An Information System Model with Two Processing Units

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This paper represents a new information system model as the development of [1]. The system model consists of Processing module with two processing untis, Memory module, Control module and Passive module.

First we give some assumptions and definitions of the system that can be a model of multi-processing systems.

Second we analyze the behavior of the system theoretically. Specially the transitions between memory module and control module or passive module are investigated.

1. Introduction

In recent years, much effort has gone into the developments of information systems. Specialy T.C. Lowe [1] had proposed a general model of information systems consisting of control units and passive units. In [1], partitioned systems are defined formally and illustrated. The fundamental results are expressions for the expected number of boundary crossing. Problems of synthesizing partitions in order to minimize the expected number of boundary crossings are also discussed.

In this paper, we add processing module having two processing units and memory module to the system model of [1]. The model of this paper may be useful in both analysis and design of such organizations that have two processing parts. For example, computer systems having two processing units can be considered as one of such organizations.

Following this introduction, a new information system model is defined formally. Then we analyze the behavior and movements of the system theoretically. Specially the transitions between memory module and control module or passive module are investigated.

2. Definitions and Explanations of an Information System Model

In this section, we give some definiticns of a new information system model presented here.

[**Definition. 1**] An Information System Model (***ISM***) consists of ten factors ($\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{H}$, $\boldsymbol{E}$, $\boldsymbol{E}'$, $\boldsymbol{P}$, $\boldsymbol{P}'$, $\boldsymbol{Q}$) as follows,

$\boldsymbol{A}$: *Processing module* having two *processing units* a_1 and a_2. $\boldsymbol{A}=(a_1, a_2)$

$\boldsymbol{B}$: *Memory module*

$\boldsymbol{C}$: *Control module* having n *control* units, $\boldsymbol{C}=(c_i;\ i=1, 2, \cdots\cdots, n)$

$\boldsymbol{D}$: *Passive module* having m *passive units*, $\boldsymbol{D}=(d_u;\ u=1, 2, \cdots\cdots, m)$

$\boldsymbol{G}$: Vector of control-unit volumes in which g_i is the value of the volume measure of c_i; $\boldsymbol{G}=[g_i;\ i=1, 2, \cdots\cdots, n]$

$\boldsymbol{H}$: Vector of passive-unit volumes in which h_u is the value of the volume measure of d_u, $\boldsymbol{H}=[h_u;\ u=1, 2, \cdots\cdots, m]$

$\boldsymbol{E}$: Vector of probabilities that control-unit enters a_1 first. $\boldsymbol{E}=[e_i;\ i=1, 2, \cdots\cdots, n]$

$\boldsymbol{E}'$: Vector of probabilities that control-unit enters a_2 first. $\boldsymbol{E}'=[e_i';\ i=1, 2, \cdots\cdots, n]$

$\boldsymbol{P}$: Matrix of control-unit transition probabilities to a_1. $\boldsymbol{P}=[p_{ij};\ i=1, 2, \cdots\cdots, n;\ j=1, 2, \cdots\cdots, n]$

$\boldsymbol{P}'$: Matrix of control-unit transition probabilities to a_2. $P'=[p'_{ij};\ i=1, 2, \cdots\cdots, n;\ j=1, 2, \cdots\cdots, n]$

$\boldsymbol{Q}$: Matrix of passive-unit reference in which q_{iu} is the zero-one variable indicating a relationship between c_i and d_u.

$\boldsymbol{Q}=[q_{iu};\ i=1,\ 2,\cdots\cdots,\ n;\ u=1,\ 2,\cdots\cdots,\ m]$

To make the image of the system more clear, a figure of the Information System Model can be depicted as in Fig. 1.

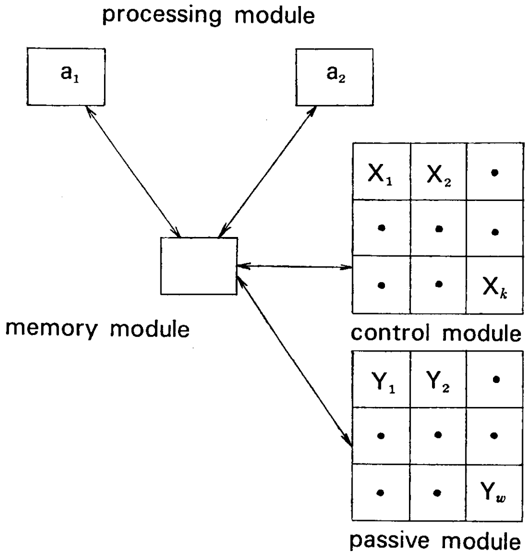

Fig. 1 Illustration for an information system model

Next we define a partition of the control module C and the passive module D in what follows,

[**Definition.** 2] If each subset $X_i(X_i \neq \phi)$ of control-module C satisfies the next Eq. (1),

$$\bigcup_{i=1}^{k} X_i = C \quad \text{and} \quad X_i \cap X_j = \phi \ \text{for any } i \neq j \tag{1}$$

then a set $\{X_i \mid i=1,\cdots\cdots,k\}$ is called a partition of C. Each X_i is also called a group of the partition. Similarly, a group $Y_i(i=1,\cdots\cdots,w)$ of passive module can be defined.

To represent each group of a given partition on C, we use the following zero-one matrix R in Eq. (2)

$$\boldsymbol{R}= \begin{array}{c} \\ c_i \end{array} \begin{array}{c} c_j \\ \left| \begin{array}{c} \vdots \\ \cdots r_{ij} \\ \\ \end{array} \right| \end{array} \tag{2}$$

where,

$$r_{ij}=\begin{cases} 1; \text{ If } c_i \text{ and } c_j \text{ belong to a same group.} \\ 0; \text{ otherwise} \end{cases}$$

As the same to Eq. (2), we define S for groups of passive module as follows,

$$\boldsymbol{S}= \begin{array}{c} \\ d_u \end{array} \begin{array}{c} d_v \\ \left| \begin{array}{c} \vdots \\ \cdots\cdots s_{uv} \\ \\ \end{array} \right| \end{array} \tag{3}$$

where,

$$s_{uv}=\begin{cases} 1\ ; \text{ If } d_u \text{ and } d_v \text{ belong to a same group.} \\ 0\ ; \text{ otherwise} \end{cases}$$

The Explanation of the ISM

Control units enter processing units through the memory module. If control units enter processing units, the passive units related with those control units must enter the processing units. Here we assume that control units and passive units are transferred as group between memory module and control or passive module. We also assume that memory module can store at most two groups of control units. Once a group is stored in one of memory module, it can be kept until a new group is transferred on the one. And control units and passive units are transferred as individual between processing module and memory module. More than one control unit cannot enter a_1 or a_2.

Now assume that there exists no control unit in a_1 at time t_0 and a control unit c_{i1} enters the a_1 at the next time t_1. At some later time control unit c_{i1} goes out from a_1 and immediately c_{i2} enters a_1. This continues for a finite time, until at time t_{f+1}, control unit c_{if} goes out of a_1 and no other unit enters from t_{f+1}. The time interval from t_1 to t_f is called an *a_1-busy period.* As the same, we can define an *a_2-busy period.*

If, for an a_1-busy period, a control unit c_i is the first control unit to enter a_1, then the c_i is called as a first entrance unit of a_1. The probability that c_i is the first entrance unit of a_1, is represented by e_i, which is an element of vector E. where $\sum_{i=1}^{n} e_i=1$. Similarly the probability of first entrance to a_2 expressed by a vector E'. During an a_1-busy period (or, a_2-busy period), suppose that control unit $c_i(c_s)$ enters a_1 at some time, the probability that $c_j(c_k)$ enters a_1 at the next time is $p_{ij}(p'_{sk})$; an element of the probability transition matrix $P(P')$. It is required on the matrix P or P' that $\lim_{k\to\infty} p^k=0$, $\lim_{k\to\infty} p'^k=0$ and that $P$and P' not be a function of time. Whenever a control unit enters a_1 or a_2, it references some passive units. Suppose that control unit c_i enters a_1 or a_2 at time t_k, the passive units referred at that time are those d_u for which $q_{iu}=1$, where q_{iu} is an element of the zere-one passive-unit reference matrix Q.

3. Trasfers among Different Groups of *C* and *D*

In this section, we define Transfers among different groups of C and D (for simplicity, written as **TDG** of C and D) to study the movement of the system.

Assumption

Before defining **TDG**, we assume that any a_1-busy period and any a_2-busy period start and finish at the same time. Moreover it is assumed that each control unit of a_1 and a_2 are transferred at same interval.

For convenience' sake, we define the following four functions $\triangle(t)$, $\triangle'(t)$, $\sigma(c_i)$ and $\sigma'(c_i)$. The former two functions $\triangle(t)$ and $\triangle'(t)$ describe the history of control flow in a system. The latter two functions $\sigma(c_i)$ and $\sigma'(c_i)$ are concerned with describing one aspect of a system, indicating Y_w containing passive unit d_u referred by a control unit c_i.

Suppose that during an a_1-busy period and a_2-busy period, the sequence of control units that enter a_1 and a_2 are respectively ; c_{i1}, c_{i2}, ……, c_{it}……, c_{if} and c_{j1}, c_{j2}, ……, c_{jt}……, c_{jm}. Then the functions $\triangle(t)$ and $\triangle'(t)$ for the sequences of control units are shown in Eq. (4).

$$\begin{aligned}&\triangle(t)=c_{it} \text{ for } t=1,\ 2,\ \cdots\cdots,\ f\ ;\ a_1\text{- processing unit}\\ &\triangle'(t)=c_{jt} \text{ for } t=1,\ 2\cdots\cdots,\ m\ ;\ a_2\text{- processing unit}\end{aligned} \tag{4}$$

If a control unit c_i is transferred to the set of some groups Y_w containing d_u referenced by the c_i. Then the set can be obtained as Eq. (5). [See the reference [1]]

$$\begin{aligned}\sigma(c_i)&=\{Y_w\,|\,{}^{\exists}d_u(q_{iu}=1\wedge d_u\in Y_w) \text{ for } c_i\in C\\ \sigma'(c_i)&=\{Y_w\,|\,{}^{\exists}d_u(q_{iu}=1\wedge d_u\in Y_w) \text{ for } c_i\in C\end{aligned} \tag{5}$$

Definition of T D G for the Control Module C

If a partition of control module is given, a number $D(t)$ of transfers among different groups at the time t, for a busy period, can be defined as follows ; At the time $t=1$, if a group contains both the entrance unit of a_1 and that of a_2, let $D(t)=1$. In another case, let $D(t)=2$. At the time $t=2,\ 3,\cdots\cdots$, f, $D(t)$ is the number of the groups entering the memory module at the time t which were not stored in it at the previous time. $(t-1)$.

Then we can formulate $D(t)$ by Eq. (6) and (7) as follows,

$$\underset{t=1}{D(1)}=\begin{cases}1\ ;\ \text{If there exists a group } X_k \text{ which satisfies } \triangle(1)\in X_k \text{and } \triangle'(1)\in X_k' & (6)\\ 2\ ;\ \text{elsewhere} & (6)'\end{cases}$$

$$\underset{t=2,\cdots,f}{D(t)}=\begin{cases}2\ ;\ \text{If there exist } X_k,\ X_k',\ X_k'',\ X_k''' \text{ which satisfy } \triangle(t-1)\in X_k\wedge \\ \quad \triangle'(t-1)\in X_k'\wedge\triangle(t)\in X_k''\wedge\triangle'(t)\in X_k'''\wedge(X_k\neq X_k''\neq X_k''')\wedge & (7)\\ \quad (X_k'\neq X_k''\neq X_k''') \\ 1\ ;\ \text{If there exist } X_k,\ X_k',\ X_k'',\ X_k''' \text{ which satisfy } \triangle(t-1)\in X_k\wedge \\ \quad \triangle'(t-1)\in X_k'\wedge\triangle(t)\in X_k''\wedge\triangle'(t)\in X_k'''\wedge(X_k\neq X_k'')\wedge(X_k'\neq X_k'')\wedge \\ \quad (X_k''=X_k''')\vee(X_k'=X_k'')\wedge(X_k\neq X_k''\neq X_k''')\vee(X_k\neq X_k''\neq X_k''')\wedge & (7)' \\ \quad (X_k'=X_k''')\vee(X_k'\neq X_k''\neq X_k''')\wedge(X_k=X_k'')\vee(X_k'\neq X_k''\neq X_k''')\wedge \\ \quad (X_k=X_k''') \\ 0\ ;\ \text{elsewhere} & (7)''\end{cases}$$

Definition of TDG for the passive Module D

If a partition of passive module is given, a number of **TDG** of passive module at the time can be defined, as follows; At the time $t=1$, $D(1)$ is number of groups of passive unit referred by the first control unit of a_1 or a_2. At the time $t=2,\ 3,\cdots$, f, $D(t)$ is number of groups of passive unit referred at the time t which were not referred by the control unit at the previous time $(t-1)$. Then we can obtain the following equation.

$$D(1)=|\sigma(\triangle(1))|+|\sigma'(\triangle'(1))\cap\overline{\sigma(\triangle(1))}|$$
$$\begin{aligned}D(t)=&|\sigma(\triangle(t))\cap\overline{\sigma(\triangle(t-1))}\cap\overline{\sigma'(\triangle'(t-1))}|\\&+|\sigma'(\triangle'(t))\cap\overline{\sigma'(\triangle'(t-1))}\cap\\&\overline{\sigma(\triangle(t))}\cap\overline{\sigma(\triangle(t-1))}|\end{aligned} \tag{8}^*$$
$t=2, \cdots,$ f

4. Control Unit-activity

In this section the fundamental parameters used to describe the movement of control units are discussed.

Flow of Control

It is assumed that the behavior of control units is described by an absorbing Markov chain. It is necessary to analyze control flow for studying the movement of the system. Many studies have been done in [1]. We shall cite these results in what follows which are necessary for our new model.

4.1 Preparations

Since both of P and P′ describe the transient states of absorbing Markov chain, it holds for P and P′.

$$\lim_{k\to\infty} P^k=0 \tag{9}$$

and

$$F=\sum_{k=0}^{\infty} P^k \tag{10}$$

is bounded. Each f_{ij} expresses the expected number of times that c_j enters a_1 during an a_1-busy period, if control unit c_i is assumed to be the first entrance unit of a_1. Then

$$\gamma_j=\sum_{i=1}^{n} e_i f_{ij} \tag{11}$$

is the number of times that c_j is expected to enter a_1. Thus the vector $\Gamma=[\gamma_1, \gamma_2, \cdots, \gamma_n]$ gives the expected number of times that each control unit will enter a_1 during an a_1-busy period. It is also shown in [1] that

$$\Gamma=E(I-P)^{-1} \tag{12}$$

Since the total expected number of times that c_i enters a_1 is γ_i, the expected number of control transfers c_i to c_j is given as

$$\tau_{ij}=\gamma_i p_{ij} \tag{13}$$

4.2 Definition of the Joint Transition

Let the probability that c_i (or, c_s) is the first entrance unit of a_1 (a_2) is e_i (e_j'). Then the probability of the first joint entrance that c_i is the first entrance unit of a_1 and c_s is the first entrance unit of a_2, can be obtained as $e_i\times e_j'$. Then the vector Π is given as follows

$$\Pi=[\pi_{11'}, \pi_{12'}, \cdots, \pi_{1n'}, \cdots \pi_{nn'}] \tag{14}$$

Assume that c_i and c_s enter a_1 and a_2 respectively, the probability that c_j (or, c_k) enters a_1 (or, a_2) at the next time is p_{ij} (or, p'_{sk}) The probability of joint transition $l_{is', jk'}$ that c_i is in a_1 and c_s is in a_2 and the next time c_j enters a_1 and c_k enters a_2, is defined as $l_{is', jk'}=p_{ij}\times p'_{sk}$, Then the matrix $\boldsymbol{L}$ can be given as follows,

$$\boldsymbol{L}=\begin{vmatrix} l_{11', 11'}\cdots l_{11', 1n'}\cdots l_{1n', 1n} \\ \vdots \qquad\qquad \vdots \\ l_{11', n1'}\cdots l_{11', nn'} \qquad \vdots \\ \vdots \\ l_{n1', n1'}\cdots\cdots\cdots\cdots l_{nn', nn'} \end{vmatrix} \tag{15}$$

As the prpbability of transition to a_1 and a_2 is Markov chain, the joint transition is Markov chain.

$$\lim_{k\to\infty} \boldsymbol{L}^k=0 \tag{16}$$

and

$$\boldsymbol{T}=\sum_{k=0}^{\infty} \boldsymbol{L}^k \tag{17}$$

is bounded. A $t_{is', jk'}$ is the expected number of times that c_j enters a_1 and c_k enters a_2 at the same time, if $c_i(c_s)$ are the first joint entrance units of $a_1(a_2)$. As the probability that c_i and c_s are the first joint entrance units is π_{is}, the expected number of times that c_j enters a_1 and c_k enters a_2 at the same time during a busy period, is given in Eq. (18)

$$j_{jk'}=\sum_{i=1}^{n}\sum_{s=1}^{n} \pi_{is'}\cdot t_{is', jk'} \tag{18}$$

A vector $\boldsymbol{J}$ is given as follows,

$$\boldsymbol{J}=[j_{11'}, j_{12'}, \cdots, j_{1n'}, \cdots, j_{nn'}] \tag{19}$$

The vector $\boldsymbol{J}$ expresses the expected number of times that each pair of control units enter processing module at the same time.

Here we define the joint control transfers that $c_i(c_s)$ enters $a_1(a_2)$ and at the next time $c_j(c_k)$ enters $a_1(a_2)$. Then the expected number of joint control transfers can be obtained,

$$m_{is', jk'}=j_{is'}\cdot l_{is', jk'} \tag{20}$$

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*where, |S| denotes the number of elements of S.

5. Total Expected Number of Transfers Between Different Groups

In this section we will examine the total expected number of *TDG*, based on the results obtained the preceding section.

Now assume that a partitions of control module C and passive module D are given. Then we can express the total number of $TDG(C)$ in Eq. (21).

$$\mathbf{D}(1)+\sum_{t=2}^{f}\mathbf{D}(\mathbf{t}) \tag{21}$$

The expected number T of $TDG(C)$ can be given by summing all possible events.

$$T=\Pr[(6)]+2\times\Pr[(6')]+2\times\sum_{t=2}^{f}\Pr[(7)]+\sum_{t=2}^{f}\Pr[(7')] \tag{22}$$

where, the Pr [(6)] (or, Pr [(6')]) is the expected number of times that some c_i and c_s are first joint entrance units under the condition that c_i and c_s are included in same (different) group.

Therefore, based on Eq. (2) and Eq. (14), it holds that

$$\Pr[(6)]=\sum_{i=1}^{n}\sum_{s=1}^{n}r_{is}\,\pi_{is}' \tag{23}$$

and

$$\Pr[(6')]=\sum_{i=1}^{n}\sum_{s=1}^{n}(1-r_{is})\pi_{is}' \tag{24}$$

The Pr[(7)] is the expected number of times that $c_i(c_s)$ enter $a_1(a_2)$ and at the next time $c_j(c_k)$ enter $a_1(a_2)$ under the condition that two groups entering the memory module are different from those of previous time. From Eq. (2) and Eq. (20), it can be get as follows,

$$\sum_{t=2}^{f}P_r[(7)]=\sum_{i=1}^{n}\sum_{s=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}m_{is',jk'}(1-r_{ij})\cdot(1-r_{ik})(1-r_{jk})(1-r_{sj})(1-r_{sk}) \tag{25*}$$

Simiarly,

$$\begin{aligned}\sum_{t=2}^{f}Pr[(7')]=\sum_{i=1}^{n}\sum_{s=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}m_{is',jk'}\{&(1-r_{ij})(1-r_{sj})r_{jk}\\
&+(1-r_{ij})(1-r_{ik})(1-r_{jk})(r_{sj}+r_{sk}-r_{sj}r_{sk})\\
&+(1-r_{sj})(1-r_{sk})(1-r_{jk})(r_{ij}+r_{ik}-r_{ij}r_{ik})\\
&-(1-r_{ij})(1-r_{sj})(1-r_{ik})(1-r_{jk})(r_{jk}r_{sj}+r_{jk}r_{sk}-r_{jk}r_{sj}r_{sk})\\
&-(1-r_{ij})(1-r_{sj})(1-r_{sk})(1-r_{jk})\\
&\cdot(r_{jk}r_{ij}+r_{jk}r_{ik}-r_{jk}r_{ij}r_{ik})-(1-r_{ij})(1-r_{ik})\\
&\cdot(1-r_{jk})(1-r_{sj})(1-r_{sk})(r_{sj}r_{ij}+r_{sj}r_{ik}+r_{sk}r_{ij}+r_{sk}r_{ik}-r_{jk}r_{sj}r_{ij}\\
&-r_{jk}r_{sj}r_{ik}-r_{jk}r_{sk}r_{ij}-r_{jk}r_{sk}r_{ik}-r_{sj}r_{sk}r_{ij}-r_{sj}r_{sk}r_{ik}-r_{sj}r_{ij}r_{ik}\\
&-r_{sk}r_{ij}r_{ik}+r_{jk}r_{sj}r_{sk}r_{ij}+r_{jk}r_{sj}r_{sk}r_{ik}+r_{jk}r_{sk}r_{ij}r_{ik}\\
&+r_{sj}r_{sk}r_{ij}r_{ik}-r_{jk}r_{sj}r_{sk}r_{ij}r_{ik})\}\end{aligned} \tag{26}$$

Combinating above four equations yields

$$\begin{aligned}T=\sum_{i=1}^{n}\sum_{s=1}^{n}\pi_{is}'(2-r_{is})+\sum_{i=1}^{n}\sum_{s=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}m_{is',jk'}\{&(1-r_{ij})\cdot(1-r_{sj})r_{jk}\\
&+(1-r_{ij})(1-r_{ik})(1-r_{jk})(r_{sj}+r_{sk}-r_{sj}r_{sk})\\
&+(1-r_{sj})(1-r_{sk})(1-r_{jk})(r_{ij}+r_{ik}-r_{ij}r_{ik})\\
&-(1-r_{ij})(1-r_{sj})(1-r_{jk})(1-r_{ik})(r_{jk}r_{sj}+r_{jk}r_{sk}-r_{jk}r_{sj}r_{sk})\\
&-(1-r_{ij})(1-r_{sj})(1-r_{jk})(1-_{sk})(r_{jk}r_{ij}+r_{jk}r_{ik}-r_{jk}r_{ij}r_{ik})\\
&-(1-r_{ij})(1-r_{ik})(1-r_{jk})(1-r_{sj})(1-r_{sk})(r_{sj}r_{ij}+r_{sj}r_{ik}+r_{sk}r_{ij}\\
&+r_{sk}r_{ik}-r_{jk}r_{sj}r_{ij}-r_{jk}r_{sj}r_{ik}-r_{jk}r_{sk}r_{ij}-r_{jk}r_{sk}r_{ik}-r_{sj}r_{sk}r_{ij}\\
&-r_{sj}r_{sk}r_{ik}-r_{sj}r_{ij}r_{ik}-r_{sk}r_{ij}r_{ik}+r_{jk}r_{sj}r_{sk}r_{ij}+r_{jk}r_{sj}r_{sk}r_{ik}\\
&+r_{jk}r_{sk}r_{ij}r_{ik}+r_{sj}r_{sk}r_{ij}r_{ik}-r_{jk}r_{sj}r_{sk}r_{ij}r_{ik}-2)\}.\end{aligned} \tag{27}$$

Total Expected Number of Transfers Between different groups of D

We can get total number of the $TDG(D)$ in Eq. (28), by using Eq. (12), (13).

*Proof is omitted here.

$$|\sigma(\triangle(1))|+|\sigma(\triangle'(1))\cap\overline{\sigma(\triangle(1))}|+\sum_{t=2}^{f}|\sigma(\triangle(t))\cap\overline{\sigma(\triangle(t-1))}\cap\overline{\sigma(\triangle'(t-1))}|$$
$$+\sum_{t=2}^{f}|\sigma(\triangle'(t))\cap\overline{\sigma(\triangle'(t-1))}\cap\sigma(\triangle(t))\cap\overline{\sigma(\triangle(t-1))}| \tag{28}$$

The expected number T' of $TDG(D)$ can be determined by including the probability of events leading to $TDG(D)$ of Eq. (28) and summing over all possible events;

$$T'=\sum_{j=1}^{n}P_r[\triangle(1)=C_j]|\sigma(C_j)|+\sum_{j=1}^{n}\sum_{k=1}^{n}P_r[\triangle(1)=C_j\wedge\triangle'(1)=C_k]|\sigma'(C_k)\cap\overline{\sigma(C_j)}|$$
$$+\sum_{t=2}^{f}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{n}P_r[\triangle(t)=C_j\wedge\triangle(t-1)=C_i\wedge\triangle'(t-1)=C_s]|\sigma(C_j)\cap\overline{\sigma(C_i)}\cap\overline{\sigma'(C_s)}|$$
$$+\sum_{t=2}^{f}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{n}\sum_{k=1}^{n}P_r[\triangle'(t)=C_k\wedge\triangle(t)=C_j\wedge\triangle'(t-1)=C_s\wedge\triangle(t-1)=C_i]|\sigma'(C_k)$$
$$\cap\overline{\sigma(C_j)}\cap\overline{\sigma'(C_s)}\cap\overline{\sigma(C_i)}| \tag{29}$$

As a c_j enters a_1 or a_2, it references some groups of passive units. Then the number of passive units which is referenced is

$$\sum_{u=1}^{m}q_{ju} \tag{30}$$

Now we consider next function to determine $|\sigma(C_j)|$

$$\left[1-\min\left(1,\ \sum_{v=1}^{u-1}q_{ju}S_{uv}\right)\right] \tag{31}$$

The Eq. (31) has zero if there exist a Y_w and d_v such that both d_u and d_v are elements of Y_w and are referenced by C_j, and v<u; the value is one otherwise. If Eq. (29) is multiplied by q_{ju} and summed over $u=1, 2, \cdots$, m, one term of unity is added to the sum for each Y_w containing at least one element referenced by c_j. The term added corresponds to the d_u possesing the smallest subscript u, which d_u is an element of Y_w. Thus, each Y_w containing any d_u referenced by c_j is counted once, and

$$|\sigma(c_j)|=\sum_{u=1}^{m}q_{ju}\left[1-\min\left(1,\ \sum_{v=1}^{u-1}q_{ju}S_{uv}\right)\right] \tag{32}$$

The second term in Eq. (29) is evaluated by first noting that

$$|\sigma(c_j)\cap\overline{\sigma(c_i)}|=|\sigma(c_j)|-|\sigma(c_j)\cap\sigma(c_i)| \tag{33}$$

In order to obtain $|\sigma(c_j)\cap\sigma(c_i)|$, the Eq. (32) is modified to include a factor that is unity whenever the d_u contributing to the sum in Eq. (32) is an element of and zero otherwise:

$$|\sigma(c_j)\cap\sigma(c_i)|=\sum_{u=1}^{m}q_{ju}\left[1-\min\left(1,\ \sum_{v=1}^{u-1}q_{ju}S_{uv}\right)\right]\left[\min\left(1,\ \sum_{v=1}^{u}q_{iv}S_{uv}\right)\right] \tag{34}$$

Combinating the above three equations yields

$$|\sigma(c_j)\cap\sigma(c_i)|=\sum_{u=1}^{m}q_{ju}\left[1-\min\left(1,\ \sum_{v=1}^{n-1}q_{ju}S_{uv}\right)\right]\left[1-\min\left(1,\ \sum_{v=1}^{m}q_{iv}S_{uv}\right)\right] \tag{35}$$

Furthermore, to get $|\sigma(c_j)\cap\overline{\sigma(c_i)}\cap\sigma(c_s)|$, the Eq. (35) is modified to include a factor that is unity whenever the d_u contributing to the sum in Eq. (35) is an element of $\sigma(c_s)$, and zero otherwise:

$$|\sigma(c_j)\cap\overline{\sigma(c_i)}\cap\sigma(c_s)|=\sum_{u=1}^{m}q_{ju}\left[1-\min\left(1,\ \sum_{v=1}^{u-1}q_{ju}S_{uv}\right)\right]$$
$$\cdot\left[1-\min\left(1,\ \sum_{v=1}^{m}q_{iv}S_{uv}\right)\right]\cdot\left[\min\left(1,\ \sum_{v=1}^{m}q_{sv}S_{uv}\right)\right] \tag{36}$$

Therefore,

$$|\sigma(c_j)\cap\overline{\sigma(c_i)}\cap\overline{\sigma(c_s)}|=\sum_{u=1}^{m}q_{ju}\left[1-\min\left(1,\ \sum_{v=1}^{u-1}q_{ju}S_{uv}\right)\right]$$
$$\cdot\left[1-\min\left(1,\ \sum_{v=1}^{m}q_{iv}S_{uv}\right)\right]\cdot\left[1-\min\left(1,\ \sum_{v=1}^{m}q_{sv}S_{uv}\right)\right] \tag{37}$$

Similarly,

$$|\sigma(c_k)\cap\overline{\sigma(c_j)}\cap\overline{\sigma(c_s)}\cap\overline{\sigma(c_i)}|=\sum_{u=1}^{m}q_{ju}\Big[1-\min\Big(1,\sum_{v=1}^{u-1}q_{jv}S_{uv}\Big)\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{iv}S_{uv}\Big)\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{sv}S_{uv}\Big)\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{kv}S_{uv}\Big)\Big] \tag{38}$$

Based on Eq. (12), (14) and (19), we can get the next expression.

$$e_j=\gamma_j-\sum_{i=1}^{u}\tau_{ij} \tag{39}$$

and

$$\pi_{jk}'=J_{ik}'-\sum_{i=1}^{n}\sum_{s=1}^{n}m_{is}',{}_{jk}' \tag{40}$$

From Eq. (26) we can get next equatin.

$$\sum_{t=2}^{f}P_r[\triangle(t-1)=c_i\wedge\triangle'(t-1)=c_s\wedge\triangle(t)=c_j\wedge\triangle'(t)=c_k]=m_{is}',{}_{jk}' \tag{41}$$

Therefore, substitution of Eq. (35), (36), (37), (38), (39), (40), (41) into (29) yields

$$\begin{aligned}T'=&\sum_{j=1}^{n}\Big[\gamma_i-\sum_{i=1}^{n}\tau_{ij}\Big]\cdot\Big[\sum_{u=1}^{m}q_{iu}\Big[1-\min\Big(1,\sum_{v=1}^{u-1}q_{jv}S_{uv}\Big)\Big]\Big]+\sum_{j=1}^{n}\sum_{k=1}^{n}\Big[J_{ik}'-\sum_{i=1}^{n}\sum_{k=1}^{n}m_{is}',{}_{jk}'\Big]\\&\cdot\Big[\sum_{u=1}^{m}q_{ku}\Big[1-\min\Big(1,\sum_{v=1}^{u-1}q_{ku}S_{uv}\Big)\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{ju}S_{uv}\Big)\Big]\Big]\\&+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{n}\sum_{k=1}^{n}m_{is}',{}_{jk}'\cdot\sum_{u=1}^{m}q_{ju}\Big[1-\min\Big(1,\sum_{v=1}^{u-1}q_{jv}S_{uv}\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{iv}S_{uv}\Big)\Big]\\&\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{sv}S_{uv}\Big)\Big]+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{n}\sum_{k=1}^{n}m_{is}',{}_{jk}'\sum_{u=1}^{m}q_{ku}\Big[1-\min\Big(1,\sum_{v=1}^{u-1}q_{kv}S_{uv}\Big)\Big]\\&\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{jv}S_{uv}\Big)\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{sv}S_{uv}\Big)\Big]\cdot\Big[1-\min\Big(1,\sum_{v=1}^{m}q_{iv}S_{uv}\Big)\Big]\end{aligned} \tag{42}$$

6. Conclusions

In this paper, a new information system model is proposed. Some fundamental analyses for the system are examined. More research is required to extend the basic results obtained in the analyses. The study of the analyses under the different conditions, is also desirable. Here we do not deal with the syntheses problems of the system. So many problems in the syntheses remain unsolved. Above all, the investigations for the partition of control module that gives the minimum expected number of joint control transfers, will be examined in the future.

References

〔1〕 T. C. Lowe, Analysis of an Information System Model with Transfer Penalties, IEEE Trans. Comput. vol. c-22. No. 5 pp. 469-480 May. 1973.

〔2〕 J. Kral, One way of estimating frequencies of jumps in a program, Commun. Ass. Comput. March., vol 11, pp. 475-480, July 1968.

〔3〕 S. W. Golombs and L. d. Baumbert, Backtrack programming, J. Ass. Comput. Mach., vol. 4, pp. 516-524, Oct. 1965.

〔4〕 W. Feller, An Introduction to Probability Theory and Its Applications. New York: Wiley, 1950.