The Modified MHD Couette Flow

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Introduction

The Modified Hartmann flow, i.e., the Hartmann flow with Hall effects is given by Sherman and Sutton[1]. They found a surprising result that the velocity profile changes from the characteristic Hartmann profile to the typical poiseuille profile as the Hall parameter $\omega_0\tau$ increases.

This note was carried out practically with the similar analysis for MHD Couette flow with Hall effect, i.e., the modified MHD Couette flow, and the analogous interesting result was found that the velocity profile approaches to the typical, linear velocity pıcfile of Couette flow as $\omega_0\tau$ increases.

Nomenclature

a=height of the channel
B_0=applied magnetic field
E_x, E_y=electric field in x and y directions
j_x, j_y=electric current density in x and y directions
p_y=pressure gardient in y direction
u=velocity in x direction
u_∞=velocity of the moving wall
v=velocity in y direction
z=coordinate parallel to B_0, and origin is taken on the stationary wall
η=viscosity of fluid
ρ=mass density
$\omega_0\tau$=Hall parameter
Subscripts
x=main flow
y=crossflow

Theory and Results

We define the following dimensionless variables and parameters:

$U=u/u_\infty$, $V=v/u_\infty$, $Z=z/a$, $e_x=E_x/u_\infty B_0$, $K=E_y/u_\infty B_0$, $H_a=aB_0(\sigma/\eta)^{1/2}$,
$R_e=u_\infty a\rho/\eta$, $P_y=p_y/\rho u_\infty^2$, $J_x=j_x/\sigma u_\infty B_0$, $J_y=j_y/\sigma u_\infty B_0$.

The governing, dimensionless momentum equations of the Couette flow for incompressible, laminar and electrically conducting fluild with Hall effect are given by

$$d^2U/dZ^2-H_a^2U/\{1+(\omega_0\tau)^2\}+H_a^2(\omega_0\tau)V/\{1+(\omega_0\tau)^2\}+A_1=0 \quad (1)$$

$$d^2V/dZ^2-H_a^2V/\{1+(\omega_0\tau)^2\}-H_a^2(\omega_0\tau)U/\{1+(\omega_0\tau)^2\}+A_2=0 \quad (2)$$

where

$$A_1=H_a^2(K+\omega_0\tau e_x)/\{1+(\omega_0\tau)^2\}$$
$$A_2=-R_eP_y-H_a^2(e_x-\omega_0\tau K)/\{1+(\omega_0\tau)^2\}$$

The dimensionless boundary conditions for the present case are given as follows:

at $Z=0$, $U=V=0$,
at $Z=1$ $U=1$, $V=0$.

Once U and V are obtained the current densities are calculated by the following relations:

$$J_x=\{e_x+V-\omega_0\tau(K-U)\}/\{1+(\omega_0\tau)^2\} \quad (3)$$
$$J_y=\{K-U+\omega_0\tau(e_x+V)\}/\{1+(\omega_0\tau)^2\} \quad (4)$$

Equations (1) and (2) contain five parameters, i.e., H_a, $\omega_0\tau$, K, R_eP_e, and e_x. Of these parameters R_eP_y and e_x are determined by the following physical conditions:

$$\int_0^1 V dZ=0 \quad (5)$$
$$\int_0^1 J_x dZ=0 \quad (6)$$

Equation (5) implies the condition of the no net cross flow, and equation (6) shows that the segmented electrodes are used.

The numerical calculations are carried out for $H_a=8$ and $K=0.25$ (open circuit condition) for several values of $\omega_0\tau$; the results are presented for U, V, J_x, and J_y in Figs. 1 and 2. When $\omega_0\tau\rightarrow 0$, j_x and j_y tend to zero, so that **J**×**B** force becomes zero over the channel. Therefore an originally linear velocity profile should again be linear. Other physical characteristics of the velocity distributions can be explained on the basis of current flow in almost same manner as is done for the modified Hartmann flow[2].

Referrences

1) Sherman, A., and Sutton, G.W., "The combined effect of tensor conductivity and viscosity on an MHD generator with segmented electrodes", in A.B. Cambel, T.P. Anderson, and M.M. Slawsky (eds.), Proc. 4th Biennial Gas Dynamics Symp., Northwestern Unversity Press, Evanston, Ill., 1962.

2) Sutton, G.W., and Sherman, A., "Engineering Magnetohydrodynamics", Mc Graw-Hill, New York, 1965, 370.

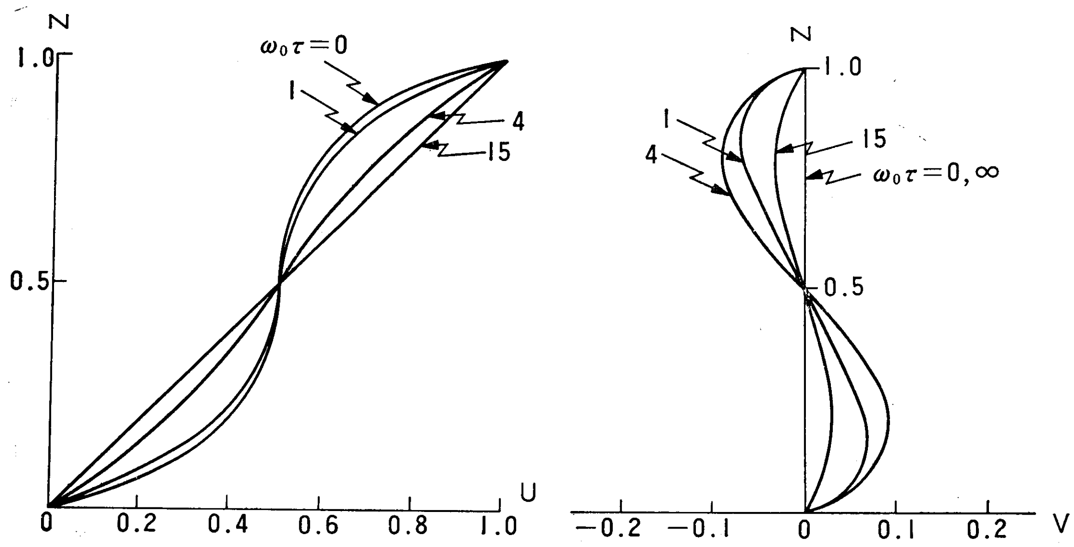

Fig. 1 The flow pattern in the modified Couette flow.

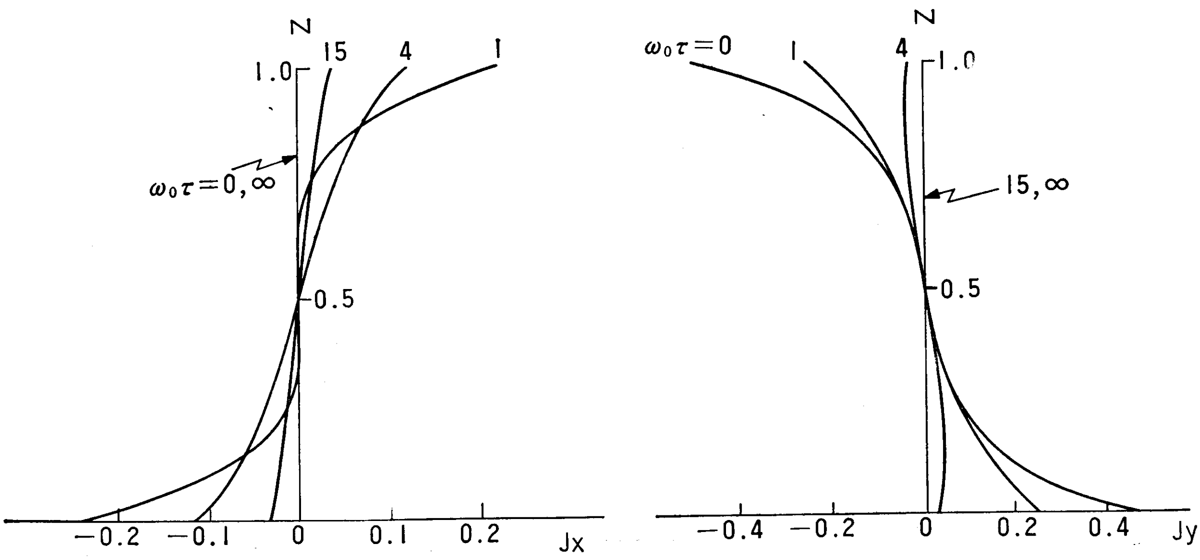

Fig. 2 Current distributions in the modified Couette flow.