ANALYSIS OF PRODUCTION SYSTEMS WITH ADVANCE DEMAND INFORMATION

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Chapter 1

Introduction

1.1 Introduction

Nowadays there are a lot of problems in which Japanese enterprises are afflicted. If these problems are not dealt with at all, Japanese economy tapers and becomes small.

Prices of crude oil, metals and foods have kept rising for several years. This makes Japanese economy sluggish. Japan is poor in natural resources, so relies on import from foreign countries for much of these. Rises of prices of raw materials give serious damage to Japanese economy. Enterprises cannot raise prices of products similarly when prices of raw materials increase. This is because if enterprises raise prices of products, then a consumer tends not to buy the products and profit of enterprises decreases. In these situations, it is very important for enterprises to try to absorb increment of costs of raw materials by cutting down costs which occur in manufacturing processes and raising productivity.

Moreover, a menace to Japanese industry is the growth of the emergent countries of Asia such as China and India. In China, since the year of 2003, the GDP is growing up at the rate which exceeds ten percents a year and Chinese economy will develop more and more in the future. The growth rate of India is also the second highest after that of China in BRICs (Brazil, Russia, India and China) countries. In these emergent countries, low-priced products are mass-manufactured by using the cheap labor force. For example, China is called “the factory of the world”, and products with the tag of “made in China” can be seen all over the world. A lot of products which are manufactured in these emergent countries are imported to Japan. Technological development power and improvement of the performance of production systems are keys for Japanese economy to survive in the world.

In the environment which surrounds Japanese manufacturer, a feature of production method has changed from a large quantity production of a few kinds of products to a small quantity production of a lot of kinds of products. Recently, consumers have come to request various kinds of products and the time interval between an order placement and the delivery date has become short. Enterprises always have to cope with these changes.

For problems mentioned above, a common matter for solutions is "improvement of performance of production systems." It is important for manufacturers to produce high quality products at a low cost with meeting their due dates. On manufacturing stages, various costs occur, such as holding costs and backlog costs. To cut down these costs, a lot of production policies have been considered, such as kanban, CONWIP and base stock policies. Recently, there are some production models in which the idea of advance demand information is combined with these production policies. Advance demand information is information on demand for the next several days which is obtained in advance, and by using this information the amount of production at each machine can be decided every day. If we use advance demand information effectively, then the performance of production systems can be improved.

In this thesis, we analyze production/inventory systems with advance demand information.

In section 1.2, we explain various production policies such as kanban, CONWIP and base stock policies.

In section 1.3, we introduce advance demand information and related previous studies. We explain advantages and effects on the use of advance demand information.

In section 1.4, we explain a join-type production line briefly.

In section 1.5, we state the objective of this study.

In section 1.6, we describe the outline of this thesis.

1.2 Production Control Policies

To improve the performance of production systems, it is important for manufacturers to process items into finished products with a lower cost. To do so, it is desirable that the numbers of work-in-processes and backlogs are reduced. A reduction of the number of work-in-processes is, however, in conflict with a reduction of the number of backlogs. To realize low cost production, a lot of production control policies have ever been considered and discussed well. MTO (Make-To-Order), kanban, CONWIP (constant work-in-process), base stock and MRP (Material Requirements Planning) policies are well-known production control policies.

Under MTO, production starts after the demand arrival. That is, a production order placement is triggered by the demand arrival.

Under a kanban policy, the withdrawal kanbans specify the kind and quantity of the parts which the subsequent stage should withdraw from the preceding stage, while the production-ordering kanbans specify the kind and quantity of parts which the preceding stage must produce.

Under a CONWIP policy, the number of work-in-processes in the production line is kept at a constant. That is, the number of work-in-processes in the line is pre-determined and a new item enters in the initial stage every time finished goods leave from the final stage.

Under a base stock policy, a base stock level of each machine is predetermined. The inventory position at each machine is defined as the total amount of inventories that exist in the machine and machines of its downstream. If the inventory position at each machine is less than the base stock level of the machine, the machine produces products until the inventory position reaches the base stock level.

MRP is developed in USA in the 1960's. Under a bill of material and master production schedule which provides the production quantities of independent demand articles, the amount of parts which are required is calculated and the production order is placed according to an available amount of each stock.

Production control policies mentioned above are classified into pull type and push type controls. In pull type policies, the timings of preparations for work, preparations of raw materials and production order placements are informed from downstream to the upstream machines. In push type policies, on the other hand, they are informed from upstream to the downstream machines. MTO and MRP are push type policies, and base stock, kanban and CONWIP policies are pull type policies.

In previous studies on production control policies, for example, Clark and Scarf (1960) have analyzed a tandem line in which each stage sends products to downstream within a deterministic and positive lead time and shown that the base stock policy is optimal. Huh and Janakiraman (2008) have presented a new proof of the optimality of echelon order-up-to policies in serial inventory systems, first proved in the seminal paper by Clark and Scarf (1960). Their proof is based on a simple-path analysis as opposed to the original proof, based on dynamic programming induction. Daniel and Rajendran (2005) have presented simulation-based heuristic methodologies to compute installation base-stock levels in a serial supply chain, which minimize the total supply chain cost. Dogru, van Houtum and de Kok (2008) have dealt with a stochastic serial inventory system with a given fixed batch size per stage and linear inventory holding and penalty costs. They have generalized newsvendor equations for the optimal reorder levels. In Buzacott and Shanthikumar (1993), a unified production line, which includes constant work-in-process (CONWIP), kanban and base stock controls, has been proposed. Dallery and Liberopoulous (2000) have proposed extended kanban production controls, in which a base stock control combined with kanbans is proposed. Veatch and Wein (1996) have analyzed kanban and base stock controls in a two-stage tandem line by assuming exponentially distributed processing times and using the theory of Markov decision processes. They have shown that a base stock control is near-optimal in this line when workload in a upper stage is heavy and the discount cost rate is small. Chen and Song (2001) have analyzed a multi-stage serial inventory system with Markov-modulated demand. They have shown that the optimal policy is an echelon base stock policy with state-dependent order-up-to levels. Shang (2008) has dealt with a multi-stage serial inventory system in continuous-review. They have presented a heuristic for finding base order quantities for stochastic inventory models and shown the heuristic is near optimal. Axsater and Marklund (2008) have considered a continuous-review two-echelon inventory system with one central warehouse and a number of non-identical retailers. They have presented a new policy for warehouse ordering and shown the presented policy is optimal in the broad class of position-base policies based on complete information about retailer inventory positions.

1.3 Advance Demand Information

In most previous studies on production control, it is assumed that each demand requires a finished product at the same time as the arrival of information on demand. In many facilities, however, information on demand for the next several days has been obtained in advance, and the amount of production at each machine is decided every day by using this information. This information is referred to as *advance demand information*, which is abbreviated as ADI. If ADI is used effectively, then the performance of production/inventory systems can be improved. By combining the concept of ADI with existing production control policies such as a base stock policy, a production order can be placed at an appropriate timing and, as a result, the amounts of work-in-processes and backlogs can be reduced. The interval between an arrival of information on demand and the time at which the customer requires a finished product is referred to as *demand lead time*. In the case that demand lead time is longer than production lead time, if the production order is placed at the same time as the arrival of information on demand, then the finished products are completed before the due date. In this case, by delaying the timing of the production order placement afterwards, the amount of finished goods inventories can be reduced. The interval between a production order placement and the due date is referred to as *release lead time*. In the case that demand lead time is shorter than production lead time, if the production order is placed after the arrival of information on demand, then demand can not be met without inventory stocks. In this case, by having inventory stocks, backlogs can be reduced.

Recently, production control problems with ADI have been discussed to improve performance of production-inventory systems. Liberopoulos and Tsikis (2003) have formulated production controls including base stock and kanban control in the tandem production line with ADI. In this model, the order date of each demand at each machine is decided from the predicted production lead time and the due date of the demand. They have expanded the framework for a tandem production line with lot sizing and ADI, and presented hybrid policies that combine an installation kanban policy and an installation stock policy or an echelon stock policy with ADI. Gallego and Ozer (2001) have shown that the state-dependent (s,S) and base stock policies are optimal for stochastic inventory systems with and without fixed costs, respectively, where the state of the system is composed of a modified inventory position which consists of the known requirement and observed demands beyond the protection period (the lead time plus a review period). They have shown that the management need not obtain ADI beyond the protection period for inventory control purposes. Tan, Gullu and Erkip (2007) have developed a model that incorporates imperfect ADI with inventory policies and shown the optimal ordering policy is of state-dependent order up-to type, where the optimal order level is an increasing function of the ADI size. In Karaesmen, Liberopoulos and Dallery (2004), a single-stage M/M/1 make-to-stock production/inventory system is considered and the value of ADI is investigated. The optimal base stock level and release lead time, which minimize the total expected average inventory and backorder related cost, are derived for given demand lead time.

1.4 A Join-Type Production Line

Production controls in serial production lines have been studied by many researchers. Production lines, however, usually have join-type, fork-type or network-type figures. Recently, to cut down costs, a part is used for many products in common. In this case, parts processed at one stage are utilized at the several immediate downstream stages. This type of production line has a fork-type figure. In finished products manufacturing line, on the other hand, several parts from different machines or facilities are assembled into a new product. For example, in an engine assembly line, cylinder blocks are manufactured at an upstream machine. Then at the next machine pistons from another upstream machine are built into the cylinder block. Similarly, at the last machine camshafts from the other upstream machine are built into the cylinder block where pistons are attached, and the engine assembly is completed. This type of production line has a join type figure, which is shown in Fig.1.1. A network-type production line includes join-type and fork-type lines.

Production controls in production lines mentioned above have not been considered well before. The reason is considered as follows. In analysis of these complicated types of production lines, the computation size becomes too large to analyze mathematically

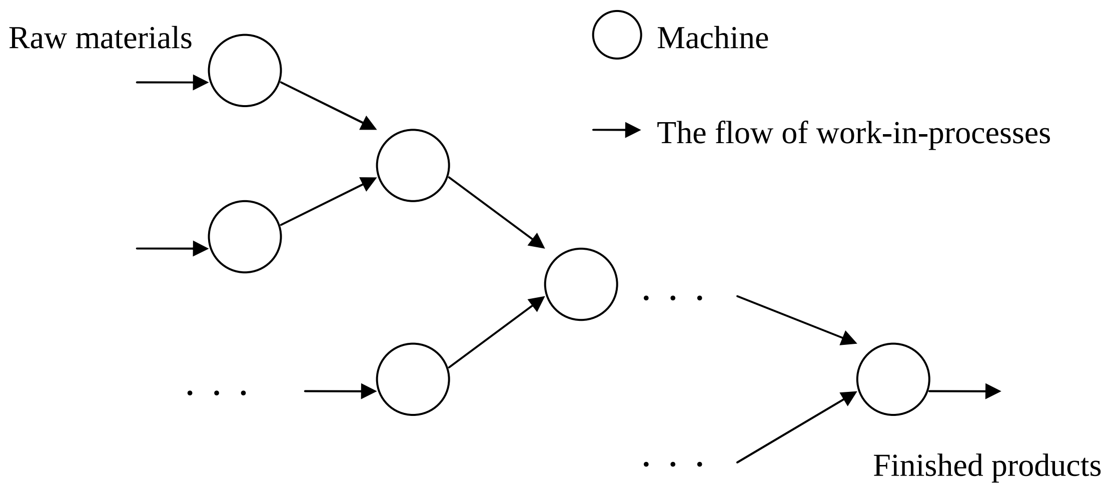

Fig. 1.1 A join-type production line

since the number of random variables increases. In fork-type production line, it is also difficult to determine the sequence in which parts processed at one machine are delivered to the several immediate downstream machines.

To evaluate the performance of production control policies in more practical production lines, it is valuable to analyze production lines such as join type, fork-type and network type lines. Join type production lines are seen at many practical production systems. We deal with multi-stage join-type production lines in chapters 3 and 4 in this thesis.

1.5 Objective of This Study

In many previous studies on production control policies, under given distributions of time intervals of demand arrivals and processing time, production systems are analyzed under the assumption that each demand requires a finished product at the same time as the arrival of information on demand. Recently, the means of conveyance of information on demand and production order placements are changing from physical ways to the way using information technology (IT) and network technology. Therefore all machines have been able to easily obtain information on demand and the amount of inventory stocks at the same time. Therefore, in a lot of practical production lines, the amount of daily production is determined with using information on future demand. By introducing the use of ADI into traditional production policies such as kanban and base stock policies, the performance of traditional production systems can be improved. In this thesis, we analyze production systems with ADI. We analyze a single stage production-inventory system with ADI and derive the optimal base stock level and release lead time theoretically. We also deal with production line with ADI under the base stock policy and propose the simulation-based heuristic algorithm for finding appropriate base stock levels of all machines. Moreover, we deal with a join-type production line with batch production, kanban, and ADI. Since the mechanism of base stock and kanban policies is simple, there are a lot of practical production lines controlled by these existing production control policies without ADI. Since ADI can be easily introduced into these existing production control policies, our models are able to be applied into practical production lines and results of our studies are helpful to solve practical problems on production control.

1.6 Outline

In this section, we outline this thesis.

In chapter 2, we deal with a single stage production-inventory system with a single product and continuous review with ADI. For a given demand lead time, we derive the optimal release lead time and base stock level which maximize the total expected average profit the manufacture receives theoretically. We also investigate a relation between demand lead time and the total expected average profit under the optimal release lead time and base stock level with changing the demand arrival rate function and the ratio of inventory and backlog cost rates.

In chapter 3, we analyze a join type (assembly) production line under base stock control with ADI in discrete time. We propose the simulation-based heuristic algorithm for finding appropriate base stock levels of all machines at short time for determined information delay period, and evaluate the performance. We also show the relations between information delay period and base stock levels found by the algorithm.

In chapter 4, we consider a join-type production line with batch production, kanban and ADI in continuous time. Recursive equations on release times of products at all machines are derived. In numerical examples, the line is simulated on a personal computer by using these equations, and we examine the average inventory and the fraction of backlogs by changing estimated production lead time and initial inventories of each machine.

In chapter 5, we give the conclusion of this thesis and discuss future research.

Chapter 2

Analysis of a Single Stage Production/Inventory System with Advance Demand Information

2.1 Introduction

In this chapter a single stage production/inventory system with advance demand information is analyzed.

The demand lead time usually affects the number of demand. The appropriate length of demand lead time is different between part suppliers and manufacturers. If demand lead time is long, then a manufacturer has to wait for a long time after the order placement. If the manufacturer is not able to wait, he results in having stocks in his warehouse and holding costs are incurred. Therefore, if there are alternative suppliers with short demand lead time, then the manufacturer reduces orders to the supplier with long demand lead time and increases orders to different suppliers with short demand lead time. In the model of this chapter, it is assumed that the arrival rate is strictly decreasing with respect to the demand lead time.

Demand lead time τ is also assumed to be fixed and constant among demands in this model. Fixed demand lead time is often seen between a part supplier and a manufacturer in the group companies. In group companies, a part supplier supplies parts into a single manufacturer, where demand lead time is often fixed and constant. Our model can be applied into such a case.

In this chapter, for a fixed demand lead time τ, we derive theoretically the optimal release lead time and base stock level which maximize the total expected average profit over an infinite horizon. We also investigate how the total expected profit under the optimal release lead time and base stock level changes when the length of demand lead time is changed.

In the next section we introduce a single stage production-inventory system and show the expression on the total expected average profit. In section 2.3 the optimal release lead time and base stock level which maximize the total expected average profit are derived. In section 2.4 we investigate a relation between demand lead time and the total expected average profit under the optimal release lead time and base stock level. In section 2.5 we conclude the study of this chapter..

2.2 A Single Stage Production-Inventory System

2.2.1 Model Assumption

A single stage production-inventory system with a single product and continuous review is considered. The information on demand which requires a finished product at time t is obtained at time $t-\tau$, where τ is referred to as the *demand lead time*. Demand follows a Poisson arrival process and the arrival rate depends on the demand lead time. The rate is denoted by $\lambda(\tau)$. The processing time follows an exponential distribution with rate μ. It is assumed that $0<\lambda(\tau)<\mu$ and $\lambda'(\tau)<0$ for all $\tau \geq 0$. A parameter L, which is the margin between a production order placement and the delivery time, is defined as the *release lead time*. It is assumed that $L \leq \tau$. If the system obtains information on demand at time $t-\tau$, then the production order is placed at time $t-L$. A production cost for one product is c. A holding cost is incurred for finished products as long as they remain in the system. The holding cost for one product per unite time is $h\ (>0)$. When a finished product is delivered to the customer, the manufacturer receives a reward r. When demand is not filled, it is backlogged and the backlog cost $b\ (>0)$ is incurred for one backlog per unit time. The parameter S denotes a base stock level of the system and is assumed to be non-negative.

2.2.2 Notations

τ : demand lead time,

$\lambda(\tau)$: a demand arrival rate when demand lead time is τ,

μ: a service rate of a machine,

$\rho(\tau)$: $\dfrac{\lambda(\tau)}{\mu}$,

L: release lead time,

c: a production cost for one product,

h: a holding cost for one product per unit time,

r: a reward for one product which the manufacture receives,

b: a backlog cost for one product per unit time,

S: a base stock level of the system,

X_n: a random variable which denotes a time interval between the $n-S$ th and n th demand arrivals,

W_n: a random variable which denotes the time interval from the $n-S$ th production order placement to finishing production of the corresponding product,

$p(\tau,S,L)$: the total expected average profit,

$L^*(\tau)$: the optimal release lead time,

$S^*(\tau)$: the optimal base stock level,

$L^*(\tau,S)$: a solution of $\dfrac{\partial p(\tau,S,L)}{\partial L}=0$,

$\hat{S}(\tau)$: a solution of $\dfrac{\partial p(\tau,S,\tau)}{\partial S}=0$,

τ_1: a positive solution of $L^*(\tau,0)=\tau$,

τ_2: a positive solution of $\hat{S}(\tau)=0$,

$p(\tau)$: $p(\tau)=p(\tau,S^*(\tau),L^*(\tau))$,

$reward(\tau)$: rewards under $L=L^*(\tau)$ and $S=S^*(\tau)$ when demand lead time is τ,

$cost(\tau)$: $cost(\tau) = p(\tau) - reward(\tau)$,

τ^* : demand lead time τ at which $p(\tau)$ is maximized,

$\bar{S}(\tau)$: a solution of $L^*(\tau, S) = 0$,

$\tilde{S}(\tau)$: a solution of $L^*(\tau, S) = \tau$.

2.2.3 Total Expected Average Profit

The expected average profit is derived in the similar way to Karaesmen, et al.(2004). The relation between the demand arrival and the production order placement is shown in Fig.2.1.

Let X_n be a random variable which denotes a time interval between the $n-S$ th demand arrival and the n th one. Since the time interval of the demand arrival is exponentially distributed, the random variable X_n follows the Erlang distribution with parameters $(\lambda(\tau), S)$. Then the probability density function of random variable X_n

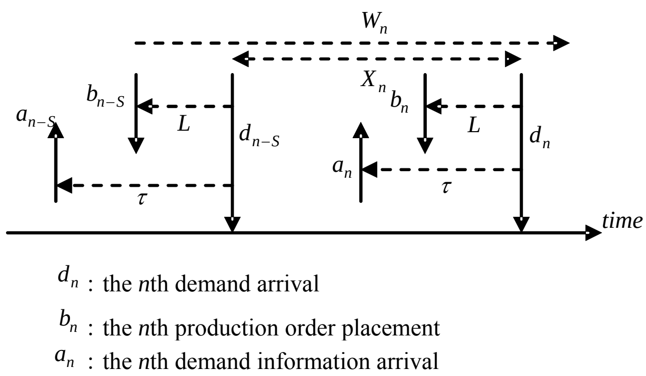

Fig. 2.1 The relation between the demand arrival and the production order placement

can be expressed as

$$f_X(x) = \frac{\lambda(\tau)^S x^{S-1}}{(S-1)!} e^{-\lambda(\tau)x}.$$

Since the base stock level of the system is S, the finished product which the n th customer receives corresponds to the one whose processing is triggered by the $n-S$ th production order placement. A random variable W_n denotes the time interval from the $n-S$ th production order placement to finishing production of the corresponding product. Then, the limiting distribution of W_n is equal to the limiting sojourn time distribution of an M/M/1 queue with arrival rate $\lambda(\tau)$ and service rate μ, and the distribution function is given by

$$F_W(w) = 1 - e^{-\mu(1-\rho(\tau))w},$$

where $\rho(\tau) = \frac{\lambda(\tau)}{\mu}$. From assumptions of $\lambda(\tau)$ we have $0 < \rho(\tau) < 1$ and $\rho'(\tau) < 0$ for all $\tau \geq 0$. Note that W_n and X_n are independent.

If $W_n < X_n + L$, the production is finished before the n th demand arrival and a holding cost is incurred for the finished product. If $W_n > X_n + L$, the production is not finished by the n th demand arrival and the demand is backlogged and a backlog cost is incurred. Let random variables X and W follow the same distribution as X_n and W_n, respectively. Then the total expected average profit $p(\tau,S,L)$ over an infinite horizon is given by

$$\begin{aligned} p(\tau,S,L) &= \lambda(\tau)\Big(r - c - hE\big[(X+L-W)^+\big] - bE\big[(W-X-L)^+\big]\Big) \\ &= \lambda(\tau)\Big(r - c - hE\big[(X+L-W) + (W-X-L)^+\big] - bE\big[(W-X-L)^+\big]\Big) \qquad (2.1) \\ &= \lambda(\tau)\Big(r - c - hE[X+L-W] - (h+b)E\big[(W-X-L)^+\big]\Big), \end{aligned}$$

where $a^{+} = \max\{a, 0\}$. Then

$$E[X + L - W] = \frac{S}{\lambda(\tau)} + L - \frac{1}{\mu(1-\rho(\tau))}, \tag{2.2}$$

and

$$E\left[(W - X - L)^{+}\right] = \int_0^\infty \int_{x+L}^\infty (w - x - L) f_W(w) dw f_X(x) dx$$
$$= \int_0^\infty \frac{e^{-\mu(1-\rho(\tau))(x+L)}}{\mu(1-\rho(\tau))} f_X(x) dx = \frac{\rho(\tau)^S}{\mu(1-\rho(\tau))} e^{-\mu(1-\rho(\tau))L}. \tag{2.3}$$

From Eqs. (2.1) through (2.3), the total expected average profit $p(\tau, S, L)$ over an infinite horizon can be calculated as follows:

$$p(\tau, S, L) = \lambda(\tau)(r - c) - h\left(S + \lambda(\tau)L - \frac{\rho(\tau)}{1-\rho(\tau)}\right) - (h+b)\frac{\rho(\tau)^{S+1}}{1-\rho(\tau)} e^{-\mu(1-\rho(\tau))L}. \tag{2.4}$$

2.3 Optimal Release Lead Time and Base Stock Level

For given demand lead time τ, we derive the release lead time $L^*(\tau)$ and the base stock level $S^*(\tau)$ which maximize the expected average profit.

First, we show the following proposition.

Proposition 2.1 For given S and τ, $p(\tau, S, L)$ is concave with respect to L, and $p(\tau, S, L)$ is maximized at $L = L^*(\tau, S)$ on $L \in (-\infty, \infty)$, where

$$L^*(\tau, S) = -\frac{1}{\mu(1-\rho(\tau))}\left(\log\frac{h}{h+b} - S\log\rho(\tau)\right). \tag{2.5}$$

For given τ, $p(\tau, S, L^*(\tau, S))$ is strictly decreasing with respect to S. □

Proof of Proposition 2.1:

From Eq. (2.4),

$$\frac{\partial p(\tau,S,L)}{\partial L}=-h\lambda(\tau)+(h+b)\mu\rho(\tau)^{S+1}e^{-\mu(1-\rho(\tau))L},$$

and

$$\frac{\partial^2 p(\tau,S,L)}{\partial L^2}=-(h+b)\mu^2(1-\rho(\tau))\rho(\tau)^{S+1}e^{-\mu(1-\rho(\tau))L}<0. \tag{2.6}$$

From Eq.(2.6), for given S and τ, $p(\tau,S,L)$ is a concave function with respect to L. Since $L^*(\tau,S)$ satisfies $\frac{\partial p(\tau,S,L)}{\partial L}=0$, $p(\tau,S,L)$ is maximized at $L=L^*(\tau,S)$ on $L\in(-\infty,\infty)$.

From Eqs.(2.4) and (2.5),

$$p(\tau,S,L^*(\tau,S))=\lambda(\tau)(r-c)-h\left\{S-\frac{\rho(\tau)}{1-\rho(\tau)}\left(\log\frac{h}{h+b}-S\log\rho(\tau)\right)\right\}. \tag{2.7}$$

From Eq.(2.7),

$$\frac{\partial p(\tau,S,L^*(\tau,S))}{\partial S}=h\left(-1+\frac{\rho(\tau)\log\rho(\tau)}{1-\rho(\tau)}\right)<0. \tag{2.8}$$

From Eq. (2.8), for given τ, $p(\tau,S,L^*(\tau,S))$ is strictly decreasing in S. □

For given τ and S, $p(\tau,S,L)$ is concave with respect to L. For given τ and S, $p(\tau,S,L)$ is strictly increasing on $L\in\left[-\infty,L^*(\tau,S)\right]$ and strictly decreasing on $L\in\left(L^*(\tau,S),\infty\right]$. Therefore, for given τ and S, on $L\in[0,\infty]$, $p(\tau,S,L)$ is maximized at $L=\tau$ when $\tau\le L^*(\tau,S)$, at $L=L^*(\tau,S)$ when $0\le L^*(\tau,S)<\tau$ and at $L=0$ when $L^*(\tau,S)<0$.

Let $\hat{S}(\tau)$ satisfy $\frac{\partial p(\tau,S,\tau)}{\partial S}=0$ for given τ. Then from Eq.(2.4) it is uniquely given by

$$\hat{S}(\tau)=\frac{1}{\log\rho(\tau)}\log\left\{\frac{h}{h+b}\cdot\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}\right\}. \quad (2.9)$$

Proposition 2.2 The equation $L^*(\tau,0)=\tau$ has a unique positive solution τ_1. If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)}<\frac{h+b}{h}$, then the equation $\hat{S}(\tau)=0$ has a unique positive solution τ_2 and it holds that $0<\tau_2<\tau_1$. If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)}=\frac{h+b}{h}$, then the equation $\hat{S}(\tau)=0$ has a unique solution $\tau_2=0$. If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)}>\frac{h+b}{h}$, then the equation $\hat{S}(\tau)=0$ has no non-negative solutions. □

Proof of Proposition 2.2:

If $L^*(\tau,0)=\tau$, then from Eq.(2.5) we have

$$-\frac{1}{\mu(1-\rho(\tau))}\log\frac{h}{h+b}-\tau=0. \quad (2.10)$$

It holds that

$$\frac{d}{d\tau}\left(-\frac{1}{\mu(1-\rho(\tau))}\log\frac{h}{h+b}-\tau\right)=\frac{-\rho'(\tau)}{\mu(1-\rho(\tau))^2}\log\frac{h}{h+b}-1<0 \quad (2.11)$$

since $\rho'(\tau)<0$ and $b>0$.

Since $\rho'(\tau)<0$, $0<\rho(\tau)<1$ for $\tau>0$ and $b>0$, we have

$$\lim_{\tau\to\infty}\left(-\frac{1}{\mu\left(1-\rho(\tau)\right)}\log\frac{h}{h+b}-\tau\right)=-\infty, \tag{2.12}$$

and

$$\lim_{\tau\to 0}\left(-\frac{1}{\mu\left(1-\rho(\tau)\right)}\log\frac{h}{h+b}-\tau\right)=-\frac{1}{\mu\left(1-\rho(0)\right)}\log\frac{h}{h+b}>0. \tag{2.13}$$

From Eqs.(2.11) through (2.13), Eq.(2.10) has a unique positive solution τ_1.

If $\hat{S}(\tau)=0$, then from Eq.(2.9) we have

$$\frac{h}{h+b}\cdot\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}=1. \tag{2.14}$$

Since $\rho'(\tau)<0$ and $0<\rho(\tau)<1$, $\frac{d}{d\tau}e^{\mu(1-\rho(\tau))\tau}=\mu\left(-\rho'(\tau)\tau+1-\rho(\tau)\right)e^{\mu(1-\rho(\tau))\tau}>0$ and $\lim_{\tau\to\infty}e^{\mu(1-\rho(\tau))\tau}=\infty$. We consider $\frac{x-1}{x\log x}$ for $0<x<1$ to show that $\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}$ is strictly increasing with respect to τ. We have $\frac{d}{dx}\left(\frac{x-1}{x\log x}\right)=\frac{\log x-x+1}{\left(x\log x\right)^2}$. Since $\frac{d}{dx}\left(\log x-x+1\right)=\frac{1}{x}-1>0$ for $0<x<1$, $\lim_{x\to+0}\left(\log x-x+1\right)=-\infty$ and $\lim_{x\to 1}\left(\log x-x+1\right)=0$, it holds that $\frac{d}{dx}\left(\frac{x-1}{x\log x}\right)<0$ for $0<x<1$. Since $\rho'(\tau)<0$ and $0<\rho(\tau)<1$, $\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}$ is strictly increasing with respect to τ. It holds that $\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}>0$, and so $\frac{h}{h+b}\cdot\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}$ is strictly increasing with respect to τ. We also have

$$\lim_{\tau\to\infty}\frac{h}{h+b}\cdot\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}=\infty \quad ,$$

and

$$\lim_{\tau\to 0}\frac{h}{h+b}\cdot\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}=\frac{h}{h+b}\cdot\frac{\rho(0)-1}{\rho(0)\log\rho(0)}.$$

If $\frac{h}{h+b}\cdot\frac{\rho(0)-1}{\rho(0)\log\rho(0)}<1$, then Eq.(2.14) has a unique positive solution τ_2. If $\frac{h}{h+b}\cdot\frac{\rho(0)-1}{\rho(0)\log\rho(0)}=1$, then Eq.(2.14) has a unique solution $\tau_2=0$. If $\frac{h}{h+b}\cdot\frac{\rho(0)-1}{\rho(0)\log\rho(0)}>1$, then Eq.(2.14) has no non-negative solutions.

We discuss the relation between τ_1 and τ_2 when $\frac{h}{h+b}\cdot\frac{\rho(0)-1}{\rho(0)\log\rho(0)}<1$.

Since $L^*(\tau_1,0)=\tau_1$, from Eq.(2.5) we have

$$\rho(\tau_1)=\frac{1}{\mu\tau_1}\log\frac{h}{h+b}+1.$$

Since $\hat{S}(\tau_2)=0$, from Eq.(2.9) we have

$$\rho(\tau_2)=\frac{1}{\mu\tau_2}\log\frac{h}{h+b}+1+\frac{1}{\mu\tau_2}\log\frac{\rho(\tau_2)-1}{\rho(\tau_2)\log\rho(\tau_2)}.$$

Since $\frac{d}{dx}\left(\frac{x-1}{x\log x}\right)<0$ and $\lim_{x\to 1}\frac{x-1}{x\log x}=1$ for $0<x<1$, it holds that

$$\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}>1.$$

Therefore $\frac{1}{\mu\tau}\log\frac{h}{h+b}+1<\frac{1}{\mu\tau}\log\frac{h}{h+b}+1+\frac{1}{\mu\tau}\log\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}$ for $\tau>0$. Since $\rho'(\tau)<0$ and $\frac{d}{d\tau}\left(\frac{1}{\mu\tau}\log\frac{h}{h+b}+1\right)=-\frac{1}{\mu\tau^2}\log\frac{h}{h+b}>0$, it holds that $\tau_2<\tau_1$. □

The release lead time $L^*(\tau)$ and the base stock level $S^*(\tau)$ which maximize $p(\tau,S,L)$ are expressed as follows.

Theorem 2.1 If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)}\geq\frac{h+b}{h}$,

$L^*(\tau)=\tau$ and $S^*(\tau)=0$, for $\tau\in[0,\tau_1)$ and

$L^*(\tau)=L^*(\tau,0)$ and $S^*(\tau)=0$, for $\tau\in[\tau_1,\infty)$.

If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)}<\frac{h+b}{h}$,

$L^*(\tau)=\tau$ and $S^*(\tau)=\hat{S}(\tau)$, for $\tau\in[0,\tau_2)$,

$L^*(\tau)=\tau$ and $S^*(\tau)=0$, for $\tau\in[\tau_2,\tau_1)$ and

$L^*(\tau)=L^*(\tau,0)$ and $S^*(\tau)=0$ for $\tau\in[\tau_1,\infty)$. □

Proof of Theorem 2.1:

Let $f(\tau)=L^*(\tau,0)-\tau$. Then

$$f'(\tau)=\frac{-\rho'(\tau)}{\mu(1-\rho(\tau))^2}\log\frac{h}{h+b}-1<0.$$

Therefore if $L^*(\tau,0)\le\tau$, then $\tau\ge\tau_1$, and if $L^*(\tau,0)>\tau$, then $\tau<\tau_1$.

Assume $L^*(\tau,0)\le\tau$. From Eq. (2.5) $L^*(\tau,0)\ge 0$. Since $L^*(\tau,S)$ is strictly decreasing with respect to S from Eq. (2.5), $L^*(\tau,S)\le\tau$ for $S\in[0,\infty)$. Let $\bar{S}(\tau)$ satisfy $L^*(\tau,S)=0$. From proposition 2.1, for $L=0$, $p(\tau,S,L)$ is maximized at $S=\bar{S}(\tau)$ on $S\in\left[\bar{S}(\tau),\infty\right)$. Therefore from proposition 2.1, for $L\in[0,\tau]$ and $S\in[0,\infty)$, $p(\tau,S,L)$ is maximized at $(L,S)=\left(L^*(\tau,0),0\right)$ for given $\tau\in[\tau_1,\infty)$.

Assume $L^*(\tau,0)>\tau$. From Eq.(2.5) $\tilde{S}(\tau)$ which satisfies $L^*(\tau,S)=\tau$ is uniquely given by

$$\tilde{S}(\tau)=\frac{1}{\log\rho(\tau)}\left(\log\frac{h}{h+b}+\mu\left(1-\rho(\tau)\right)\tau\right). \tag{2.15}$$

From proposition 2.1, for $S\in\left[\tilde{S}(\tau),\infty\right)$, $p(\tau,S,L)$ is maximized at $(L,S)=\left(L^*\left(\tau,\tilde{S}(\tau)\right),\tilde{S}(\tau)\right)=\left(\tau,\tilde{S}(\tau)\right)$ for given $\tau\in[0,\tau_1)$. Since $L^*(\tau,S)$ is strictly decreasing with respect to S, $L^*(\tau,S)>\tau$ for given $S\in\left[0,\tilde{S}(\tau)\right)$. Therefore from proposition 2.1, for given $S\in\left[0,\tilde{S}(\tau)\right)$, $p(\tau,S,L)$ is maximized at $L=\tau$ on $L\in[0,\tau]$. We investigate the relation between $p(\tau,S,\tau)$ and S. Let $p(\tau,S)=p(\tau,S,\tau)$. Then

$$p(\tau,S)=\lambda(\tau)(r-c)-h\left(S+\lambda(\tau)\tau-\frac{\rho(\tau)}{1-\rho(\tau)}\right)-(h+b)\frac{\rho(\tau)^{S+1}}{1-\rho(\tau)}e^{-\mu(1-\rho(\tau))\tau}.$$

We have

$$\frac{\partial p(\tau,S)}{\partial S}=-h-(h+b)\frac{\rho(\tau)}{1-\rho(\tau)}e^{-\mu(1-\rho(\tau))\tau}\rho(\tau)^{S}\log\rho(\tau), \tag{2.16}$$

and

$$\frac{\partial^2 p(\tau,S)}{\partial S^2}=-(h+b)\frac{\rho(\tau)}{1-\rho(\tau)}e^{-\mu(1-\rho(\tau))\tau}\rho(\tau)^{S}\left(\log\rho(\tau)\right)^2<0. \tag{2.17}$$

From the definition of $\hat{S}(\tau)$, $p(\tau,S)$ is maximized at $S=\hat{S}(\tau)$ on $S\in(-\infty,\infty)$.

Since $\dfrac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}>1$, which is shown in proof of proposition 2.2, from Eqs.(2.15) and (2.16) we have

$$\left.\frac{\partial p(\tau,S)}{\partial S}\right|_{S=\tilde{S}(\tau)}=-h\left(1-\frac{\rho(\tau)\log\rho(\tau)}{\rho(\tau)-1}\right)<0.$$

Therefore $\hat{S}(\tau)<\tilde{S}(\tau)$. Note that $\dfrac{h}{h+b}\cdot\dfrac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}$ is strictly increasing with respect to τ from proof of proposition 2.2 and $\log\rho(\tau)<0$. From proposition 2.2, if $\dfrac{\rho(0)-1}{\rho(0)\log\rho(0)}\geq\dfrac{h+b}{h}$, then $\dfrac{h}{h+b}\cdot\dfrac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}\geq 1$ for $0\leq\tau<\tau_1$.

Therefore, from Eq.(4.9) $\hat{S}(\tau)\leq 0$ for $0\leq\tau<\tau_1$. If $\dfrac{\rho(0)-1}{\rho(0)\log\rho(0)}<\dfrac{h+b}{h}$, then $\dfrac{h}{h+b}\cdot\dfrac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}<1$ for $0\leq\tau<\tau_2$ and $\dfrac{h}{h+b}\cdot\dfrac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}\geq 1$ for $\tau_2\leq\tau<\tau_1$. Therefore, from Eqs.(2.9) and (2.17), $\hat{S}(\tau)>0$ for $0\leq\tau<\tau_2$ and $\hat{S}(\tau)\leq 0$ for $\tau_2\leq\tau<\tau_1$.

From above, theorem 2.1 can be derived. □

From theorem 2.1, we get the following insights. When τ is small, the arrival time interval between demand information and the corresponding demand is small. Therefore the production order is placed at the same time as the arrival of demand information in order to complete processing a product by the corresponding demand arrival. When τ is large, the production order is placed at a few periods after the arrival of demand information in order to reduce holding cost. If b is large, then it tends to hold that $\frac{\rho(0)-1}{\rho(0)\log\rho(0)} < \frac{h+b}{h}$. Then, the optimal base stock level is positive for small τ. This reason is as follows. When τ is small, the processing of a product may not be completed by the arrival of corresponding demand with no finished products in stock. Hence the number of backlogs is reduced by having finished products in stock.

From theorem 2.1, $L^*(\tau) = L^*(\tau, 0)$ when $\tau \geq \tau_1$. Then $L^*(\tau)$ is decreasing for increase of τ from Eq.(2.5) and the assumption of $\rho(\tau)$. If τ is large, a demand arrival rate per unit time decreases from assumption. It reduces possibility that processing of the preceding product is not completed when a production order is placed. This makes the optimal release lead time decrease.

Let $p(\tau) = p(\tau, S^*(\tau), L^*(\tau))$. From the theorem 2.1, $p(\tau)$ can be expressed as follows:

If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)} \geq \frac{h+b}{h}$,

$$p(\tau) = \rho(\tau)(r-c)\mu - h\left\{\mu\rho(\tau)\tau - \frac{\rho(\tau)}{1-\rho(\tau)}\right\} - (h+b)\frac{\rho(\tau)}{1-\rho(\tau)}e^{-\mu(1-\rho(\tau))\tau}$$

$$\text{for} \quad \tau \in [0, \tau_1),$$

and

$$p(\tau) = \rho(\tau)(r-c)\mu + h\frac{\rho(\tau)}{1-\rho(\tau)}\log\frac{h}{h+b}, \qquad \text{for} \quad \tau \in [\tau_1, \infty).$$

If $\frac{\rho(0)-1}{\rho(0)\log\rho(0)} < \frac{h+b}{h}$,

$$p(\tau) = \rho(\tau)(r-c)\mu - h\left\{\frac{1}{\log\rho(\tau)}\log\left(\frac{h}{h+b}\cdot\frac{\rho(\tau)-1}{\rho(\tau)\log\rho(\tau)}\cdot e^{\mu(1-\rho(\tau))\tau}\right)+\mu\rho(\tau)\tau-\frac{\rho(\tau)}{1-\rho(\tau)}\right\}+\frac{h}{\log\rho(\tau)}$$

$$\text{for} \quad \tau\in[0,\tau_2),$$

$$p(\tau) = \rho(\tau)(r-c)\mu - h\left\{\mu\rho(\tau)\tau-\frac{\rho(\tau)}{1-\rho(\tau)}\right\}-(h+b)\frac{\rho(\tau)}{1-\rho(\tau)}e^{-\mu(1-\rho(\tau))\tau}$$

$$\text{for} \quad \tau\in[\tau_2,\tau_1),$$

and

$$p(\tau) = \rho(\tau)(r-c)\mu + h\frac{\rho(\tau)}{1-\rho(\tau)}\log\frac{h}{h+b}, \qquad \text{for} \quad \tau\in[\tau_1,\infty).$$

2.4 Numerical Examples

In this section, for given $\rho(\tau)$, we provide some insights into the relation

Table 2.1 Numerical results

b	a	τ_1	τ_2	τ^*	$p(\tau^*)$
2	0.01	10.8	10.5	31.6	7.98
	0.1	3.62	3.30	3.43	7.95
	1	1.44	0.95	0.32	7.54
10	0.01	16.1	15.8	51.1	5.40
	0.1	5.60	5.22	5.48	5.38
	1	2.59	1.72	0.54	4.47

between $p(\tau)$ and τ. Let $\rho(\tau)=e^{-a\tau}$, where a is constant and positive. Then $\rho(\tau)$ satisfies $0<\rho(\tau)<1$ and $\rho'(\tau)<0$. We use the following combination of parameters:

$$h=1,\ r=20,\ c=5,\ \mu=1,$$

and deal with cases of $b=2$ and 10. Note that under these parameters and $\rho(\tau)$, it

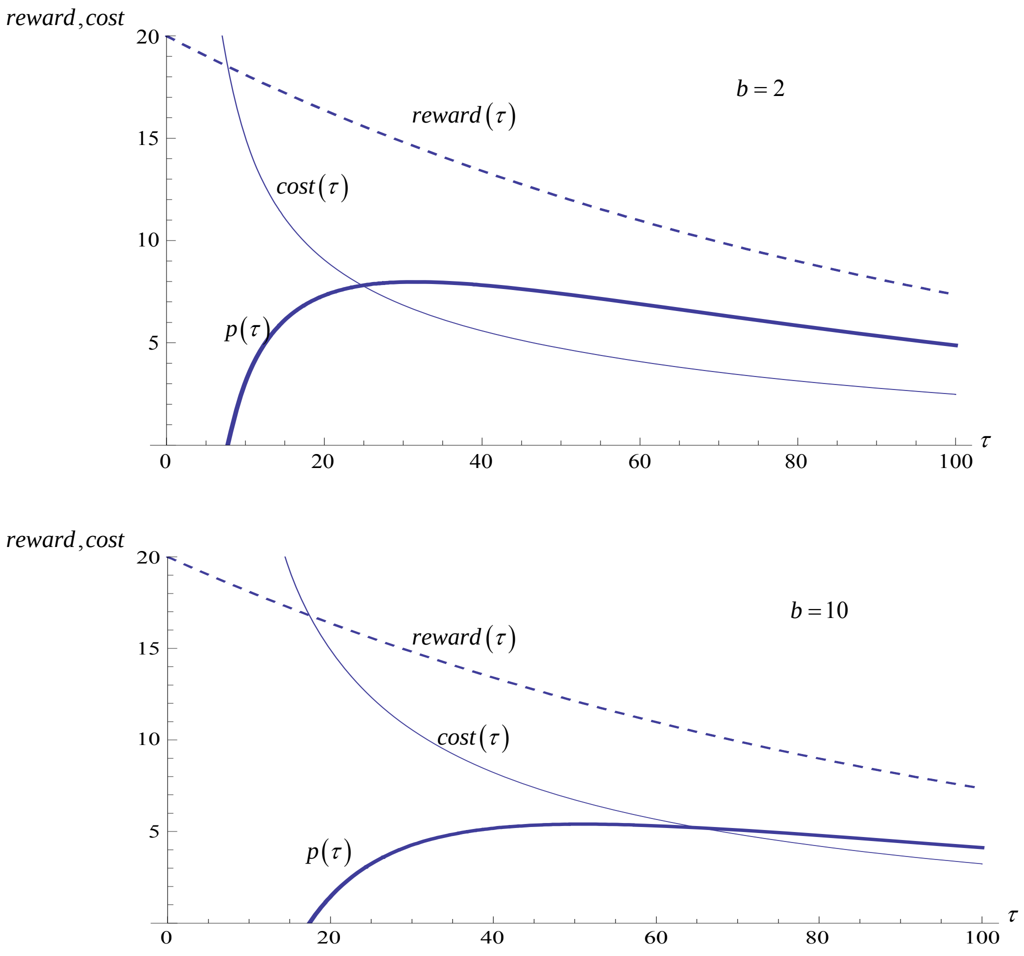

Fig. 2.2 The relation between the total expected profit and demand lead time under $a=0.01$

holds that $\dfrac{\rho(0)-1}{\rho(0)\log\rho(0)} < \dfrac{h+b}{h}$ since $\lim_{\tau\to 0}\dfrac{e^{-a\tau}-1}{e^{-a\tau}\log e^{-a\tau}} = 1$.

First, for the cases of $a=1$, 0.1 and 0.01, we investigate τ_1, τ_2, τ^* and $p(\tau^*)$, where the total expected average profit becomes the largest at $\tau=\tau^*$. See Table 2.1. We also show the relation between $p(\tau)$ and τ in Figs.2.2-2.4, where $reward(\tau)=\rho(\tau)\mu r$ and $cost(\tau)=p(\tau)-reward(\tau)$. When $a=0.01$, it holds that

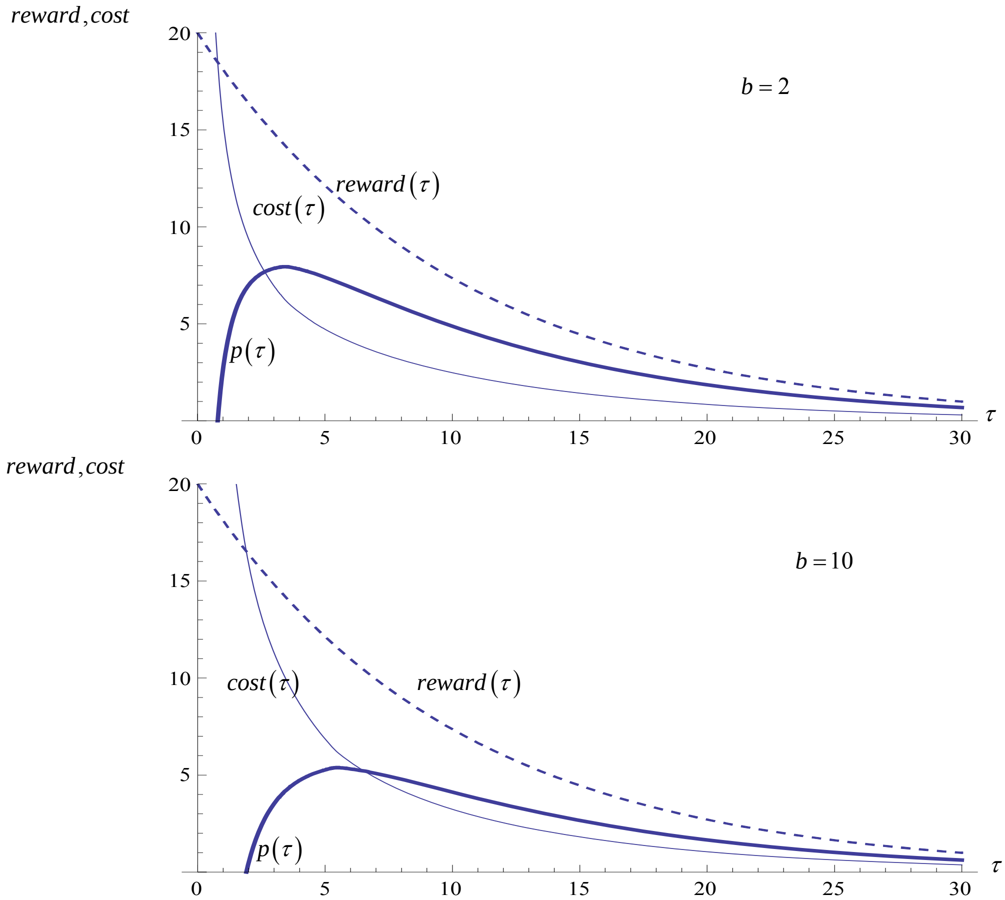

Fig. 2.3 The relation between the total expected profit and demand lead time under $a=0.1$

$\tau_1 < \tau^*$, when $a = 0.1$, $\tau_2 < \tau^* < \tau_1$, and when $a = 1, 0 < \tau^* < \tau_2$. From Table 2.1 and Fig.2.2-2.4, we get the following insights about the relation between a and τ^*.

(1) When a is large, that is, when the arrival rate of demand is rapidly decreasing for increase of demand lead time, the total expected average profit becomes the largest for short demand lead time.

(2) When a is small, that is, when the arrival rate of demand is slowly decreasing for

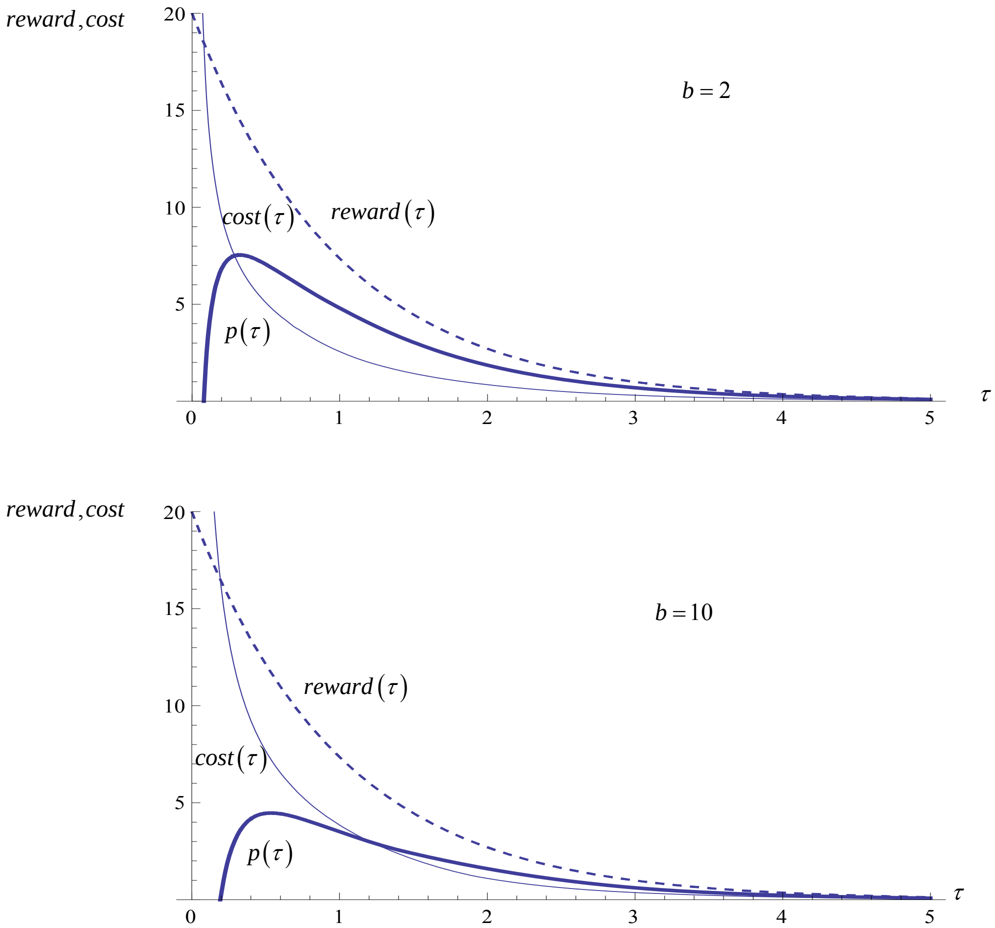

Fig. 2.4 The relation between the total expected profit and demand lead time under $a = 1$

increase of demand lead time, the total expected average profit becomes the largest for long demand lead time.

From Table 2.1, the smaller a becomes, the longer τ^* which maximizes $p(\tau)$ becomes. When a is large, the increase of demand lead time leads to the great decrease of the number of demand. This causes a great decrease of a reward which the manufacturer receives. Therefore a small demand lead time increases the total expected average profit. When a is small, the number of demand is slowly decreasing for increase of demand lead time. Then a reward which the manufacturer receives does not change so much for increase of demand lead time. Increase of demand lead time reduces backlogs without increase of base stock levels. Hence a large demand lead time reduces backlog costs and increases total expected average profit.

Next, we investigate with changing the ratio of b and h. We compare the cases of $b = 2$ and 10 with keeping $h = 1$. From Table 2.1 and Fig.2.2-2.4, we get the following insights about the relation between b and τ^*.

(3) For a given τ, when b is increased, the total expected average profit becomes small.

If b becomes large, a backlog cost in itself increases but a reward does not change for a given τ. Hence, the total expected average profit becomes small.

(4) When b is small, τ^* becomes short, that is, demand lead time at which the total expected average profit is maximized becomes short.

From Figs.2.2-2.4, for short τ, $cost(\tau)$ under $b = 2$ decrease faster than that under $b = 10$. This reason is as follows. From Table 2.1, when $b = 10$, τ_2 is larger than when $b = 2$. Therefore, from proposition 2.1, when $b = 10$, there are more stocks than when $b = 2$ for short τ. Hence, in the case of $b = 10$, $cost(\tau)$ decreases more slowly for short τ. On the other hand, $reward(\tau)$ of $b = 10$ are the same as that of

$b=2$ for a given τ. Therefore, when $b=2$, the difference between $reward(\tau)$ and $cost(\tau)$ is maximized at shorter τ than when $b=10$.

2.5 Conclusion

We consider a single stage production-inventory system with a single product and continuous review. For fixed demand lead time, the optimal release lead time and base stock level which maximize the total expected average profit are derived theoretically. In the case that a backlog cost rate is not so large for a holding cost rate, the optimal release lead time equals to demand lead time and the optimal base stock level is zero for short demand lead time, and the optimal release lead time decreases and the optimal base stock level remains zero for long demand lead time. In the case that a backlog cost rate is so large for a holding cost rate, the optimal release lead time equals to demand lead time and the optimal base stock level is positive for very short demand lead time, and otherwise the same as the former case. In numerical examples, we show when the arrival rate of demand is slowly decreasing for increase of demand lead time, the total expected average profit becomes the largest for long demand lead time. We also show that when backlog cost rate is small demand lead time at which the total expected average profit is maximized becomes short.

Chapter 3

Base Stock Policy in a Join-Type Production Line with Advance Demand Information

3.1 Introduction

In this chapter, a join-type (assembly) production line under base stock control with advance demand information in discrete time is analyzed. The time interval between the arrival of ADI and the production order placement for informed ADI is referred to as *information delay period*. In the base stock policy, base stock levels of upstream machines are greater than ones of downstream. In machines in which the number of machines existing in the downstream is the same, the base stock levels of the machine which has upstream machines are desired to be greater than ones of the machine which has no upstream machines. Using these properties, we propose the simulation-based heuristic algorithm for finding appropriate base stock level of each machine for a given information delay period of each machine. In numerical examples the relation between information delay period and base stock level found by the algorithm is shown.

In section 3.2 a multi-stage join-type production line is described. In section 3.3 the base stock policy with advance demand information is explained and the recursive equations are derived. In section 3.4 the heuristic algorithm for appropriate base stock levels is proposed. In section 3.5 the model is simulated on a personal computer with using proposed algorithm, and the relation between information delay period and base stock level found by the algorithm is discussed. In section 3.6 we conclude the study of this chapter.

3.2 A Multi-Stage Join-Type Production Line

3.2.1 Model

A join-type production line with K machines is shown in Fig. 3.1. A set of machines in the production line is denoted by $\hat{K} = \{1, 2, \cdots, K\}$, and machine K produces finished products. At each machine parts from multiple preceding processes one by one are processed or assembled as a single new product, and it is sent to the next machine. Let the machine following machine i be denoted by $d(i)$ for $i \in \hat{K} - \{K\}$. Products processed at machine i for $i \in \hat{K}$ are placed in the buffer for finished products of machine i, which is denoted by FG_i. The quantity of finished products in FG_i at the beginning of period n for $n \in \{1, 2, \cdots\}$ is defined as $J_i(n)$. At the end of period n for $n \in \{1, 2, \cdots\}$, $Q_i(n)$ products leave FG_i for $i \in \hat{K} - \{K\}$, and at the end of period $n + T_i$ they arrive at the buffer for unfinished products, which is denoted by ST_i for $i \in \hat{K} - \{K\}$. The maximal amount of products delivered from FG_i to ST_i for $i \in \hat{K} - \{K\}$ is denoted by U_i. The amount of unfinished products in ST_i at the beginning of period n is denoted by $I_i(n)$ for $i \in \hat{K} - \{K\}$ and $n \in \{1, 2, \cdots\}$. The

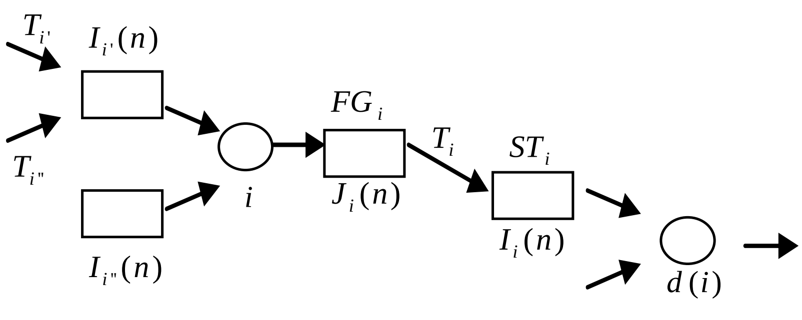

Fig. 3.1 A Join-type production line

capacities of FG_i for $i \in \hat{K}$ and ST_i for $i \in \hat{K} - \{K\}$ are denoted by B_i and A_i, respectively. At machine K, finished products are placed in FG_K, and the amount of finished products in FG_K at the beginning of period n is denoted by $J_K(n)$ for $n \in \{1, 2, \cdots\}$. When $J_K(n) > 0$, there are $J_K(n)$ products. When $J_K(n) < 0$, $-J_K(n)$ demands are backlogged. At the beginning of period n for $n \in \{1, 2, \cdots\}$, production order quantity $P_i(n)$ of machine i for $i \in \hat{K}$ is decided. Production capacity of machine i for $i \in \hat{K}$ in period n for $n \in \{1, 2, \cdots\}$ is distributed with $p_k^i = P(C_i(n) = k)$ for $k = 0, 1, \cdots, C_i$ and independent among periods, where $C_i(n)$ denotes the production quantity of machine i for in period n and C_i denotes the maximal quantity of production of machine i. The amount of products processed at machine i for $i \in \hat{K}$ in period n for $n \in \{1, 2, \cdots\}$ is denoted by $P_i'(n)$. Demand information informed in period n for $n \in \{1, 2, \cdots\}$ is denoted by $D(n)$, which has the distribution $q_k = P(D(n) = k)$ for $k = 0, 1, \cdots$ and is independent among periods. $D(n)$ products are required in period $n + L$, where L is demand lead time and deterministic. A set of machines in a downstream of machine i for $i \in \hat{K}$ is denoted by $S(i)$, which does not include machine i. A set of machines directly preceding machine i for $i \in \hat{K}$ is defined as $R(i)$.

3.2.2 System Evaluation

The performance measure of this model consists of a holding cost and the amount of backlogs. A total holding cost in period n is denoted by $Z(n)$. The holding cost rate at machine i is defined as h_i, and the holding cost is incurred for the products in FG_i, in ST_i and under transportation from FG_i to ST_i. Since several parts from upstream machines one by one are assembled into a new product it is assumed that $\sum_{k \in R(i)} h_k \le h_i$.

Then it holds that

$$Z(n) = \sum_{i \in \hat{K} - K} h_i \left(J_i(n) + \sum_{l=n-T_i}^{n-1} Q_i(l) + I_i(n) \right) + h_K \left[J_K(n) \right]^+ . \tag{3.1}$$

When $J_K(n) < 0$, $-J_K(n)$ demands are backlogged, therefore the amount of backlogs in period n is equal to $\left[-J_K(n) \right]^+$, where $[a]^+ = \max(0, a)$.

3.2.3 Notations

We give notations which are not defined above.

$d^n(i) = d\left(d^{n-1}(i)\right)$ for $n = 1, 2, 3, \cdots$, $d^1(i) = d(i)$ for $i \in \hat{K}$,

$w(i)$: the number of elements of $S(i)$ for $i \in \hat{K}$.

3.3 Base Stock Policy with Advance Demand Information

The inventory position at each machine is the total amount of inventory that exists in the machine and all machines in a downstream from the machine. In base stock system the base stock level of each machine is predetermined. The base stock level of machine i is denoted by N_i. If the inventory position at each machine is less than the base stock level of the machine, the machine produces products until the inventory position reaches the base stock level.

The time interval between the arrival of ADI and the production order placement for informed ADI is referred to as *information delay period*. Information delay period at machine i is denoted by l_i, where it is assumed that l_i satisfies $l_i \le L + 1$. In this model, the inventory position at machine i in period n is defined as the total amount of inventory that exists in machine i and its downstream machines from the machine minus the amount of demand which will be required at the system from n to $\left(n+L-l_i\right)$ periods. Note that machine i produces without advance demand information when $l_i=L+1$. It is assumed that $I_i\left(0\right)=N_i-N_{d(i)}$ for $i\in\hat{K}-\left\{K\right\}$, $J_i\left(0\right)=0$ for $i\in\hat{K}-\left\{K\right\}$ and $J_K\left(0\right)=N_K$.

The inventory position at machine i at the beginning of period n is $\sum_{j\in S(i)}\left\{J_j\left(n\right)+\sum_{l=n-T_i}^{n-1}Q_j\left(n\right)+I_j\left(n\right)\right\}+J_K\left(n\right)-\sum_{l=n-L}^{n-l_i}D\left(l\right)$. Under the base stock policy, the production order quantity of machine i must be no more than the difference between the base stock level of machine i and the inventory position at machine i. It is also no more than the minimal numbers of products of each machine directly preceding machine i, and the machine can produce products until the empty space of FG_i becomes full. Since production capacity is C_i, the production order quantity at machine i in period n is given by

$$P_i\left(n\right)=\min\left\{\begin{array}{l}N_i-\left[\sum_{j\in S(i)}\left\{J_j\left(n\right)+\sum_{l=n-T_i}^{n-1}Q_j\left(n\right)+I_j\left(n\right)\right\}+J_K\left(n\right)-\sum_{l=n-L}^{n-l_i}D\left(l\right)\right],\\ \\ \min_{j\in R(i)}I_j\left(n\right),B_i-J_i\left(n\right),C_i\end{array}\right\},\quad \text{for } i\in\hat{K}-\{K\}$$

(3.2)

At the last machine, it holds that

$$P_K\left(n\right)=\min\left\{N_K-\left[J_K\left(n\right)-\sum_{l=n-L}^{n-l_i}D\left(l\right)\right],\min_{j\in R(K)}I_j\left(n\right),B_K-\left[J_K\left(n\right)\right]^+,C_K\right\}. \qquad (3.3)$$

When $l_i=1$ in Eqs.(3.2) and (3.3), machine i uses full advance demand information. When $l_i=L+1$, machine i uses no advance demand information.

Since the production quantity at machine i in period n is defined as the minimum of the production order quantity at machine i in period n and the production capacity at machine i in period n, we have

$$P_i'(n) = \min\{P_i(n), C_i(n)\}. \tag{3.4}$$

Hence we have

$$P(P_i'(n) = k) = \begin{cases} \sum_{l=P_i(n)}^{C_i} p_l^i & k = P_i(n), \\ p_k^i & k = 0,1,2,\cdots,P_i(n)-1. \end{cases} \tag{3.5}$$

It is assumed that the amount of products which leave FG_i at the end of period n is determined at the beginning of period n. Since the products under transportation are finally delivered to ST_i, the amount of products which leave FG_i at the end of period n can not exceed the number of the empty buffers of ST_i minus the amount of products under transportation. Only the products existing in FG_i at the end of period n can be delivered, and the maximal delivery quantity at machine i is U_i. Therefore, the amount of products which leave FG_i at the end of period n is given by

$$Q_i(n) = \min\left\{A_i - I_i(n) - \sum_{l=n-T_i}^{n-1} Q_i(l), J_i(n) + P_i'(n), U_i\right\}. \tag{3.6}$$

Using the above equations, the amount of products in each stage at the beginning of the period $n+1$ is derived.

In period n, unfinished products in ST_i are assembled at machine $d(i)$. $Q_i(n-T_i)$ products which leave FG_i at the end of period $n-T_i$ arrive in ST_i at the end of period n. Therefore the amount of unfinished products in ST_i at the beginning of the period $n+1$ is given by

$$I_i(n+1) = I_i(n) - P_{d(i)}'(n) + Q_i(n-T_i) \qquad (i = 1,2,\cdots,K-1) \tag{3.7}$$

In period n, $P_i'(n)$ products which are assembled at machine i are placed in FG_i and $Q_i(n)$ finished products leave FG_i to ST_i. Therefore the amount of products in FG_i at the beginning of period $n+1$ is given by

$$J_i(n+1) = J_i(n) + P_i'(n) - Q_i(n) \quad (i = 1, 2, \cdots, K-1) \tag{3.8}$$

At machine K, products assembled are sent to FG_K, and the products corresponding to the demand which arrived before $n-L$ periods leave the system. Therefore it holds that

$$J_K(n+1) = J_K(n) + P_K'(n) - D(n-L). \tag{3.9}$$

$J_K(n)$ can take a negative value, which implies backlogs. When $J_K(n) < 0$ $-J_K(n)$ demands are backlogged, therefore the amount of backlogs becomes $[-J_K(n)]^+$.

3.4 Algorithm for Computing Appropriate Base Stock Levels

In the base stock policy, base stock levels of upstream machines are greater than ones of downstream. Among machines having the same number of their downstream machines, the base stock levels of the machine which has upstream machines are desired to be greater than ones of the machine which has no upstream machines, because machines which have upstream machines are affected by variations of productions of upstream machines. In this section a simulation-based algorithm which can find appropriate base stock levels minimizing the average holding cost is proposed, under the condition that the number of backlogs must be no more than a certain value. Since the holding cost rate is set as $\sum_{k \in R(i)} h_k \le h_i$, the total holding cost decreases when base stock levels are reduced.

In the proposed algorithm it is assumed that the number of backlogs must be no more than $\hat{N}$ in m periods. After base stock levels of upstream machines are determined, base stock levels of the downstream machines are determined. Also, among the machines having the same number of their downstream machines, after base stock levels of the machines having its upstream machines are determined, base stock levels of the machines having no upstream machines. Base stock levels which are not yet determined are reduced while the amount of backlogs is more than $\hat{N}$ by simulation. If the amount of backlogs is more than $\hat{N}$, the base stock levels are determined. Repeating these procedures, base stock levels of all machines are determined. Finally base stock levels which reduce the average inventory cost are found by simple local search.

The notation $w(i)$ represents the number of elements of $S(i)$. Let $E(k)=\{i;\{w(i)=k\}\cap\{R(i)=\phi\}\}$ and $F(k)=\{i;\{w(i)=k\}\cap\{R(i)\neq\phi\}\}$ for $0\le k\le \bar{k}$, where $\bar{k}=\max_{i\in\hat{K}} w(i)$. It is noted that $F(\bar{k})=\phi$, $F(k)\neq\phi$ $(0\le k\le \bar{k}-1)$, and $E(0)=\phi$. Let $\vec{N}=\{N_1,N_2,\cdots,N_K\}$. The neighborhood of $\vec{N}$ in simple local search is denoted by $H(\vec{N})$. $H(\vec{N})$ is defined as

$$H(\vec{N})=\left\{(n_1,n_2,\cdots,n_K);n_i\in\left\{\begin{matrix}\max(0,N_i-t),\max(0,N_i-t)+1,\\ \cdots,\min_{j\in R(i)}\{\min(N_j,N_i+t)\}\end{matrix}\right\}\right\}, \qquad (3.10)$$

where t is an arbitrary positive integer which denotes the size of neighborhood.

***Algorithm* :**

Step 1.

Let $M=\{1,2,3,\cdots,K\}$. Set $k=\bar{k}$, $N=\max_{i\in M-\{K\}}\left[\sum_{k\in\{i\cup S(i)\}}(A_k+B_k)+B_K\right]$ and $N_i=N\ (i\in M)$, and compute $Z(n)$ by simulation. If the amount of backlogs is more than $\hat{N}$, then stop. Otherwise $N=N-1$.

Step 2.

Set $N_i = N\ (i \in M)$, and compute $Z(n)$. If the amount of backlogs is no more than $\hat{N}$, then set $N = N - 1$ and go to Step 2.

Step 3.

Set $N = N + 1, N_i = N\ (i \in M)$ and $M = M - E(k)$, and then set $k = k - 1$.

Step 4.

Compute $Z(n)$. If the amount of backlogs is no more than $\hat{N}$, then set $N = N - 1$, $N_i = N\ (i \in M)$ and go to Step 4.

Step 5.

Set $N = N + 1$, $N_i = N\ (i \in M)$ and $M = M - F(k)$. If $E(k) = \phi$ and $M \neq \phi$, then $k = k - 1$ and go to Step 4. If $M = \phi$, go to Step8.

Step 6.

Compute $Z(n)$. If the amount of backlogs is no more than $\hat{N}$, then set $N = N - 1$, $N_i = N\ (i \in M)$ and go to Step 6.

Step 7.

Set $N = N + 1$, $N_i = N\ (i \in M)$ and $M = M - E(k)$, and then set $k = k - 1$. Go to Step 4.

Step 8.

Find improvement solution of $\vec{N}$ by simple local search and stop.

3.5 Numerical Examples

In this section the performance of algorithm proposed in Section 3.4 is evaluated and the property of base stock system with advance demand information mentioned in Section 3.3 is examined by simulation. Program was written by C language and run on a personal computer with 2.4GHz Pentium CPU and 512 Mbytes memories.

The join-type production line for numerical examples is illustrated in Fig. 3.2. It is

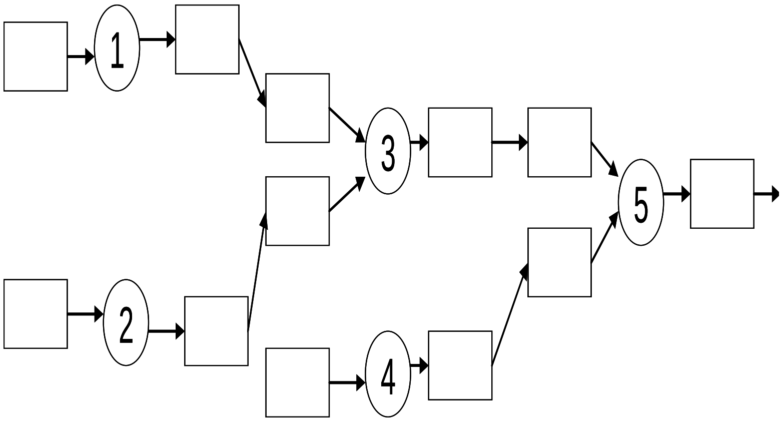

Fig. 3.2 A Join-type production line with 5 machines

assumed that demand follows a binominal distribution with $n=10$ and $p=\frac{1}{2}$.

Demand lead time L is set as 3 and it is assumed that $T_i = 0$ $(i=1,2,\cdots,4)$. A_i $(i=1,2,\cdots,4)$ and B_i $(i=1,2,\cdots,5)$ are large enough compared with production capacity and demand.

Cost coefficient h_i $(i=1,2,\cdots,5)$ is set as $h_1 = h_2 = 0.15$, $h_3 = 0.35$, $h_4 = 0.30$ and $h_5 = 0.75$. The positive integer t which denotes the size of neighborhood is set as 2.

For the following two cases, average inventory costs are computed. Excluding first 1000 periods, average inventory cost over 300000 periods is computed, that is $m = 300000$. It is assumed that the number of backlogs over 300000 periods must be no more than 300, that is $\hat{N}$ is set to be 300. In numerical examples, optimal base stock levels that minimize the average inventory cost are also found. For all possible combinations of base stock levels of all machines, average inventory costs are computed.

Case 1:

This is the case that there is machine failure. The probability distribution of production capacity is set as $p_j^i = \begin{cases} 0.90 & (j = 10), \\ 0.02 & (j = 5,6,7,8,9), \\ 0 & (j = 0,1,2,3,4). \end{cases}$

Case 2:

This is the case that there is no machine failure. The probability distribution of production capacity is set as $p_j^i = \begin{cases} 1 & (j = 10), \\ 0 & (j = 0,1,\cdots 9). \end{cases}$

For two cases mentioned above, base stock levels found by using algorithms proposed in Section 3.4 and optimal base stock levels are shown in Tables 3.1 and 3.2.

Table 3.1 Base stock level, average cost and backlogs for Case1

l_1,l_2,l_3,l_4,l_5	N_1,N_2,N_3,N_4,N_5	backlogs	avarage cost
1,1,1,1,1	5,5,0,0,0	259	9.99
2,2,2,2,2	13,13,5,5,0	229	10.1
1,1,2,2,3	6,5,5,5,5	242	9.77
2,2,1,1,1	14,12,0,0,0	294	10.6
4,4,4,4,4	27,27,20,20,13	289	11.6

Table 3.2 Base stock level, average cost and backlogs for Case2

l_1,l_2,l_3,l_4,l_5	N_1,N_2,N_3,N_4,N_5	backlogs	avarage cost
1,1,1,1,1	0,0,0,0,0	0	7.8
2,2,2,2,2	10,10,0,0,0	0	7.05
1,1,2,2,3	0,0,0,0,0	0	5.92
2,2,1,1,1	10,10,0,0,0	0	9.49
4,4,4,4,4	26,26,19,19,10	208	10.8

For 22 combinations of $(l_1, l_2, l_3, l_4, l_5)$ including the combinations not shown in tables, appropriate base stock levels found by using the proposed algorithm equal to the optimal base stock levels. The computation time for finding appropriate base stock levels using the proposed algorithm is 5 to 10 minutes. Almost part of computation time is consumed for simple local search. On the other hand the computation time for finding optimal base stock levels is several days.

When the demand arrives at the system, raw material is assembled into new parts at machines 1 and 2. In the next period, the parts are assembled into new parts at machine 3 and raw material is assembled into new parts at machine 4. In the next period the parts are assembled into finished products. Machines 1 and 2 have two machines in their downstream, machines 3 and 4 one machine and machine 5 no machines, respectively. The number of periods required for the assembling into finished products is 3 at machines 1 and 2, 2 at machines 3 and 4, and 1 at machine 5, respectively. Since the maximal amount of demand is 10 and the average production capacity of each machine is 9.7 in Case 1 and 10 in Case 2, respectively, parts of the same number as the demand for 1 period are assembled into new parts at one machine in 1 period in almost cases. When $(l_1, l_2, l_3, l_4, l_5) = (1,1,2,2,3)$, machines 1 and 2 utilize ADI for 3 periods, machines 3 and 4 for 2 periods and machine 5 for 1 period . Since due date of demand is 3 periods, assembling parts into finished products is just finished on the due date. On the other hand, when $(l_1, l_2, l_3, l_4, l_5) = (1,1,1,1,1)$, all machines begin the assembling parts at the same time as the arrival of the demand. Since assembling the parts in machines 3, 4 and 5 into finished products is finished before the due date, the numbers of products increase at downstream machines. Therefore the average inventory cost under $(l_1, l_2, l_3, l_4, l_5) = (1,1,1,1,1)$ is greater than that under $(l_1, l_2, l_3, l_4, l_5) = (1,1,2,2,3)$.

From Tables 3.1 and 3.2, it is found that the average inventory cost under $(l_1, l_2, l_3, l_4, l_5) = (2,2,2,2,2)$ is greater than that under $(l_1, l_2, l_3, l_4, l_5) = (1,1,1,1,1)$ in Case 1, whereas in Case 2, the average inventory cost under $(l_1, l_2, l_3, l_4, l_5) = (1,1,1,1,1)$ is greater than that under $(l_1, l_2, l_3, l_4, l_5) = (2,2,2,2,2)$. The reason is as follows. In Case

1, when $(l_1, l_2, l_3, l_4, l_5) = (1,1,1,1,1)$, machines 1 and 2 utilize ADI for 3 periods. If the production begins at machines 1 and 2 after the demand arrives at the system, assembling into finished products is finished before the due date in almost cases. When $(l_1, l_2, l_3, l_4, l_5) = (2,2,2,2,2)$, machines 1 and 2 utilize ADI for 2 periods. If the production begins at machines 1 and 2 after the demand arrives at the system, assembling parts in machines 1 and 2 into finished products is not finished before the due date. To complete the assembling before the due date, machines 1 and 2 have work-in-processes. This is why the values of the base stock levels of machines 1 and 2 are positive as shown in Table 3.1. Since the system has a lot of work-in-processes because of the base stock levels, the average inventory cost increases. In Case 2, when $(l_1, l_2, l_3, l_4, l_5) = (1,1,1,1,1)$, all machines begin the assembling at the same time as the arrival of the demand. Assembling the parts in machines 3 and 4 into finished products is finished in 1 period before the due date. Similarly, assembling parts in machine 5 into finished products is finished in 2 periods before the due date. Therefore the amount of products increases at downstream machines. When $(l_1, l_2, l_3, l_4, l_5) = (2,2,2,2,2)$, machines 1 and 2 have work-in-processes because the value of the base stock level is positive as shown in Table 3.2. Since the values of base stock levels of machines 3, 4 and 5 are 0, work-in-processes tend not to flow into downstream machines. Since the inventory cost in upstream machine is smaller than that of downstream, the average inventory cost decreases.

The number of backlogs is zero for 4 results in Table 3.2. The reason is as follows. The production capacity is 10 in all machines and the maximal amount of demand is 10. When $l_1 = l_2 = l_3 = l_4 = l_5 = 2$, for example, all machines utilize ADI for 2 periods. Assembling parts in machines 3, 4 and 5 into finished products is finished before the due date. The values of base stock levels of machines 1 and 2 are positive as shown in Table 3.2. To complete assembling parts in machines 1 and 2 into finished products before the due date, machines 1 and 2 have work-in-processes.

In Tables 3.1 and 3.2, the average inventory cost increases when $(l_1, l_2, l_3, l_4, l_5) = (4,4,4,4,4)$. The utilization of ADI is efficient for reducing the average inventory cost.

3.6 Conclusion

In this chapter, a join-type production line with ADI under the base stock policy is analyzed. The time interval between the arrival of ADI and the production order placement for informed ADI is referred to as *information delay period*. We propose the simulation based heuristic algorithm for finding appropriate base stock levels of all machines for determined information delay period and evaluate the performance. By using this proposed algorithm, appropriate base stock levels of all machines are found in a short time. For numerical results computed, the base stock levels found by this algorithm equal to the optimal base stock levels. We show the relation between information delay period and the appropriate base stock level found by using the algorithm. We also show that the average holding cost and backlogs in the case that ADI is not utilized are both greater than those in the case that ADI is utilized.

Chapter 4

Production Control with Advance Demand Information in a Join-Type Production Line

4.1 Introduction

In this chapter, production control with advance demand information is introduced into the join-type (assembly) production lines, in which at each machine multiple items from multiple preceding processes are processed or assembled as a single new product, and it is sent to the successive machine. The products are produced with a batch, and multiple items from each machine are used as parts of one product at the next machine. For a given demand lead time the timing of the production order placement for informed ADI is determined by estimating production lead time of each machine. It is assumed that time intervals for demand and processing time are random. The control method of the system is closer to hybrid policy C reported in Liberopoulos and Tsikis (2003). The synchronization mechanism in the model is associated with simultaneous extended kanban control system proposed in Liberopoulos and Dallery (2000). The model is formulated by recursive equations on the release time of products at each machine. These equations reduce the computation times for computer simulation on the join-type production lines with ADI. Sensitivity on parameters of production control such as estimated production lead time and initial inventory is analyzed.

In the next section the join-type production controlled by kanban policy with ADI and batch production is described. In section 4.3 the recursive equations on the release dates of products at each machine are derived. In section 4.4 the model is simulated on a personal computer and we examine the average inventory and the fraction of backlogs by changing estimated production lead time and initial inventories of each machine. In section 4.5 we conclude the study of this chapter.

4.2 A Join-Type Production Line with Batch Productions, Kanban and ADI

4.2.1 Model Assumption

A join-type production line with N machines is shown in Fig. 4.1. A set of machines is $M = \{1, 2, \cdots, N\}$, and machine N produces finished products. For machine $n \in M$, $U(n)$ denotes a set of machines directly preceding machine n, and $d(n)$ represents a machine succeeding machine n.

Figures 4.2 and 4.3 show flows of information tags, kanban and products. For each product processed at machine n, it is assumed that a_k products processed at machine k are needed for each k in $U(n)$. Therefore, for one finished product at machine N, b_n products are needed at machine n, where b_n is $a_n a_{d(n)} \cdots a_N$. Each demand has deterministic demand lead time T. A lot size of machine n is denoted by Q_n. When demand for each product occurs, b_n demand information tags are sent to the tag store,

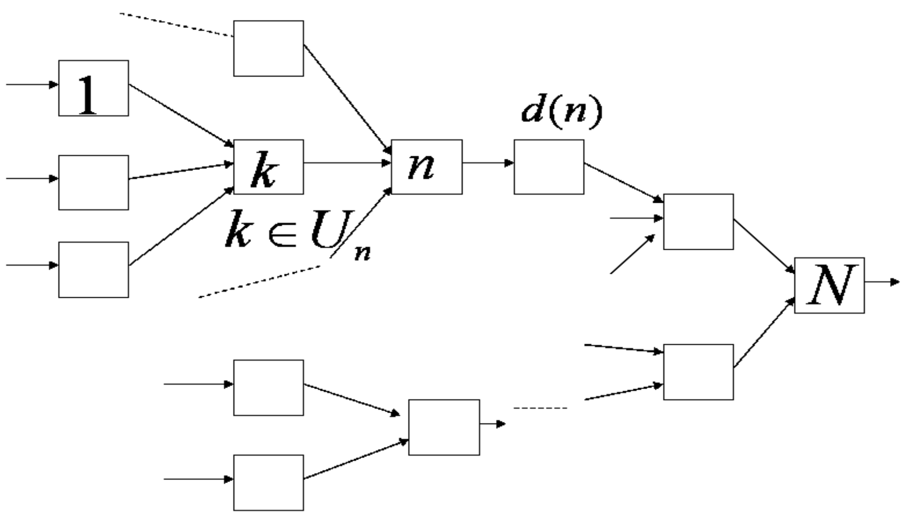

Fig. 4.1 A join type production line

named SD_n for all $n \in M$. When the number of demand information tags in SD_n attains Q_n, they are collected and replaced with a requisition tag, and it is sent to OD_n. The information delay for demand at machine n is denoted by T_n. The requisition tag stays at OD_n during T_n, and after then it is sent to BD_n. T_n satisfies $0 \le T_n \le T$. For example, we set T_n as

$$T_n = \max(T - L_n, 0),$$

where L_n means the estimated production lead time from the time when requisition tag

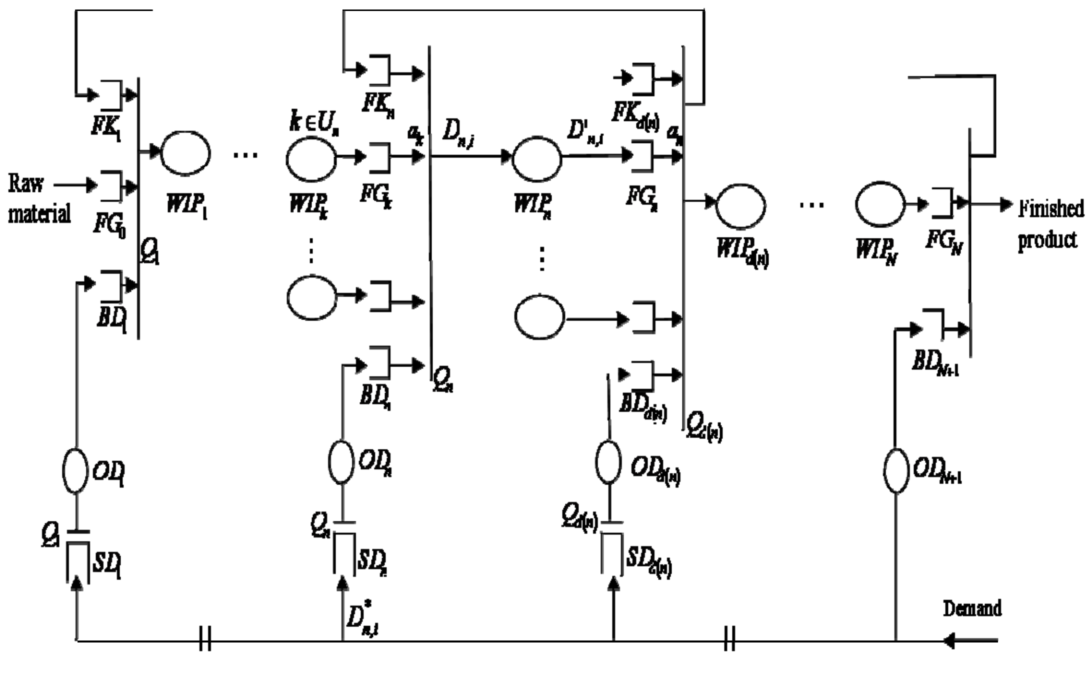

Fig. 4.2 Flow of information and products

is sent to BD_n at machine n to the time when the corresponding finished product is released from machine N. This setting means that at machine n satisfying $T_n > 0$ the unfinished product is processed as the corresponding finished product is released into the inventory of finished products just on due date for the demand, and at upper stream machines satisfying $T_n = 0$, the products are processed as the base stock policy with ADI. Initially, it is assumed that there are no demand information tags in SD_n and no

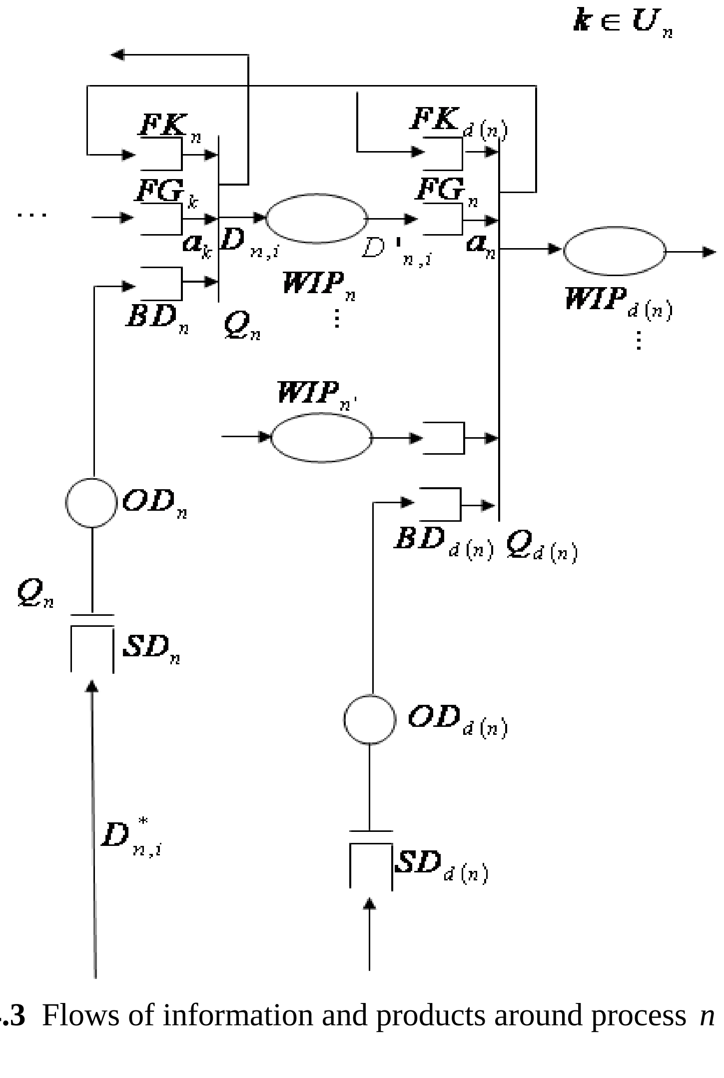

Fig. 4.3 Flows of information and products around process n

requisition tags in OD_n and BD_n.

FG_n in the figure is the inventory of products finishing process at machine n, and FK_n is the space for the production kanbans for machine n. The number of kanbans of machine n is denoted by K_n. When $a_k Q_n$ products are in FG_k for all $k \in U(n)$, one kanban is placed in FK_n, and one requisition tag and machine n exist in BD_n, they are collected and move to WIP_n, where WIP_n represents an inventory buffer and machine n. At machine n, collected products are processed in a FIFO order. Then they are released to FG_n as a finished product, and one kanban is moved to FG_n every time when Q_n products are released to FG_n. When Q_n products in FG_n is released to $WIP_{d(n)}$, one kanban is returns to FK_n. It is assumed that $Q_n / (a_n Q_{d(n)})$ is a positive integer, and it is denoted by j_n.

Initially, i_n^0 products which complete processing at machine n are placed at FG_n. The i_n^0 means safety stocks, and it is assumed that i_n^0 satisfies $l_n Q_n$, where l_n is an integer and $0 \le l_n \le K_n$. Therefore, initially there is $K_n - l_n$ kanbans in FK_n. At the last machine N, L_N is set to be 0 and for convenience we set $Q_{N+1} = 1, a_N = 1$, and $d(N) = N + 1$. When a demand occurs, a demand tag for machine N stays during T. After that, it is sent as a requisition tag to BD_{N+1}. If there is a finished product in FG_N, then the demand is satisfied and leaves the system with the product. If there is no product when the tag arrives at BD_{N+1}, the demand waits for the arrival of a finished product from machine N, and the delay for its due date occurs.

4.2.2 Notations

$d(n)$: the machine succeeding machine n,

$d^0(n) = n$, $d^z(n) = d\left(d^{z-1}(n)\right)$ for $z = 1, 2, \cdots,\ \ n \in M$,

e_n : the number of successive machines from machine n to machine N, excluding

machine n for $n = 1, 2, \cdots, N-1$, that is, $d^{e_n}(n) = N$,

$e_N = 0$,

a_n : the number of products required for processing one product in $d(n)$ for

$n = 1, 2, \cdots, N-1$,

$a_N = 1$,

$b_n = a_n a_{d(n)} \cdots a_N$ for $n = 1, 2, \cdots, N-1$,

T : demand lead time,

Q_n: lot size of machine n for $n = 1, 2, \cdots, N-1$,

SD_n : the space for demand information for machine n for $n = 1, 2, \cdots, N$,

OD_n : the space in which requisition tag stay for T_n for $n = 1, 2, \cdots, N+1$,

BD_n : the space for requisition tag for machine n for $n = 1, 2, \cdots, N+1$,

FG_n : the inventory of products processed at machine n for $n = 0, 1, \cdots, N$,

FK_n : the space for production kanbans for machine n for $n = 1, 2, \cdots, N$.

$D^*_{N+1,i}$: the i th arrival epoch of demand,

$\sigma_{n,i}$: the i th processing time at machine n, which is a random variable for $n \in M$ and $i = 1, 2, \cdots$,

$D'_{n,i}$: the i th release time from WIP_n to FG_n at machine n for $n \in M$ and $i = 1, 2, \cdots$,

$D^*_{n,i}$: the arrival epoch of i th demand information tag at SD_n for $n \in M$,

$D_{n,i}$: the ith release time of products from FG_k $(k \in U_n)$ to WIP_n for $n \in M$ and $i = 1, 2, \cdots$.

Decision variables.

L_n : the estimated production lead time for $n = 1, 2, \cdots, N-1$,

$T_n = \max(T - L_n, 0)$ for $n = 1, 2, \cdots, N-1$,

i^0_n : the number of initial inventory in FG_n for $n = 1, 2, \cdots, N-1$,

K_n : the number of kanbans of machine n for $n = 1, 2, \cdots, N-1$.

4.3 Recursive Equations on Release Times of Products

In this section, we derive the recursive equation on release time of products at all machines. Since the arrival epoch of the i th demand information tag for machine n is the arrival epoch of $\left\lceil \frac{i}{b_n} \right\rceil$ th demand for finished products, it holds that

$$D^*_{n,i} = D^*_{N+1, \left\lceil \frac{i}{b_n} \right\rceil} \qquad \text{for } n \in M \text{ and } i = 1, 2, \cdots ,$$

where $\lceil x \rceil$ is the smallest integer which is more than or equal to x.

Since the machine processes products one by one in a FIFO order, it holds that

$$D'_{n,i} = \max\left(D'_{n,i-1}, D_{n,i}\right) + \sigma_{n,i} \qquad \text{for} \quad n \in M \quad \text{and} \quad i = 1, 2, \ldots \tag{4.1}$$

Now we discuss the release time of products for the i th production at machine $n \in M$ from preceding inventories to WIP_n. First, for each $k \in U_n$, products completing process at machine k for the i th product at machine n must be in FG_k. Since the size of products for each batch at machine n is $a_k Q_n$ and there are i_k^0 products initially, the $\left(\left\lceil \frac{i}{Q_n} \right\rceil a_k Q_n - i_k^0\right)$ th products must be in FG_k for the release. Next, the requisition tag for the lot including the i th demand must be located in BD_n. Therefore, from the model description, the arrival epoch of the requisition tag at BD_n is given by $D^*_{n,\left\lceil \frac{i}{Q_n} \right\rceil Q_n} + T_n$.

Lastly, the kanban for i th product at machine n must be located in FK_n. Initially, there are $K_n - l_n$ kanbans in FK_n. Since the kanban is one for one batch with size Q_n, the $\left(\left\lceil \frac{i}{Q_n} \right\rceil - (K_n - l_n)\right)$ th kanban must be returned to FK_n. To do this, $\left(\left\lceil \frac{i}{Q_n} \right\rceil - (K_n - l_n)\right) Q_n$ products must be released from FG_n to $WIP_{d(n)}$. One product at machine $d(n)$ needs a_n products in FG_n. Therefore, when products necessary for the $\frac{\left(\left\lceil \frac{i}{Q_n} \right\rceil - (K_n - l_n)\right) Q_n}{a_n} = \frac{\left\lceil \frac{i}{Q_n} \right\rceil Q_n - (K_n Q_n - i_n^0)}{a_n}$ th process at machine $d(n)$ are released into $WIP_{d(n)}$, the kanban for the i th product at machine n returns to FK_n. From these results, we have the following equations for $D_{n,i}$:

$$D_{n,i}=\max\left(\max_{k\in U_n}\left(D'_{k,\left\lceil \frac{i}{Q_n}\right\rceil a_k Q_n - i_k^0}\right), T_n + D^*_{n,\left\lceil \frac{i}{Q_n}\right\rceil Q_n}, D_{d(n),\frac{\left\lceil \frac{i}{Q_n}\right\rceil Q_n-(K_nQ_n-i_n^0)}{a_n}}\right) \quad n=1,2,\cdots,N. \tag{4.2}$$

Let $D_{N+1,i}$ be the epoch when a finished product leaves the system with the i th demand. It occurs when the i th demand arrives at BD_{N+1} and the corresponding finished product is placed at FG_N. Since there are i_n^0 finished products in FG_N initially, we have

$$D_{N+1,i}=\max\left(D'_{N,i-i_N^0}, T+D^*_{N+1,i}\right). \tag{4.3}$$

To derive the recursive equations on $D'_{n,i}$, we introduce the following function:

$$g(n,i,0)=\left\lceil \frac{i}{Q_n}\right\rceil,$$

$$\begin{aligned} g(n,i,z)=g(n,i,z-1)\,j_{d^{z-1}(n)}-K_{d^{z-1}(n)}j_{d^{z-1}(n)} \\ +l_{d^{z-1}(n)}j_{d^{z-1}(n)} \end{aligned} \qquad \text{for } i\ge 1, z=1,2,\cdots,e_n.$$

$g(n,i,z)$ corresponds to the lot number of semi-finished product at machine $d^z(n)$ whose process must be finished at machine $d^z(n)$ for the i th processing at machine n. Using this function we have the next recursive equations.

$$D_{n,i}=\max\left(\begin{array}{l}\max_{z=0}^{e_n}\left(\max_{k\in U_{d^z(n)}}\left(D'_{k,g(n,i,z)a_kQ_{d^z(n)}-i_k^0}\right)\right),\\ \max_{z=0}^{e_n}\left(T_{d^z(n)}+D^*_{d^z(n),g(n,i,z)Q_{d^z(n)}}\right),\\ D'_{N,g(n,i,e_n)Q_N-K_NQ_N}, T+D^*_{N+1,g(n,i,e_n)Q_N-(K_NQ_N-i_N^0)}\end{array}\right) \qquad \text{for} \quad i\ge 1, n\in M \tag{4.4}$$

From equations (4.1) , (4.3)and (4.4), recursive equations on $D'_{n,i}$ is give by

$$D'_{n,i} = \max\left(\begin{array}{l} D'_{n,i-1}, \\ \max_{z=0}^{e_n}\left(\max_{k \in U_{d^z(n)}} \left(D'_{k,g(n,i,z)a_k Q_{d^z(n)} - i_k^0} \right) \right), \\ \max_{z=0}^{e_n}\left(T_{d^z(n)} + D^*_{d^z(n),g(n,i,z)Q_{d^z(n)}} \right), \\ D'_{N,g(n,i,e_n)Q_N - K_N Q_N}, T + D^*_{N+1,g(n,i,e_n)Q_N - (K_N Q_N - i_N^0)} \end{array} \right) + \sigma_{n,i} \qquad (i \geq 1, n \in M)$$

(4.5).

In (4.5), we note that $g(n,i,z)Q_{d^z(n)}$ can be computed as

$$g(n,i,z)Q_{d^z(n)} = \left(\begin{array}{l} \left\lceil \frac{i}{Q_n} \right\rceil J_n(0,e_n) - \sum_{x=0}^{z-1} K_{d^x(n)} J_n(x,e_n) \\ + \sum_{x=0}^{z-2} l_{d^x(n)} J_n(x+1,e_n) + l_{d^{z-1}(n)} J_n(z,e_n) \end{array} \right) \times A_n(z,e_n),$$

where

$$J_n(x,y) = j_{d^x(n)} j_{d^{x+1}(n)} j_{d^{x+2}(n)} \cdots j_{d^{y-1}(n)} j_{d^y(n)}$$

and

$$A_n(x,y) = a_{d^x(n)} a_{d^{x+1}(n)} a_{d^{x+2}(n)} \cdots a_{d^{y-1}(n)} a_{d^y(n)}.$$

Note that in the model, controllable parameters are T_n, K_n and i_n^0. The lot size Q_n can be also controlled, but in this paper it is assumed to be predetermined. In the following section we mainly discuss the relation between T_n and performance measure such as the amount of finished products inventories and the fraction of backlogs.

4.4 Numerical Examples

In this chapter the recursive equations are utilized to simulate the model and discuss the relation between estimated production lead time and the amount of finished goods inventories or the fraction of backlogs. Program was written by C language and run on a personal computer with 2.4GHz Pentium CPU and 512 Mbytes memories.

The join-type production line for numerical experiments is illustrated in Fig. 4.4. We assume that the demand process is Poisson with rate 1/8. All of a_n and j_n are set as 1 and K_n is set as 5. l_n is also set as 1 except for $n = 5$. We also set $T_n = \max(T - L_n, 0)$ where $L_n = x \times (e_n + 1)$. We change x and T, and discuss the effect of T , T_n and l_5 on the average inventory of finished products and the fraction of backlogs against demand. The service time at each machine is uniformly distributed on [3,4]. In each simulation 105000 items are produced and divided into 21 batches with 5000 items, and derive the 95% confidence interval by discarding the first batch. The interval is, however, very small and in the following numerical results only the expectations are presented.

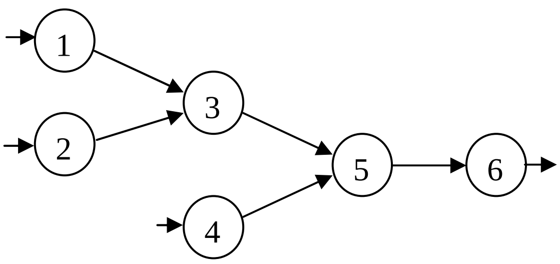

Fig. 4.4 Join-type production line for numerical experiments

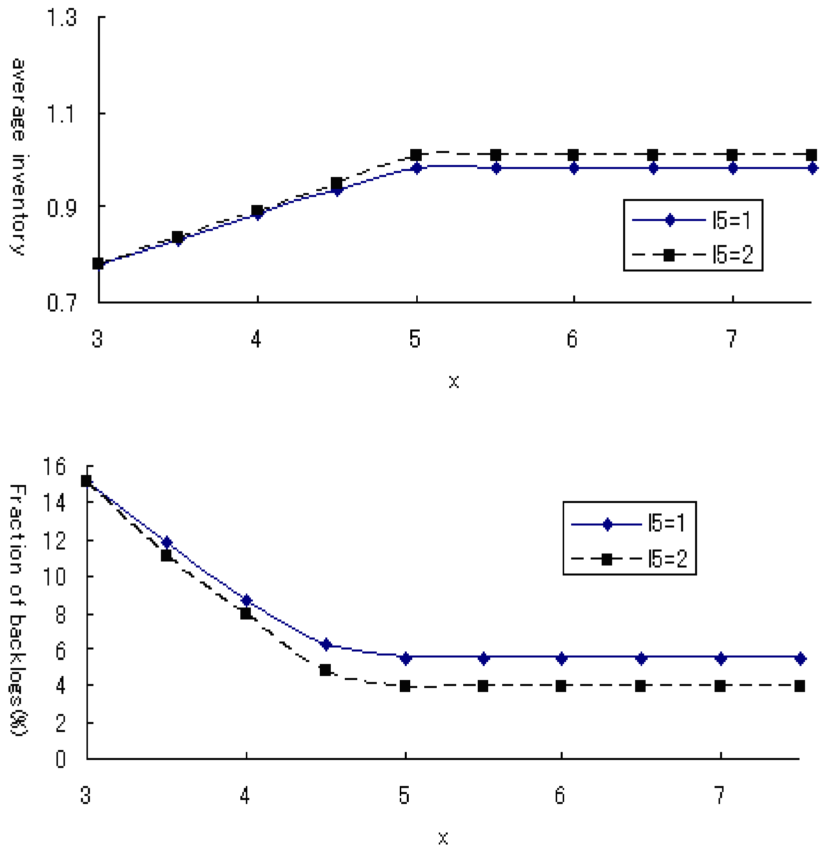

Fig. 4.5 Average inventory and fraction of backlogs ($T = 5$, $l_5 = 1$ or 2)

Note that for each simulation the computation time is only 3 or 4 seconds. This means that by equation (4.5) the performance under given parameters can be easily obtained.

Figures 4.5 and 4.6 show the average amount of finished products in inventory and the fraction of backlogs against demand when $T = 5$ and $T = 7$, respectively. From the figures we have the following insights.

1. When T is long, that is, demand lead time is long, the fraction of backlogs decreases with a little increase on the amount of finished products.

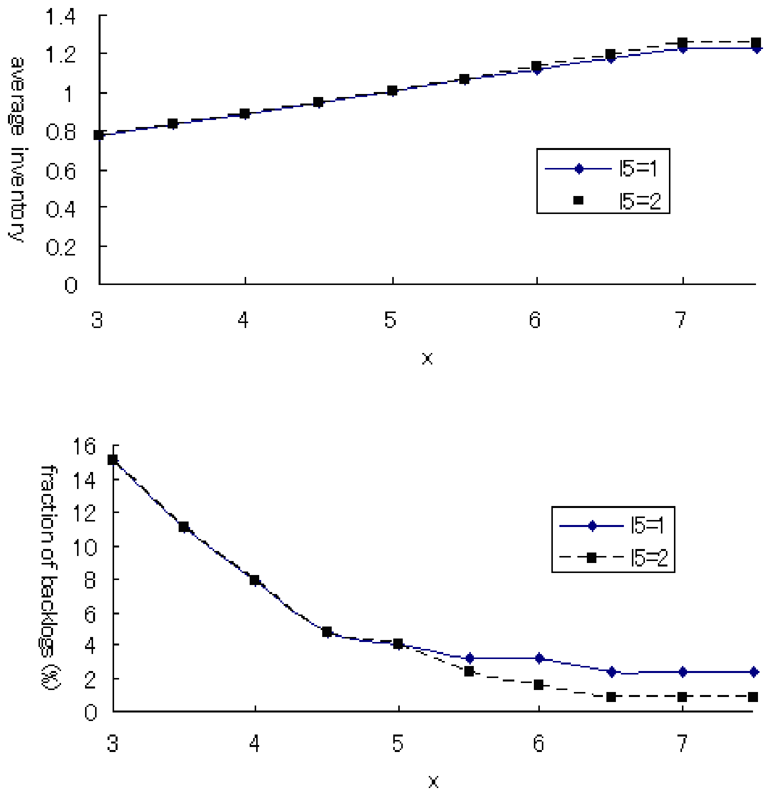

Fig. 4.6 Average inventory and fraction of backlogs ($T = 7$, $l_5 = 1$ or 2)

2. When x increases (that is, L_n increases and T_n decreases), the fraction of backlogs decreases rapidly whereas the average inventory increases slightly. We note that for $x > T$, $T_n = 0$ for all n, that is, the performance measure is the same for all $x > T$.

3. When l_5 increases, that is, the initial work-in-process at machine 5 (which is the preceding machine of the last processing machine) increases, and x (or T_5) is small, the fractions of backlogs decreases although the average inventory of finished products is almost the same. That is, having the appropriate safety stock on the

preceding machine of the last processing machine decreases the backlogs, as the amount of finished products does not increase.

4.5 Conclusion

In this chapter we analyze the join-type production line with batch productions, kanbans and ADI. For given demand lead time, the timing of the production order placement for demand informed by ADI is determined according to estimated production lead time of each machine. The recursive equations on the release time of products at all machines are derived, and by these equations the system can be simulated on a personal computer in a few seconds. In numerical examples, for a given demand lead time, the amounts of inventory and the fraction of backlogs are examined with changing estimated production lead time and initial inventory at each machine. As estimated production lead time at each machine is increased, the fraction of backlogs decreases rapidly whereas the average inventory increases slightly. This property is especially remarkable when demand lead time is long. We show that backlogs can be reduced with little increase of inventory by appropriate settings of estimated production lead time and initial inventory at each machine for given demand lead time.

Chapter 5

Conclusion

In this thesis, we deal with production systems with ADI.

In chapter 2, we analyze a single stage production-inventory system with ADI on continuous review theoretically. We derive the optimal release lead time and base stock levels which maximize the total expected profit for a given demand lead time theoretically. We get the following theoretical result. In the case that a backlog cost rate is not so large for a holding cost rate, when demand lead time is short, the optimal release lead time equals to demand lead time and the optimal base stock level is zero, and as demand lead time becomes long, the optimal release lead time decreases and the optimal base stock level remains zero. In the case that a backlog cost rate is so large for a holding cost rate, the optimal release lead time equals to demand lead time and the optimal base stock level is positive for very short demand lead time, and otherwise the same as the former case. We also show the relation between demand lead time and the total expected profit under the optimal release lead time and base stock level.

In chapter 3, we deal with a join-type production line with ADI under a base stock policy in discrete time. For determined information delay periods of all machines, a simulation-based heuristic algorithm which can find appropriate base stock levels of all machines is proposed, and the performance is evaluated. For numerical results computed, the base stock levels found by using this proposed algorithm are equal to the optimal base stock levels. We show the relation between information delay period and base stock levels found by using the proposed algorithm. In the case that ADI is not used at all machines the average cost and backlogs are both greater than those in the case that ADI is used at any machines.

In chapter 4, we consider a join-type production line with batch productions, kanbans and ADI in continuous time. For a given demand lead time, the timing of the production order placement for informed ADI is determined by estimating production lead time of each machine. We derive recursive equations on release times of products at all machines. With using these recursive equations, the model is simulated on a personal computer. We investigate the amount of average inventory and the fraction of backlogs with changing estimated production lead time and initial inventory at each machine. We show that when estimated production lead times of all machines are increased, the fraction of backlogs decreases rapidly whereas the average inventory increases slightly. This property is especially remarkable when demand lead time is long.

In models dealt with in this thesis, to reduce holding costs, appropriate timings of production order placements are determined by setting release lead time or information delay period appropriately for given demand lead time. Moreover, by setting the base stock levels or the number of kanbans appropriately, backlogs are reduced with little increase of holding costs. That is, if demand lead time is shorter than production lead time at a machine, then the machine has inventory stocks and starts the production at the same time as the arrival of ADI. This is because the demand can not be met without inventory stocks if the production starts after the arrival of ADI. If demand lead time is longer than production lead time at a machine, then the machine delays the start of the production afterwards from the arrival of ADI. This is because the finished product is completed before the due date and a holding cost is incurred if the production starts at the same time as the arrival of ADI. The advantage of using ADI is that the timing of the production order placement for final demand can be determined appropriately with taking demand lead time and production lead time into consideration. Though a reduction of the amount of inventory is generally in conflict with a reduction of the number of backlogs, we show that backlogs can be reduced with little increase of work-in-processes by using ADI. There are a lot of production lines controlled by existing production policies without ADI because of the simple control mechanism. Our studies on production controls with ADI are very valuable in a sense that the use of ADI can be easily introduced into production lines controlled by existing production control mechanisms.

In models dealt with in this thesis, there are a lot of hypotheses which are different from practical production lines. For example, it is assumed that demand lead time is constant among demands. In practical production lines, however, there are cases where the length of demand lead time is different among demands. Analyses for these cases remain in future researches. In the theoretical analysis in chapter 2, it is also assumed that arrival intervals of demand and processing times of products follow exponential distributions because of easiness of the analysis. To apply our studies into practical cases, it is necessary to analyze the case where they follow general distributions.

Though the model dealt with in chapter 2 is analyzed theoretically, the models dealt with in chapters 3 and 4 are analyzed in the simulation-based methods because of the difficulty of theoretical analyses. It is valuable to analyze these models theoretically in future. Especially, in chapter 3 we propose simulation-based algorithm for finding appropriate base stock levels. The performance of this algorithm can be evaluated more certainly by the theoretical analysis.

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Quoted Papers

Chapter 2

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Chapter 3

Hiraiwa, M., Tsubouchi, S. and Nakade, K. (2007), Base Stock Policy in a Join-Type Production Line with Advanced Demand Information, *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, Vol. 1, No. 3, pp. 399-407.

Chapter 4

Nakade, K. and Hiraiwa, M. (2006), Production Control with Advance Demand Information in a Join-Type Production Line, *Information Control Problems in Manufacturing 2006*, Dolgui, A. et al. (eds.), Vol. 2, pp. 323-328, England, Elsevier.