Stress at the Joint of a Semi-Infinite Plate and a Strip Plate

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A semi-infinite plate with a strip mound is analyzed as a plane elastic problem and a thin plate bending problem. A rational mapping function of a sum of fractional expressions is used for the analysis. The exact solution can be obtained for the shape represented by a rational mapping function. The analytical results, four cases for a plane elastic problem and three cases for a thin plate bending problem, are discussed.

1. Introduction

Stresses in the neighbourhood of a joint of a semi-infinite plate and a strip plate are investigated. The analyzed plate is a sheet of plate made from a semi-infinite plate and a strip plate with same elastic property, that is, a semi-infinite plate with a strip mound. Such a plate is analyzed as a plane elastic problem and a thin plate bending problem. The rational mapping function for the stress analysis is used. If a mapping function is rational, the closed solution can be obtained without solving Fredform's integral equation. Therefore the solution is exact for the shape which is represented by the rational mapping function. The rational mapping function is formed as a sum of fractional expressions. The mapping function of a polynominal expression has extensively been used till now and plates with a hole in the majority of cases have been analyzed[1,2]. However since a power series of a conformal mapping function for shape with strip or convexity has very slow convergency, the mapping function of a polynominal expression can not been used with good accuracy for such a shape. However by using a rational mapping function of a sum of fractional expression, the analytical solution can be obtained even for the shape with a strip or convexity.

Okabayashi[3] and Hasebe[4,5] have analyzed some plane elastic problems and thin plate bending problems by the rational mapping function of a sum of fractional expressions. The complex variable method[6,1] is used for stress analysis.

It seems that exact solution for a semi-infinite plate with a strip mound which is dealed in this paper hardly has been obtained. A semi-infinite plate with a strip mound can be found in some part of the joint of structure. It is considered to be one of the fundamental elements of structure. Stress in the neighbourhood of the corner is of special interest.

2. Conformal Mapping Function

A conformal mapping function which maps a semi-infinite region with a strip mound of infinite length into a unit circle can be obtained by Schwarz-Christoffel's transformation. The conformal mapping function is

$$z=K\int\frac{\sqrt{1+\zeta^2}}{(1-\zeta)\,(1+\zeta)^2}d\zeta. \qquad (2.1)$$

The shape on z plane and the corresponding unit

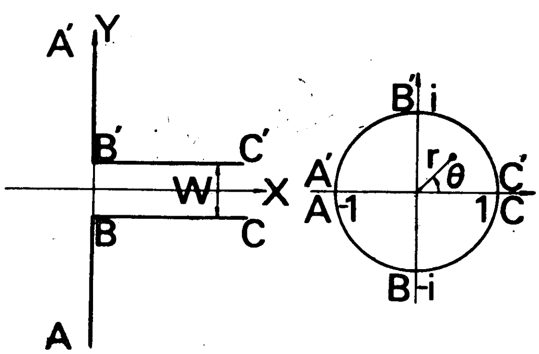

Fig. 1　Semi-infinite region with a strip on the z plane and the unit circle on the ζ plane

circle are shown in Fig. 1. If the width of the strip is w, the coefficient K of (2.1) is $K=2\sqrt{2}\,\mathrm{w}/\pi$.

A rational mapping function of a sum of fractional expressions can be formed, since (2.1) is an irrational function. The method to form the rational mapping function is discussed as follows. If (2.1) is separated into term which converges rapidly and terms which converge slowly, the following equation can be obtained,

$$z\pi/\mathrm{w}=\int\frac{d\zeta}{1-\zeta}-\frac{2}{1+\zeta}+2\sqrt{2}\int\left\{\frac{\sqrt{1+\zeta^2}}{(1-\zeta)(1+\zeta)^2}-\frac{\sqrt{2}}{4(1-\zeta)}-\frac{\sqrt{2}}{2(1+\zeta)^2}\right\}d\zeta. \tag{2.2}$$

The first term corresponds to a strip with an infinite length and the second term corresponds to a semi-infinite region. Rational functions are formed for the respective terms in (2.2). When the first term of (2.2) is expanded into a power series, we get,

$$\int\frac{d\zeta}{1-\zeta}=\sum_{n=1}^{\infty}\zeta^n/n\equiv\sum_{n=1}^{\infty}a_n\zeta^n, \tag{2.3}$$

where $a_n=1/n$.
On the other hand, the following fractional expression to k terms is considered,

$$\sum_{j=1}^{k}\frac{A_j}{1-\alpha_j\zeta}=\sum_{n=0}^{\infty}\sum_{j=1}^{k}A_j\,\alpha_j^n\zeta^n. \tag{2.4}$$

The coefficients A_j and α_j of (2.4) are determined in such a way that (2.4) is approximately equal to (2.3). For this purpose, equating the coefficients of the power series in (2.3) and (2.4), we can get,

$$\sum_{j=1}^{k}A_j\alpha_j^n=a_n. \tag{2.5}$$

In order to determine the values of A_j and α_j, it is required to choose 2k numbers of a_n. The convergence of the power series in (2.3) is very slow and the coefficients a_n decrease monotonously when n increases. In this paper, the value of k is taken as 12 and the following 24 terms of a_n are selected, a_2,a_3,a_4,a_5; $a_{2M},a_{3M},a_{4M},a_{5M}$; $a_{2M^2},a_{3M^2},a_{4M^2},a_{5M^2}$; $a_{2M^3},a_{3M^3},a_{4M^3},a_{5M^3}$; $a_{2M^4},a_{3M^4},a_{4M^4},a_{5M^4}$; $a_{2M^5},a_{3M^5},a_{4M^5},a_{5M^5}$. Here the value of M is taken as 8. The terms a_n and $M=8$ are selected in such a way that the terms in (2.4) agree with those in (2.3) from the low order to the high order. The terms are also selected to make the numerical calculation of (2.5) convenient. Therefore M is not always equal to 8. Equation (2.5) is solved by repeating the calculation[4]. In order to make (2.4) regular in the unit circle, $|\alpha_j|<1$ must be confirmed. The last term, $a_{5M^5}=a_{5\cdot 8^5}$ is the coefficient of the 163840th order of ζ. Further, the terms with order higher than 163840 exist in (2.4) and also agree with the fairly high order terms of (2.3). Therefore we can conclude that (2.4) is a fairly good approximate expression of (2.3). Since the second term of (2.2) is rational, the fractional expressions for the third term of (2.2) is formed next.
The following expression is considered,

$$\sum_{j=1}^{m}\frac{B_j}{1-\beta_j\zeta}=\sum_{n=0}^{\infty}\sum_{j=1}^{m}B_j\beta_j^n\zeta^n. \tag{2.6}$$

The third term of (2.2) is expanded into a power series. This series converges rapidly and the decrease in the coefficients of this series is not monotonous. Therefore B_j and β_j in (2.6) are determined in such a way that the coefficients of the power series of (2.6) agree with the coefficients in the power series expanded from the third term in (2.2) from the second to the 2m+1th term. Then the coefficients B_j and β_j can be determined by solving the equation of m degrees and m linear simultaneous equations.[7] In this paper, m is taken as 14. Also $|\beta_j|<1$ ($j=1, 2\ldots, 14$) must be satisfied. The terms with order of ζ higher than 30 exist in (2.6) and fairly agree with the power series expanded from the third term in (2.2). Therefore (2.6) is a fairly good approximate expression of the third term in (2.2).
From the above discussion, the rational mapping function for (2.2) becomes

$$z=\omega(\zeta)=\frac{\mathrm{w}}{\pi}\left\{\zeta+\sum_{j=1}^{12}\left(\frac{A_j}{1-\alpha_j\zeta}-A_j-A_j\alpha_j\zeta\right)\right\}-\frac{2\mathrm{w}}{\pi(1+\zeta)}+\frac{2\sqrt{2}\,\mathrm{w}}{\pi}\left\{x_1\zeta+\sum_{j=1}^{14}\left(\frac{B_j}{1-\beta_j\zeta}-B_j-B_j\beta_j\zeta\right)\right\}+\frac{2\mathrm{w}}{\pi}\equiv E_{-1}+E_0\zeta+\sum_{k=1}^{26}\frac{E_k}{\zeta_k-\zeta}+\frac{E_{27}}{1+\zeta}. \tag{2.7}$$

In the above equations, x_1 is the coefficient of the first term of power series expanded from the third term in (2.2). The last constant term, $2\mathrm{w}/\pi$, is added in such a way that the rim of the semi-infinite region coincide with the y axis (see Fig.1).

If the values on the unit circle are substituted into (2.7), a diagram can be drawn. Then the accuracy of (2.7) can be investigated. The length of the strip is finite, since the mapping function is rational. The length of the strip is about 4.5 times its width. It is assumed that the length of the strip is long enough for the investigation of the strip in the neighbourhood of the corner. The radius of curvature at the point B' and B shown in Fig.1 is $2.053 \cdot 10^{-4}$w. Therefore we can say that the radius of curvature is very small and the corner is sharp. The y-coordinate of the strip rim exists between 0.4987w and 0.5003w, which form a boundary line wave. However the amplitude of the wave is very small as compared to the width of the strip w. Therefore we can say that the boundary line of the strip is a fairly straight line.

The above discussion shows that the accuracy of the rational mapping function is good.

3. Method of Analysis

The methods of analysis for a plane elastic problem and thin plate bending problem by the complex variable method[6,1] are briefly discussed in the following.

A) Plane elastic problem

If the regular complex stress function in the unit circle are denoted by $\phi(\zeta)$ and $\psi(\zeta)$, the stress components are

$$\begin{aligned}&\sigma_x+\sigma_y=4\mathrm{Re}[\phi'(\zeta)/\omega'(\zeta)]\\&\sigma_y-\sigma_x+2i\tau_{xy}=2[\overline{\omega(\zeta)}\{\phi'(\zeta)/\omega'(\zeta)\}'+\psi'(\zeta)]/\omega'(\zeta)\\&\sigma_r+\sigma_\theta=\sigma_x+\sigma_y\\&\sigma_\theta-\sigma_r+2i\tau_{r\theta}=e^{2i\beta}(\sigma_y-\sigma_x+2i\tau_{xy}),\end{aligned} \tag{3.1}$$

where Re[ ] is the real part of [ ]. σ_r, σ_θ and $\tau_{r\theta}$ denote the stress components in the curvilinear coordinates expressed by a mapping function $z=\omega(\zeta)$ and $\exp(i\beta)=\zeta\omega'(\zeta)/|\zeta\omega'(\zeta)|$. $\overline{\omega(\zeta)}$ denotes the conjugate complex function of $\omega(\zeta)$.

The boundary condition on the unit circle $\sigma=e^{i\theta}$ is represented by

$$\begin{aligned}&\phi(\sigma)+\frac{\omega(\sigma)}{\overline{\omega'(\sigma)}}\overline{\phi'(\sigma)}+\overline{\psi(\sigma)}=i\int(p_x+ip_y)\,ds\\&\equiv H(\sigma),\end{aligned} \tag{3.2}$$

where the integral with respect to s shows the integration along the boundary line. p_x and p_y are forces in the direction of the respective x and y axes. When (3.2) is multiplied by $d\sigma/[2\pi i(\sigma-\zeta)]$ and the Cauchy integral is carried out on the unit circle, the following equation can be obtained,

$$\begin{aligned}&\phi(\zeta)+\frac{1}{2\pi i}\int\frac{\omega(\sigma)}{\overline{\omega'(\sigma)}}\,\frac{\overline{\phi'(\sigma)}}{\sigma-\zeta}d\sigma+\overline{\psi(0)}\\&=\frac{1}{2\pi i}\int\frac{H(\sigma)}{\sigma-\zeta}\,d\sigma\equiv A(\zeta).\end{aligned} \tag{3.3}$$

On the other hand, the mapping function (2.7) is regular inside the unit circle, and has poles of the first order at $\zeta=\zeta_k$ ($k=1,2,\cdots,26$) outside the unit circle, and at $\zeta=-1$ on the unit circle. The first derivative, $\omega'(\zeta)$, has no zeros as inside the unit circle and so $\overline{\omega'}(1/\zeta)$ has no zeros outside the unit circle.

Since the first derivative of the complex stress function, $\phi'(\zeta)$, is regular inside the unit circle, $\phi'(\zeta)$ can be expanded into a Taylor series. Therefore when the point of reflection of ζ_k with respect to the unit circle is represented by ζ_k' $(=1/\overline{\zeta_k})$ ($k=1,2,\cdots,26$), $\phi'(\zeta)$ may be written as

$$\phi'(\zeta)=A_k+A_{k1}(\zeta-\zeta_k')+A_{k2}(\zeta-\zeta_k')^2+\cdots \tag{3.4a}$$

and at the origin,

$$\phi'(\zeta)=A_0+A_{01}\zeta+A_{02}\zeta^2+\cdots. \tag{3.4b}$$

$\overline{\phi'}(1/\zeta)$ is regular outside the unit circle.

From the above discussion, we can write the following expression for the region outside the unit circle,

$$\frac{\omega(\zeta)}{\overline{\omega'}(1/\zeta)}\,\overline{\phi'}(1/\zeta)=\bar{C}_0\bar{A}_0\zeta+\sum_{k=1}^{26}\frac{\bar{C}_k\bar{A}_k}{\zeta_k-\zeta}+R(1/\zeta).$$

$R(1/\zeta)$ is a regular function outside the unit circle. $\bar{C}_0=E_0/\overline{\omega'(0)}$ and $\bar{C}_k=E_k/\overline{\omega'(\zeta_k')}$. In the above expression, the point $\zeta=-1$ is regular. $\bar{\sigma}=1/\sigma$ holds on the unit circle. When the above expression is substituted into the integral term on the left-hand side of (3.3), the following equation can be obtained by carring out the Cauchy integral,

$$\phi(\zeta)+\bar{C}_0\bar{A}_0\zeta+\sum_{k=1}^{26}\frac{\bar{C}_k\bar{A}_k}{\zeta_k-\zeta}+\text{constant}=A(\zeta). \tag{3.5}$$

When (3.5) is differentiated once, and then $\zeta=\zeta_k'$ ($k=1,2,\cdots,26$) and $\zeta=0$ are substituted, 27 linear equations with unknown quantities of A_0, $\bar{A}_0$, A_k and $\bar{A}_k$ can be obtained since $\phi'(\zeta_k')=A_k$ and $\phi'(0)=A_0$ hold in (3.4a) and (3.4b). When these equations are separated into the real and imaginary part, we can obtain 54 linear equations. Thus the real and imaginary parts of A_0 and A_k are determined by solving these linear simultaneous equations. However, if loading condition is symmetric about one axis (in this paper, x axis), A_0 and A_k can be determined by solving 27 linear simutaneous equations.

From the above discussion, $\phi(\zeta)$ is determined. When the conjugate equation of (3.2) is multiplied by $d\sigma/[2\pi i(\sigma-\zeta)]$ and the Cauchy integral is carried out on the unit circle, the complex stress function $\psi(\zeta)$ can be obtained as follows,

$$\psi(\zeta)=\frac{1}{2\pi i}\int\frac{\overline{H(\sigma)}}{\sigma-\zeta}d\sigma-\frac{\overline{\omega}(1/\zeta)}{\omega'(\zeta)}\phi'(\zeta)$$

$$+C_0A_0/\zeta+\sum_{k=1}^{26}\frac{C_kA_k\zeta_k'^2}{\zeta-\zeta'_k}-\overline{\phi(0)}. \quad (3.6)$$

The constant terms in (3.5) and (3.6) can be determined from the condition of displacement. However the constant terms are independent in the calculation of the stress components since the derivatives of $\phi(\zeta)$ and $\psi(\zeta)$ are used in (3.1).

B) Thin plate bending problem

If the regular complex stress functions in the unit circle are denoted by $\phi(\zeta)$ and $\psi(\zeta)$, bending moments M_x, M_y, twisting moment M_{xy} and shearing forces N_x, N_y are

$$\begin{aligned}&M_x+M_y=-4D(1+\upsilon)Re[\phi'(\zeta)/\omega'(\zeta)]\\&M_y-M_x+2iM_{xy}=2D(1-\upsilon)[\overline{\omega(\zeta)}\{\phi'(\zeta)/\omega'(\zeta)\}'\\&\quad+\psi'(\zeta)]/\omega'(\zeta)\\&N_x-iN_y=-4D\{\phi'(\zeta)/\omega'(\zeta)\}'/\omega'(\zeta)\\&M_r+M_\theta=M_x+M_y \quad (3.7)\\&M_\theta-M_r+2iM_{r\theta}=e^{2i\beta}(M_y-M_x+2iM_{xy})\\&N_r-iN_\theta=e^{i\beta}(N_x-iN_y),\end{aligned}$$

where "D" is the flexural rigidity and υ is the poisson's ratio. M_r, M_θ, $M_{r\theta}$, N_r and N_θ denote the stress components in the curvilinear coordinates expressed by a mapping function $z=\omega(\zeta)$.

The boundary condition on the unit circle $\sigma=e^{i\theta}$ is represented by

$$\frac{\upsilon+3}{\upsilon-1}\phi(\sigma)+\frac{\omega(\sigma)}{\overline{\omega'(\sigma)}}\overline{\phi'(\sigma)}+\overline{\psi(\sigma)}$$

$$=\frac{1}{D(1-\upsilon)}\int_0^S[m(s)+i\int_0^S p(s)ds]dz\equiv M(\sigma), \quad (3.8)$$

where the integral with respect to s shows integration along the boundary line. $m(s)$ is the bending moment and $p(s)$ is the bending forces per unit length along the boundary line. The complex functions $\phi(\zeta)$ and $\psi(\zeta)$ can be obtained in the same manner as the plane elastic problem by obtaining $\phi(\zeta)$ and $\psi(\zeta)$ from (3.2). In this case, $\phi(\zeta)$ and $\psi(\zeta)$ are

$$\frac{\upsilon+3}{\upsilon-1}\phi(\zeta)+\bar{C}_0\bar{A}_0\zeta+\sum_{k=1}^{26}\frac{\bar{C}_k\bar{A}_k}{\zeta_k-\zeta}+\text{constant}$$

$$=\frac{1}{2\pi i}\int\frac{M(\sigma)}{\sigma-\zeta}d\sigma \quad (3.9)$$

$$\psi(\zeta)=\frac{1}{2\pi i}\int\frac{\overline{M(\sigma)}}{\sigma-\zeta}d\sigma-\frac{\overline{\omega}(1/\zeta)}{\omega'(\zeta)}\phi'(\zeta)+C_0A_0/\zeta$$

$$+\sum_{k=1}^{26}\frac{C_kA_k\zeta_k'^2}{\zeta-\zeta_k'}-\frac{\upsilon+3}{\upsilon-1}\phi(0). \quad (3.10)$$

The values of C_0, C_k and ζ_k' are the same as those in the plane elastic problem. The unknown quantities $A_0,\bar{A}_0,A_k$ and $\bar{A}_k$ can be determined by solving linear simultaneous equations which can be obtained by substituting $\zeta=0$ and $\zeta=\zeta_k'$ $(k=1, 2, \cdots, 26)$ into the first derivative of (3.9) since $\phi'(0)=A_o$ and $\phi'(\zeta_k')=A_k$ hold. The constant terms in (3.9) and (3.10) can be determined from the condition of displacement. However these constant terms are independent in the calculation of the stress components.

4. Analytical Result

Four loading conditions for the plane elastic problem and three loading conditions for the thin plate bending problem are considered. The analytical results for the respective loading condition are discussed in the following.

(a) Uniaxial tension in the plane

The concentrated load P, as shown in Fig. 2, acts at the tip of the strip. In this case, $A(\zeta)$ in the right-hand side of (3.3) is

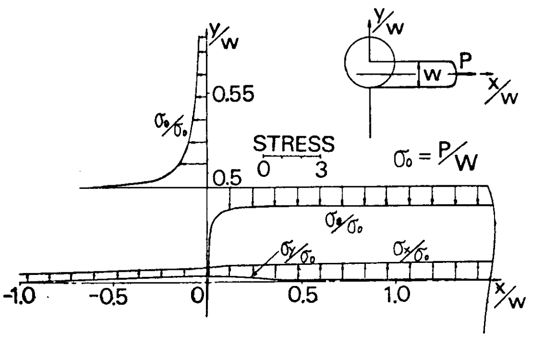

Fig. 2 Stress distributions along the boundary and on the x axis under a concentrated load P at the tip of the strip

$A(\zeta)=P/(2\pi)\log[(\zeta+1)/(\zeta-1)]$

and if the first term in the right-hand side of (3.6) is denoted by $B(\zeta)$, we can write $B(\zeta)=-A(\zeta)$. The range of the stress concentration at the corner is small and the stress, σ_θ/σ_0, along the boundary in the neighbourhood of $x/w=0.2$ is nearly equal to 1.0. The region of influence of the stress, σ_x/σ_0, on the x axis exerts far into the semi-infinite plate.

(b) Bending in the plane

A couple $M=P\cdot\varepsilon$, as shown in Fig. 3, acts at the tip of the strip. In this case, $A(\zeta)=M/(2\pi i)\log[(\sigma_2-\zeta)/(\sigma_1-\zeta)]$ and $B(\zeta)=-A(\zeta)$, where σ_1 and σ_2 are the points on the unit circle which correspond to the loading points.

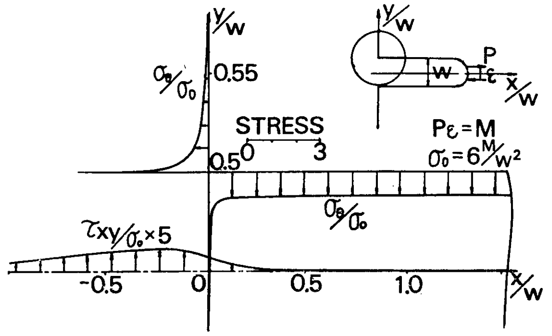

Fig. 3 Stress distributions along the boundary and on the x axis under a couple M at the tip of the strip

The value of the stress, a small distance away from the corner and along the boundary of the strip, agrees with $\sigma_x=My/I=6M/w^2$. The value of $\sigma_x=My/I$ can be calculated from beam theory. The shearing stress on the x axis exists below $x/w\fallingdotseq0.5$. The maximum shearing stress occurs at $x/w\fallingdotseq-0.18$.

(c) Concentrated load $P=1.0$ acting on the side of the strip

In this case, $A(\zeta)=P/(2\pi i)\ \log[(\zeta+1)/(\zeta-\sigma_1)]$, and $B(\zeta)=A(\zeta)$, where σ_1 is the point on the unit circle which corresponds to the loading point. In this paper, three cases of load acting at the respective $x/w=0.5$, 2.5 and 4.5 as shown in Fig.4 are considered. The dashed line in Fig. 4 shows the stress distributions obtained from beam theory. We can find the applicable range of beam theory from Fig.4. The loading point at $x/w=0.5$ is not applicable for beam theory since it is too near the corner.

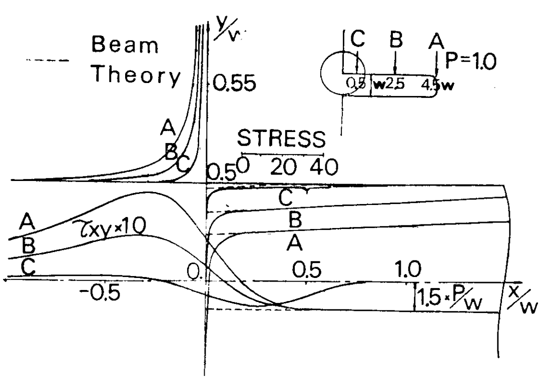

Fig. 4 Stress distributions along the boundary and on the x axis under a concentrated load P on the side of the strip

(d) Uniform tension of the semi-infinite plate

Uniform tension of the semi-infinite plate, as shown in Fig.5, is considered. The required complex stress function, $\phi(\zeta)$ and $\psi(\zeta)$, are represented by

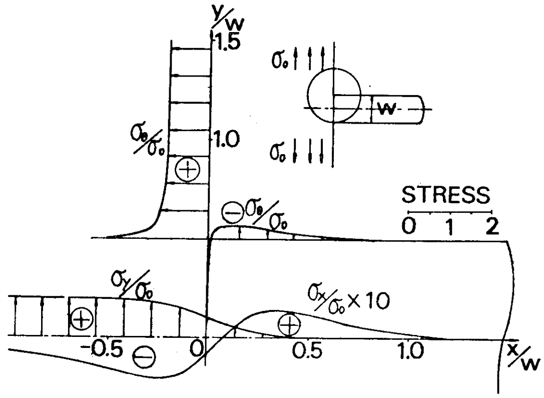

Fig. 5 Stress distributions along the boundary and on the x axis under uniform tension σ_0

$$\phi(\zeta)=\phi_0(\zeta)+\phi_1(\zeta)\ \text{and}\ \psi(\zeta)=\psi_0(\zeta)+\psi_1(\zeta), \quad (4.1)$$

where $\phi_1(\zeta)$ and $\psi_1(\zeta)$ are the complex stress functions due to the uniform tension σ_0 in the direction of the y axis, and are given by

$\phi_1(\zeta)=\sigma_0\omega(\zeta)/4$ and $\psi_1(\zeta)=\sigma_0\omega(\zeta)/2$.

The external load does not act on the boundary. When (4.1) is substituted into (3.3), we get the function

$A(\zeta)=-\sigma_0\ [2\omega(\zeta)-(E_0+\bar{E}_0)/(1+\zeta)]/4$

which is required to determine $\phi_0(\zeta)$. $B(\zeta)$ which is required to determine $\psi_0(\zeta)$ is equal to $A(\zeta)$.

The stress σ_θ/σ_0 along the boundary of the strip suddenly decreases at a point a small distance away from the corner and becomes the compressive stress.

(e) Transverse bending of the strip

A moment M, as shown in Fig.6, acts at the tip of the strip. If the right-hand side of (3.9) and the first term on the right-hand side of (3.10) are respectively denoted by $C(\zeta)$ and $D(\zeta)$, we can write

$C(\zeta)=M/[2\pi D(1-\nu)]\ \log[(\zeta+1)/(\zeta-1)]$ and $D(\zeta)=-C(\zeta)$.

The bending moments along the boundary and on the x axis for the respective poisson's ratio 0.0 and 0.5 are shown in Fig.6. The relation between the

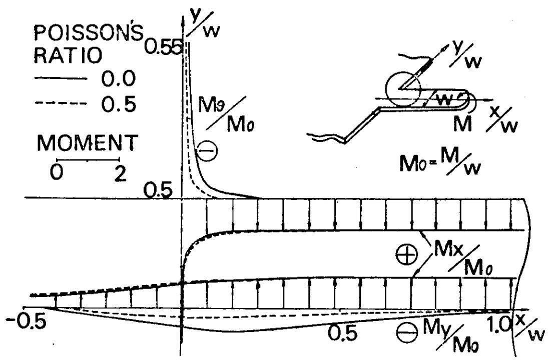

Fig. 6 Bending moment distributions along the boundary and on the x axis under a moment M at the tip of the strip

poisson's ratio and the bending moment may be considered to be linear[5]. Therefore the stress distribution for poisson's ratio between 0.0 and 0.5 can be found by the method of proportional allotment. The bending moment which produce tension at the back of the plate, as shown in Fig.6, is positive. The maximum bending moment for poisson's ratio 0.5 at the corner is larger than that for poisson's ratio 0.0. The values of the bending moment suddenly change from positive to negative at the corner. The values of bending moment along the boundary of the semi-infinite plate for poisson's ratio 0.0 are larger than those poisson's ratio 0.5.

(f) Torsion of the strip

A torsional moment T, as shown in Fig.7, acts at the tip of the strip. In this case, $C(\zeta)$ and $D(\zeta)$ are

$C(\zeta)=T/[\pi iD(1-\nu)]\log[(\zeta+1)/(\zeta-1)]$ and $D(\zeta)=C(\zeta)$.

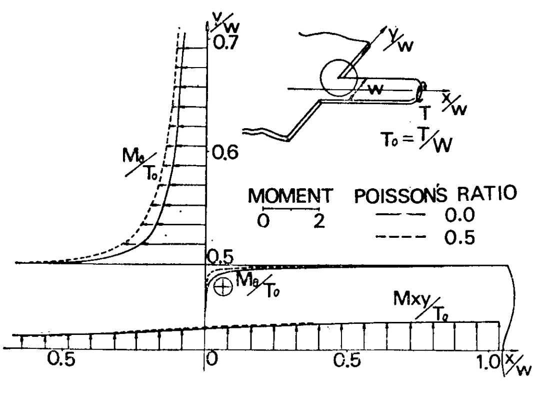

Fig. 7 Bending moment along the boundary and the twisting moment on the x axis under a torsional moment T

The bending moment along the boundary of the semi-infinite plate is fairly large, whereas that along the boundary of the strip is not so.

(g) Uniform bending of the semi-infinite plate

Uniform bending of the semi-infinite plate, as shown in Fig.8, is considered. The required complex stress functions, $\phi(\zeta)$ and $\psi(\zeta)$, are represented by

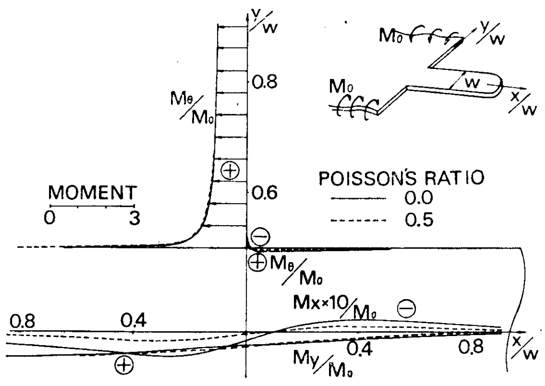

Fig. 8 Bending moment along the boundary and on the x axis under uniform bending moment M_0

$$\phi(\zeta)=\phi_0(\zeta)+\phi_1(\zeta) \quad \text{and} \quad \psi(\zeta)=\psi_0(\zeta)+\psi_1(\psi), \quad (4.2)$$

where $\phi_1(\zeta)$ and $\psi_1(\zeta)$ are the complex stress functions due to the uniform bending moment M_0 about the x axis, and are given by

$\phi_1(\zeta)=-M_0\omega(\zeta)/[4D(1+\nu)]$ and $\psi_1(\zeta)=M_0\omega(\zeta)/[2D(1-\nu)]$.

The external load does not act on the boundary. When (4.2) is substituted into (3.8), we get the function

$$C(\zeta)=-M_0[2\omega(\zeta)-(E_0+\bar{E}_0)/(1+\zeta)]/[4D(1-\nu)]$$

which is required to determine $\phi_0(\zeta)$. $D(\zeta)$ which is required to determine $\psi_0(\zeta)$ is equal to $C(\zeta)$. The very high stress concentration occurs at the side of the semi-infinite plate, but at the side of the strip the bending moment suddenly decreases and becomes the negative bending moment.

5. Conclusion

A rational mapping function of a sum of fractional expressions can be formed for a comparatively arbitrary shape. It can be formed even for a shape with convexity or strip which can not be analyzed by a mapping function of a polynominal expression. Once the coefficients of fractional expressions for a strip (A_j and α_j of (2.4)) are determined, the coefficients can always be used for the analysis of the shape with a strip. Accordingly to form the fractional expression is required only for a term with quick convergency (see the third term on the right-hand side of (2.2)). If the mapping function is formed in the type of (2.7), the stress functions for a plane elastic problem or a thin plate bending problem are given by (3.5) and (3.6), or (3.9) and (3.10) respectively. Therefore once the mapping function is formed, it is easy to calculate the stress components. The method using the rational mapping function is particularly efficient for obtaining the stress distributions at the place of stress concentration since the solution is analytically obtained in the closed form. This method can be applied for crack problem, and has been used for obtaining stress distributions in the neighbourhood of a crack and stress intensity factor which is important in fracture mechanics[8,9].

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