Effect of Impeller Blade Number on Power Input in Agitated Vessel with Highly Viscous Liquid

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An effect of the impeller blade number on the power input is analized numerically for the agitated vessel with paddle impeller. The power input at very small Reynolds number of $Re_G \leqq 1$ is roughly proportional to one-third power of the impeller blade number. At higher Reynolds number of $Re_G > 1$ the power input increases slightly with increase of Reynolds number, then the Reynolds number dependency of the power input is high at small impeller blade number. The average shear rate for non-Newtonian fluid is almost independent of Reynolds number and of the flow behaviour index, whereas it is roughly proportional to one-third power of the impeller blade number. This fact confirms the availability of the assumption that the average shear rate is equivalent to the average shear rate at the impeller tip radius. The eddy kinematic viscosity estimated from the model by Hiraoka and Fan is almost independent of the impeller blade number.

Introduction

The two dimensional model for the laminar flow in an agitated vessel has been developed by authors, and the resultant power input was in good agreement with the experimental one. The model, however, was restricted only to the vessel with two blade paddle impeller. In this paper, the effect of the impeller blade number on the power input is discussed by using the same model as that in the previous papers[1,2].

1. Governing Equations

The dimensionaless vorticity equation for the incompressible fluid can be written by using the moving coordinate system fixed on the impeller, as follows:

$$Re\cdot\frac{1}{r}\cdot\frac{\partial(\psi,\omega)}{\partial(r,\theta)}=\mu^*\nabla^2\omega+F(\psi,\omega,\mu^*) \tag{1}$$

where

$$\left.\begin{aligned}F(\psi,\omega,\mu^*)&=2\left(\frac{\partial\mu^*}{\partial\theta}\right)\left(\frac{\partial\omega}{\partial\theta}\right)+r\left(\frac{\partial\mu^*}{\partial r}\right)\left\{2r\left(\frac{\partial\omega}{\partial r}\right)+\omega\right\}\\&-4\left\{\frac{1}{r}\left(\frac{\partial^2\psi}{\partial r\partial\theta}\right)-\frac{1}{r^2}\left(\frac{\partial\psi}{\partial\theta}\right)\right\}\left\{r\left(\frac{\partial^2\mu^*}{\partial r\partial\theta}\right)-\left(\frac{\partial\mu^*}{\partial\theta}\right)\right\}\\&+\left\{\left(\frac{\partial^2\mu^*}{\partial\theta^2}\right)-r^2\left(\frac{\partial^2\mu^*}{\partial r^2}\right)\right\}\left\{\omega+2\left(\frac{\partial^2\psi}{\partial r^2}\right)\right\}\\&+2\left(\frac{\partial\mu^*}{\partial r}\right)\left(\frac{\partial^2\psi}{\partial r^2}\right)\\\nabla^2&=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2}{\partial\theta^2}\end{aligned}\right\} \tag{2}$$

The vorticity is defined by using the stream function, ψ, as follows:

$$\omega=-\nabla^2\psi \tag{3}$$

and the stream function is defined by the following relations satisfying the continuity equation.

$$u_\theta=\frac{\partial\psi}{\partial r},\quad u_r=-\frac{1}{r}\cdot\frac{\partial\psi}{\partial\theta} \tag{4}$$

In Eqs. (1) through (4) all variables are reduced to dimensionless ones by using the rotational velocity of vessel wall, V, the vessel radius, $D/2$, and the specified viscosity, μ_0. μ^* and Re mean the dimensionless local viscosity and Reynolds number of $DV\rho/2\mu_0$, respectively.

For Newtonian fluid at constant temperature, the dimensionless viscosity, μ^*, is unity and the term of $F(\psi,\omega,\mu^*)$ in Eq. (1) is zero.

For non-Newtonian power law fluid, the viscosity is expressed as

$$\mu^*=\frac{\mu}{\mu_o}=\left|\sqrt{\frac{1}{2}(\Delta:\Delta)}\right|^{n-1} \tag{5}$$

The specified viscosity, μ_o, is here selected as

$$\mu_o=K(2V/D)^{n-1} \tag{6}$$

where K and n are the fluid consistency and the flow behaviour index of power law fluid, respectively. The double dot product in Eq.(5) is related to the strain rate components.

$$\frac{1}{2}(\Delta:\Delta)=\Delta_{\theta\theta}^2+\Delta_{r\theta}^2 \tag{7}$$

where

$$\Delta_{\theta\theta}=2\left(\frac{1}{r}\cdot\frac{\partial^2}{\partial r\partial\theta}-\frac{1}{r}\cdot\frac{\partial}{\partial\theta}\right)\psi \tag{8}$$

$$\Delta_{r\theta}=\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\cdot\frac{\partial}{\partial r}-\frac{1}{r^2}\cdot\frac{\partial^2}{\partial\theta^2}\right)\psi$$

Applying Eqs. (1) through (8) to the flow domain in an agitated vessel with paddle impeller, we obtain the following boundary conditions.

i) $u_\theta=\frac{\partial\psi}{\partial r}=0,\ u_r=-\frac{1}{r}\cdot\frac{\partial\psi}{\partial\theta}=0$

at impeller and shaft

ii) $u_\theta=\frac{\partial\psi}{\partial r}=1,\ u_r=-\frac{1}{r}\cdot\frac{\partial\psi}{\partial\theta}=0$

at vessel wall (9)

iii) $\frac{d}{dr}\int_0^{2\pi}r^2(\tau_{r\theta}+Re\cdot u_r u_\theta)d\theta=0$

$(d/D<r<1)$

(conservation of angular moment at steady state)

The third condition can be rewritten as

$$\frac{d}{dr}\int_0^{2\pi}r^2\tau_{r\theta}\,d\theta-Re\cdot r\int_0^{2\pi}\left(\frac{\partial\psi}{\partial\theta}\right)\omega d\theta=0 \tag{10}$$

2. Calculation Procedure

The numerical algorithm is the same as that in the previous papers[1,2]. First the flow domain is devided into $M\times N$ subdomains and at each nodal point the governing equations and the associated boundary conditions are expressed by the five-point difference forms. Then, the resultant difference equations and allied boundary conditions are solved by the S.O.R. method. In this calculation, number of divisions, $M\times N$, is 10×10, 20×20 or 20×10 depending on the convergence speed. For more details of the calculation procedures refer to the previous papers[1,2].

3. Results and Discussion

Numerical calculations are made for all cases of the combination of $d/D=0.3$, 0.5, 0.8 and $n_p=2$, 4, 8. For non-Newtonian fluid, the flow behaviour index, n, is varied in the range of 0.6 — 1.2.

A calculation result of the power input for Newtonian fluid is shown in **Fig. 1**, where the power input is modified to the product of friction factor and modified Reynolds number. The power input at very small Reynolds number is approximately proportional to one-third power of impeller blade number, as already reported in previous paper[1], i.e.,

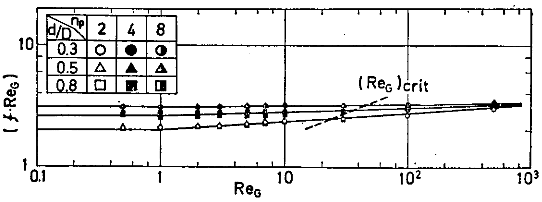

Fig. 1 Dependency of power input on both modified Reynolds number and impeller blade number.

$$f\cdot Re_G=2(n_p/2)^{1/3} \tag{11}$$

$$(Re_G\leqq1;2\leqq n_p\leqq8)$$

For high Reynolds number range of $Re_G>1$, the power input is shown to increase slightly with increasing Reynolds number. However, the dependency of the power input on Reynolds number decreases with the increase of impeller blade number. The solid line in Fig. 1 can be expressed approximately with the following empirical equation.

$$f\cdot Re_G=2(n_p/2)^{1/3}(Re_G)^{0.247/(n_p^2+11)^{1.3}} \tag{12}$$

$$(Re_G>1;\ 2\leqq n_p\leqq8)$$

Stream lines in the vessel with paddle of $d/D=0.8$ are shown in **Fig. 2** for the cases of $n_p=2$, 4 and 8, respectively. The large circulation flows in the sector constructed with impeller blades separate into small ones and disappear with increase of the impeller blade number.

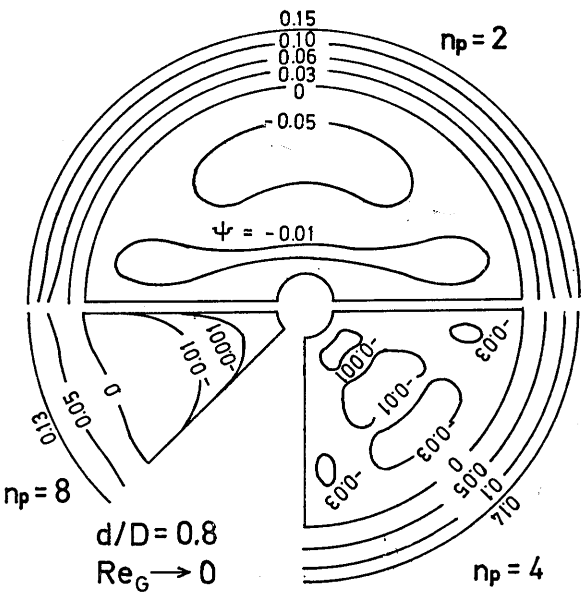

Fig. 2 Effect of impeller blade number on flow pattern.

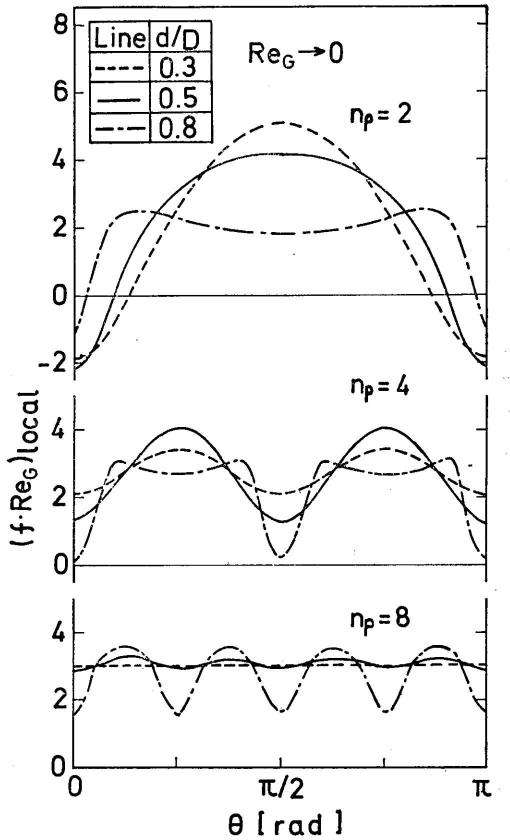

Fig. 3 Effect of impeller blade number on shear stress distribution at vessel wall.

Shear stress distribution in θ direction on the vessel wall at $Re_G \to 0$ is shown in **Fig. 3** for the impeller of $d/D = 0.3$, 0.5, 0.8 and $n_p = 2$, 4, 8. The shear stress distribution becomes flat and has no negative value, as the impeller blade number increases.

For non-Newtonian power law fluid, the apparent viscosity is usually estimated with the following equation.

$$\mu_a = K(\dot{\gamma}_{av})^{n-1} \tag{13}$$

where the average shear rate, $\dot{\gamma}_{av}$, is related to the rotational speed, N, as follows:

$$\dot{\gamma}_{av} = B \cdot N \tag{14}$$

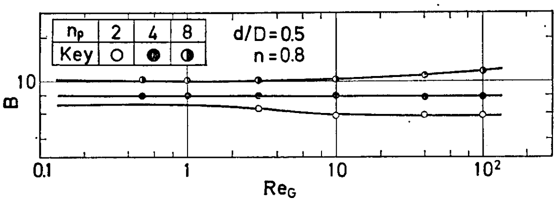

Fig. 4 Dependency of B-value on both modified Reynolds number and impeller blade number.

The proportional constant, B, is the empirical one which depends on the impeller geometry.

An calculated value of B by using the same manner as that in previous paper[2)] is shown in **Fig. 4** for the combination of the fluid of $n = 0.8$ and the impeller of $d/D = 0.5$. Figure 4 shows that the B-value is almost independent of Reynolds number, whereas it increases with increase of impeller blade number. The increment of B-value is roughly proportional to one-third power of the impeller blade number. This fact confirms the availability of the assumption that the average shear rate is equivalent to the average shear rate at the impeller tip radius; in other words, the B-value is expressed as[2)]

$$B = \{(f/2) \cdot Re_G\} \frac{2\pi}{\eta} \cdot \frac{1}{1-(d/D)^2} \tag{15}$$

or

$$B = \frac{2\pi}{\eta} \cdot \frac{(n_p/2)^{1/3}}{1-(d/D)^2} \tag{16}$$

from the combination between Eqs. (11) and (15).

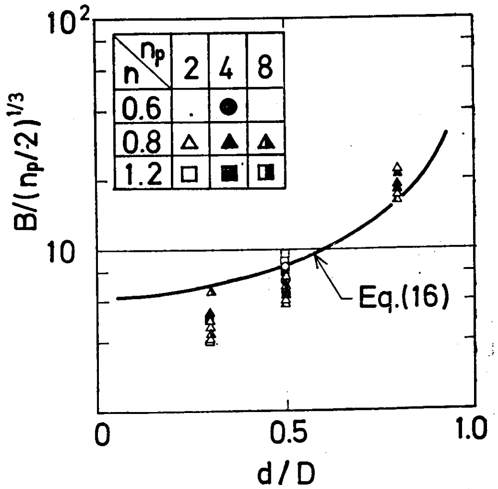

Fig. 5 Relationship between B-value and impeller geometries.

The calculated value of B for the different impeller sizes and different flow behaviour indices are plotted in **Fig. 5**, where the solid line expresses the empirical equation of Eq.(16). Equation (16) satisfies well the calculated values of B for every combinations of impeller size, impeller blade number and flow behaviour index.

The eddy kinematic viscosity is analized for the impeller blade number of 2, 4 and 8, by using the model developed by Hiraoka and Fan[3)]. Then, a calculated results of the eddy kinematic viscosity is shown in **Fig. 6**,

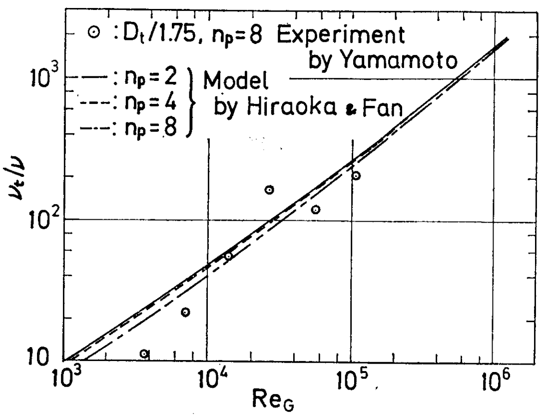

Fig. 6 Dependency of eddy kinematic viscosity on both modified Reynolds number and impeller blade number.

where the eddy kinematic viscosity is almost independent of the impeller blade number, and coincides well with the experimental result of the eddy diffusivity by Yamamoto[4].

Conclusion

An effect of the impeller blade number on the power input is analized numerically for the agitated vessel with paddle impeller. The power input at very small Reynolds number of $Re_G \leqq 1$ is roughly proportional to one-third power of the impeller blade number. At higher Reynolds number of $Re_G > 1$ the power input increases slightly with increase of Reynolds number, then the Reynolds number dependency of the power input is high at small impeller blade number. The average shear rate for non-Newtonian fluid is almost independent of Reynolds number and of the flow behaviour index, whereas it is roughly proportional to one-third power of the impeller blade number. This fact confirms the availavility of the assumption that the average shear rate is equivalent to the average shear rate at the impeller tip radius. The eddy kinematic viscosity estimated from the model by Hiraoka and Fan is almost independent of the impeller blade number.

Nomenclature

B	=dimensionless average shear rate defined by Eq. (14)	[−]
D	=vessel diameter	[m]
D_t	=eddy diffusivity	[m^2/s]
d	=impeller diameter	[m]
f	=friction factor	[−]
K	=fluid consistency	[kg/m·s^{2-n}]
L	=characteristic length(=$(D\eta/2)\cdot\ln(D/d)$)	[m]
M,N	=number of division	[−]
N	=rotational speed	[s^{-1}]
n	=flow behaviour index	[−]
n_p	=impeller blade number	[−]
Re	=Reynolds number (=$DV\rho/2\mu_0$)	[−]
Re_G	=modiffed Reynolds number (=$Lv_\theta\rho/\mu_a$)	[−]
r	=dimensionless radius	[−]
u_r,u_θ	=dimensionless velocity components	[−]
V	=rotational velocity of vessel wall	[m/s]
v_θ	=characteristic velocity (=$(\pi/2)Nd\beta$)	[m/s]
β	=correction factor in v_θ (=$2\cdot\ln(D/d)/\{(D/d)-(d/D)\}$)	[−]
$\dot{\gamma}_{av}$	=average shear rate	[s^{-1}]
Δ_{ij}	=tensor components of deformation rate	[s^{-1}]
η	=correction factor in L (=$1+\exp[-10\{(D/d)-1\}]$)	[−]
θ	=angle in cylindrical coordinates	[rad]
μ	=viscosity or non-Newtonian viscosity	[Pa·s]
μ^*	=dimensionless non-Newtonian viscosity (=μ/μ_o)	[−]
μ_a	=apparent viscosity	[Pa·s]
μ_o	=specified viscosity in Eq. (6)	[Pa·s]
ν	=kinematic viscosity	[m^2/s]
ν_t	=eddy kinematic viscosity	[m^2/s]
ρ	=fluid density	[kg/m^3]
τ_{ij}	=dimensionless shear stress component	[−]
ψ	=dimensionless stream function	[−]
ω	=dimensionless vorticity	[−]

References

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