

# Heat Transfer of Non-Newtonian Fluid in Anchor Agitated Vessel

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(Received September 6, 1977)

The heat transfer coefficient at the wall of anchor agitated vessel in non-Newtonian fluid is measured and correlated with the modified Reynolds number, where the apparent viscosity is calculated with the simple estimation method proposed by the authors. The experimental data of non-Newtonian fluid in turbulent range agree well with the correlation equation of Newtonian fluid. This concludes that the analogy relation between heat and momentum transfer at the vessel wall holds in non-Newtonian fluid, and that the simple estimation method of the apparent viscosity is successful for the correlation of the heat transfer coefficient.

## Introduction

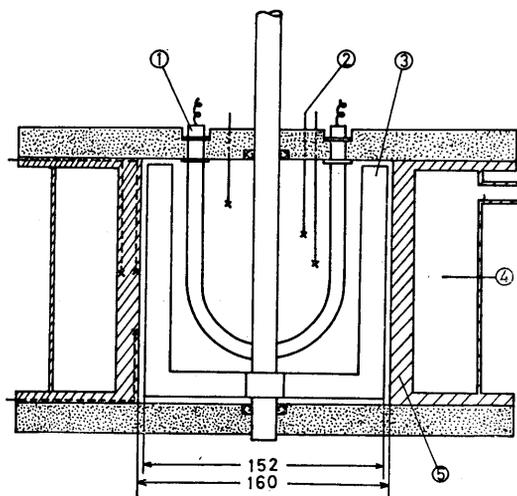
In chemical and biochemical industries, the mixing and heat transfer operations of non-Newtonian fluid in agitated vessel are often encountered and it is necessary to examine their performance, as these strongly affect on the yield and quality of the products. But the generalized rules of those performances in non-Newtonian fluid have not been satisfactorily established yet. For the power consumption of non-Newtonian fluid in laminar range, Metzner<sup>2)</sup> and other many workers have proposed the concept of the average shear rate and correlated the power input with the apparent viscosity. The authors have also reported the simple estimation method of the average shear rate for the correlation of power input in non-Newtonian fluid<sup>1)</sup>.

This paper deals with the extension of this simple estimation method to the correlation of heat transfer coefficient of non-Newtonian fluid in agitated vessel, and with the establishment of the analogy relation between momentum and heat transfer at the vessel wall for the agitation of non-Newtonian fluid.

## Experimental Apparatus and Procedures

The agitated vessel, 16.0 cm inner diameter and 16.0 cm depth, is constructed with the stainless steel cooling wall and the upper and bottom walls made

adiabatically from polyvinyl chloride, as shown in Fig. 1. The anchor agitator used, 15.2 cm diameter, 15.2 cm height and 1.5 cm width, is made of brass.



① Electric heater ② Thermocouple ③ Anchor agitator  
④ Cooling jacket ⑤ Cooling wall

Fig. 1 Agitated vessel with anchor agitator

The agitated fluids used are 0–70 wt% glycerol aqueous solutions as Newtonian fluid and 0.5–2.0 wt% C. M. C. (Carboxy Methyl Cellulose) aqueous solutions as non-Newtonian fluid, respectively. For the glycerol aqueous solution, the viscosity<sup>6)</sup>, the density<sup>6)</sup>, and the specific heat<sup>5)</sup> are listed as the

functional forms of temperature in Table 1, and the thermal conductivity<sup>7)</sup> is obtained from the following equation.

$$\left. \begin{aligned} k &= x \cdot k_g + (1-x) \cdot k_w \quad [\text{cal/cm} \cdot \text{sec} \cdot ^\circ\text{C}] \\ k_g &= 0.000695 \{1 + 0.00262(t-20)\} \\ k_w &= 0.001403 \{1 + 0.00380(t-20)\} \end{aligned} \right\} \quad (1)$$

where  $k_g$  and  $k_w$  are the thermal conductivities of glycerol and water, respectively,  $x$  means the weight fraction of glycerol, and  $t$  is expressed in the unit of  $^\circ\text{C}$ . For the C.M.C. aqueous solution, the density, the specific heat<sup>5)</sup>, the thermal conductivity<sup>5)</sup> and the fluid consistency and flow behaviour index of power model are also listed in Table 2.

The heat transfer coefficient at the vessel wall is obtained from the measurements of the heat flux at the cooling wall and the temperature difference between the bulk fluid and the cooling wall surface. The heat flux at the vessel wall is calculated from the heat generation of electric heater in the vessel. The bulk temperature is obtained from averaging arithmetically the local temperatures measured by three thermocouples, which are placed at the cross marks shown in Fig. 1. The temperature of cooling wall surface is indirectly measured by five thermocouples inserted into the vessel wall. Four of them are near the agitated fluid side and the rest near the cooling water side. The measured temperatures are

Table 1 Physical properties of glycerol aqueous solution

glycerol wt%	viscosity [g/cm <sup>3</sup> · sec] $\mu = A + B \cdot t$		density [g/cm <sup>3</sup> ] $\rho = C + D \cdot t$		specific heat [cal/g · °C] $c_p = E + F \cdot t$	
	A	B	C	D	E	F
20.88	0.02662	-0.000442	1.724	-0.027	0.9248	-0.000144
35.75	0.04768	-0.000823	1.793	-0.028	0.8742	-0.000488
50.56	0.09956	-0.00187	1.856	-0.029	0.8046	-0.000494
58.92	0.16055	-0.00318	1.897	-0.030	0.7715	-0.000765
69.73	0.3894	-0.00840	1.920	-0.030	0.7184	-0.000716

(20 < t [°C] < 50)

Table 2 Physical properties of C.M.C. aqueous solution

C.M.C. wt%	density [g/cm <sup>3</sup> ] $\rho = A + B \cdot t + C \cdot t^2$			specific heat [cal/g · °C] $c_p = D + E \cdot t + F \cdot t^2$		
	A	B	C	D	E × 10 <sup>2</sup>	F × 10 <sup>4</sup>
0.5	0.9977	0.00027	-0.00001	0.9457	0.0488	0.0404
1.0	1.0028	0.00017	-0.00001	0.9163	0.4219	0.0905
1.5	1.0068	0.00008	-0.00001	0.8870	1.0189	0.3350
2.0	1.0153	-0.00032	0.0	0.8576	1.8397	0.7740

(20 < t [°C] < 50)

(30 < t [°C] < 80)

C.M.C. wt%	thermal conductivity [cal/cm · sec · °C] $k = A + B \cdot t + C \cdot t^2$			fluid consistency [g/cm · sec <sup>2-n</sup> ] $K = D + E \cdot t$		flow behaviour index [-] n
	A	B × 10 <sup>5</sup>	C × 10 <sup>7</sup>	D	E	
0.5	0.001278	0.6434	-0.3211	0.0796	-0.0014	0.98
1.0	0.001268	0.6187	-0.2933	0.3804	-0.0068	0.93
1.5	0.001256	0.5937	-0.2656	0.927	-0.0147	0.90
2.0	0.001246	0.5687	-0.2378	7.818	-0.150	0.80

(20 < t [°C] < 70)

(10 < t [°C] < 40)

extrapolated to give the cooling wall surface temperature. The heat transfer coefficient is measured at the impeller rotational speed in the range of 40–150 rpm for each agitated fluid.

**Results and Discussion**

The heat transfer coefficient,  $h$ , is reduced to the  $j$ -factor and correlated with the modified Reynolds number. In this manner, the  $j$ -factor and the modified Reynolds number are defined as<sup>3)</sup>

$$\left. \begin{aligned} j_H &= (h/\rho c_p v_\theta) \cdot Pr^{2/3} \\ Re_G &= (Lv_\theta \rho/\mu) \end{aligned} \right\} \quad (2)$$

where  $v_\theta$  and  $L$  are the characteristic velocity and length, respectively, already defined as<sup>1),3)</sup>

$$\left. \begin{aligned} v_\theta &= (\pi/2) Nd\beta \\ \beta &= 2 \cdot \ln(D/d) / \{(D/d) - (d/D)\} \\ L &= \{(D/2) \cdot \ln(D/d)\} \cdot \eta \\ \eta &= 1 + \exp[-10\{(D/d) - 1\}] \end{aligned} \right\} \quad (3)$$

For the non-Newtonian fluid, the apparent viscosity,  $\mu_{av}$ , is estimated from the following equation proposed by the authors<sup>1)</sup>.

$$\left. \begin{aligned} \mu_{av} &= K \cdot (\gamma_{av})^{n-1} \\ \gamma_{av} &= \frac{7.1}{\eta} \cdot \frac{1}{1 - (d/D)^2} \end{aligned} \right\} \quad (4)^*$$

This estimation method has already been confirmed

to correlate well the power consumption of non-Newtonian fluid in the turbulent range by the same empirical equation as that for Newtonian fluid, i.e.,<sup>1)</sup>

$$f/2 = 0.121 Re_G^{-1/3} \quad (5)$$

where  $f$  is the friction factor defined by using the average shear stress at the wall and the characteristic velocity, as<sup>3)</sup>

$$f/2 = \tau_w / \rho v_\theta^2 \quad (6)$$

On the other hand, it has been reported that the Newtonian fluid satisfies the analogy relation between heat and momentum transfer at the vessel wall in the turbulent range, expressing as<sup>3),4)</sup>

$$j_H \cdot (1.4d/\beta D) = f/2 \quad (7)$$

Then, the heat transfer coefficient is plotted as the relation of the modified  $j$ -factor expressed by Eq. (7) versus the modified Reynolds number, as shown in Fig. 2. The experimental data of both Newtonian and non-Newtonian fluids in the turbulent range agree well with the empirical form of Eq. (5). This means that the analogy relation between heat and momentum transfer at the vessel wall holds in non-Newtonian fluid, and that the simple estimation method of the apparent viscosity defined by Eq. (4) is successful for the correlation of the heat transfer coefficient, too.

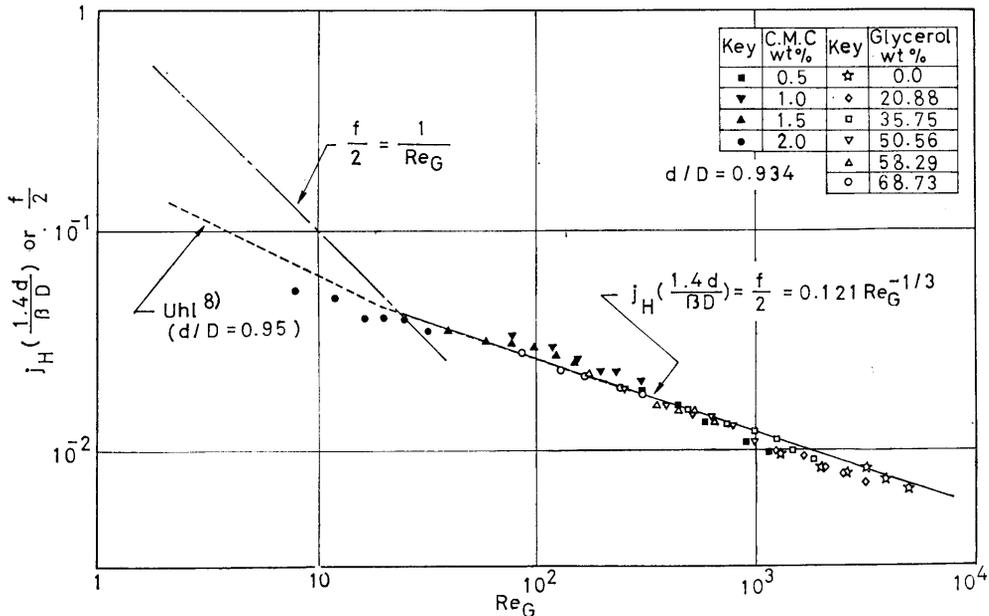


Fig. 2 Relationship between  $j$ -factor and modified Reynolds number

\*) The estimation equation of  $\mu_{av}$  in reference 1 is corrected with the parameter  $\eta$  to give Eq. (4) for the proximity impeller.

For the laminar range, the experimental data are so a few that the correlation equation cannot be obtained to compare with that of Newtonian fluid.

### Concluding Remarks

The heat transfer coefficient at the vessel wall of the anchor agitated vessel in non-Newtonian fluid is measured and correlated with the modified Reynolds number, where the apparent viscosity is calculated with the simple estimation method proposed by the authors. The experimental data of non-Newtonian fluid in the turbulent range agree well with the correlation equation of Newtonian fluid. This concludes that the analogy relation between heat and momentum transfer at the vessel wall holds in non-Newtonian fluid, and that the simple estimation method of the apparent viscosity is successful for the correlation of the heat transfer coefficient.

### Nomenclature

$d$ = impeller diameter	[cm]	$n$ = flow behaviour index	[—]
$D$ = vessel diameter	[cm]	$Re_G$ = modified Reynolds number	[—]
$f$ = friction factor	[—]	$v_\theta$ = characteristic velocity	[cm/sec]
$h$ = heat transfer coefficient	[cal/cm <sup>2</sup> ·sec·°C]	$x$ = weight fraction of glycerol	[—]
$j_H$ = $j$ -factor for heat transfer	[—]	$\beta$ = correction factor defined by Eq. (3)	[—]
$k$ = thermal conductivity	[cal/cm·sec·°C]	$\gamma_{av}$ = average shear rate	[sec <sup>-1</sup> ]
$k_g$ = thermal conductivity of glycerol	[cal/cm·sec·°C]	$\eta$ = correction factor defined by Eq. (3)	[—]
$k_w$ = thermal conductivity of water	[cal/cm·sec·°C]	$\mu$ = viscosity	[g/cm·sec]
$K$ = fluid consistency	[g/cm·sec <sup>2-n</sup> ]	$\mu_{av}$ = apparent viscosity	[g/cm·sec]
$L$ = characteristic length	[cm]	$\rho$ = density	[g/cm <sup>3</sup> ]
		$\tau_w$ = average shear stress at wall	[g/cm·sec <sup>2</sup> ]

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