# Chaotic Behaviour in Ferroelectric Crystals

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In this paper the chaotic behaviour in nonlinear electric circuits is discussed, and as a modification of the L-R-Diode nonlinear system, the chaotic behaviour in R-L-ferroelectric system is analyzed.

## § 1. Introduction

Nonlinear dynamics have been a concern of many theoretical and experimental scientfic researchers in recent years. [1-10] There are many nonlinear phenomena observed in the electrical circuits. They are known to be the dynamical systems named chaos and the bifurcation phenomena. It is also reconfirmed by extensive computer simulation of the mathematical model; the 3-rd order autonomous ordinary differential equation.

Matsumoto et al.[3-9] reported the nonlinear 5-elements electrical circuit system. Despite the simplicity of the circuit, it shows the great variety of the bifurcation phenomena. By changing the capacitance values, many phenomena including Hoph bifurcation, periodic-double cascade, Rossler's spiral-type and screw-type attractors, boundary crisis, Shilnikov-type phenomana, etc, have been observed. These systems are now very important for the basis of the chaos-type nural computer.[10]

In the ferroelectric or piezoelectric crystal, these nonlinear dynamics have been investigated by many researchers.[11,12] D. J. Jefferies has been reported the periodic multiplication and chaotic behaviour in the Rochelle salt crystal. The piezoelectric active vibrational resonances were tuned over a 2:1 range by an applied voltage.[11] At high driving amplitudes, subharmonics of the driving frequency occur, accompanied by regions of chaotic behaviour. Huang and Kim observed the characteristics of the nonlinear response of a driven $KH_2PO_4$ crystals at temperature near the ferroelectric transition.[12]

This paper reports the review of the chaotic behaviour in electric circuits. In §2, the results of the calculation of the chaotic behaviour for the nonlinear effects of the iterated system. The nonlinear dynamics in electric circuits are discussed in §3 and §4. In §5, the chaotic behaviour in ferroelectrics is described.

## §2. Chaotic Behaviour of the Iterated System

There are many mathematical models of chaotic bifurcations. The example is the iterated eqution of forms such as

$$x_{n+1}=4\lambda x_n\ (1-x_n) \qquad (1)$$

It shows onset threshfolds in for non-convergent behaviour and quiet bands where the period can be multiplied by an integer number. The figure 1 shows the behaviour of the bifurcation as the function of $\lambda$.

For the double chaotic model for two variables is given by

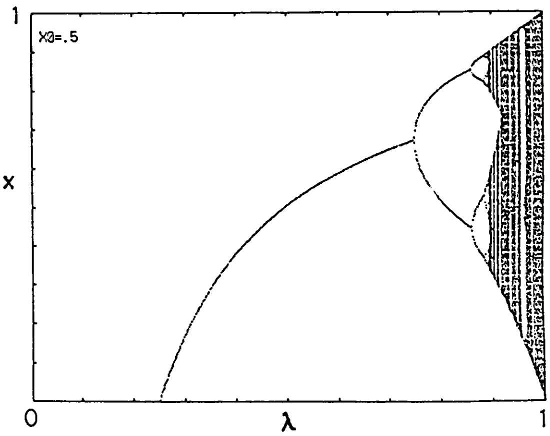
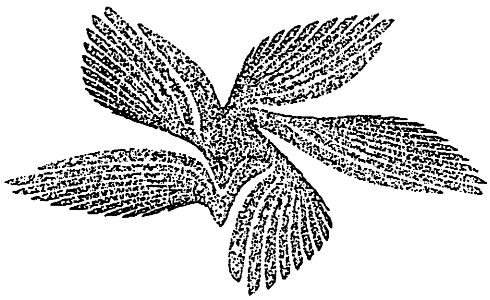


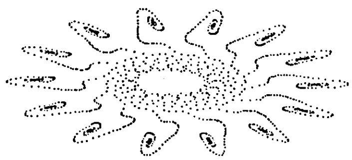

**Fig. 1** Bifurcation diagram of the eq. (1).

$$x_{n+1}=y_n+0.008(1-0.05y_n^2)y_n+f(x_n)$$
$$y_n=-x_n+f(x_{n+1})$$
$$f(x)=\mu x+2(1-\mu)x^2/(1+x^2). \qquad (2)$$

There are many interesting cases for various values of $\mu$. The plot for $\mu=-0.8$ and $\mu=0.9$ are shown in figure 2 (a) and (b).



(a)



(b)

**Fig. 2** 2-D map of eq.(2). (a) $\mu=-0.8$, (b) $\mu=0.9$.

## § 3. The Double Scroll Bifurcation

T. Matsumoto et al. reported the double scroll bifurcation system observed in the electrical circuit. [7] They used a simple circuit as shown in Fig. 3. They observed the extremely complicated non-periodic waveforms. The $v$-$i$ characteristic of the nonlinear register $R$ is described by

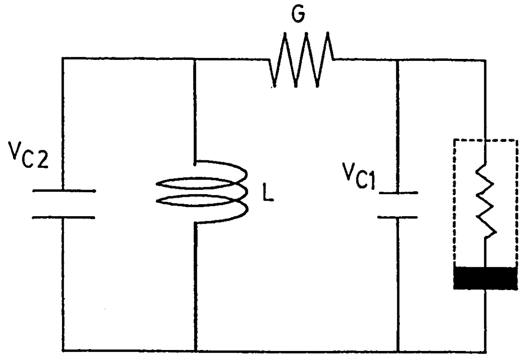
$$C_1(dV_{c1}/dt)=G(V_{c2}-V_{c1})-g(V_{c1})$$
$$C_2(dV_{c2}/dt)=G(V_{c1}-V_{c2})+i_L$$
$$L\ (di_L/dt)=-V_{c2} \qquad (3)$$

where the function $g(x)$ is given by figure 3(b), and $G$ is the conductance chosen as the bifurcation parameter.

The numerical simulation of the bifurcation phenomena observed from the physical circuit Figure 3.

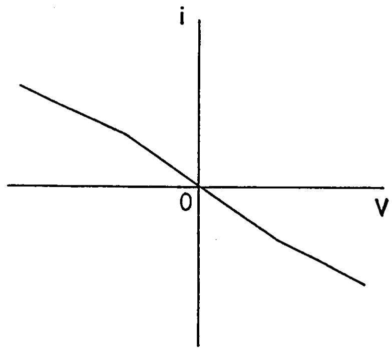
Equilibria of eq. (3) are given by

$$G(V_{c2}-V_{c1})\ -g(V_{c1})\ =0$$





(a)





(b)

**Fig. 3** Simple uncoupled circuit with chaotic attractors; (a) circuitry, (b) $v$-$i$ characteristic of the non-linear register.

$$G(V_{c1}-V_{c2})\ +i_L=0$$
$$V_{c2}=0 \qquad (4)$$

Therefore for fixed values of $G$, these are three equilibria:

$$P+: V_{c1}=k,\ V_{c2}=0,\ i_L=-Gk$$
$$0\ : V_{c1}=V_{c2}=i_L=0$$
$$P-: V_{c1}=-k,\ V_{c2}=0,\ i_L=Gk \qquad (5)$$

where $k$ and $-k$ are the positive and negative solutions of

$$GV_{c1}+g(V_{c1})\ =0. \qquad (6)$$

Many bifurcation were derived from various parameter values of $c_1$, such as Hopf bifurcation, Periodic doubling, etc. The result for the case of Hopf bifurcation is shown in figure 4.

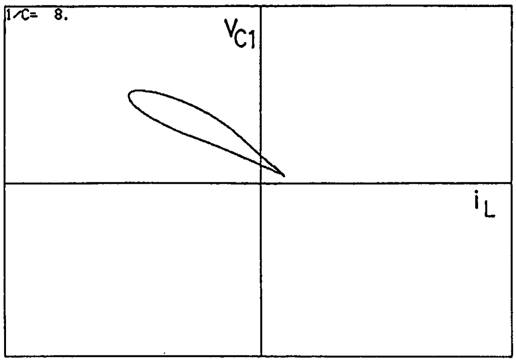



**Fig. 4** Periodic orbits on the $(i_L, V_{c1})$ −plane.

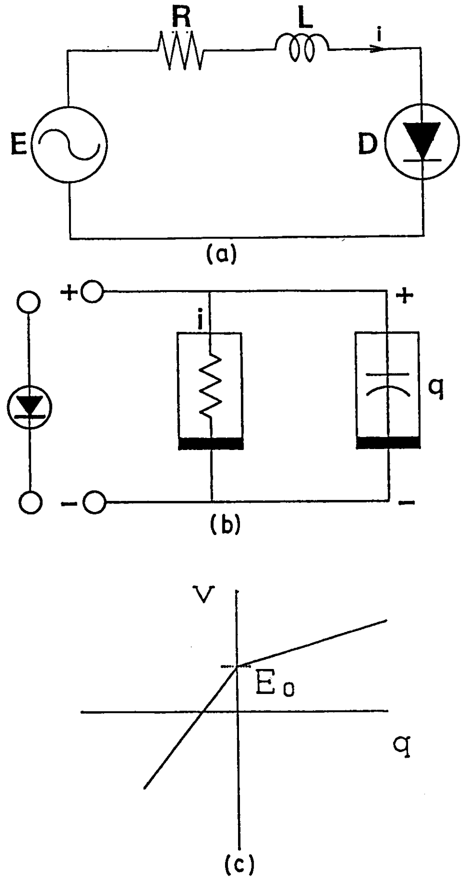



**Fig. 5** Driven $R$-L-Diode circuit; (a) circuitry, (b) equivalent circuit, and (c) simplified capacitor characteristic.

## §4. The driven R-L-Diode circuit

The bifurcation in a driven $R$-L-Diode circuit was discussed by Tanaka et al. [9]. The used driven circuit is shown in figure 5. Despite of its simplicity, the circuit exhibits a rich variety of interesting bifurcation phenomena. The diode is replaced by a parallel conection of a nonlinear registor and a capacitor given by figure 5(b). The relation $V$ vs. $q$ is rather complicated, therefore the nonlinear capacitor is replaced with the piecewise capacitor given by figure 5 (c). For the simplicity, the sinusoidal electric field was replaced by a square wave voltage source of the period $T=1/f$. Then the dynamics of the circuit is described by

$$dq/dt=i$$

$$L\,(di/dt)=-Ri-\begin{Bmatrix} q/C_1 \text{ if } q>=0 \\ q/C_2 \text{ if } q<0 \end{Bmatrix} -E_0\begin{cases} +E \text{ if } nT<t<(n+1)T \\ -E \text{ if } (n+1/2)T<t<(n+1)\ T \end{cases} \qquad (7)$$

where $q$ is the charge stored in the capacitor, $i$ is the

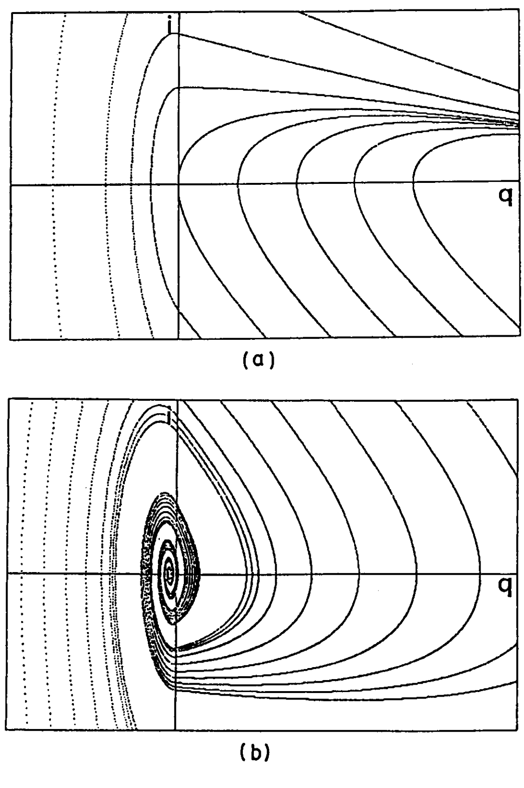



**Fig. 6** Projected orbits on $(q, i)$ −plane; (a) $0<t<1/2$, (b) $1/2<t<1$.

current of the circuit, $R$ is the registor, L is the inductance and $C_1$ ($C_2$) is the capacitance of the capacitor.

Tanaka et al. analysed the circuit of figure 5 driven by sinusoidal voltage source. They observed the bifurcation tree which represents a one-dimensional Poincarè section taken at each fundamental period, $T=1/f$ of the sinusoidal source.

The results are easily simulated by the 2-D map which mimics the point transformation. They plotted the $q$ vs. $i$ curves as shown in figure 6. On the trajectory, which passes through the origin, a trapezoid A at $t=0$ is deformed along the flow as $t$ increases.

A simple 2-D map model was proposed to describe the point transformation, and to exhibits the same bifurcation phenomena as those observed experimentally from the circuit in figure 5. The bifurcation phenomena was described by

$$x_{n+1}=y_n-1 \begin{cases} +a_1x_n & \text{if } x_n>0 \\ -a_2x_n & \text{if } x_n<0 \end{cases}$$

$$y_{n+1}=bx_n \tag{8}$$

For the $R$-L-Diode circuit, Tanaka et al. showed the one-parameter bifurcation diagram of $x$ with $a_1=0.7$, $b=-0.13$ and $a_2$ is varied over range $0<a_2<20$. (Fig. 7)

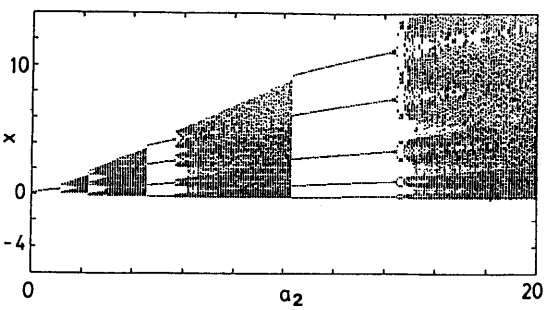



**Fig. 7** One-parameter bifurcation diagram of $x$ for the 2-D map model.

## §5. Bifurcation Response in Ferroelectrics

Huang and Kim showed a real-time display a periodic doubling response observed in KDP. [12] When the frequency was used as a bifurcation parameter, they observed a complete periodic-doubling sequence to chaos. Jeffries showed the result observed in Rochelle salt. [11]

In this paper we report its simulation of the nonlinear behaviour effects in ferroelectrics. The circuit

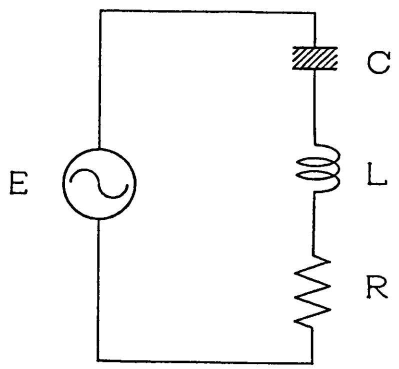



**Fig. 8** Driven $R$-L-Ferroelectric crystal (C) circuit.

used for the computer simulation is shown in figure 8, which is quite same as Fig. 5. The Gibbs free energy appropriate for the second order phase transition in ferroelectric crystal is

$$G=(1/2)\alpha P^2+(1/4)\beta P^4 \tag{9}$$

It gives the electric field $E=\mathrm{d}G/\mathrm{d}P$ as function of the polarization $P$;

$$E=\alpha P+\beta P^3. \tag{10}$$

The dynamics is described by

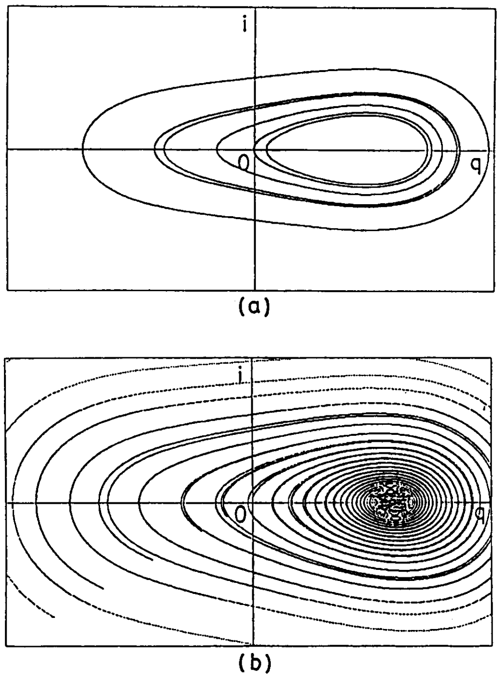



**Fig. 9** Periodic orbits on the $(q,\ i)$ −plane; (a) $a=-0.0001$, $b=-0.9845$, $c=0.1$, (b) $a=-0.102$, $b=-0.9845$, $c=0.2$.

(a) Period-two signal, (b) period-three signal.

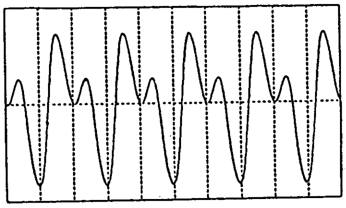
$$dq/dt = i$$
$$L\,di/dt = V - V_R - V_c \quad (11)$$

where $V_c = aq + bq^3$.

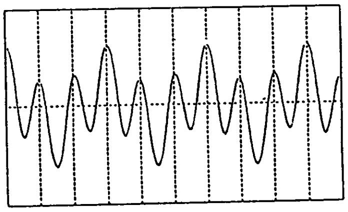
Therefore we get the differential equation correspond to eq.(11) as

$$dq/dt = i$$
$$di/dt = -ai - bq^3 + c \quad (12)$$

From the equation we get the flow ($q$ - $i$ curve), which is shown in figure 9. For the system shown in Fig. 9, we get the multiplication of the frequency voltage imposed. The result calculated from eq. (12) are shown in Fig. 10. The 3-rd harmonic oscilation of the frequency is found. The detailed analysis is now on progress.



(a)



(b)

**Fig. 10** Multiplication characteristic of the circuit shown in Fig. 8 (a)Period-two signal, (b)period-three signal.

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