

# Notes on Logical Semantics (1)

— enjoying Montague's theories —

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In referring to my notes on semantics as 'logical', I would like to stress my interest in the semantic analysis of natural languages which tends to be focussed predominantly on 'formal' aspects of meaning. Some students would frankly call it 'formal' semantics (for example, Shirai(1985)), but I prefer the adjective 'logical' to 'formal'. Though the term 'natural logic' was once used (e.g. in Lakoff(1972)), it seems to have long been out of use. Anyway I wouldn't hesitate to call 'logical' any attempt to analyse the meaning of natural languages as explicitly as possible, from linguistic as well as logical points of view.

For linguists an initiation to this kind of semantics would probably best be made by studying Ricnard Montague's works; at least my interest in this aspect was first aroused by him, though I had long been fairly interested in some of the logical analyses of natural language from so-called 'philosophical' points of view. (My latent discontent with many proposals made in traditional semantics may be attributed to their lack of 'explicit' formalization.) Montague's actual analyses of English, based upon his own methodology, have excited me much, making me realize the promising possibility of studying the meaning of English by following his rigorous and explicit approaches to the natural language. In my first notes on logical semantics presented here I will therefore be concerned with Montague's theories, especially in their formal aspects.

1.1 Before addressing myself to Montague's approaches to the

semantic problems of natural languages, a brief mention would be not inappropriately made of those works done by some modern logicians.

Starting with G.Frege(1879), logical analyses made by so-called 'symbolic' logicians, which might be said to have been almost completed by Russell-Whitehead (1910-13), seemed to me to be stimulating enough in their explicitness and endeavour towards clear-cut exploration of the meaning of natural languages, as contrasted with those of traditional grammarians.

There seemed to be, at the same time, too many restrictions imposed upon their range of analyses as applied to the treatment of natural language; their target is almost exclusively directed at the analysis of declarative sentences, excluding imaginative or belief contexts. Sentences containing what Russell calls 'ego-centric' particulars are simply ignored; within their framework it seems to be out of the question to treat emotional sentences. Their explicit analyses of some syntactically ambiguous sentences are appealing enough, and no doubt one of their chief motives lies in constructing a formal language in order to avoid philosophical paradoxes (e.g. Frege's or Russell's), but when it comes to semantic analyses, their superficial nature cannot be concealed. Their methods are apparently limited to certain 'logically' well-formed expressions, which might also be reflected in the present state of affairs in the realm of machine translation.

1.2 It would be too pessimistic to be discouraged by such criticisms. To those who feel at home in the niche of 'scientific' or 'analytic' routine the most important thing should be the first step to be taken at all towards the establishment of a certain firm position. Once forward steps have been taken, only further efforts are waiting to be made, modifying their methods according to the criticisms offered.

In passing, I would like here to mention a highly favourable tendency recently observed in the field of linguistic theories. I mean the rapid obsolescence of authoritarianism; no more bound

by any kind of orthodoxies, many researchers in this field seem to be studying more freely without any undue respect to their predecessors' doctrines. To quote from Pollard & Sag(1987), 'with the development of an expressive and formally precise *lingua franca*, essentially the full range of current theories can be composed, decomposed, compared, recombined, and generally tinkered with.' I agree with them in saying that they are constrained 'only by the individual researcher's *aesthetic sense, philosophical predispositions, and responsibility to get the facts rights*. (italics mine). (I am here, however, anticipating the paper to be presented on the next occasion.)

As early as Frege (1892) a major difference is pointed out between extensional and non-extensional contexts; he emphasizes, besides the reference of the sign, the need to consider the 'sense' of the sign, wherein 'the mode of presentation' is contained. Or as another historical example, Russell's introduction of the concept 'definite description' (Russell(1905)) may be mentioned as an approximate remedy for the semantic analysis of English proper names. In a wider perspective, Lukasjewiz's proposition for multi-valued logic seemed to be encouraging.

Other logicians were also conscious of these defects in the logical analyses of natural language in terms of (at least first-order) predicate logic. (It must of course be admitted that formal languages have been developing for some time among philosophers that can treat fairly efficiently imperatives, questions, and other nondeclarative constructions (e.g. Lewis(1972)). Many find out that inadequacy often lies in their exclusive dependency on the 'extensional' aspect of symbolic logic; 'intensional' contexts should necessarily be taken into consideration. There seem to be two approaches to the intensional contexts: one is by differentiating surface forms from deeper ones, which can be seen in the Russellian theory of description or earlier Chomskyan theories, and the other is by positing an intermediate language such as that of intensional logic into

which expressions containing deceptive meaning can be translated; and it is undoubtedly the latter approach that has been adopted by R.Montague. To mention one more example, in traditional symbolic logic the analysis of the verb 'be' is made in terms of multiple logical relational structures (e.g. membership, identity and copula). Even here Montague tries to analyze the verb without making recourse to any so-called deep structure differences but by introducing differentiating concepts of intensional logic.

2.1. Two decades have almost passed since Montague's premature death, and he himself would not perhaps have foreseen that his seemingly formidable theories would come to exert such a profound influence. Having nothing particularly innovative in my notes, I would first of all like to describe my personal feelings of congeniality when perusing his theories, confessing that I am allowed, at least secretly in my heart, to enjoy the beauty of elegance and refinement which his thoroughly consistent logical theories afford me. To a man who, half in despair of seeking the 'truth', wished to find the value of beauty at least in an elegance of logical consistency, Montague's approaches sounded like a gospel.

2.2. His theory of an abstract 'Universal grammar' might as well be applied to 'real' natural language data as many highly theoretical assumptions in mathematics have been freely utilized to clarify the actual phenomena of (natural) sciences. Indeed it was his writings dealing with an actual natural language, English, that first attracted the attention of linguists. To many of them Montague may still be remembered as the author of 'English As a Formal Language' (Montague [3] ) or 'The Proper Treatment of Quantification in Ordinary English' (Montague [5] ). That was also the reason why his theory was first introduced to the linguistic world as Montague 'Grammar' .

Subsequently, however, it has come to be realized that his grammar should be understood in a much wider sense: unlike

that of N.Chomsky, his 'Universal Grammar', aiming at finding substantive and formal universals, is an attempt to treat, within the same strictly defined and precise framework, not only natural languages but also artificial ones, mathematics (or even a symbolic system of bees' dancing). For example, at the beginning of Montague [4], he asserts that 'there is no important theoretical difference between natural languages and artificial languages' and considers it possible 'to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory.'

2.3. Apparently his 'universal' constructs might be interpreted as roughly equivalent to those of Chomskyan 'competence' possessed by ideal speakers, but there are evidently no psychological elements presupposed which have been equipped innately (in support of the 'autonomy of syntactics'). His insistence upon (meta-)mathematical elegance has been laid without any claim to such psycho-physiological constructs.

In Montague [4] he denies the possibility of working without a distinction between sense and denotation, and, while agreeing with Chomsky and his associates on this point, criticizes them of not having constructed any adequate and comprehensive semantic theories. There he defines the basic aim of semantics as characterizing the notions of a true sentence (under a given interpretation) and of entailment, while the various syntactical categories, especially the set of declarative sentences, are to be characterized in syntax. He insists that he fails to see any great interest in syntax except as a preliminary to semantics, and predicts that such syntactical analyses of particular fragmentary languages as suggested by the Chomskyan school, even if successful in correctly characterizing the sentences of languages, will prove to lack semantic relevance. Thus Montague in his paper tries to develop an adequate and comprehensive theory of universal syntax and semantics.

3.1. In presenting notes on Montague's theories below, I will

concern myself mainly with Montague [4], because Montague [5] can be regarded as an application of his more general theory developed in Montague [4], with several new considerations added to discuss actual fragments of English.

In his 'Universal Grammar' (Montague [4]) and especially in Montague [5], he analyzes the meaning of English sentences through an intermediate artificial language, that of intensional logic, and very comprehensive portions of natural languages can be adequately interpreted by way of the translation into the system of intensional logic, which will finally give the meaning. There his concern is restricted almost solely to a mere statement of definitions, avoiding almost all discussions and intuitive amplifications, deferring a more extended development to a book which he intended to write under the presumed title of *The Analysis of Language*, and which was regrettably never completed.

Montague's use of the term 'Universal Grammar' is evidently quite different from that usually employed in linguistics, where it is used to describe those features common to all natural languages, as distinguished from artificial languages of mathematics or computer programming. He uses the term to refer to a logical or mathematical framework sufficiently general to comprehend any system of communication. As stated above, his introduction of intensional logic together with translation rules can be said to set up three 'algebras' — syntactic, semantic and transformational, where structure-preserving mappings are defined among them, those mappings being mathematically called 'homomorphisms.'

In his scheme of Universal Grammar the syntactic algebra is first described, then followed by the construction of semantic algebra, explicating the homomorphic mapping between them. As a very succinct example his presentation of the semantic algebra, defined as  $\langle B, G_r \rangle_{r \in \Gamma}$ , is made in Montague [4], corresponding completely to the syntactic algebra,  $\langle A, F_r \rangle_{r \in \Gamma}$ . Here B, a universe of meanings, and G, a set of semantic operations are respectively

specified by an interpretation, meanings being assigned to basic expressions of L (language) by a function. Finally the algebra of translation should be described. Here, like many other other philosophers and linguists, he wishes to interpret ambiguous meanings of a sentence by assigning each of them to a corresponding (syntactically) different unambiguous expression. His introduction of intensional logic can safely be regarded as a strategy for this purpose, where an unambiguous ('disambiguated') language is constructed for pairing ambiguous expressions with unambiguous ones. For Montague, the language L is thus to be defined as the pair DL and R, where DL stands for 'disambiguated' language and R for 'ambiguating relation' by which unambiguous expressions in DL are related to ambiguous (polysemous) ones observed in ordinary natural languages.

Although, as he himself contends in Montague [5], inducing an interpretation by translation into an intermediate language (like intensional logic) cannot be necessarily essential, the details of development observed in Montague [5] 'possess certain aesthetic merits, of coherence and conceptual simplicity.' This has been an argument affording the keenest delight to me.

3.2. As is often pointed out, a substantial part of Montague's theories is 'semantics' in a wider sense (including 'pragmatics' or rather the latter comprising the former, as described below). His is defined, furthermore, as 'truth-conditional', 'modeltheoretic' and 'possible world' semantics.

3.2.1 We might call a kind of semantics 'truth-conditional' when the meaning of a sentence is initially determinable by its truth conditions. Semantically by a truth condition is to be meant the relationship holding between a sentence and the state of affairs of a certain world. Notice that we have already here been presupposing the theory of 'possible worlds' which will be explained below.

In traditional logical conception truth conditions primarily refer to (declarative) sentences, thus excluding apparently other

expressions or units smaller than sentences. Mostly, to accommodate these expressions, Frege's so-called Principle of Compositionality is invoked: the meaning of the whole is a function of the meanings of the parts with their mode of combination. This principle is tentatively to be adopted when a unit smaller than a sentence is syntactically well-formed in the light of a certain rule such as that of a phrase structure grammar. Anyway it seems to be urgent to provide such rules to specify an infinite number of truth conditions by means of recursive devices.

Of course, just as in other approaches, in truth-conditional semantics, the famous distinction should be clearly born in mind between an object language and its meta-language in which the former target is being described. Even if we seem to be explaining the meaning of some expression of the object language (e.g. English) in terms of the same language (in this case in English), it does not incur any vicious circularity, so long as the latter is being used as the meta-language. It is primarily in this sense that Montague's use of extensional plus intensional logic as the meta-language is not essentially necessary; it is just for clarity, or, to put it in a favourite parlance, for the 'elegance' of description.

3.2.2 In speaking of model-theoretic semantics, a predominantly abstract logico-mathematical model is being conceived to explain the semantic values of expressions in the object language; hence set-theoretic constructs are employed in such models. A model is usually introduced by defining what sorts of things there are in the world in question with a specified interpretation of them. So-called logical connectives are supposed to remain invariant in any model.

3.2.3. What evidently makes Montague's semantics surpass any other traditional logical semantics is the introduction of the notion of 'possible worlds' ; here a deep contribution might well be attributed to the works of S.A. Kripke, the topmost



philosopher especially in modal logic, (cf. Kripke (1980), where he gives a clear and succinct explanation of this concept). In short a possible world may be conceived as containing everything that a sentence can be about i.e. everything that could affect the truth value of any sentence.

Montague utilizes the concepts of a possible world together with indices of time (more precisely, a possible world regarded by him as an ordered pair of a possible world and a point of time) to be included in his system of intensional logic. His recourse to the 'type' theory, which has already been introduced by Russell, makes it possible to ensure at least the homomorphism (though not isomorphism) between syntax and semantics. He tries to correspond the categories of syntax in a one-to-one fashion to semantic types; thus the two basic syntactic categories, sentence and name, correspond to the two basic semantic types. The semantic types are further divided into extensional and intensional types, the latter being introduced to explicate chiefly the intensional contexts.

In Montague ([4] & [5]), his type theory becomes more complicated because he tries there to explain linguistic meaning via the meta-language of intensional logic. One main reason for this complication is in line with his desire to treat uniformly NP's, proper names, and others even in intensional contexts. Thus, for example, corresponding to the syntactic category  $t/e$  (a usual VP), the semantic type  $\langle\langle s, e \rangle, t \rangle$  is allotted, that is, a function from the intension (which itself is a function from a world 'saeculum' [Lat., according to McCawley (1981)] to an entity) of an entity to the truth value. Predicates are in his theory predicated of intensions because doing semantics both completely extensionally and completely compositionally is an impossibility, making it difficult to base the translation of an item exclusively on the extensions of its immediate constituents.

Thus, as seen immediately above, even a usual proper name is given the category,  $t/(t/e)$ , just as in other NP's; it is only when

a meaning postulate is provoked for a proper name, making it what Kripke (1980) calls 'a rigid designator,' that it is guaranteed as designating the same individual always in every possible world. By the same token, another postulate is offered to extensional verbs so that they can be given simpler traditional predicate-logic interpretation, their truth values being confirmed to depend only on the extension (but not on the intension) of their objects in the world in question.

3.2.4 In speaking of Montague's works, his considerations of pragmatic aspects of natural language are also to be highly evaluated. In Montague ([1], [2]), the relation between pragmatics and intensional logic is revealingly discussed. Morris (1938) is well-known for referring, by the word 'pragmatics,' to the study of language which involves, besides linguistic expressions and their referents, also the users or the possible contexts of use of the expressions.

In order to preserve the Fregean principle of compositionality, what he first called 'indices' are forced to include all complexes of relevant aspects of contexts, e.g. demonstratives, Russellian 'ego-centric particulars', tense operators and so on. Though many of them may be included under the notion of possible worlds, they do suggest the need to establish tense logic and modal logic in general, with perhaps several other branches (e.g. deontic or epistemic logic), where such closely connected and linguistically significant problems will be discussed as 'pre-supposition' or 'propositional attitudes.'

4. Finally a very brief mention will be made of possible contributions which Montague's theories have made to linguistic analyses in their narrower sense. Syntactically his insistence upon the iso- (or homo-) morphism between syntax and semantics has done much to develop certain syntactic theories which claim the possibility of constructing a syntactical theory utilizing only (context-free) phrase structure grammar without any recourse to such grammatical devices as transformations or (technically

speaking) unrestricted (type 0) grammar

As a powerful and important doctrine exemplifying such theories can undoubtedly be mentioned Generalized Phrase Structure Grammar(GPSG) proposed by Gazdar and others (Gazdar et al.(1985)), which are now developing under the name of Head-driven Phrase Structure Grammar [HPSG] ). Their criticism of Generative Semantic approaches, which they cannot maintain as offering a truly semantic analysis, seems to be fairly convincing, and backed up with syntactic analysis in terms of GPSG, semantic analyses essentially in line with Montague's method have been attempted extensively, with recent developments (e.g. 'situation' semantics) proposed for inadequacies found in Montague's original treatment of natural language. In principle, however, it may safely be said that these semantic analyses are being made within the framework of Montague's possible world semantics. Or more linguistically oriented, information- [unification-] based theories which are now being explored by several scholars (e.g. Pollard & Sag(1987)) are also to be regarded as deeply influenced by the Montague's initial conception of semantics.

5. Summing up in a manner of conclusion these rather desultory notes, what I dare to call the enjoyment of Montague's theories could be sought in the clear-cut attitude in which he tries to present a correlation between linguistic expressions and external world(s), grasping the world of meaning which is homomorphic to that of syntax intensionally in terms of mapping from possible worlds to linguistic entities (or categories with various types), without any assertion of theoretical constructs of mental process which might constitute psychological reality.

The 'elegance' of his theories, of which I speak so admiringly above, is undoubtedly due to an extremely terse and strict definition and description of his system. Contrast his original presentation of 'Universal Grammar' with the most elucidating and explanatory notes by Halvorsen & Ladusaw (1979), which

follow, and interpret his theory literally from section to section, taking almost double the number of pages to do so.

No doubt, in understanding his theories, forced upon us are rigorous drills in operating (logical) symbols and learning of a new language of intensional logic, but, in spite of this rather arduous mental labour, even a weakening mind (like mine) finds them intellectually very stimulating and rewarding; hence the word 'enjoyment.'

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