General Correlation Method of Power Consumption in Agitated Vessels

—Correction of characteristic velocity—

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By using the characteristic velocity corrected in the general correlation method, the power consumptions in agitated vessels are well correlated for the wide range of d/D as compared with the previous correlations. And when the variable $\gamma'(n_p b/D)$ for the similarity condition of impeller is larger than 0.25, the flow in non-baffled agitated vessel falls into the fully turbulent condition.

Introduction

It has been reported that analogy for transport phenomena at the wall of agitated vessel leads to the general correlation method for power input and heat transfer coefficient by introducing the modified Reynolds number defined by the characteristic velocity v_θ and length L, i.e., $(\pi/2)Nd$ and $(D/2)ln(D/d)$, respectively.[3,4] But this correlation method was limited in the range of $d/D>0.6$, due to the difference of the transport mechanisms of momentum and heat in rotational flow field.

This paper deals with the development of the correlation method into the range of $d/D<0.6$ by modifying the characteristic velocity, and discusses the applicability of this method to laminar, transition and turbulent regions.

Correction of Characteristic Velocity

Characteristic velocity is derived from the analysis of velocity and temperature distributions in laminar rotating flow in coaxial cylinders.

First, the temperature distribution in laminar flow can be expressed with the heat flux q_w at the outer wall and the diameter D of the outer cylinder.

$$T-T_w=\frac{q_w}{\lambda}\cdot\frac{D}{2}ln\left(\frac{D}{2r}\right) \tag{1}$$

where T_w is the wall temperature at the outer cylinder.

Next, the tangential velocity distribution in laminar region is also expressed with the shear stress τ_w at the outer wall and diameter D.

$$v=\frac{\tau_w}{\mu}\cdot\frac{D}{4}\left(\frac{D}{2r}-\frac{2r}{D}\right) \tag{2}$$

It is indicated that the temperature distribution is linear to the variable $(D/2)ln(D/2r)$, however the velocity is not proportional to this variable. But when the right hand side of Eq. (2) is expanded in a Taylor series, the velocity distribution is expressed as

$$v=\frac{\tau_w}{\mu}\cdot\frac{D}{2}ln\left(\frac{D}{2r}\right)\cdot\left[1+\frac{1}{3!}\left\{ln\left(\frac{D}{2r}\right)\right\}^2+\frac{1}{5!}\left\{ln\left(\frac{D}{2r}\right)\right\}^5+\cdots\right] \tag{3}$$

For the range of $ln(D/2r)<0.5$, as the Taylor series is well represented by the first term, the velocity distribution can be approximated as

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$$v=\frac{\tau_w}{\mu}\cdot\frac{D}{2}ln\left(\frac{D}{2r}\right) \quad (ln(D/2r)<0.5) \quad (4)$$

Eq. (4) gives the same function as Eq. (1), and the error by the approximation is shown in **Fig. 1.** Here it is concluded that the analogy for momentum and heat transfer holds for the range of $ln(D/2r)<0.5$. On the other hand, for the range of $ln(D/2r)>0.5$ the velocity must be modified to apply the similarity to different sizes of d/D.

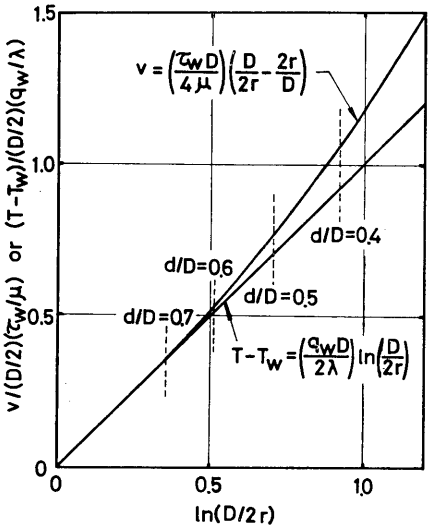

Fig. 1 Velocity and temperature distributions in laminar rotational flow in coaxial cylinders.

$$v'=\beta_r\cdot v=\frac{\tau_w}{\mu}\cdot\frac{D}{2}ln\left(\frac{D}{2r}\right) \quad (5)$$

where

$$\beta_r=2ln(D/2r)/(D/2r-2r/D) \quad (6)$$

Similarly, the characteristic velocity must be also corrected by the correction factor β.

$$v_\theta'=\beta v_\theta=\beta\cdot\frac{\pi}{2}Nd \quad (7)$$

where

$$\beta=2ln(D/d)/(D/d-d/D) \quad (8)$$

The correction factor β_r indicates the ratio of two curves in Fig. 1.

New Correlation Equations

When the characteristic velocity is corrected with Eq. (7), the new correlation equations for laminar, transition and turbulent regions are to be expressed in **Table 1**, in which a prime is added to the new variables. Table 1 shows that the new correlation equations are corrected only at the numerical con-

Table 1 Correlation equations

Already reported equations (*J. Chem. Eng. Japan*, **6**, 464 (1973))	Corrected equations
$v^\theta=(\pi/2)Nd$, $L=(D/2)ln(D/d)$ (A. 1)	$v_\theta'=(\pi/2)Nd\beta$, $L=(D/2)ln(D/d)$ (B. 1)
$f/2=\tau_w/\rho v_\theta^2$, $Re_G=Lv_\theta\rho/\mu$ (A. 2)	$f'/2=\tau_w/\rho v_\theta'^2$, $Re_G'=Lv_\theta'\rho/\mu$ (B. 2)
(i) Laminar region	
$f=C(Re_G)^{-1}$ (A. 3)	$f'=C'(Re_G)^{-1}$ (B. 3)
$C'=C/\beta$	
$N_P\cdot Re_d=2\pi^3(1+\alpha)\overline{C}\times(n_pb/D)(D/d)^2/ln(D/d)$ (A. 4)	$N_P\cdot Re_d=2\pi^3(1+\alpha)\overline{C'}\times(n_pb/D)(D/d)^2/(D/d-d/D)$ (B. 4)
$\overline{C'}=2\overline{C}/\beta$	
(ii) Transition region	
$(n_pb/D)\{(d/D)^5ln(D/d)\}^{1/3}=\text{const.}$ (A. 5)	$(n_pb/D)\{(d/D)^5ln(D/d)/\beta^5\}^{1/3}=\text{const.}$ (B. 5)
$\gamma=\{(d/D)^5ln(D/d)\}^{1/3}$ (A. 6)	$\gamma'=\{(d/D)^5ln(D/d)/\beta^5\}^{1/3}$ (B. 6)
(iii) Turbulent region	
$f=C_t(Re_G)^{-m}$ (A. 7)	$f'=C_t'(Re_G')^{-m}$ (B. 7)
$C_t'=C_t/\beta^{2-m}$	

stants by the correction factor β.

Application of Correlation Equations

(i) Laminar Region In laminar region the friction factor is inversely proportional to the modified Reynolds number as shown by Eq. (B. 3). Assumed that the proportional constant C' in Eq. (B. 3) is proportional to $(n_P b/H)$, Eq. (B. 4) for N_P is easily derived from Eq. (B. 3) and $N_P \cdot Re_d$ depends on the variable $(n_P b/D)(D/d)^2/(D/d-d/D)$ given by only impeller dimensions.

The experimental data of power input for laminar region by Nagata et al.[6] are plotted in **Fig. 2** based on Eq. (B. 4). Here number of impeller blade n_P is two. Fig. 2 shows that Eq. (B. 4) holds for the nonproximity impellers of

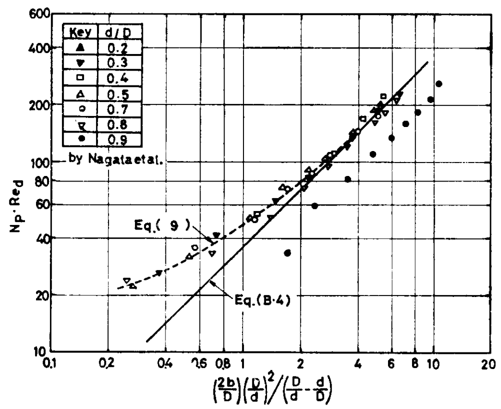

Fig. 2 Correlation of power input in laminar region for two-blade paddles.

$d/D \leqq 0.8$ and large width impellers. Whereas for the small width impellers $N_P . Re_d$ seems to be a constant value asympototically. Taking into account this fact, the experimental equation is obtained as

$$N_P \cdot Re_d = 13 + 34\left(\frac{n_P b}{D}\right)\left(\frac{D}{d}\right)^2 \Big/ \left(\frac{D}{d} - \frac{d}{D}\right) \qquad (9)$$

This equation is very well for the nonproximity impellers as shown in Fig. 2.

The experimental data for different number of impeller blade by Rushton et al.[7] and Chapman and Holland[1] are plotted in **Fig. 3** according to Eq. (B. 4), where the data include partly for the baffled agitated vessels, and for the piched blade impeller, the effective blade width of which is calculated as $b \sin\theta$. Fig. 3 shows that the power input for all impellers seem to be correlated with Eq. (9).

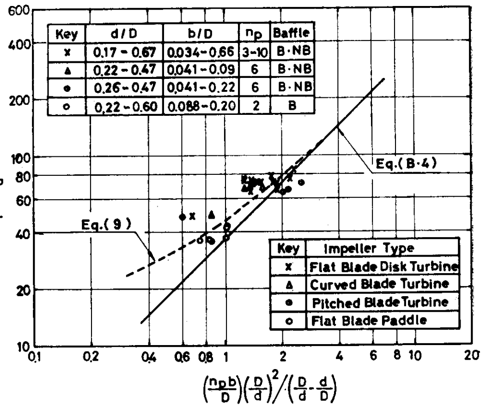

Fig. 3 Correlation of power input in laminar region for impellers with different blade number.

(ii) Transition Region Nagata et al.[6] already reported an empirical equation of power comsumption in agitated vessels with free surface, which could be applied to wide Reynolds number range and to various impeller dimensions.

The similarity condition of Eq. (B. 5) is examined by using the Nagata's equation for impellers which may scarecely affect the shape of free surface. According to the definitions, Power number and impeller Reynolds number are transformed into friction factor and modified Reynolds number respectively.

$$\left.\begin{aligned} \frac{f'}{2} &= N_P \cdot \frac{4(D/d)^3}{\pi^4 \beta^2 (1+\alpha)} \\ Re_G' &= Re_d \cdot \left(\frac{\pi}{4}\beta \cdot \frac{D}{d}\, ln \frac{D}{d}\right) \end{aligned}\right\} \qquad (10)$$

The correlation of $f'/2$ vs. Re_G' for a set of impellers satisfying Eq. (B. 5) is derived from Nagata's equation and is shown in **Fig. 4.** It is apparent that the friction factor for transition region $(30 < Re_G' < 10^3)$ are well correlated with a single curve for the wide range of d/D comparing to the previously reported results[4]. Fig. 4 proves that the correction for characteristic velocity brings a success to the correlation of power input.

(iii) Turbulent Region In Fig. 4 for turbulent region the power curves calculated from Nagata's equation for a set of impellers satisfying Eq. (B. 5) are not coincident with each other and disagree with the following equation for the fully turbulent non-baffled agitated vessel[3,4].

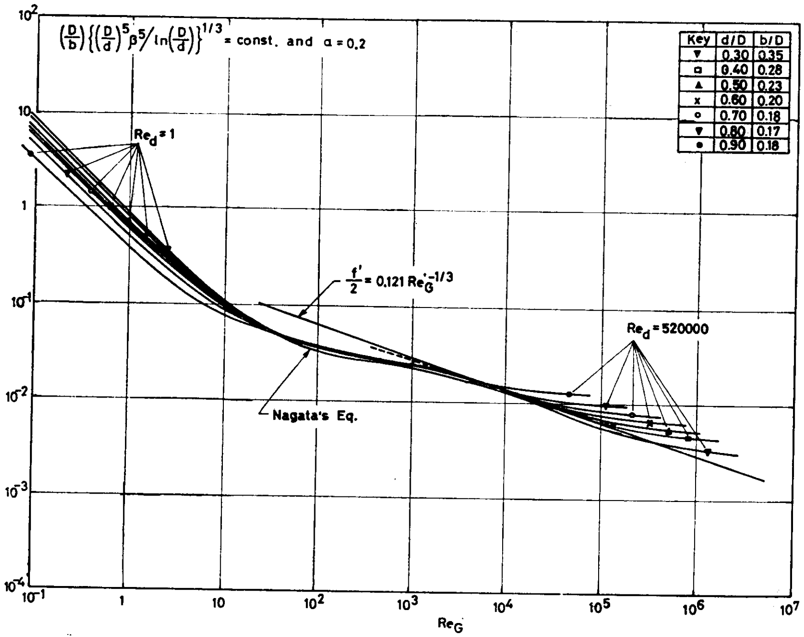

Fig. 4 Correlation of friction factor with modified Reynolds number for a set of impellers satisfying the similarity condition of Eq. (B. 5).

$$\frac{f'}{2}=0.121\ (Re_G')^{-1/3} \tag{11}$$

This means that the power input is affected by the shape of free surface or Froude number in this region. Whereas the correlation equations given in Table 1 do not take account of Froude number. So it is impossible to compare the former with the latter in turbulent region, where the shape of free surface affects on the power consumption. Then in this region the power consumption in an agitated vessel without free surface is to be measured and compared with the correlation equations in Table 1.

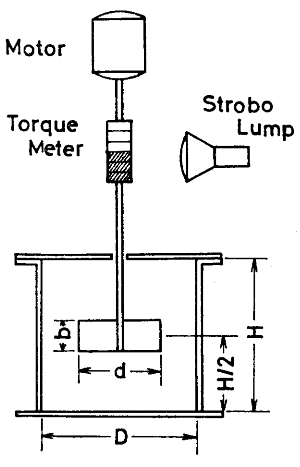

Fig. 5 Experimental apparatus to measure the power comsumption in a closed type agitated vessel.

Experimental Apparatus and Procedure

The experimental apparatus is shown schematically in **Fig. 5**. The vessel used for the almost experiments is 20cm inner diameter and 20cm depth acrylic acid resin tank, having the paddle impeller at the centre of it. The another vessel for the rest expriments is 17cm inner diameter and 17cm depth. The impellers used in this experiment are made from polyvinyl chloride resin, and these dimensions are shown at the first three columns of **Table 2**. The fluids used are water and aqueous solution of glycerol of 20, 40, 65, 70wt. per cent.

The impeller is driven by the variable speed motor, and the rotational speed is measured with the stroboscopic tachometer. The power consumption in an agitated vessel is measured with Yamazaki rotary torque meter (Model SS-1R, 500 G-cm) set on the driving shaft of impeller.

Experimental Results The experimental range and results are summarized in Table 2. And the results are correlated with Eqs. (A. 7) and (B. 7), where the value of α is assumed to be 0. 2. [2] When the characteristic velocity

Table 2 Dimensions of impeller, experimental range and results

No.	d/D	b/D	np	Re_G	C_t	C_t'	m
1	0.41	0.10	2	1100—120000	0.03	0.04	0.22
2	0.41	0.20	2	1000—110000	0.09	0.11	0.30
3	0.41	0.40	2	900—100000	0.13	0.16	0.32
4	0.40	0.10	8	900—100000	0.12	0.15	0.29
5	0.41	0.20	6	1000—90000	0.16	0.20	0.33
6	0.41	0.20	8	1000—90000	0.17	0.21	0.33
7	0.41	0.50	4	1000—95000	0.15	0.19	0.33
8	0.41	0.40	6	1000—90000	0.18	0.23	0.33
9	0.40	0.40	8	1100—90000	0.18	0.22	0.33
10	0.51	0.10	2	2100—76000	0.05	0.06	0.24
11	0.51	0.15	2	1300—76000	0.09	0.10	0.28
12	0.51	0.20	2	1100—70000	0.13	0.15	0.31
13	0.51	0.10	4	1200—70000	0.10	0.11	0.28
14	0.50	0.30	2	1000—68000	0.17	0.20	0.33
15	0.51	0.15	4	800—56000	0.17	0.19	0.31
16	0.51	0.10	6	1300—66000	0.15	0.17	0.31
17	0.51	0.40	2	750—66000	0.19	0.21	0.33
18	0.51	0.20	4	600—60000	0.19	0.22	0.33
19	0.51	0.10	8	600—55000	0.19	0.22	0.33
20	0.50	0.15	6	650—55000	0.20	0.23	0.33
21	0.50	0.50	2	600—55000	0.19	0.22	0.33
22	0.50	0.10	10	650—58000	0.19	0.22	0.33
23	0.51	0.30	4	550—60000	0.21	0.23	0.33
24	0.50	0.20	6	550—60000	0.22	0.24	0.33
25	0.50	0.15	8	600—60000	0.21	0.24	0.33
26	0.61	0.10	2	650—48000	0.05	0.05	0.22
27	0.60	0.20	2	650—45000	0.14	0.15	0.30
28	0.60	0.40	2	650—45000	0.20	0.21	0.33
29	0.60	0.10	8	800—42000	0.21	0.23	0.33
30	0.60	0.20	6	700—40009	0.23	0.25	0.33
31	0.60	0.41	4	700—40000	0.22	0.23	0.33
32	0.72	0.12	2	250—34000	0.07	0.07	0.22
33	0.71	0.24	2	200—30000	0.17	0.17	0.31
34	0.71	0.47	2	160—30000	0.23	0.23	0.33
35	0.71	0.47	4	190—28000	0.25	0.26	0.33
36	0.71	0.12	6	190—28000	0.25	0.25	0.33
37	0.71	0.24	6	190—?8000	0.29	0.29	0.33
38	0.71	0.12	8	190—28000	0.27	0.28	0.33

expressed by Eq. (A.1) without correction is used, the coefficient C_t in Eq. (A.7) depends

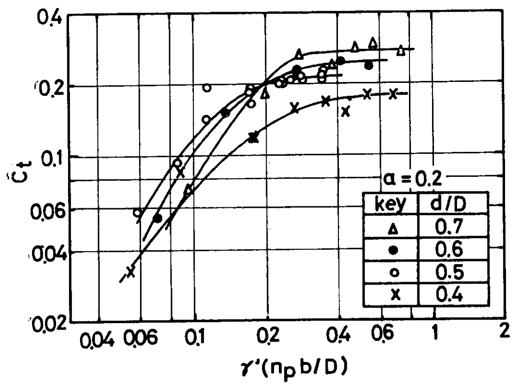

Fig. 6 Correlation of the coefficient in Eq. (A.7) with the similarity condition for the paddles with different d/D.

on the impeller diameter d/D even for the set of impellers satisfying the similarity condition, as shown in **Fig. 6**. On the other hand when the characteristic velocity corrected by Eq. (B.1) is used, the coefficient C_t' in Eq. (B.7) seems to be correlated with a single curve shown in **Fig. 7**. Besides for the abscisa defined by Eq. (B.5) being larger than 0.25, the coefficient C_t' and the power of Reynolds number m seem to be constant values of 0.25 and 0.33 respectively. These asympototic values are well coincident with that in Eq. (11).

The restlts in Figs. 6 and 7 are given by the measured values of total torque which are the sum of the torque at the cylindrical wall and that at the upper and lower bottom walls of the vessel. Whereas the correlation equations in Table 1 are derived from the analysis

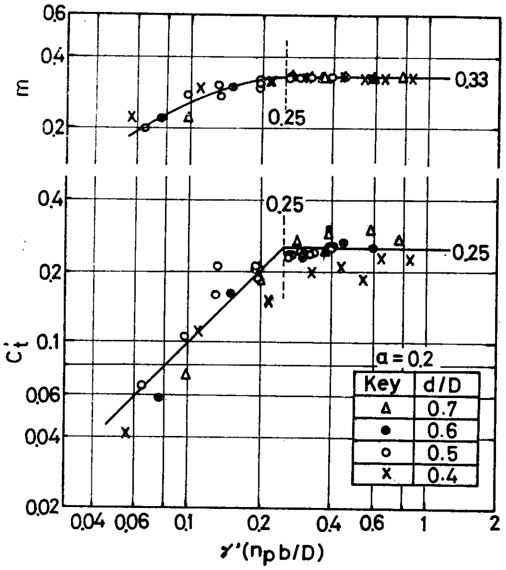

Fig. 7 Correlation of the coefficient and power consant in Eq. (B. 7) with the similarity condition for paddles with different d/D.

of transport phenomena at the side wall, so it is better to measure the torque at the side wall directly. The apparatus and procedure to measure the torque at the side wall of vessel have been already reported[5]. The results obtained by this apparatus for the various paddle impellers of $n_p=2$ are plotted in **Fig. 8**, where the coefficient C_t' is directly obtained without the

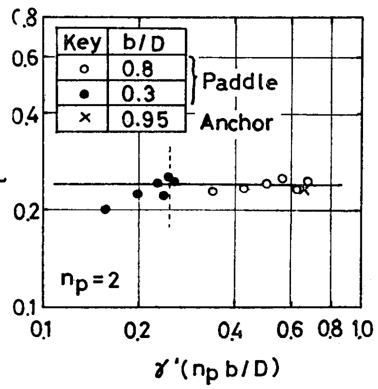

Fig. 8 Correlation of the coefficient in Eq. (B. 7) directly obtained from the side wall torque with the similarity condition for two-blade paddles.

assumption to α value. The results in Fig. 8 are coincident with that in Fig. 7 and Eq. (11).

Conclusive Remark

Correcting the characteristic velocity in the general correlation method, the power consumption in an agitated vessel can be well correlated for the wide range of d/D as compared with the previously proposed correlations. And when the variable $\gamma'(n_p b/D)$ for the similarity condition of impeller is larger than 0.25, the flow in non-baffled agitated vessel falls into the fully turbulent condition.

Nomenclature

b = impeller blade width	[cm]
C = constant in Eqs. (A. 3) and (B. 3)	[—]
C_t = constant in Eqs. (A. 7) and (B. 7)	[—]
D = vessel diameter	[cm]
d = impeller diameter	[cm]
f = friction factor	[—]
H = vessel height	[cm]
L = characteristic length $(=(D/2)ln(D/d))$	[cm]
m = power constant in Eqs. (A. 7) and (B. 7)	[—]
N = rotational speed	[sec^{-1}]
n_p = number of impeller blade	[—]
N_p = Power number	[—]
q = heat flux	[cal/cm^2sec]
Re_d = impeller Reynolds number $(=Nd^2\rho/\mu)$	[—]
Re_G = modified Reynolds number $(=v_\theta L\rho/\mu)$	[—]
r = radius	[cm]
T = temperature	[°C]
v = tangential velocity	[cm/sec]
v_θ = characteristic velocity $(=(\pi/2)Nd)$	[cm/sec]
α = ratio of torque at bottom wall to that at side	[—]
β = correction factor defined by Eq. (8)	[—]
γ = variable defined by Eq. (A. 6)	[—]
λ = thermal conductivity	[cal/cm sec°C]
μ = viscosity	[g/cm sec]
ρ = density	[g/cm^3]
τ = shear stress	[$g/cm\ sec^2$]

supper-and sub-script

$'$ = corrected by β

$_w$ = at vessel wall

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