The System Diagnoses by the Test S-gates

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This paper describes the fault diagnoses of systems represented by the equivalent SEC-graphs (6) with a single input vertex v_1 and a single output vertex v_2. On the system diagnoses, W. Mayeda (4) introduces a *test S-gate* which will stop the transmission by shorting the vertex where the *test S-gate* is inserted to the ground terminal.

In this paper, we investigate the system diagnoses by means of *test S-gates*. First, we study the properties of Ω_V for a given set V of vertices to which *test S-gates* are assigned. Second, we discuss the algorithm for obtaining the set V of vertices under which the fault vertex can be distinguished uniquely.

Above all, we examine the algorithm for nearly optimum set of vertices.

Third, we consider which order should they be activated after the set of vertices have obtained.

1. Introduction

About fault diagnoses of system associated with connected units, many papers were published. (1)-(6) In the previous papers (1)-(2) the authors have shown that faults can be detected and located by means of placed test points within the system. But by this method a signal will pass through test points which introduces additional unfavorable delay to the system of desired speediness.

Hence in that paper (4), instead of test points, W. Mayeda introduced a *test S-gate* which will stop transmission by shorting the units where it is inserted to the ground terminal. By using the *test S-gate*, there is no additional delay in normal operation and they tried to do system diagnoses efficiently.

In the present paper we investigate the system diagnoses by means of the *test S-gates*. First, we study the properties of Ω_V for a given set V of units to which the *test S-gates* are assigned. Second, we discuss the algorithm for obtaining the set V of units under which the fault unit can be distinguished uniquely.

In the large system, it is difficult and unpractical to seek the optimum set V. In this paper, we examine the algorithm for a nearly optimum set V. Third, we investigate the optimum order of detection to minimize the average testing time.

2. Definition and Assumption

A system can be represented by the **SEC**-*graph* (4) such that each unit will be indicated by vertex and a connection between two units will be indicated by a directed edge. On the system diagnoses, we first give assumptions and necessary definitions in the following.

〔**Assumption 1**〕 We assume a single fault only among vertices.*

〔**Assumption 2**〕 We can apply the test inputs only to the vertex v_1 and observe the output only at the vertex v_2.

〔**Assumption 3**〕 We assume that the output, which passes a fault vertex, can be distinguished from the output which passes only faultless vertices.

〔**Definition 2-1**〕 When *test S-gate* is assigned to a vertex v_i and when it is activated, all outgoing signals from the vertex v_i will be stopped.

*It means that there exists at most one faulty unit in the system.

〔**Definition 2-2**〕 In an ***SEC-graph***, $\sigma(v_i)$ denotes the set of vertices which can be reached from v_i by a directed edge. Conversely $\sigma^{-1}(v_i)$ shows the set of vertices which can reach to v_i by a directed edge.

〔**Definition 2-3**〕 For a given set of vertices V, Ω_V is a set of all vertices which are in at least one directed path from v_1 to v_2 which does not pass any vertex of V.

For example, we choose V as $V=\{v_4, v_6\}$ in a ***SEC-graph*** in ***Fig. 1***. Then Ω_V is a

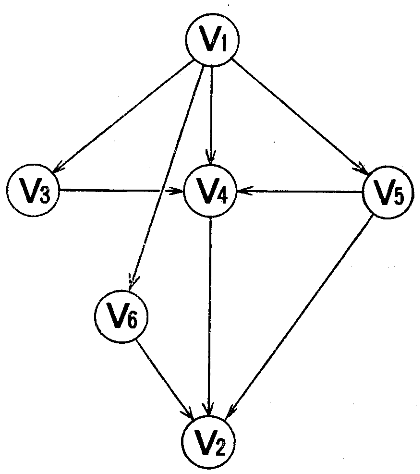

Fig. 1 An SEC-graph

set of vertices which satisfies the above condition. That is, Ω_V becomes $\Omega_V=\{v_1, v_2, v_5\}$.

As easily known, we can show the next Lemma.

〈**Lemma 2-1**〉 $\Omega_V \subseteq V(G)-V$, for any V, $V \subseteq V(G)$.

According to the assumption on the diagnoses, we can apply the test inputs to the vertex v_1 and observe the output at the vertex v_2. Hence, if the faulty output are observed at v_2, we can conclude that a faulty vertex is in Ω_V. Contrary, if we observe the normal output at v_2, then all vertices included in Ω_V will be proved normal.

〔**Definition 2-4**) Let V(G) be a set of all vertices in an ***SEC-graph*** G. For a given set B_i where $B_i \subseteq V(G)$, we define the partition of vertices V(G) as follows,

$$V(G)=\bigcup_i B_i \qquad (1)^*$$

〔**Definition 2-5**〕 For a set of *test S-gates* V, define P_V and k as follows,

$$P_V=\prod_{V'\subseteq V} \Omega_{V'}=\{B_1 \cdots, B_i, \cdots B_m\} \qquad (2)^{**}$$

$k=max|B_i|$, where, $|B_i|$ means the order of B_i after deleting the vertex v_2. Then we call the system is ***k-distinguishable*** under the set V.

As easily shown from *Def. 2-5*, we can locate the faulty vertex at most within k pieces of vertices under the use of *test S-gates* assigned to the vertices in V. So that, if v_i and v_j belong to different blocks of P_V each other, v_i and v_j are distinguishable under V.

Under the assumptions (1)-(3), in order to locate a faulty vertex of the system, we must gain the proper *test S-gates* V under which the system is ***1-distinguishable.***

But by next Lemma, we know that for some system there exists no V under which system is 1-distinguishable.

〈**Lemma 2-2**〉 If $\Omega\{v_i\} \not\ni v_j$ and $\Omega\{v_j\} \not\ni v_i$ are satisfied for a system, then there is no V under which a fault of v_i and v_j is distinguishable. Except that, there is at least a set V under which system is 1-distinguishable.

(proof) Assume that a fault of v_i and v_j is distinguishable, there exists a set V′ which satisfy $\Omega_{V'} \ni v_i$ (or v_j) and $\Omega_{V'} \not\ni v_j$ (or v_i). If $\Omega_{V'} \not\ni v_j$, then $\Omega_{\{v_j\}} \supseteq \Omega_{V'}$. Hence if $\Omega_{\{v_j\}} \not\ni v$ and $\Omega_{\{v_i\}} \not\ni v_j$ then $\Omega_{V'} \not\ni v_i$. Similarly, if $\Omega_{V'} \not\ni v_i$, then $\Omega_{V'} \not\ni v_j$. Hence, in this case, a fault of v_i and v_j is undistinguishable. Except that, for example, if $\Omega_{\{v_i\}} \ni v_j$, then choose V′ as $V'=\{v_i\}$ and if $\Omega_{\{v_j\}} \ni v_i$, then set $V'=\{v_j\}$. Then it is clear that they are distinguishable under the V′ Q. E. D.

If $\Omega_{\{v_i\}} \not\ni v_j$ and $\Omega_{\{v_j\}} \not\ni v_i$, than v_i and v_j indicate the equivalent fault. In this paper, we study the only system which don't have the equivalent faults.

〔**Definition 2-6**〕 In a set of all vertices V, if $\Omega_{V'} \subseteq \Omega_{\{v_i\}}$, where $V' \not\ni v_i$ and $V' \subseteq V$ and $v_i \in V$, then v_i is a redundant vertex in V.

〔**Definition 2-7**〕 In an ***SEC-graph*** G, a vertex semicut from v_1 to v_2 is a minimal set of vertices such that the deletion of all vertices in the set destroy all directed path from v_1 to v_2. [4]

〈**Lemma 2-3**〉 An ***SEC-graph*** from v_1 to

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*B_i is called a block.

**The operation π denotes the partition.

v_2 is not 1-distinguishable under V if V contains no *vertex semicut* from v_1 to v_2. (4)

3. The Ω_V and the Vertex Semicut

In this section, we discuss the method of obtaining the Ω_V for a given subset of vertex V and furthermore investigate the algorithm for seeking the all vertex semicuts of an ***SEC-graph***.

3.1 The Algorithm for Seeking the Ω_V

Algorithm 1:

step 1) Given an ***SEC-graph***, make the vertex matrix C as follows,

$$C = \begin{array}{c|ccccc|} & v_l \cdots & \cdots & v_j & \cdots & v_n \\ v_1 & & & \vdots & & \\ \vdots & & & \vdots & & \\ v_j & \cdots & \cdots & c_{ij} & \cdots & \cdots \\ \vdots & & & \vdots & & \\ v_n & & & \vdots & & \end{array} \qquad (3)$$

, where,

$$c_{ij} = \begin{cases} 0: \sigma(v_i) \not\ni v_j \\ 1: \sigma(v_i) \ni v_j \end{cases}$$

step 2) In the vertex matrix C, let all the elements of the v_i-*row* and v_i-*column* satisfying $v_i \in V$, be *zero*.

step 3) If all elements of v_i-*row* (or v_i-*column*) are *zero*,* then make the v_i-column (-or v_i-row) be **0**-*column* (or **0**-*row*) *vector*, where $i \neq 1, 2$.

Let the matrix obtained above be M. In the matrix M, m is the number of columns which are not **0**-*column vector*.

step 4) We compute K of *Eq.*(4), where $\sum$ is the sum of matrix and $M_i = M \cdot M \cdots\cdots M$.

$$K = \sum_{i=0}^{m} M^i = \begin{array}{c|ccccc|} & l \cdots & \cdots & j & \cdots & n \\ l & & & \vdots & & \\ \vdots & & & \vdots & & \\ i & \cdots & \cdots & K_{ij} & \cdots & \\ \vdots & & & \vdots & & \\ n & & & \vdots & & \end{array} \qquad (4)$$

step 5) In K, we define a and b as follows,

$a = v_1$-*column-vector*$(K_{11}, \cdots\cdots, K_{1n})$

$b = v_2$-*row-vector* $(K_{12}, \cdots\cdots\cdots, K_{n2})$

we compute $K_{1i} \cdot K_{i2}$, where $i = 1, 2, \cdots\cdots, n$.

step 6) If, for each $K_{1i} \cdot K_{i2}$, $K_{1i} \cdot K_{i2} = 0$, then $\Omega_V \ni v_i$. Otherwise set $\Omega_V \not\ni v_i$.

By the algorithm mentioned above, we can gain the Ω_V for a given set V effectively.**

3.2. A Few Comments on Ω_V

In this section we give some results about Ω_V.

〈**Lemma 3-1**〉 If $V' \subseteq V$, then $\Omega_{V'} \supseteq \Omega_V$ and if $V' \cup V'' = V$, then $\Omega_{V'} \cap \Omega_{V''} \supseteq \Omega_V$.

(proof) It is obvious from the definition.

〈**Lemma 3-2**〉 If $\sigma^{-1}(v_j) = \{v_i\}$, then $\Omega_{\{v_i\}} \subseteq \Omega_{\{v_j\}}$.

(proof) From the assumptions, there is no edge which entry to v_j except edge from v_i. Hence, if v_i is *test S-gate*, there is no directed path from v_1 to v_j and $\Omega_{\{v_i\}} \ni v_j$. That is, if v_i is *test S-gate*, then $\Omega_{\{v_i\}} = \Omega_{\{v_i, v_j\}}$. Hence, by *Lemma 3-1*, $\Omega_{\{v_i\}} = \Omega_{\{v_i, v_j\}} \subseteq \Omega_{\{v_j\}}$.

Q.E.D.

But the converse of *Lemma 3-2* does not always hold good. For example, in *Fig.1*

$\Omega_{\{v_3\}} = \{v_1, v_2, v_4, v_5, v_6\}$

$\Omega_{\{v_4\}} = \{v_1, v_2, v_5, v_6\}$

then, $\Omega_{\{v_3\}} \supseteq \Omega_{\{v_4\}}$. But $\sigma^{-1}(v_3) \neq \{v_4\}$.

Based on *Lemma2-1* and *3-2*, we can obtain the next *Lemma*.

〈**Lemma 3-3**〉 For any v_i and v_j $(i, j \neq 1, 2)$ which satisfy $\sigma^{-1}(v_j) = \{v_i\}$, if $\Omega_{\{v_j\}} \ni v_i$, then the fault of v_i and v_j can be distinguished. Otherwise, they can not be distinguished.

(proof) The necessary and sufficient condition for a fault of v_i and v_j being distinigusable, is $\Omega_{\{v_i\}} \ni v_j$ or $\Omega_{\{v_j\}} \ni v_i$. But from the assumption and *Lemma 3-2*,

$\Omega_{\{v_i\}} \subseteq \Omega_{\{v_j\}}$. Hence, *Lǝmma 3-3* was verified.

3.3 The Method for Seeking All Vertex Semicuts

As the necessary condition for 1-distinguishable under V, V must contain a *vertex semicut* from v_1 to v_2. Hence, it is important to search all *vertex semicuts* in the system. In this section, we consider the technique for obtaniing all *vertex semicuts*.

〔**Definition 3-1**〕 Let $\sigma^{-1}(v_2)$ be the *initial vertex semicut* in a system. $\sigma^{-1}(v_2)$ is not always a vertex semicut in a system. But it

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*For brevity, we call it ***o***-*row-vector* (*or* ***o***-*column-vector*)

**We have programmed this algorithm by means of the FORTRAN LANGUAGE.

contains at least a vertex semicut in itself. Then we can indicate the lemma as,

⟨Lemma 3-4⟩ If V contains at least a set of *vertex semicut*, then

$$\{V' \cup \sigma^{-1}(V'') \mid V' \cup V'' = V,$$
$$V' \cap V'' = \phi,\ \sigma^{-1}(V'') \not\ni v_1\}$$

contains at least a set of *vertex semicut.*

(proof) The proof is obvious from *Lemma 3-2.*

Next, we introduce an algorithm for obtaining all *vertex semicuts* in a system.

Algorithm 2:

step 1) We seek the *initial vertex semicut* in the system. Let it be V_0, for brevity.

step 2) Separate V_0 as follows,

$$V_0 = \vec{V}_0 \cup \overleftarrow{V}_0,\ \vec{V}_0 \cap \overleftarrow{V}_0 = \phi$$

where, if $\sigma^{-1}(v_i) \ni v_1$ then $v_i \in \vec{V}_0$, otherwise, $v_i \in \overleftarrow{V}_0$.

step 3) Set $i=1$

step 4) We seek V_i shown in Eq. (5)

$$V_i = \{V_i^1, \cdots V_i^j, \cdots V_i^l\}$$
$$= \{V_i^j \mid V_i^j = \vec{V}_{i-1}^t \cup (\overleftarrow{V}_{i-1}^t - V') \cap \sigma^{-1}(V'),$$
$$V' \neq \phi,\ V' \subseteq \overleftarrow{V}_{i-1}^t\} \qquad (5)$$

Where, if $V_i^j \supseteq V_{i'}^{i'}$, then we delete V_i^j from V_i $(i \geqq i')$. Moreover in the case $V_i = \phi$, then go to stop 6 and in the case $V_i \neq \phi$, then go to step 5.

step 5) Set $i = i+1$ and go to step 4.

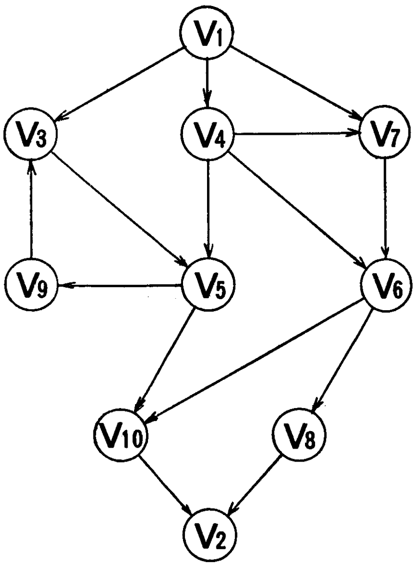

Fig. 2 SEC-graph for a system

step 6) Let $\{V_i \mid V_i \in V_k$ or $V_i = V_0\}$ be $\boldsymbol{V}$, where V_i, $V_j \in \boldsymbol{V}$ and $V_i \subseteq V_j$, delete V_j from $\boldsymbol{V}$. Because V_j is not a vertex semicut.

step 7) Replace the element of $\boldsymbol{V}$ as follows,

$$\boldsymbol{V} = \{V_1, V_2, \cdots\cdots V_j, \cdots V_l\} \qquad (6)$$

, where $|V_j| \leqq |V_{j+1}|$*, $i=1, 2, \cdots\cdots, l-1$

By the above procedure we can obtain all the vertex semicuts.

Now, we explain briefly the *algorithm 2* mentioned above with the use of the next example,

〔**Example 1**〕 Apply the above algorithm to the *SEC-graph* represented by *Fig. 2*, we obtain the result, shown *in Eq.*(7).

$$\boldsymbol{V} = \begin{cases} \{v_8, v_{10}\} \\ \{v_6, v_{10}\} \\ \{v_5, v_6\} \\ \{v_4, v_7, v_{10}\} \\ \{v_4, v_5, v_7\} \\ \{v_3, v_4, v_6\} \\ \{v_3, v_9, v_7\} \end{cases} \qquad (7)$$

4. Decision of Test S-gate

4.1 Definition and Lemma

In this section, we discuss how to search the test S-gates V under which a system is 1-distinguishable. We use the algorithm 1 and 2 mentioned in the previous sections. We give Definition and Lemma as follows,

〔**Definition 4-1**〕 For a given ***SEC-graph***, ***F-matrix*** is defined as follows,

$$F = \begin{array}{c} \\ \Omega_{\{v_3\}} \\ \vdots \\ \Omega_{\{v_i\}} \\ \vdots \\ \Omega_{\{v_n\}} \end{array} \begin{array}{c} v_3 \cdots\cdots v_j \cdots\cdots v_n \\ \left| \begin{array}{ccc} & \vdots & \\ & \vdots & \\ \cdots\cdots & f_{ij} & \cdots\cdots \\ & \vdots & \\ & \vdots & \end{array} \right| \end{array} \qquad (8)$$

, where

$$f_{ji} = \begin{cases} 0 & \text{iff } v_j \notin \Omega_{\{v_i\}} \\ 1 & \text{iff } v_j \in \Omega_{\{v_i\}} \end{cases}$$

〔**Definition 4-2**〕 Define g as follows, $g = min|V_j|$, where the system is 1-distingushable under V_j. Then the V_j which gives g, is called the optimum set of test *S-gates* for the system.

⟨**Lemma 4-1**⟩ For any v_i and $v_j (i, j \neq 1, 2)$ if

*$|V_j|$ denotes the number of elements in the set V_j

there exists v_m in V, which satisfies $f_{mi} \oplus f_{mj}=1$, then a fault of v_i and v_j can be distinguished under this V.

(Lemma 4-2) $\Omega_{\{v_{i_1}\}} \cap \Omega_{\{v_{i_2}\}} \cap \cdots\cdots \cap \Omega_{\{v_{i_s}\}} \supseteq \Omega_{\{v_{i_1}, v_{i_2}, \cdots\cdots, v_{i_s}\}}$

(proof) The proof is obvious from Def. 2-3.

⟨Lemma 4-3⟩ For any v_i and v^j, if $f_{mi} \oplus f_m{}^j=0$ is satisfied for any $v_m \in$ V, a fault of v_i and v^j can not always be undistinguished under V.

(proof) We prove it by indicating the example in *Fig. 3* as follows, if set $V=\{u_3, v_4, v_5\}$,

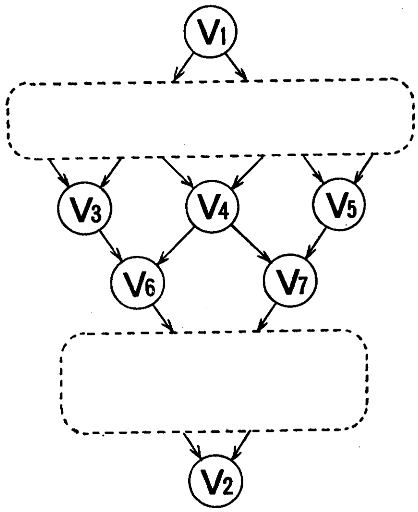

Fig. 3 Example for Lemma 4-3

then $f_{36} \oplus f_{37}=0$, $f_{46} \oplus f_{47}=0$ and $f_{56} \oplus f_{57}=0$ are hold. But if $V_t=\{v_3, v_4\}$, then $f_{t5} \oplus f_{t7}=1$. Hence, v_6 and v_7 can be distinguished under V.

Q. E. D.

⟨Theorem 4-1⟩ For any v_i and v_j which satisfy $f_{mi} \oplus f_{mj}=0$ for any $v_m \in$ V, v_i and v_j can be distinguished under V, if there exist V′ which satisfies both $\Omega_{V'} \ni v_i$ (or v_j) and $\Omega_{V'} \not\ni v_j$ (or v_i). Where,

$$V' \subseteq \widetilde{V}=\{v_k \mid f_{ki} \cdot f_{kj}=1,\ v_k \in V\}$$

(proof) It is obvious that sufficient condition is proved by *Def. 2-6* and *2-7*. Next, we will prove necessary condition. We assum that v_i and v_j are distinguished under V. Then, there exist V′ which satisfy $\Omega_{V'} \ni v_i$ (or v_j) and $\Omega_{V'} \not\ni v_j$ (or v_i). By assumption, $f_{mi} \oplus f_{mj}=0$ is satisfied for any $v_m \in \widetilde{V'}$, so $f_{mi}=f_{mj}=0$ or $f_{mi}=f_{mj}=1$. If $f_{mi}=f_{mj}=0$, then $\Omega_{\{v_m\}} \not\ni v_i$ and $\Omega_{\{v_m\}} \not\ni v_j$. By Lemma 3-1, $\Omega_{\{v_m\}} \supseteq \Omega_{V'}$. Hence, $\Omega_{V'} \not\ni v_i$ and $\Omega_{V'} \not\ni v_j$ are satisfied. This is contrary to the assumption. After all, $f_{mi} \cdot f_{mj}=1$ is satisfied for any $v_m \ni V'$. Finally we gain $V' \subseteq \widetilde{V}$.

Q. E. D.

From *Lemma 4-1* and *Theorem 4-1*, in order that we examine if v_i and v_j are distinguished under *test S-gates* V, first we apply *Lemma 4-1* to *F-matrix* obtained by *algorithm 1*. If there exists a *test S-gate* v_m in V which satisfies $\Omega_{\{v_m\}} \ni v_j$ (or $\Omega_{\{v_m\}} \ni v_j$) and $\Omega_{\{v_m\}} \not\ni v_j$ ($\Omega_{\{v_m\}} \not\ni v_i$), we can conclude v_i and v_j are distinguishable under the V. However, if there exists no *test S-gates* v_m in V, we can apply the *Theorem 4-1* to the V efficiently.

4.2. *Some Criteria for Nearly Optimum Test S-gates*

In a given ***SEC-graph***, in order to seek the optimum *test S-gates* under which the system is 1-distinguished, we may examine if the system is 1-distinguished from *Lemma 4-1* and *Theorem 4-1* under all V which contains vertex semicuts. Therefore, the *algorithm 2* is effective for obtaining the optimum *test S-gates*. We know that it is sufficient enough to investigate ***V*** obtained in *section 3.3.*. But when it is a large scale system and the number of units is enormous, it is difficult and unpractical to obtain the optimum *test S-gates*. Hence, we discuss the algorithm for obtaining the nearly optimum *test S-gates* in what follows.

Now, assume that the number of pair of undistinguishable vertices is ***h*** under the *test S-gates* V. Then, when we add a *test S-gate* v_i to the V we assume that the number of pair of vertices which is distinguished under v_i, but undistinguished under V, is ***k***. Then it is clear that if we choose $V \cup \{v_i\}$ as the ***new*** *test S-gates*, the number of undistinguishable pair of vertices are less than $h-k$.

Hence, according to the criteria mentioned below, we add a new *test S-gate* v_i to V.

Criterion 1) Choose a v_i which gives the maximum ***k***. Then if there are some vertices which give maximum ***k***, select one of them based on the next criterion.

Criterion 2) Choose the v_i such that in the $(h-k)'$,s remaining pair, there exists many pairs whose both elements are contained in $\Omega_{\{v_i\}}$.

According to the *criterion 1* and *2*, let us seek the nearly optimum *test S-gates* by the *algorithm 3.*

4.3. *Algorithm for Nearly Optimum Test S-gates*

Algorithm 3:

step 1) Set $i=1$

step 2) By *algorithm 2*, we search all sets of vertex semicuts.

step 3) We examine if the system is 1-distinguished under V_i. If it is 1-distinguished, V_i is the optimum *test S-gates* V. Otherwise, go to step 4

step 4) From *criterion* 1 *and* 2, we continue to add a new vertex v_k to V_i and set $V_i=V_i\cup\{v_k\}$ until the system is 1-distinguished. Set $V_E=V_i$.

step 5) Set $i=i+1$

step 6) Set w as $w=|V_E|-|V_i|$. If $w\leq 0$, then go to step 8. If $w>0$, go to step 7.

step 7) Add new vertices to V_i until the system is 1-distinguished, where added number of vertices are less than w-1. If the system is 1-distinguished, then let V_i be V_E and go to step 5. Otherwise, go to step 5.

step 8) V_E is the nearly optimum set V.

By the above procedure, we can gain the nearly optimum set V under which a system is 1-distinguished.

4.4. *Consideration for the Equivalent Fault*

As we referred *in section* 2, input unit and output unit are not distinguished under any *test S*-gates. Furthermore, a system represented by *Fig. 4* is not 1-distinguished under any V. However, in this case we can distinguished these equivalent faults, modifying the systems by one of the following procedures.

1) By adding some edges to the system
2) By adding some vertices to the system
3) By placing some input vertices except v_1*
4) By placing some output vertices except v_2**

In this section, we discuss about *procedure 1.* We propose two methods on concerning *procedure 1* as follows,

1) First, we add an edge from $\{S_1(G)\cup v_i\}$ to $S_2(G)$, shown in *Fig. 5*.

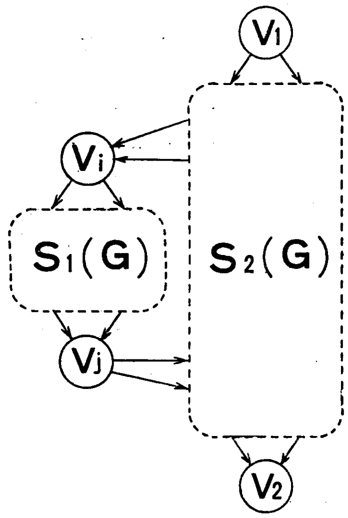

Fig. 4 Example for the equivalent fault

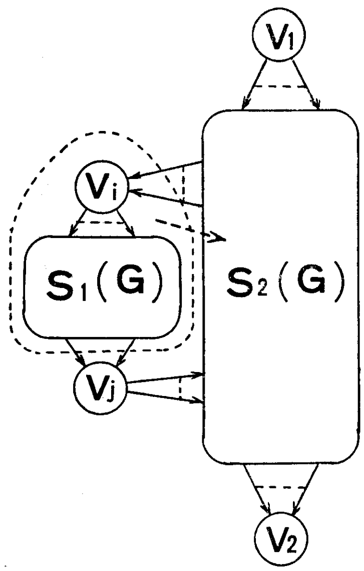

Fig. 5 Example for procedure 1

2) Second, we add an edge from $S_2(G)$ to $\{S_1(G)\cup v_j\}$.

Above all, one of future problem is to find an algorithm which minimize the number of additional edges.

5. Minimization of the Average Testing Time

Supposing, by the procedure introduced in the previous section, we have gained the *test*

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*We can apply the test inputs only to the input vertices.

**We can observe the output only at the output vertices.

S-gates V under which the system is 1-distinguishable. In this section, the fault probability of each vertex have been known experimently. Then we aim to decide the *testing procedure* for minimizing the average *testing time.*

5.1. Definition and Assumption

〔**Definition 5-1**〕 If the event 〔$X=x_i$〕 has probability p_i, we define the information measure that X has taken on the value x_i, is - log p_i.

〔**Definition 5-2**〕 Let X take on a finite number of possible value $x_1, \cdots\cdots, x_n$ with probabilities $p_1, \cdots\cdots, p_n$. Then the average information measure **I** conveying which event occured, is $I=-\sum_{i=1}^{n} p_i \log p_i$, where $\sum_{i=1}^{n} p_i=1$.

This is the average information measure which is required in order to decide the uncertain event.

〔**Definition** 5-3〕 Assume the event 〔$X=x_i$〕 has probability p_i, where $p_1, \cdots\cdots, p_n$ be arbitray positive numbers with $\sum_{i=1}^{n} p_1=1$. Then we define the uncertainty as,

$$\boldsymbol{H}(p_1, p_2, \cdots\cdots, p_n)=-\sum_{i=1}^{n} p_i \log p_i \qquad (10)$$

That is, the information decreases the uncertainty of situation and the information measure wili be obtained by calculating the rate of the decrease of the uncertainty. The quantity H(X), has been called the entropy of X.

〔**Definition 5-4**〕 If the entropy of situation changes from **H** to **H**$'$ by gaining the information, then the average information measure $\boldsymbol{I}$ can be defined as $\boldsymbol{I}=\mathbf{H}-\mathbf{H}'$.

Before we discuss the testing procedure we give some assumptions.

Assumption 1) In a system associated with N units v_i $(i=1, 2, \cdots\cdots, N)$, we have ever known the probability p_i that each unit be faulty, where $\sum_{i=1}^{N} p_i=1$.

Assumption 2) We assume that the testing time required in each test is equal.

5.2. Decision of Testing Procedure

Generally two different procedure, the *preset testing procedure* and *the adaptive testing procedure*, are developed for the optimal fault location procedure of the system. The former is a test program under the assumption that the choice of the succession of test operations spplied to the system does not depend upon the outcomes of the tests. The latter is a test program under the assumption that the choice of the succession of test operations depends upon the outcomes of the tests.

In future, we expect that a system will have the function of self-diagnosis. Hence, in this paper we diagnose the system by the adaptive testing procedure which is more effective for self-diagnosable system.

Now, if we gained the test S-gates V as shown in *Eq.* (11)

$$V=\{v_{i_1}, \cdots\cdots\cdots, v_{i_m}\} \qquad (11)$$

Then there are $\boldsymbol{M}$ ways of activating them.

$$M={}_mC_1+\cdots\cdots\cdots+{}_mC_m$$
$$=\sum_{i=1}^{m} {}_mC_i=2^m-1 \qquad (12)$$

We can apply some of (2^m-1)'s tests to the system.* Hence it is important to find the testing procedure for minimizing the average testing time.

As easily know, by applying one test V$'$ to the system, all units in a system can be separated into two blocks $\Omega_{V'}$ and $\overline{\Omega}_{V'}$. Then the information we can know in a test, conveys in which block the faulty vertex is contained.

Now, assume the faulty vertex is v_f, then $v_f \in \Omega_{V_1}$ or $v_f \in \overline{\Omega}_{V_1}$ exists, where the probability p_1 and p_2 can be shown as, $p_1=\sum p_i(v_i \in \Omega_{V_1})$, $p_2=\sum p_j(v_j \in \overline{\Omega}_{V_1})$, $p_1+p_2=1$. In this case, the average information measure $\boldsymbol{I}$ conveying in which block v_f is contained, is $\boldsymbol{I}=-\sum_{i=1}^{2} p_i \log p_i$. $\boldsymbol{I}$ is rewritten as,

$$\boldsymbol{I}=-\{p_1 \log p_1+(1-p_1)\log(1-p_1)\} \qquad (13)$$

We differenciate $\boldsymbol{I}$ by p_1 to seek the test which maximize the average information measure $\boldsymbol{I}$.

$$\frac{dI}{dp_1}=\frac{d[-\{p_1 \log p_1+(1-p_1)\log(1-p_1)\}]}{dp_1}$$
$$=-\{\log p_1-\log(1-p_1)\} \qquad (14)$$

If $p_1=p_2=\frac{1}{2}$, then the average information measure $\boldsymbol{I}$ becomes maximum.

Next, we generally discuss which test $\mathbf{V}_i$ should be activated in the i-th test. Now assume $v_f \in \Omega_{V_i}$. Then the faulty vertex v_f satisfies

*Each test corresponds to each subset V$'$ uniquely (V$'\subseteq$V), so that we designate the test as V$'$ for brevity.

$v_f \in \Omega_{V_i} \cap (\bigcap_{j=1}^{i-1} \tilde{\Omega}_{V_j})$.*

Next, in the $(i+1)$-th test, the v_f implies either *Eq.* (15) or (16).

$$v_f \in \Omega_{V_{i+1}} \cap \Omega_{V_i} \cap (\bigcap_{j=1}^{i-1} \tilde{\Omega}_{V_j}) \quad (15)$$

$$v_f \in \overline{\Omega}_{V_{i+1}} \cap \Omega_{V_i} \cap (\bigcap_{j=1}^{i-1} \tilde{\Omega}_{V_j}) \quad (16)$$

Let the sum of fault probability of vertices which is contained in two blocks as mentioned above, be p_{i_1} and p_{i_2}. Then p_{i_1} and p_{i_2} are shown in Eq. (17).

$$\left.\begin{array}{l} p_{i_1} = P_r\{v_a | v_a \in \Omega_{V_{i+1}} \cap \Omega_{V_i} \cap (\bigcap_{j=1}^{i-1} \tilde{\Omega}_{V_j})\} \\ p_{i_2} = P_r\{v_b | v_b \in \overline{\Omega}_{V_{i+1}} \cap \Omega_{V_i} \cap (\bigcap_{j=1}^{i-1} \tilde{\Omega}_{V_j})\} \\ p_{i_1} + p_{i_2} = p_i (\neq 1) \end{array}\right\} \quad (17)$$

, where $p_i = \sum p_k (v_k \in \Omega_{V_i} \cap (\bigcap_{j=1}^{i-1} \tilde{\Omega}_{V_j}))$

In this case, by changing p_{i_1} and p_{i_2} Eq. (18) yields,

$$\frac{p_{i_1}}{p_i} + \frac{p_{i_2}}{p_i} = 1 \quad (18)$$

Then the expectation value of information measure I is $-\sum_{j=1}^{2} \frac{p_{i_j}}{P_i} \log\left(\frac{p_{i_j}}{P_i}\right)$ (19)

Finally, p_{i_1} and p_{i_2} which maximize I are to satisfy $p_{i_1} = p_{i_2} = \frac{p_i}{2}$

The best i-th st test is to give $V_{i+1} (V_{i+1} \subseteq V)$ which satisfies *Eq.* (19).

By the above consideration, we can obtain the testing procedure which minimize the average testing time approximately. We have also programmed this testing procedure.

6. Conclusion

In this paper, we have presented a new approach to the diagnoses problem of digital system, which has provided several interesting results and may help to gain further insights into this general area.

The main results obtained in this paper, are as follows,

1) For any v_i and v_j, if there aro no v_i and v_j which satisfy $\Omega_{\{v_i\}} \not\ni v_j$ and $\Omega_{\{v_j\}} \not\ni v_i$, the system is 1-distinguished under the set of *test S-gates* V. We can know it by examining ***F-matrix*** easily.**

2) By the use of properties of *vertex semicuts*,*** we gained a nearly optimum set V.**** This algorithm is shown to be effective for large scale systems and easily programmed.

3) We have investigated *the order* of activating the *test S-gates* which minimize the average testing time.

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*$\tilde{\Omega}_{V_i}$ is either Ω_{V_i} or $\overline{\Omega}_{V_i}$ depend on whether or not the output indicates the existence of a fault vertex.

***F-matrix* is obtained by *algorithm* 1.

***The vertex semicuts are obtained by *algorithm* 2.

****Nearly optimum set V are obtained by *algorithm* 3.