Transport Phenomena and Flow Behaviour in Agitated Vessels*

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Defining the characteristic velocity, length and friction velocity respectively, which specify the performances in agitated vessel, it is implied that the flow behaviour and transport phenomena at the wall are well expressed with the simple correlation equations for the all Reynolds number ranges, laminar to turbulent. This method is useful to correlate the other performances in agitated vessels.

Introduction

Agitated vessels have been widely used in chemical processes, and its performances have been studied by many workers. However, due to the complex behaviour of fluid flow in agitated vessels, the general correlation connecting various performances has not been established yet. In this paper the general correlation for the various performances in an agitated vessel is proposed.

Transport Phenomena at the Wall

Definition of friction factor and j-factor

Firstly, the friction factor and j-factor are introduced to the momentum, heat and mass transfer at the wall of agitated vessel, as the same manner as that at the boundary layer flow on flat plate and pipe flow.[2,7]

$$f/2=\bar{\tau}_w/\rho v_\theta^2$$
$$j_H=(h/\rho c_p v_\theta)\cdot P_r^{2/3} \qquad (1)$$
$$j_D=(k/v_\theta)\cdot Sc^{2/3}$$

where the characteristic velocity v_θ and length L respectively are defined as follows,[4]

$$v_\theta=(\pi/2)\cdot Nd\beta$$
$$L=(D/2)\cdot \ln(D/d) \qquad (2)$$

and β is a correction factor defined as,

$$\beta=2\cdot\ln(D/d)\,/\,(D/d-d/D) \qquad (3)$$

Then, the modified Reynolds number, Re_G, can be expressed by using the foregoing variables.

$$Re_G\equiv\frac{Lv_\theta\rho}{\mu}=\left\{\left(\frac{\pi}{4}\right)\cdot\left(\frac{\beta D}{d}\right)\cdot ln\left(\frac{D}{d}\right)\right\}\cdot\left(\frac{Nd^2\rho}{\mu}\right) \qquad (4)$$

where $Nd^2\rho/\mu$ is the impeller Reynolds number, Re_d, widely used in previous papers.

Measurement of friction factor in turbulent range

The shear stress at the wall of agitated vessel with paddle impeller, $\bar{\tau}_w$, has been measured, and the friction factor was correlated with the modified Reynolds number for the fully turbulent non-baffled agitated vessel. The correlation equation is expressed as,[4]

$$f/2\equiv\bar{\tau}_w/\rho v^2_\theta=0.121Re_G^{-1/3} \qquad (5)$$

where the bar indicates the average value over vessel wall. Eq. (5) is easily rearranged to give power number N_P.

$$N_P=3.19(1+\alpha)\cdot\left(\frac{H}{d}\right)\left\{\left(\frac{\beta D}{d}\right)^5\Big/ln\left(\frac{D}{d}\right)\right\}^{1/3}\cdot Re_d^{-1/3} \qquad (6)$$

where α is the ratio of torque at the bottom wall to that at the side.

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Flow Behaviour

Universal velocity profiles

Three dimensional velocity profiles in fully turbulent non-baffled agitated vessels with paddle impellers have been measured.[3] Whereas basing on the assumption that the flow behaviour is specified by the apparent shear stress at the impeller tip, $\bar{\tau}_d$, the characteristic friction velocity for the agitated vessel, $\bar{u}^*_d$, has been defined as,

$$\bar{u}^*_d \equiv \sqrt{\bar{\tau}_d/\rho} = (D/d) \cdot \sqrt{\bar{\tau}_w/\rho} = (D/d) \cdot \bar{u}^*_w \tag{7}$$

where $\bar{u}^*_w$ is the friction velocity defined by the shear stress at the vessel wall.

At the vicinity of the vessel wall the tangential velocity and the distance from the wall are nondimensionalyzed by using the friction velocity and correction factor β.

$$v^{++} = \left(\frac{v}{\beta \bar{u}^*_d}\right) \cdot \left(\frac{\beta D}{d}\right)^2 = \left(\frac{v}{\bar{u}^*_w}\right) \cdot \left(\frac{\beta D}{d}\right) = v^+ \cdot \left(\frac{\beta D}{d}\right)$$
$$y^{++} = \frac{y(\beta \bar{u}^*_d)}{\nu} = \left(\frac{y \bar{u}^*_w}{\nu}\right) \cdot \left(\frac{\beta D}{d}\right) = y^+ \cdot \left(\frac{\beta D}{d}\right) \tag{8}$$

where v^+ and y^+ are dimensionless variables defined with $\bar{u}^*_w$. These new variables satisfy the relation of $v^{++} = y^{++}$ at the viscous sublayer.

The velocity profiles at the vicinity of the wall are correlated as shown in **Fig. 1** and are expressed as,[5]

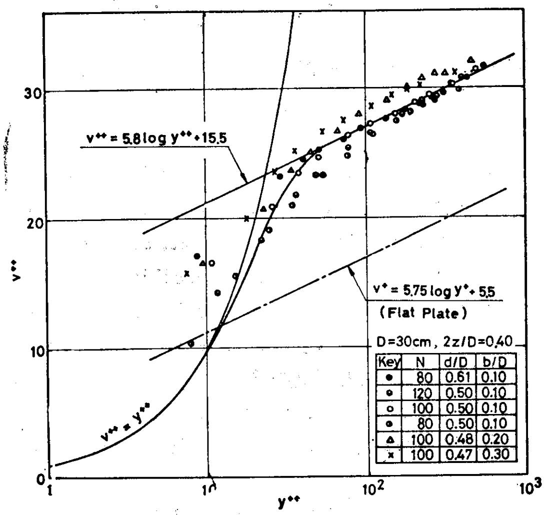

Fig. 1 Velocity profiles at the vicinity of the vessel wall

$$v^{++} = 5.8 \log y^{++} + 15.5 \tag{9}$$

This relation is regarded as the law of the wall in an agitated vessel. But this equation does not coincide with that for the flat plate even if the ratio of d/D approaches to unity.

For the potential flow region the dimensionless velocity v^{++} is also correlated with the dimensionless coordinate $2r/D$. In **Fig. 2** the tangential velocity in this region is expressed approximately as,[5]

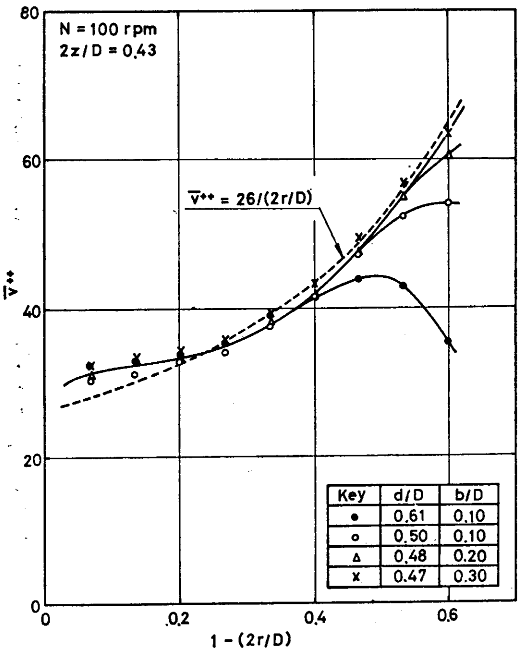

Fig. 2 Velocity profiles in potential flow region

$$v^{++} = 26/(2r/D) \doteqdot (2\sqrt{2/f} - 10\beta)/(2r/D) \tag{10}$$

On the other hand, the tangential velocity at the impeller tip radius in the fully turbulent non-baffled agitated vessel is confirmed to satisfy the following equation.

$$(v_0 - v|_{r=d/2})/\bar{u}^*_w = 10 \tag{11}$$

where v_0 is the impeller tip velocity.

Friction factor at high Reynolds number

From the law of the wall in an agitated vessel, the semi-empirical equation of friction factor at high Reynolds number is derived to express as,[6]

$$1/\sqrt{f} = 3.4 \log Re_G \sqrt{f} - 0.85 \tag{12}$$

This equation is well agreement with Eq. (5) for the range of 500 $< Re_G < 3 \cdot 10^4$ as shown in **Fig. 3**.

Analogy between j_H and f/2

From the result of Eq. (9), it is expected that the temperature distribution at the vicinity of the vessel wall is expressed as the function of y^{++}.

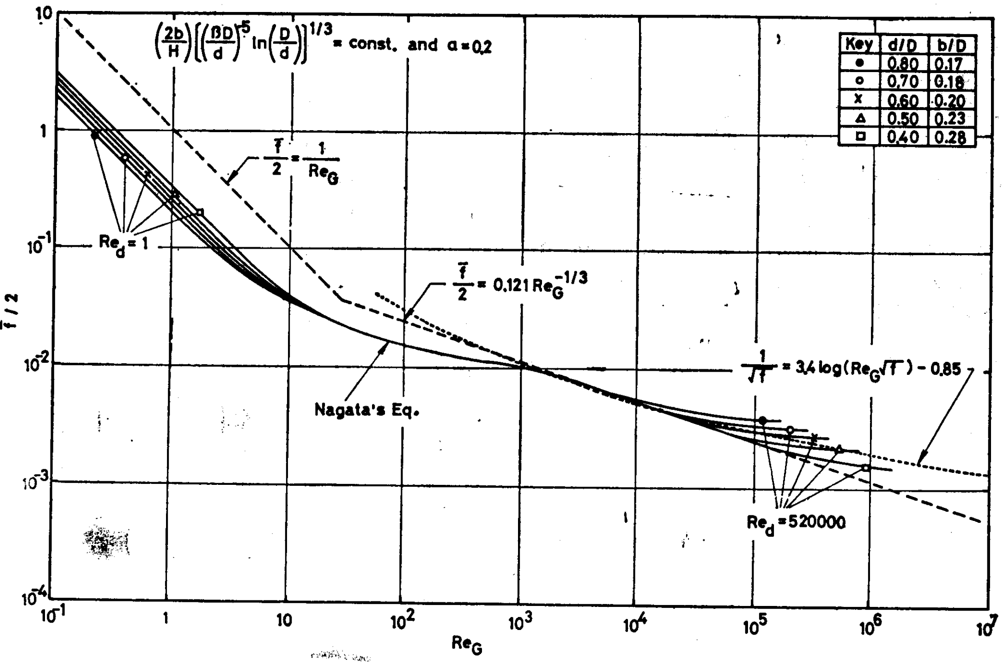

Fig. 3 Friction factor vs. modified Reynolds number

$$T^{++}=\left\{\frac{\rho c_p \bar{u}^*_w (T-T_w)}{q_w}\right\}\cdot\left(\frac{\beta D}{d}\right)=T^{++}(y^{++}) \tag{13}$$

Then assuming that the Chilton-Colburn analogy holds at the vessel wall, the velocity and temperature at the outside of the thermal boundary layer v_δ and T_δ, satisfy the following relation.

$$v_\delta^{++}=T_\delta^{++}\cdot Pr^{-2/3} \tag{14}$$

Considering that the heat transfer coefficient is defined as $h=q_w/(T_\delta-T_w)$ and that the tangential velocity at the outside of the thermal boundary layer is approximately expressed as $v_\delta \doteqdot 1.4(d/D)\cdot v_\theta$ for high Reynolds number range, the combination of Eqs. (5) and (14) gives the following analogy expression.

$$j_H\cdot\left(\frac{1.4d}{\beta D}\right)=\frac{f}{2}=0.121Re_G^{-1/3} \tag{15}$$

This relation is easily rearranged to give the ordinarily used expression.

$$\left(\frac{hD}{\lambda}\right)=\Psi\cdot\left(\frac{Nd^2\rho}{\mu}\right)^{2/3}\cdot\left(\frac{c_p\mu}{\lambda}\right)^{1/3} \tag{16}$$

where

$$\Psi=0.15\left\{\left(\frac{\beta D}{d}\right)^5\Big/ln\left(\frac{D}{d}\right)\right\}^{1/3} \tag{17}$$

The values of Ψ have been already reported by many workers, and the data are shown in Fig. 4 to compare with Eq. (17). Eq. (17) satisfies the experimental data very well.[6] Then this result leads the conclusion that the Chilton-Colburn analogy holds for the transport phenomena at the wall of agitated vessel.

For the local value of f and j_D, the analogy relationship is slso confirmed to hold experimentally.[1]

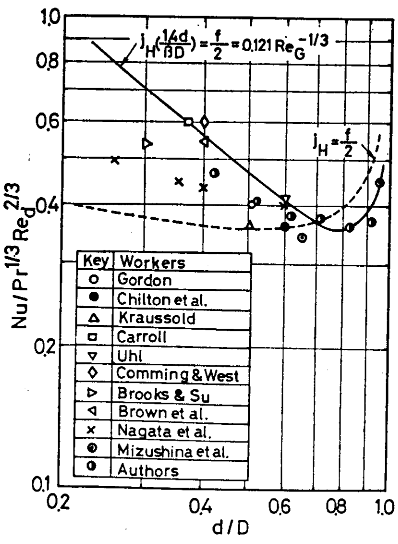

Fig. 4 Comparison of experimental results with the analogy equation

Applications

Friction factor in transition and laminar ranges of Re_G

For the transition and laminar ranges of Re_G, the foregoing simple relation in turbulent range does not hold, because the flow behaviour in these ranges depends not only on the wall condition, but also on the impeller condition. So that the friction factor in these ranges must satisfy the both conditions, i.e.,

a) wall condition

$$\frac{f}{2}=\frac{\bar{\tau}_w}{\rho v_\theta^2}=C_W\cdot\left(\frac{Lv_\theta\rho}{\mu}\right)^m \tag{18}$$

b) impeller condition

$$\frac{f}{f_{max}}=\frac{P}{P_{max}}=C_P\cdot\left(\frac{Nd^2\rho}{\mu}\right)^{m'} \tag{19}$$

where

$$P_{max}=(\pi^3/16)\cdot C_D\cdot\rho N^3 d^4 (n_p b) \tag{20}$$

On the other hand, the power input P can be connected with the shear stress at the wall.

$$P=(1+\alpha)\cdot(\pi DH)\cdot(D/2)\cdot(2\pi N).\bar{\tau}_w \tag{21}$$

Because Eq. (19) is reduced to Eq. (18) under the limiting condition of Eq. (21), the following relations must hold.

$$C_W=\left\{\frac{C_P}{4\pi C_D(1+\alpha)}\right\}\cdot\left(\frac{d}{\beta D}\right)^2\cdot\left(\frac{n_pb}{H}\right)\cdot\left\{\left(\frac{\pi}{4}\right)\cdot\left(\frac{\beta D}{d}\right)\cdot ln\left(\frac{D}{d}\right)\right\}^{-m'} \tag{22}$$

$$m=m'$$

Transition range of Re_G

In this range the flow is turbulent, so that $m=-1/3$ and C_W is a constant independent of the impeller width. Then the group of impellers, which have the same C_P value, satisfies the following relation according to Eq. (22).

$$\left(\frac{n_pb}{H}\right)\left\{\left(\frac{\beta D}{d}\right)^5\Big/ ln\left(\frac{D}{d}\right)\right\}^{-1/3}=\left(\frac{n_pb}{H}\right)\cdot\gamma=\text{const.} \tag{23}$$

This is named as the similarity condition parameter.[4]

Laminar range of Re_G

In this range, the value of m is equal to minus unity, and it is supposed that $C_W=C_W'\cdot(n_pb/H)$, then

$$\frac{f}{2}=C'_W\cdot\left(\frac{n_pb}{H}\right)\cdot\left(\frac{Lv_\theta\rho}{\mu}\right)^{-1} \tag{24}$$

or

$$N_P\cdot Re_d=\left\{\frac{2\pi^3(1+\alpha)C'_W}{C_D}\right\}\cdot\left(\frac{n_pb}{D}\right)\cdot\frac{(D/d)^2}{(D/d-d/D)} \tag{25}$$

The experimental values of $N_P\cdot Re_d$ were measured by Nagata et al. for the paddle impellers with two blades.[9] These data are plotted according to Eq. (25), as shown in **Fig. 5**. All data well satisfy the following modified equation, except $d/D=0.9$.

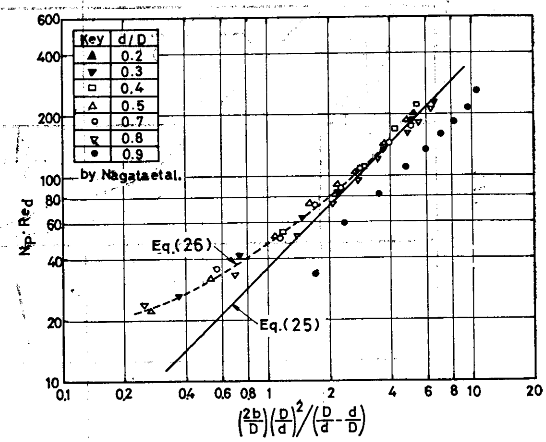

Fig. 5 Correlation of power input in laminar Reynolds number range with the new variable

$$N_P\cdot Re_d=13+34\left(\frac{n_pb}{D}\right)\cdot\frac{(D/d)^2}{(D/d-d/D)} \tag{26}$$

Impeller jet flow rate

Based on the experimental results of three dimensional velocity profiles, the following relation can be approximately derived from the angular momentum balance at the impeller tip radius.

$$\left\{\left(\frac{\pi}{2}\right)D^2H\right\}\cdot\bar{\tau}_w=\rho\left(\frac{d}{2}\right)\cdot(v_0-v|_{r=d/2})\cdot Q_d \tag{27}$$

where Q_d is the impeller jet flow rate. and the right hand side of Eq. (27) is equal to the angular momentum transferd by the impeller jet flow from the impeller tip to the potential flow region. Whereas the left hand side is equal to the torque at the side wall of vessel. Considering that $\bar{u}^*_w$ is equal to $\sqrt{\bar{\tau}_w/\rho}$, the combination of Eqs. (11) and (27) gives the following relation.

$$Q_d=(\pi/10)\cdot(D^2H/d)\cdot\bar{u}^*_w \tag{28}$$

By using the dimensionless group of N_P and N_{qd}, Eq. (28) is easily rearranged to become,

$$N_{qd}=0.1\sqrt{\left(\frac{D^2H}{d^3}\right)\cdot\frac{N_P}{1+\alpha}} \tag{29}$$

For the non-fully turbulent agitated vessel, this relation is modified by using the similarity condition parameter defined by Eq. (23).

$$N_{qd}=0.26\left\{\left(\frac{n_p b}{D}\right)\cdot\gamma\right\}^{1/3}\sqrt{\left(\frac{D^2H}{d^3}\right)\cdot\frac{N_P}{1+\alpha}} \tag{30}$$

The comparison between the calculated and experimental values is shown in **Fig. 6**. The calculated values are well agreement with the experimental ones for the various types of impeller.[8]

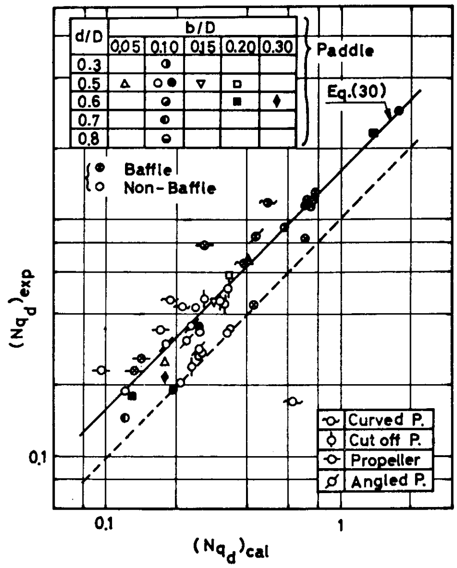

Fig. 6 Comparison between experimental and calcuated values of impeller jet flow rate for the vessel with free surface.

Conclusive Remarks

Defining the characteristic velocity, length and friction velocity respectively, which specify the performances in agitated vessel, it is implied that the flow behaviour and transport phenomena at the wall are well expressed with the simple correlation equations for the all Reynolds number ranges, laminar to turbulent. This method is useful to correlate the other performances in agitated vessels.

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