$. Hence the diversity order becomes $1 \times 3$ which is equal to the one of $4 \times (3,3)$. From Fig.5, we see that the BER’s of $4 \times (3,3)$ and $4 \times (1,1)$ coincide. On the other hand, the nullity of UE in $4 \times (2,2)$ model is calculated as $n_{t,k} = N_r - N_g + n_{u} = 4 - 4 + 2 = 2$ and the size of block channel matrix of $D_i$ becomes $2 \times 2$, thus the diversity order of 4 is obtained for the 1st eigen mode channel. We consider the reason why $4 \times (2,2)$ has better BER characteristics than $4 \times (3,3)$ even though $4 \times (2,2)$ has less receive antennas than $4 \times (3,3)$ at each UE. From the nullity equation of $n_{t,k} = N_r - N_g + n_{u}$, if the number of receive antennas at each UE is increased by $\ell$ under the constraint of $N_r$ constant, then the nullity $n_{t,k}$ is calculated as $n_{t,k} = N_r - (N_g + \ell N_r) + (n_s + \ell N_g) = N_r - N_g + n_u - \ell (N_r - 1)$ (13)

From (13), it is found that if $\ell$ is increased, the nullity $n_{t,k}$ decreases. Therefore, we can say that when the number of transmit antennas $N_r$ is constant, the increasing number of receive antennas at each UE leads to the decrease of nullity of each UE. As the size of block channel matrix is given by $D_i = n_{g,m} \times n_{t,k}$, the decrease of $n_{t,k}$ degrades the communication quality in some cases.

### V. Conclusions

In this paper, we investigated the MU-MIMO down link system in which BS and UE’s have multiple transmit and receive antennas respectively. The IUI among UE’s is pre-excluded at BS by using BD. Then the E-SDM is applied to the block channel matrix of UE to transmit multiple streams. By utilizing the fact that the size of block matrix of UE in the Block Diagonalized channel matrix becomes (number of receive antennas of UE) × (nullity of the channel matrix in which the rows of UE are excluded from the entire channel matrix), we proposed the design method of MU-MIMO down link system where the diversity order of the 1st eigen mode channel of UE is designated. As a result, we have shown by increasing the number of transmit antennas at BS, we can arbitrary improve the diversity order of eigen mode channel of UE. For some examples and through computer simulations, we verified the effectiveness of our design method. For future works, adaptive scheduling algorithms for the change of UE parameters will be considered.

### APPENDIX

**DERIVATION OF THEORETICAL BER IN FIG.2**

We derive the theoretical BER’s of BD+E-SDM $3 \times (1,1,1)$, $6 \times (1,1,1,1)$ and $6 \times (1,1)$ in Fig.2. The sizes of block matrix $D_i$ of $3 \times (1,1,1)$, $6 \times (1,1,1,1)$ and $6 \times (1,1)$ are given by $1 \times 1$, $1 \times 3$ and $1 \times 4$ respectively, where one stream is transmitted using E-SDM. One stream transmission in E-SDM is referred to as Maximum Ratio Transmission (MRT) [11] and is equivalent to the Maximum Ratio Combining (MRC) with transmit and receive weights.

We firstly derive the transmit weight $v(n_r \times 1)$ and receive weight $w(n_g \times 1)$ in MRT. The transmit signal $x$ is multiplied by the weight $v$ and transmitted to the channel through $n_r$ transmit antennas. The transmit power is expressed as

$$\left(1/2\right) \cdot E\|v^H v\|_2 = \left\|v^H E|x|^2\right\|/2 = \left(1/2\right) \cdot E|ystem in which BS and UE’s have multiple transmit and receive antennas respectively. The IUI among UE’s is pre-excluded at BS by using BD. Then the E-SDM is applied to the block channel matrix of UE to transmit multiple streams. By utilizing the fact that the size of block matrix of UE in the Block Diagonalized channel matrix becomes (number of receive antennas of UE) × (nullity of the channel matrix in which the rows of UE are excluded from the entire channel matrix), we proposed the design method of MU-MIMO down link system where the diversity order of the 1st eigen mode channel of UE is designated. As a result, we have shown by increasing the number of transmit antennas at BS, we can arbitrary improve the diversity order of eigen mode channel of UE. For some examples and through computer simulations, we verified the effectiveness of our design method. For future works, adaptive scheduling algorithms for the change of UE parameters will be considered.

### APPENDIX

**DERIVATION OF THEORETICAL BER IN FIG.2**

We derive the theoretical BER’s of BD+E-SDM $3 \times (1,1,1)$, $6 \times (1,1,1,1)$ and $6 \times (1,1)$ in Fig.2. The sizes of block matrix $D_i$ of $3 \times (1,1,1)$, $6 \times (1,1,1,1)$ and $6 \times (1,1)$ are given by $1 \times 1$, $1 \times 3$ and $1 \times 4$ respectively, where one stream is transmitted using E-SDM. One stream transmission in E-SDM is referred to as Maximum Ratio Transmission (MRT) [11] and is equivalent to the Maximum Ratio Combining (MRC) with transmit and receive weights.

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$$\left(1/2\right) \cdot E\|v^H v\|_2 = \left\|v^H E|x|^2\right\|/2 = \left(1/2\right) \cdot E|w^H w|_2 = P$$ (13)$$

where $\|v\|_2^2 = 1$ and $P$ is the transmit power. At the receiver side, the receive weight $w$ is multiplied by the received signal $\hat{x} = w^H H v x + w^H n$ (14) where $H$ is the channel matrix in MRT. The receive SNR in (14) is given by

$$S/N = P_{\text{SNR}} = \frac{w^H w}{w^H w}$$ (15)

where $\left(1/2\right)E\|n_r w^H\|_2 = \sigma_n^2 I$ and $I$ is the $n_g \times n_g$ identity matrix.

Using Lagrange multiplier method, we derive the weights $v$ and $w$ which maximize the receive SNR. By fixing the
value of numerator in (15), we minimize the value of denominator in (15). We consider the real function $h$
\[ h = w^H w + \text{Re} \left( \lambda^* \left( w^H H v - g \right) \right) + \xi^* (v^H v - 1) \] (16)
where $\lambda$ and $\xi$ are the Lagrange multipliers. The constraint conditions in (16) are $w^H H v = g$ and $\|v\|^2 = 1$. By partially differentiating $h$ with respect to $w^*$ and $v^*$, we get
\[ \frac{\partial h}{\partial w} = w + \frac{\lambda^* H v}{2} = 0 \]
\[ \frac{\partial h}{\partial v} = \frac{\lambda^* (w^H H^*)^T + \xi v}{2} - \frac{\lambda^* H^H w + \xi^* v}{2} = 0 \] (17)
From (17), we have
\[ w = -\frac{\lambda^* H v}{2}, \quad v = -\frac{\lambda^* H^H w}{2(\xi)} \] (18)
From (18), we get
\[ w = \frac{\lambda^* H H^H w}{4\xi}, \quad v = \frac{\lambda^* H^H H v}{4\xi} \] (19)
By substituting $\eta = 4\xi / \|\lambda\|^2$ into (19), the transmit and receive weights satisfy
\[ H^H H v = \eta v \]
\[ H^H H w = \eta w \] (20)
From (20), it is seen that $v$ is the eigen vector corresponding to the eigen value $\eta$ of $H^H H$ and $w$ is the eigen vector corresponding to the eigen value $\eta$ of $H H^H$. From (18) and $\|v\|^2 = v^H v = 1$, we also have
\[ w^H H w = \|w\|^2 = \xi \] (21)
From (15), the receive SNR is expressed as
\[ \frac{S}{N} = \frac{P_s}{\sigma_n^2} \frac{w^H H H^H w}{\eta \xi^2} \] (22)
As $w^H H H^H w = \eta \xi$, the receive SNR is finally obtained as
\[ \frac{S}{N} = \left( \frac{P_s}{\sigma_n^2} \right) \cdot \eta \] (23)
Accordingly, the receive SNR is maximized when the eigen value $\eta$ has its maximum value $\eta_{\text{max}}$ of $H^H H$ and $H H^H$. and the weights $v$ and $w$ are the eigen vectors corresponding to $\eta_{\text{max}}$. This ends the derivation of MRT weights.

Using (23), the BER of user $k$ in MRT transmission with QPSK signaling is given by
\[ P_{b} = \frac{1}{2} \text{erfc} \left( \frac{1}{2} \frac{S}{N} \right) \] (22)
where $\text{erfc}(\cdot)$ is the complementary error function.

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