Modeling and Analysis of Competition in a Decentralized Supply Chain with Price Sensitive Demand

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Chapter 1

Introduction

1.1 Overview

Nowadays, the business climate has considerably changed. It is marked with globalization and high competition between its different actors. In this environment, the consumers are spoiled with choices and pay considerable attention to other dimensions of product or service. For that, Contemporary business life is process driven and chain oriented where the core-seeking of companies is the reduction of costs and the maximization of their profits through a set of entities that collectively manufacture a product and sell it to an endpoint (see Stern *et al* [1]). These entities form the so called supply chain management (SCM). The SCM has a particular importance for the durability of companies and it is a tool for them to compete efficiently at all economic scales. SCM has become a necessity especially for manufacturing industry when it comes to deliver products at a competitive cost and at a higher quality than their competitors.

Two major reasons prove the importance of SCM. The first one is the existence of strong concurrence in the market and difference in companies' competence. In this case, low cost competition has become insufficient for operation of companies. New rational competences must also be developed to distinguish one company from its competitors and stand it out in the market. To do that, SCM has permitted to the companies to change their entire management operations and restructure them so that they achieve their best performances. The strategy on applying SCM will not only impact their market positioning but also strategic decision on choosing the right partners, resources and manpower. To illustrate this point of view, Chan Kim stated in the Blue Ocean Strategy the example of the Japanese automotive industries which capitalise on its resources to build small and efficient cars. These industries increase their competitiveness using supply chain in order to maximise their competencies and stand out a position in market. This strategy works well and actually Toyota Motor Corporation is considered the number one auto car maker in the world beating Ford and General Motors. The second reason concerns the achievement of mass customization instead of mass production. Martin Christopher reported in his book, Logistics and Supply Chain Management: Strategies for Reducing Cost and Improving Service, [2] "*Productivity advantage gives a lower cost profile and the value advantage gives the product or offering a differential 'plus' over competitive offerings*." SCM has permitted companies to not just have productivity advantage alone but also on value advantage. Mass production offers productivity advantage, however, through effective SCM, mass customization can be achieved. With mass customization, customers are given the value advantage through flexible manufacturing and customized adaptation instead of certain delivery delay of customized products. It is, therefore, necessary to discuss about tradeoff between stocks and lead-time based on competitions that strongly affect customer satisfactions.

The SCM has been studied extensively in management related Literature due to the various contexts in which it can be used and described. In this chapter, we will introduce the framework of SCM with its decision making models. In addition, we will present briefly the literature and models related to our newly extended works. This chapter is organized as follow: In section 1.2, we will introduce the basic of SCM structure, its different types, and its marketing flows. In section 1.3, we focus on competition between the same members of SCM, where we explain the reasons of analysis competition and we introduce the models related to this work. In section 1.4, we will present the impact of lead-time decision on SCM, where we explain the reasons of studying this decision variable and we introduce the model related to this study. In section 1.5, we define the objective of this thesis. In section 1.6, we present the different chapters of this thesis.

1.2 Supply Chain Management

1.2.1 Definition of SCM and its marketing flows

Stern et al., [1] have defined the SCM as "*A supply chain is the set off entities that collectively manufactures a product and sells it to an endpoint*". The starting point is where raw materials are being manufactured; however, the end point is where products are consumed or recycled.

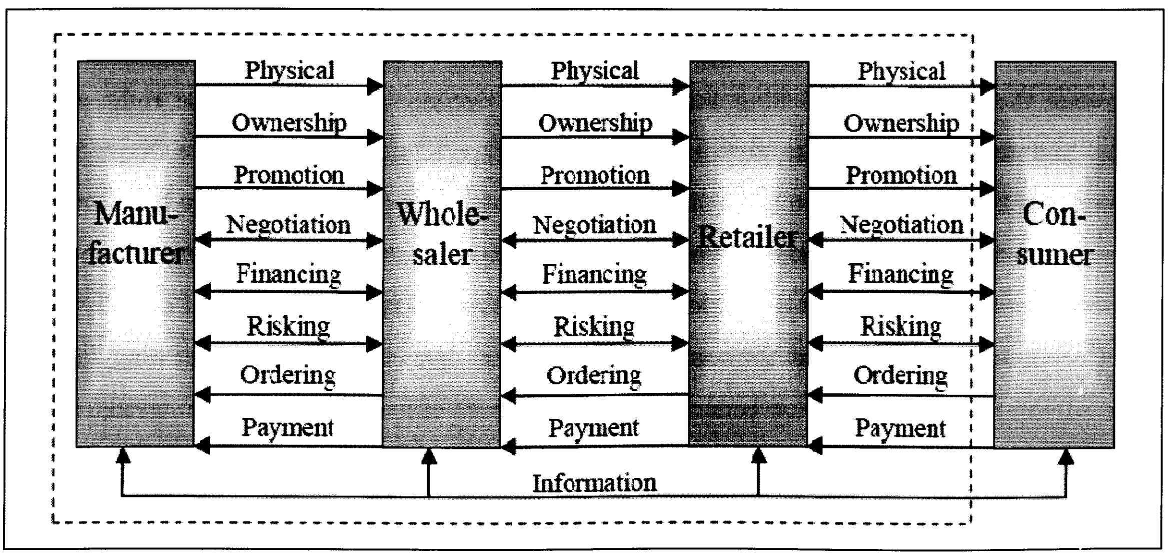

Fig. 1.1 Flows in supply chain management as reported by Stern et al., [1]

The work in a supply chain is defined with nine generic flows between the channel members as reported by in [1] and described in Fig. 1.1. Some flows move forward through the chain (physical, ownership, promotion), others move backwards (ordering and payment), whereas other flows move in both directions (negotiation, financing, information). This schematic description of a SCM is given for a single entity, however, in the market there are multiple manufactures, suppliers, retailers, and consumers.

1.2.2 Centralized and decentralized SCM

Two opposite types of SCM exist based on the decision making power. The first one is centralized in which the process of transferring and assigning decision-making authority is made at a central location of the entire supply chain. The objective of this centralized chain is to minimize the total cost of the system in order to satisfy some service level-requirements as reported by David et al., [3]. In this case, the profits are shared across the entire network using some contracts between the different members of the network. The centralized chain leads to a *global optimization* (see William J. Stevenson [4]). The second type of SCM is decentralized in which a leader decides and the other members are followers. The decentralized chain leads to a *local optimization* [4]. Theoretically, a centralized chain is at least effective as decentralized chain because the centralized decision can be the same in decentralized one even at a local position in the chain.

1.2.3 Modelling of SCM

The SCM can be formulated in different mathematical models. The easy way to understand this modelling is to consider a chain of one supplier and one retailer in a single period model or the so called *Newsvendor* or *Newsboy* problem. This model is a mathematical formulation in operations management and applied economics used to determine optimal inventory levels. It is (typically) characterized by fixed prices and uncertain demand (see Cachon and Netessine [5]). This elementary problem was studied intensively in several publications such as in Cachon and Netessine , Zhao and Atkins , and Solyali and Sural [5-7]. Although, the differences between these studies are the decision criteria and the nature of demand function, the objective is relatively the same. In first stage, the supplier and the retailer must take actions to optimize their profits. In second stage, their objective is to optimize the total supply chain. Finally, the problem is entirely solved. The actions can be taken with coordination and/or by setting a contract between the supplier and the retailer as reported by Xue *et al.* and Fugate *et al.*[8, 9]. The SCM has been modelled in different contexts based on the setting of decision variables, type and form of demands (deterministic or stochastic, linear or multiplicative), and contractual coordination between the members of the chain. Various decision variables were used in the SCM such as the wholesale price, the retail price, the inventory, the service… etc.

1.2.4 Problematic in SCM and context of this study

As given in Fig.1.1, the relationship between the members of a SCM is represented with nine generic flows, where each flow can take various variables. Then, it is difficult or may be impossible to find a complete study that gives answers to all problems in SCM, since there are a large number of models and the market is in continuous and dynamic change. Therefore, there are too many problems in SCM that are not evoked and/or completely solved. Furthermore, due to some mathematical restrictions, it is impossible to find a close form solution to some models such as existence of non-linearity multivariable equations. Thus, it is important to complete these models by building appropriates programming codes and solving them numerically.

On another hand, the degree of profitability achieved by the members of a SCM depends strongly on their competitive effectiveness. Here appears an important theme in SCM which is competition. A little knowledge about competition is known in industrial management and relatively few normative publications exist such as Greenhut and Ohta [10], Grossman and Hart [11], and Ziss [12]. The competition can be considered between multiple retailers, multiple suppliers, or multiple supply chains. Bernstein and Federgruen [13] study a dynamic inventory and pricing game for a distribution system with one supplier and two competing retailers on retail price. They actualize the study of Kirman and Sobel [14] by obtaining sufficient conditions for the existence of a unique certain equilibrium point. The same authors developed approximately the same problem except the uncertainty of the demand [15]. However, the modeling in this study was analyzed en general setting and no distribution function of demand was used. Therefore, it is important to simulate numerically this problem by setting one or more distribution functions of demand, then evaluate the performances of the chain under each distribution, and dress a comparison of results. The numerical analysis of the model presented by Bernstein and Federgruen will be presented in this thesis. In the same competition context, Zhao and Atkins [6] have introduced safety stock as a new competition factor either than retail price. This new factor is important since the consumer can move from one retailer to another in case out stock. Although the importance of this study, it lacks the setting of coordination contracts between the supplier and the multiple retailers. In this context, we have introduced buyback contact to the model of Zhao and Atkins and we have derived new conditions of existence of Nash solution. In addition, a numerical analysis of the model was carried out. On other hand, it is common for a retailer to sell products from competing suppliers. Then, the competing suppliers should manage their contract negotiations with the retailer to maximize their profits. However, in industrial management literature, the supply chain coordination have only focused on a single supplier who sells his products to a single or multiple retailers. The only recent study that discusses competition between two suppliers under different contractual forms has been reported by Cachon and Kök [16]. In this study, the authors have studied three types of contracts: wholesale-price contract, quantity-discount contract, and two-part-tariff contract in a decentralized supply chain where the demand is deterministic and sensitive to retail price. However, they did not study the sale-rebate contract and its impact on the different chain performances. Then, we will focus on this point where we will discuss competing suppliers under sale rebate contract in a decentralized supply chain, wherein the demand is sensitive to retail price.

To introduce new decision variables in competition in SCM, we have selected the lead-time due to its importance in business market. We will only present the result in a single-echelon supply chain without competition and the problem can be generalized in future to multiple retailers' competition. In this study, we will discuss the impact of lead-time decision on a decentralized supply chain for one supplier and one retailer, and wherein the demand is sensitive to retail price and lead-time.

1.3 Competition in SCM

1.3.1 Competition related literature in SCM

In actual globalized and opened market, the SCM contains multiple suppliers and multiple retailers. This reality leads to the study of newsvendor problems in multi-retailers and/or multi-suppliers supply chains as in Soares *et al.* [17]. In another term, a competition between the different actors of the chain cannot be avoided. For example, the retailers compete in the market to attract the maximum number of consumers. The recent competition related literature in SCM is limited where the important quantitative and qualitative works are cited chronologically.

1. Boyaci and Gallego [18] have studied competing two-echelon supply chains which attract Poisson demands that are proportional to their service rates.
2. Netessine, Rudi and Wang [19] have reviewed the literature in which customers substitute one product with another or switch from one retailer to another when their first-choice product or source is out of stock.
3. Bernstein and Federgruen [13] have modeled a dynamic inventory and pricing game for a distribution system with one supplier and two retailers engaged in price competition. They improve on Kirman and Sobel [14] by obtaining sufficient conditions for the existence of a unique certain equilibrium point.
4. Bernstein and Federgruen [15] have considered one manufacturer and multiple retailers who compete by choosing their retail prices. They assumed that the demand faced by each retailer is stochastic with a distribution that depends on the retail prices of all retailers.
5. Cachon and Lariviere [20] have identified a class of revenue-sharing contracts that coordinate the supply chain with one manufacturer and competing retailers.

6. Netessine and Zhang [21] have considered a supply chain with one manufacturer and competing retailers who face an exogenously determined retail price and a stochastic demand whose distribution depends on the order quantities of all retailers.
7. Zhao and Atkins [6] have modeled a competitive newsvendor problem between a single supplier and multiple retailers under simultaneous price and safety stock competition.
8. Cachon and Kök [16] have studied a competition between two suppliers who sell their products to a common retailer using wholesale-price, quantity-discount, and two-part-tariff contracts in a decentralized supply chain where the demand is deterministic and sensitive to retail price.

1.3.2 Contribution of this thesis to competition in SCM

The models described in last subsection not only differ in competing members and scenarios in supply chains but also on setting of decision variables, coordination contracts, and type of demand function (deterministic or stochastic). The small change of these points will lead in fundamental change of results such as the condition of existence and uniqueness of compromised solution (Nash solution). Furthermore, most of these models which study stochastic demand require numerical analysis since the close form solution depends on the setting of distribution function of demand. In management literature there are different distribution and demand functions which differ in their in statistic parameters and industry applications, respectively. Furthermore, the numerical analysis can be extended to simulate the effect of chain parameters, competing factors, and distribution parameters that limit generic understanding of SCM behaviours.

Therefore, we have focused on this thesis on to objectives:

The purpose of this study is to analyze numerically some competing models that were only studied analytically, in order to make them useful in some industrial management applications. The objective is study the effect of changing coordination contracts and decision variables on the results of competition in SCM through the use of some existing models.

- Three different works were studied and will be reported in chapter 2, 3 and 4. The first study reports the proprieties of Nash equilibrium retail prices in contract model with a supplier, multiple retailers and price-dependent demand. As described by Bernstein and Federgruen [15], this model is standard and can be used in several industrial management applications since retail price is the very important decision variable in the market and buyback contact is a good example of appropriate contracts, which redistributes the risk of overstocking.

- The second study discusses competition in a decentralized SCM under price and safety stock sensitive stochastic demand and buyback contract. In this model, safety stock is introduced as a new decision variable where its importance comes from its application in industry of perishable products that are sold by consumers every day. In case where a retailer faces out stock, his consumers have high probability to move to another retailer and then a decrease in customization is obtained. For that the safety-stock is necessary to keep customization, however, costs related inventory increase.

- The third model reports competing suppliers who sell their products to a common retailer and coordinate with him using sales-rebate contract. In this study, the chain is decentralized and sensitive to retail price. Suppliers' competition is very important since it is common for a retailer to sell products from competing manufacturers who should manage their contracts to maximize their profits.

1.3.3 Description of models

1.3.3.1 Multiple retailers competition for retail price

This model was analytically introduced by Bernstein and Federgruen [15] where its schematic illustration is given in Fig. 1.2. They have analyzed a contract model with single supplier and multiple retailers with price dependent stochastic demands, where retailers compete on retail prices. Each retailer decides a number of products he procures from the supplier and his retail price to maximize his own profit, given the wholesale and buy-back prices, which are determined by the supplier as the supplier's profit is maximized.

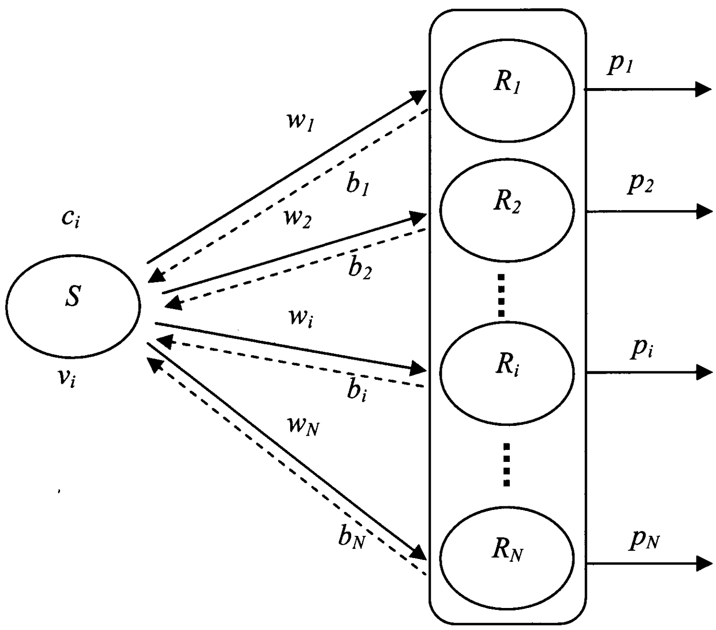

Fig. 1.2 Competing retailers' model for wholesale-buyback scheme

As the demand is stochastic and the Nash solution is non linear, it is necessary to carry out numerical analysis by setting the type of demand and its distribution in order to obtain real results that can be used in some industrial management applications, such as the video rental industry mentioned by Bernstein and Federgruen. We have analyzed analytically and numerically the model under exponential and uniform distributions and under linear and Logit demand function. This model will be described in detail in next chapter.

1.3.3.2 Multiple retailers competition for retail price and stock inventory

In this study, we discuss a model of competitive newsvendor problem between a single supplier and multiple retailers under simultaneous price and safety stock competition. The same schematic illustration of this model is given in Fig. 1.2. A price competition and a spill rate factors translate the price and safety stock competition, respectively. Our model is an extension of the problem analyzed by Zhao and Atkins [6] by the adoption of a buyback rate in the chain, which gives new Nash equilibrium conditions. Zhao and Atkins has not introduced coordination contract between the supplier and the retailers which exist in real market and also they did not deeply analyzed the effect of price and safety stock competition.

Keeping in mind the model of Bernstein and Federgruen where buyback contract was used, we will introduce this coordination contract in the model of Zhao and Atkins and we will discuss the new condition of existence and uniqueness of Nash solution. Buy-back contract is a good example of appropriate contracts, which redistributes the risk of overstocking. Furthermore, the effect of price and safety stock competition factors will be simulated numerically and compared to the case of only price competition and non-competitive model. Concerning the application of this model in management industry, it can be used in all industries perishable products that are needed by consumer every day. A detail description of this model will be given in chapter 3.

1.3.3.3 Multiple suppliers competition under Sale-rebate contract

This model focuses on competition between two independent suppliers who sell their products to a common retailer in a decentralized supply chain, under sales-rebate contract, and wherein the demand is sensitive to retail price. A schematic illustration of the model is given in Fig. 1.3. The model except the nature of coordination was studied by Cachon and Kök [16]. In their work, the authors have studied three types of contracts: wholesale-price contract, quantity-discount contract, and two-part-tariff contract in a decentralized supply chain where the demand is deterministic and sensitive to retail price. However, they did not study the sale-rebate contract and its impact on the different chain performances. Therefore, our model focuses on the study of a competition between independent suppliers who sell their products to a common retailer in a decentralized supply chain, under sales-rebate contract, and wherein the demand is sensitive to retail price. This model can be used in fields of hardware, software, and auto industries [17]. The literature of this model and its detail description will be shown in chapter 4.

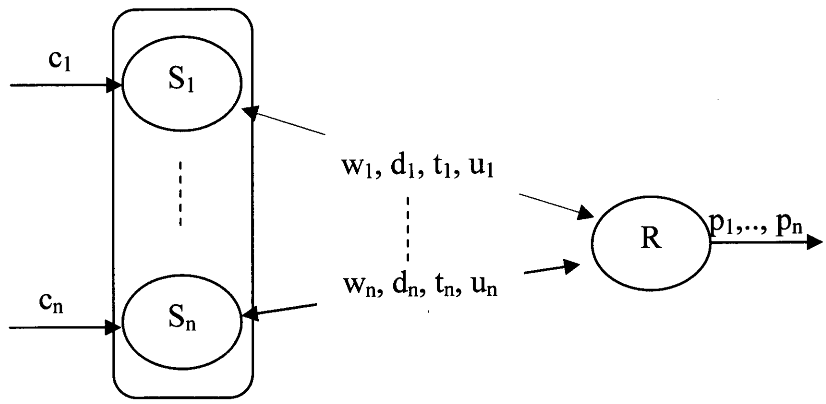

Fig. 1.3 Model of a supply chain consisting of one retailer and multiple suppliers

1.4 Impact of Lead-time Decision in SCM

1.4.1 Importance of lead-time in SCM

In actual globalized and competitive market, the consumer benefits from the variety of choices. Therefore, considering the selling price as a unique competition factor in a supply chain became insufficient. For that, the market actors have been investigating new competition criteria based on consumers' satisfaction. Sterling et al. [22] and Ballou et al. [23] reported that the rapidity and the regularity of delivery time have a particular importance in the customer service. Such delivery time is related to the so called "lead-time" factor. The related literature of this study and its objective will be reported in chapter 5.

1.4.2 Model

A schematic illustration of the model is given in Fig. 1.4. Three different seniors were studied to determine the optimal decision variables and expected profits in a two level supply chain, consisting of one supplier and one retailer. In the first scenario, the retailer decides the lead-time; however, this decision is taken by the supplier in the second scenario and centralized in the third one. One of reasons that deal with the importance of considering lead-time as a new decision variable in SCM is the inefficiency of using selling price as a unique competition factor. The results of this study can be used in several industrial management applications such as internet retailing, online selling transaction or e-retailing, post services...etc. More details of this model will be presented in chapter 5.

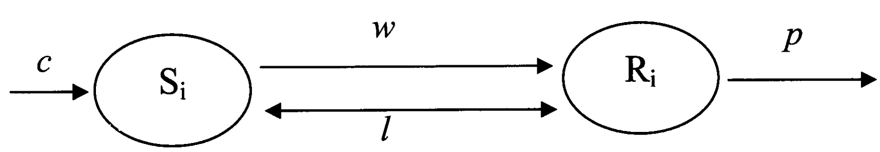

Fig. 1.4 Schematic illustration of the model

1.5 Objective of This Study

As discussed in §1.2.4, we will continue the analysis of competition in SCM by completing previous studies and through new model settings. Three independent works will be discussed. The first study reports the proprieties of Nash equilibrium retail prices in contract model with a supplier, multiple retailers and price-dependent demand, essentially numerical analysis. The second one analyses competition in a decentralized supply chain under price and safety stock sensitive stochastic demand and buyback contract. In this study, safety stock is added as a new competition factor either than retail price. In addition buyback contract between the supplier and the retailers is introduced in the supply chain. In the last work, we study competing suppliers under sales-rebate contract and price sensitive demand in a decentralized supply chain. Although these studies are independent, however, their objective is to set the conditions of existence and uniqueness of Nash solution and to analysis the behaviour of the different decision variables under various chain parameters and distribution function of demand.

In addition, we will present the results of study of impact of lead-time decision on the performances of centralized and decentralized SCM, consisting of one supplier and one retailer and wherein the demand is sensitive to lead-time, either than retail price. This work can be completed as a future work by introducing competition between multiple retailers.

1.6 Outline of The Thesis

The outline of this thesis is described as follow: In chapter 2, the proprieties of Nash equilibrium retail prices in contract model with a supplier, multiple competing retailers and price-dependent demand will be studied, where the conditions of existence and uniqueness of Nash solution will be developed. Exponential and uniform distribution functions for stochastic demand will be studied. In each case, linear and Logit demand will be used to simulate the model numerically. Finally, numerical results will be presented and discussed.

In chapter 3, we study competition in a decentralized supply chain under price and safety stock sensitive stochastic demand and buyback contract. In this model, safety stock is added in the formulation of competition. The conditions of existence and uniqueness of Nash solution will be developed and exponential distribution stochastic linear demand will be used de derive theoretical equations and numerical results. The effect of chain and demand parameters on decision variables and expected profits will be evoked.

In chapter 4, we study competition between multiple suppliers who sells their products to a common retailer where sales-rebate contract relates the chain members. The conditions of existence and uniqueness of Nash solution will be developed and numerical simulation of the model will take place. The effect of chain parameters on decision variables and expected profits will be studied.

In chapter 5, we analyse the impact of centralized and decentralized lead-time decision in a supply chain consisting of one supplier and one retailer, and wherein the demand is sensitive to retail price. Three different scenarios will be studied based on lead-time decision making, where we evaluate the optimal decision variables such as the wholesale price, the retail price, the demand, optimal lead-time, and optimal profit in each scenario. Then, the different results will be compared and discussed. In addition, the effect of chain and distribution parameters on decision variables and expected profits will be reported.

In chapter 6, the different works will be summarized and possible future problems will be discussed.

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Chapter 2

Properties of Nash equilibrium retail prices in contract model with a supplier, multiple retailers and price-dependent demand

2.1 Introduction

In this chapter, we analyze the properties of Nash equilibrium retail prices in contracting model in a decentralized supply chain consisting of one supplier, multiple competing retailers, and wherein the demand is sensitive to retail price. First, we introduce the literature related to this study, its objective, and the application of this model in industrial management. Second, we present the competing retailers model introduced by Bernstein and Federgruen [1] and discuss the sufficient conditions on the existence and the uniqueness of the Nash solution. Third, we investigate the model with exponential and uniform distribution functions and with linear and Logit demand functions. Finally, we present numerical results and discuss the behavior of Nash equilibrium solutions and properties of the profits and prices.

2.2 Literature and Objective

Recently, price contract models between suppliers and retailers with stochastic demand have been analyzed based on well-known newsvendor problems. Cachon [2] has reviewed .models with one supplier and one retailer under several types of contracts. In a market, however, many retailers exist and they compete to maximize their customization. Song et al. [3] have studied the optimal prices and the fraction of a total profit under individual optimization to that under supply chain optimization theoretically. Bernstein and Federgruen [1] have analyzed a contract model with single supplier and multiple retailers and price dependent demand, where retailers compete on retail prices. Each retailer decides a number of products he procures from the supplier and his retail price to maximize his own profit, given the wholesale and buy-back prices, which are determined by the supplier as the supplier's profit is maximized. They have proved that the retail prices become a unique Nash equilibrium solution under weak conditions on the price dependent distribution of demand. This model is very important since few publication related coordination mechanisms in decentralized supply chains with price setting or competing retailers, under demand uncertainty exist. In addition, the context of this study can be considered as a standard model which must be completely analysed. As reported by Bernstein and Federgruen, this model can be used in video rental industry which has recently incorporated "revenue-sharing" mechanisms, where the studios markedly drop their wholesale prices to store chains. However, it can be generalized to any retail supply chain since the retail price is the important decision variable in SCM and buyback coordination is good an example of appropriate contracts that redistributes the risk of overstocking. Bernstein and Federgruen [1] have not mentioned the numerical values and properties on these retail prices, the number of products and their individual and overall profits. Since the demand is stochastic and the solution cannot be obtained analytically, it is necessary to fix the distribution and the type of demand depending on industrial management application. For that, we will analyze this model analytically and numerically under exponential and uniform distribution of linear and Logit demand functions and we will discuss the results and compare between the different settings with representative values for the chain and distribution parameters.

2.3 Competing Retailers' Model

The model of competing retailers for one supplier S and N retailer $R_{1\le i\le N}$, introduced in Ref. 1, is shown in Fig 2.1. This model is set under a (w, b)-payment scheme. The supplier S incurs retailer R_i a wholesale price w_i for each product, combined with an agreement to buy back unsold inventory at b_i. The supplier has ample capacity to satisfy any retailer demand and produce products at a constant production cost rate c_i , which includes the transportation cost to retailer i. When w_i and b_i are given, each retailer R_i orders his quantity y_i and chooses his retail price p_i. A salvage rate $-\infty < v_i < +\infty$ is adopted in the supply chain. To avoid trivial setting, the model parameters are chosen as $v_i < b_i < w_i$ and $v_i < c_i$. The demand $D_i(p_i)$ is random and depends on the price vector $p = (p_1, p_2, \dots, p_N)$, with a cumulative distribution function $\tilde{G}_i(x \backslash p_1, p_2, \dots, p_N)$. It is restrained to a multiplicative form $D_i(p_i) = d_i(p)\varepsilon_i$, where ε_i is a random variable with a cumulative distribution function $G_i(.)$ and a probability density function $g_i(.)$, which is assumed to be positive only on $x \in [x^i_{min}, x^i_{max}]$. We assume that $\varepsilon_i$is independent of the price vector p, which implies that $\tilde{G}_i(x \backslash p) = G_i(x/d_i(p))$. The demand function $d_i(p)$ depends on the whole price vector. It is supposed that $d_i(p)$decreases in $p_i$and increases in $p_j$for all $i, j \in [1, \dots, N]$, that is, $\partial d_i(p)/\partial p_i < 0$ and $\partial d_i(p)/\partial p_j \geq 0$ for all $i \neq j \in [1, \dots, N]$. Let $y = (y_1, y_2, \dots, y_N)$ denote the order vector of the model. The expected profit function for the retailer R_i is given by $\pi_i(p, y) = p_i E[min\{y_i, D_i(p)\}] + b_i E[y_i - D_i(p)]^+ - w_i y_i$, where $[a]^+ = \max(0, a)$. It can be rewritten as

$$\pi_i(p, y) = (p_i - w_i)y_i - (p_i - b_i)E[y_i - D_i(p)]^+. \tag{2.1}$$

While retail prices p impact on the profit of all retailers, his order quantity affects only his own profit. Because the retailer wants to maximize his order quantity, the derivation of the retailer i's profit function on y_i is equal to zero, that is,

$$\partial \pi_i(p,y)/\partial y_i = 0\,, \tag{2.2}$$

From (2.1) and (2.2), the retailer i's optimal corresponding order is given by

$$y_i(p) = d_i(p) G_i^{-1}\left[\frac{p_i - w_i}{p_i - b_i}\right]. \tag{2.3}$$

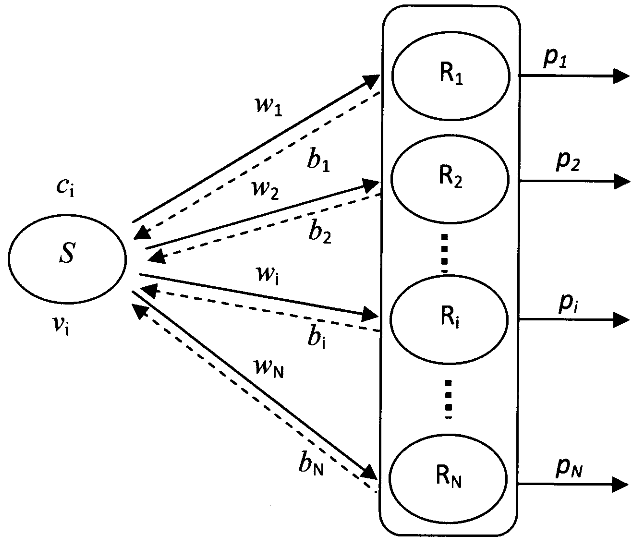

Fig. 2.1 Competing retailers model

This observation allows us to reduce the no-cooperative game in the (p,y)-space to a game in which retailers compete with one parameter p (*reduced retailer game*). From Eq. (2.1) and (2.3), we get the retailers profits as a function in p only, that is,

$$\tilde{\pi}_i(p) = d_i(p)\left[(p_i - w_i)G_i^{-1}\left[\frac{p_i - w_i}{p_i - b_i}\right] - (p_i - b_i)E\left[G_i^{-1}\left[\frac{p_i - w_i}{p_i - b_i}\right] - \varepsilon_i\right]^+\right]$$

$$= \tilde{\pi}_i^{det}(p\backslash w_i) L_i(f_i(p_i)), \tag{2.4}$$

where $\tilde{\pi}_i^{det}(p\backslash w_i) = (p_i - w_i)d_i(p)$ is the profit function with a deterministic demand $y_i = d_i(p)$, $f_i(p_i) = \frac{p_i - w_i}{p_i - b_i}$ is the critical fractile, and

$$L_i(f) = G_i^{-1}(f_i) - f_i^{-1}E\left[G_i^{-1}(f_i) - \varepsilon_i\right]^+ = \int_{-\infty}^{G_i^{-1}(f_i)} u\, g_i(u)du / f_i.$$

We define $\tilde{L}_i(p_i) \equiv \int_{-\infty}^{G_i^{-1}(f_i)} u g_i(u)du$ and we apply the logarithm to (2.4), we get for $i \in [1, \dots, N]$

$$log\tilde{\pi}_i(p) = log(p_i - b_i) + log d_i(p) + log\tilde{L}_i(p_i). \tag{2.5}$$

The supplier profit function is given by $\Pi_M = \sum_{i=1}^{N}\left((w_i - c_i)y_i - (b_i - v_i)E[y_i - D_i(p)]^+\right)$. From (2.3) we have

$$\Pi_M = \sum_{i=1}^{N} d_i(p)\left((w_i - c_i)G_i^{-1}\left[\frac{p_i - w_i}{p_i - b_i}\right] - (b_i - v_i)E\left[G_i^{-1}\left[\frac{p_i - w_i}{p_i - b_i}\right] - \varepsilon_i\right]^+\right). \tag{2.6}$$

Differentiating (2.5) on p_i for $i \in [1, \dots, N]$ $\frac{\partial log\tilde{\pi}_i(p)}{\partial p_i} = \frac{1}{d_i(p)}\frac{\partial d_i(p)}{\partial p_i} + U_i(p_i)$ with

$$U_i(p_i) = \frac{1}{p_i - b_i} + \frac{(w_i - b_i)G_i^{-1}(f_i(p_i))}{(p_i - b_i)^2 \bar{L}_i(p_i)} . \tag{2.7}$$

Bernstein and Federgruen [1] have proved that the existence of a Nash solution p^* for the reduced retailer game is assured by condition (A): For each $i \in [1, \dots, N]$, the function $logd_i(p)$ is increasing in (p_i, p_j) for all $i \neq j$. They have assumed that each retailer i chooses his price p_i from a closed interval $[p_i^{min}, p_j^{max}]$. They also proved the uniqueness of the Nash solution in the price space $\Pi_i[max(p_i^{min}, 2w - b), p_j^{max}]$ provided the following conditions (D) and (S) hold:

(D): $-\frac{\partial^2 log\pi_i^{det}(p\backslash w_i=b_i)}{\partial p_i^2} \geq \sum_{j\neq i}\frac{\partial^2 log\pi_i^{det}(p\backslash w_i=b_i)}{\partial p_i \partial p_j}$,

(S): $\psi_i(x) \doteq \left[-2x + \frac{\bar{G}_i(x)}{g_i(x)}\right]\int_{-\infty}^{x} ug_i(u)du - \bar{G}_i(x)x^2 \leq 0,$

for all $i \in [1, \dots, N]$, where$x \geq m_i$ (m_i is the median of the distribution G_i). In fact, however, the solution under the above conditions may exist on the boundary of the area $\Pi_i[max(p_i^{min}, 2w - b), p_j^{max}]$, and in this case it does not satisfy $\frac{\partial log\tilde{\pi}_i(p)}{\partial p_i} = 0$. We modify condition (S) to the following (S'): $\psi_i(x) \equiv \left[-2x + \frac{\bar{G}_i(x)}{g_i(x)}\right]\int_{-\infty}^{x} ug_i(u)du - \bar{G}_i(x)x^2 \leq 0$ for all $x \in [x_i^{min}, x_j^{max}]$. Then we have the following theorem.

Theorem If conditions (A), (D) and (S') hold, then there is a unique set of Nash equilibrium prices on $\Pi_i[w_i, \infty)$ which satisfy $\frac{\partial log\tilde{\pi}_i(p)}{\partial p_i} = 0$ for all $i \in [1, \dots, N]$.

Proof In the same way as in Bernstein and Federgruen (2005), it is shown that there is a unique Nash solution p^* in $\Pi_i[w_i, \infty)$. It also satisfies $p_i^* > 0$ for all $i \in [1, \dots, N]$, because for each $i \in [1, \dots, N]$, $\pi_i(p) = 0$ when $p_i = w_i$ whereas $\pi_i(p) > 0$ when $p_i > w_i$. It implies that $\frac{\partial log\tilde{\pi}_i(p)}{\partial p_i} = 0$ when $p = p^*$ for all $i \in [1, \dots, N]$.

In the following, the retailers sell products at these equilibrium prices, whereas the supplier knows this behavior of retailers and determines the wholesale and buyback prices to maximize his own profit. This system is called 'individual optimization. On the other hand, the problem of determining retail prices and quantities of products to maximize the entire profits of supply chain is called 'supply chain optimization.

2.4 Determination of The Nash Equilibrium

Each retailer $i \in [1,2]$ faces a random demand $D_i(p)$ where $p = (p_1, p_2)$. We assume two types of cumulative distribution functions of demand. We consider first the exponential case and then the uniform one.

2.4.1 Exponential case

The cumulative distribution function in the exponential case is given by $G_i(x) = 1 - e^{-x}$ for all $x \geq 0$, where $E\varepsilon_i$ is set as 1 without loss of generality. The inverse function of $G_i(x)$ is given by $G_i^{-1}(y) = -\log(1-y)$ for all $0 \leq y < 1$. With $f_i(p_i) = \frac{p_i - w_i}{p_i - b_i}$, we get $\hat{L}_i(f) = \frac{1}{p_i - b_i}(p_i - w_i + (w_i - b_i)\log(\frac{p_i - w_i}{p_i - b_i}))$. Then by (2.7) $U_i(p_i) = \frac{p_i - w_i}{(p_i - b_i)(p_i - w_i + (w_i - b_i)\log(\frac{w_i - b_i}{p_i - b_i}))}$.

2.4.1.1 Linear demand function

The linear demand is given by

$$d_i(p) = \alpha_i - \beta_i p_i + \sum_{j \neq i} \beta_{ij} p_j \text{ with } \alpha_i > 0, \beta_i, \beta_{ij} \geq 0 \text{ for } j \neq i, i, j \in [1,2]. \tag{2.8}$$

With this demand, we obtain the equations

$$\begin{cases} \frac{\partial \log \tilde{\pi}_1(p)}{\partial p_1} = \frac{-\beta_1}{\alpha_1 - \beta_1 p_1 + \beta_{12} p_2} + U_1(p_1) = 0 \\ \frac{\partial \log \tilde{\pi}_1(p)}{\partial p_1} = \frac{-\beta_2}{\alpha_2 - \beta_2 p_2 + \beta_{21} p_1} + U_2(p_2) = 0 \end{cases}.$$

These equations can be rewritten as

$$\begin{cases} p_1 = \frac{\beta_2 - \alpha_2 U_2(p_2) + \beta_2 p_2 U_2(p_2)}{\beta_{21} U_2(p_2)} \\ p_2 = \frac{\beta_1 - \alpha_1 U_1(p_1) + \beta_1 p_1 U_1(p_1)}{\beta_{12} U_1(p_1)} \end{cases}. \tag{2.9}$$

We can now evaluate the optimal order quantities y_1 and y_2 :

$$\begin{cases} y_1(p) = (\alpha_1 - \beta_1 p_1 + \beta_{12} p_2)\log(\frac{p_1 - b_1}{w_1 - b_1}) \\ y_2(p) = (\alpha_2 - \beta_2 p_2 + \beta_{21} p_1)\log(\frac{p_2 - b_2}{w_2 - b_2}) \end{cases}.$$

Since $E\left[G_i^{-1}\left[\frac{p_i - w_i}{p_i - b_i}\right] - \varepsilon_i\right]^+ = \log\left(\frac{p_i - b_i}{w_i - b_i}\right) + (\frac{w_i - p_i}{p_i - b_i})$, by (2.6) we get the supplier profit function $\Pi_M = \sum_{i=1}^{2} d_i(p)\left((w_i - c_i - b_i)\log\left(\frac{p_i - b_i}{w_i - b_i}\right) + b_i \frac{p_i - w_i}{p_i - b_i}\right)$. Then the retailers' profit functions are given by

$$\begin{cases} \tilde{\pi}_1(p) = d_1(p)((b_1 - w_1)\log\left(\frac{p_1 - b_1}{w_1 - b_1}\right) + (p_1 - w_1)) \\ \tilde{\pi}_2(p) = d_2(p)(b_2 - w_2)\log\left(\frac{p_2 - b_2}{w_2 - b_2}\right) + (p_2 - w_2)) \end{cases}.$$

2.4.1.2 Logit demand function

Now, we will study the problem with a logistic demand function given by

$$d_i(p) = \frac{k_i e^{-\lambda p_i}}{C_i+\sum_{j=1}^{2} k_j e^{-\lambda p_j}} \text{ for } C_i, \lambda, \text{ and } k_i > 0. \tag{2.10}$$

With this demand function we obtain the following equations

$$\begin{cases} \frac{\partial log\tilde{\pi}_1(p)}{\partial p_1} = \frac{-(C_1+k_2 e^{-\lambda p_2})}{C_1+k_1 e^{-\lambda p_1}+k_2 e^{-\lambda p_2}} + U_1(p_1) = 0 \\ \frac{\partial log\tilde{\pi}_1(p)}{\partial p_1} = \frac{-(C_2+k_1 e^{-\lambda p_1})}{C_1+k_1 e^{-\lambda p_1}+k_2 e^{-\lambda p_2}} + U_2(p_2) = 0 \end{cases}.$$

Then we have

$$\begin{cases} p_1 = -\frac{1}{\lambda} log \frac{\lambda C_2 - C_2 U_2(p_2) - k_2 e^{-\lambda p_2} U_2(p_2)}{k_1(-\lambda+U_2(p_2))} \\ p_2 = -\frac{1}{\lambda} log \frac{\lambda C_1 - C_1 U_1(p_1) - k_1 e^{-\lambda p_1} U_1(p_1)}{k_2(-\lambda+U_1(p_1))} \end{cases}.$$

The order quantities are given by

$$\begin{cases} y_1(p) = \frac{k_1 e^{-\lambda p_1}}{C_1+k_1 e^{-\lambda p_1}+k_2 e^{-\lambda p_2}} log\left(\frac{p_1-b_1}{w_1-b_1}\right) \\ y_2(p) = \frac{k_2 e^{-\lambda p_2}}{C_2+k_1 e^{-\lambda p_1}+k_2 e^{-\lambda p_2}} log\left(\frac{p_2-b_2}{w_2-b_2}\right) \end{cases}.$$

The supplier profit function and retailers' profit functions are obtained in the same way as for the linear demand function.

2.4.2 Uniform case

The cumulative distribution function in the uniform case is given by

$G_i(x) = \frac{x-(1-a_i)}{2a_i}, 1-a_i \le x \le 1+a_i,\ 0 \le a_i \le 1,\ for\ i = 1,2$,

where $E\varepsilon_i = 1$. The inverse function of $G_i(x)$ is given by $G_i^{-1}(y) 1 - a_i + 2a_i y$ for $0 \le y \le 1$. With $f_i(p_i) = \frac{p_i-w_i}{p_i-b_i}$, we get $\hat{L}_i(p_i) = \frac{p_i-w_i}{p_i-b_i}(1-a_i+a_i\,(\frac{p_i-w_i}{p_i-b_i}))$. Then, using (2.7) and $i = \{1,2\}$,

$$U_i(p_i) = \frac{1}{(p_i-b_i)}\left[1+\left(\frac{w_i-b_i}{p_i-w_i}\right)\frac{1-a_i+2a_i\left(\frac{p_i-w_i}{p_i-b_i}\right)}{1-a_i+a_i\left(\frac{p_i-w_i}{p_i-b_i}\right)}\right].$$

2.4.2.1 Linear demand function

With the linear demand given by (2.8) and $U_i(p_i)$, we obtain equations on p_1 and p_2:

$$\begin{cases} p_1 = \frac{\beta_2 - \alpha_2 U_2(p_2) + \beta_2 p_2 U_2(p_2)}{\beta_{21} U_2(p_2)} \\ p_2 = \frac{\beta_1 - \alpha_1 U_1(p_1) + \beta_1 p_1 U_1(p_1)}{\beta_{12} U_1(p_1)} \end{cases}.$$

The optimal order quantities are given by

$$\begin{cases} y_1(p) = (\alpha_1 - \beta_1 p_1 + \beta_{12} p_2)(1 - a_1 + 2a_1\left(\frac{p_1-w_1}{p_1-b_1}\right)) \\ y_2(p) = (\alpha_2 - \beta_2 p_2 + \beta_{21} p_1)(1 - a_2 + 2a_2\left(\frac{p_2-w_2}{p_2-b_2}\right)) \end{cases}.$$

The supplier profit function is equal to

$$\Pi_M = \sum_{i=1}^{2} d_i(p)\left((w_i - c_i)\left(1 - a_i + 2a_i\left(\frac{p_i-w_i}{p_i-b_i}\right)\right) - a_i b_i\left(\frac{p_i-w_i}{p_i-b_i}\right)^2\right).$$

The retailers' profit functions are given by

$$\begin{cases} \tilde{\pi}_1(p) = d_1(p)\left[(p_1 - w_1)\left(1 - a_1 + 2a_1\left(\frac{p_1-w_1}{p_1-b_1}\right)\right) - a_1(p_1 - b_1)\left(\frac{p_1-w_1}{p_1-b_1}\right)^2\right] \\ \tilde{\pi}_2(p) = d_2(p)\left[(p_2 - w_2)\left(1 - a_2 + 2a_2\left(\frac{p_2-w_2}{p_2-b_2}\right)\right) - a_2(p_2 - b_2)\left(\frac{p_2-w_2}{p_2-b_2}\right)^2\right] \end{cases}.$$

2.4.2.2 Logit demand function

With the Logit function given by (2.10), we obtain p_1 and p_2 as

$$\begin{cases} p_1 = -\frac{1}{\lambda} log \frac{\lambda C_2 - C_2 U_2(p_2) - k_2 e^{-\lambda p_2} U_2(p_2)}{k_1(-\lambda + U_2(p_2))} \\ p_2 = -\frac{1}{\lambda} log \frac{\lambda C_1 - C_1 U_1(p_1) - k_1 e^{-\lambda p_1} U_1(p_1)}{k_2(-\lambda + U_1(p_1))} \end{cases}.$$

The optimal order quantities are given by

$$\begin{cases} y_1(p) = \frac{k_1 e^{-\lambda p_1}}{C_1 + k_1 e^{-\lambda p_1} + k_2 e^{-\lambda p_2}}\left(1 - a_1 + 2a_1\left(\frac{p_1-w_1}{p_1-b_1}\right)\right) \\ y_2(p) = \frac{k_2 e^{-\lambda p_2}}{C_2 + k_1 e^{-\lambda p_1} + k_2 e^{-\lambda p_2}}\left(1 - a_2 + 2a_2\left(\frac{p_2-w_2}{p_2-b_2}\right)\right) \end{cases}.$$

The supplier profit function and retailers' profit functions are obtained in the same way as for the linear demand function.

2.4.3 Supply chain optimization

When the supplier and retailers determine the prices and order quantities as to maximize the overall profit of the supply chain, the wholesale and buyback prices are meaningless because they are payments between the supplier and retailers. As the whole of the supply chain is equivalent to a single retailer with wholesale price (c_1, c_2) and buy back (v_1, v_2), by (2.3) the optimal order quantity (the amount of products) is $y_i^I(p) = d_i(p) G_i^{-1}\left[\frac{p_i-c_i}{p_i-v_i}\right]$, and by (2.4) the overall expected profit of the supply chain is

$$\tilde{\pi}^I(p) = \sum_{i=1}^{2} (p_i - c_i)\, d_i(p) L_i\left[\frac{p_i-c_i}{p_i-v_i}\right], \tag{2.11}$$

when retail prices $(p_1, p_2)$are given. The optimal retail prices (p_1^I, p_2^I) in this integrated supply chain maximize the profit (2.11).

2.5 Numerical Examples

2.5.1 Geometric analysis of the Nash solution

We have found the equilibrium prices (p_1, p_2) that solve the profit functions for the two retailers. In the case with exponential demand and linear functions, we denote the right hand sides of two equations in (2.9) by $f_2(p_2)$ and $f_1(p_1)$, respectively. Then the equations (2.9) become $p_1 = f_2(p_2)$ and $p_2 = f_1(p_1)$. Note that in other cases the equations satisfied by $(p_1; p_2)$ form $p_1 = f_2(p_2)$ and $p_2 = f_1(p_1)$ similarly. Geometrically, to analyze the behavior of the system around the Nash solution, we plot the functions $f_i(p_i)$ for $p_1$and p_2 in Fig. 2.2. There are multiple solutions for the equations, but there is a unique Nash solution (p_1, p_2) with $p_i > w_i$ for $i = \{1, 2\}$, which has been proved in Theorem of section 2.3.

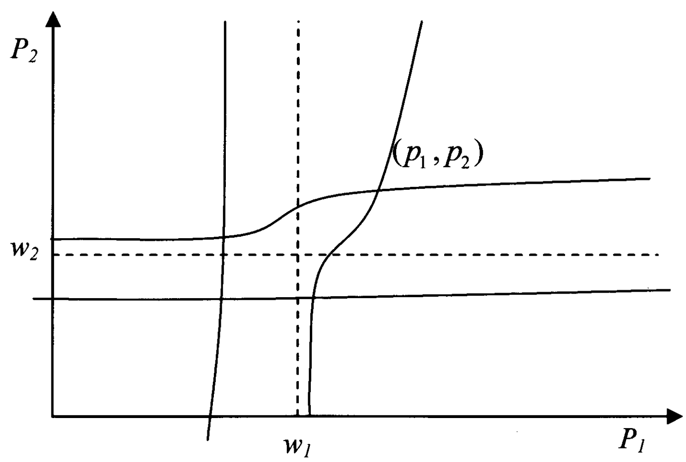

Fig. 2.2 Nash solution and system of equations

Given wholesale and buyback prices, we derive these Nash retail prices, and profits of the supplier and two retailers. We compute them for all combinations of wholesale and buyback prices, which are integers and satisfy $c_i \leq w_i \leq w_i^U$ and $v_i \leq b_i \leq w_i$, where $w_i^U$is set as the upper bound for the optimal wholesale price for the supplier, and derive optimal wholesale and buyback prices for the supplier. We also compute the overall profits and retail prices under the supply chain optimization, and compare them with the ones under individual optimization.

2.5.2 Numerical results

In numerical examples we set parameters as shown in the following: $(v_1, v_2) = (0,0)$, $(\alpha_1, \alpha_2) = (100,100)$, $(\beta_1, \beta_2) = (1,1)$, $(\beta_{12}, \beta_{21}) = (0.3, 0.3)$ (linear function), $\lambda = 0.03$, $(C_1, C_2) = (0.005, 0.005)$, $(k_1, k_2) = (1,1)$, (Logit function). Program is coded by C and the computations are done by using Fujitsu C compiler on PC. In Table 2.1, we assume exponential demand and Logit functions, whereas in Table 2.2 the linear function is assumed. In these tables two cost parameter settings are considered: $(c_1, c_2) = (30,30)$ (symmetric) and $(c_1, c_2) = (30,20)$ (anti-symmetric). The values in tables are the optimal profit for supplier, the profit for each retailer; entire expected profit (sum of supplier's and retailers' profits), optimal whole-sale and buyback prices for the supplier, Nash equilibrium retail prices and order quantities. The values in parenthesis () are the total profit, optimal retail prices and order quantities for retailers under the supply chain optimization.

Table 2.1 Exponential Demand and Logit Function

i	1	2	1	2
C_i	30	30	30	20
$\Pi_M(p)$	32.195		35.792	
$\pi_i(p_i, y_i)$	10.227	10.227	8.917	13.843
Entire expected profits	52.649 (62.430)		58.552 (70.153)	
w_i	98	98	100	88
b_i	47	47	47	47
p_i	175.420 (172.428)	175.420 (172.428)	175.376 (182.095)	168.444 (161.07)
y_i	0.311 (0.606)	0.311 (0.606)	0.276 (0.444)	0.418 (0.965)

In the cases of Tables 2.1 and 2.2, optimal whole sale prices and buybacks determined by the supplier give more profits to the supplier than retailers. In the symmetric cost cases, the optimal retail prices of two retailers become the same. Compared to supply chain optimization, the retail prices are higher and the quantities of orders are smaller in the individual optimal case. It is because under the chain optimization more amounts of demand are satisfied by decreasing retail prices and increasing order quantities, whereas in the individual optimal case the supplier wants to obtain its own profit, which leads to higher wholesale prices and as a result retail prices become higher. In the anti-symmetric cost case, the optimal wholesale price to the retailer with the smaller production cost is smaller than that to another retailer, which leads to more profits for the former retailer. The reason is that the retailer with small wholesale price sets the less retail price and more quantities of order, which implies that more amounts of demand occur in total and the supplier can sell more products to customers. In particular, with Logit demand function the demand depends on the retail prices more intensively, and the wholesale prices, retaii prices and the order quantities change more. In both cases the entire expected profits in the individual optimal cases is about 80 to 85 % of that under supply chain optimization. When the chain consists of one supplier and one retailer, it is shown in Song et al. [3] that the fraction is 3/4(in linear case) or $2/e = 0.736$ (in Logit case). The competition among retailers makes retail prices lower, which makes the fraction higher. In Table 2.3, the uniform distribution of demand is assumed with the symmetric production costs ($(c_1, c_2) = (30, 30)$), and the a_i, which corresponds to the width of the uniform distribution, is changed from 0.1 to 0.7. It implies that large a_i means the high variance of demand. As the variance increases, retail prices are higher, and profits of the supplier and retailers decrease. This is because when the variance increases, the quantity of order must be increased to apply the fluctuation of demand, whereas the retail price must be also increased to obtain profits of retailers. When a_i changes the optimal wholesale prices and buyback prices for the supplier are almost the same. Note that even if it is compared with results in the exponential case shown in Fig. 2.2, which has more variance than these uniform distributions, the difference on these prices is very small. It means that the optimal wholesale and buyback prices for the supplier are robust in the variance of the demand distribution.

Table 2.2 Exponential Demand and Linear Function

i	1	2	1	2
C_i	30	30	30	20
$\Pi_M(p)$	1200.548		1473.307	
$\pi_i(p_i, y_i)$	242.306	242.306	228.119	380.888
Entire expected profits	1685.160 (2041.22)		2082.314 (2515.01)	
w_i	89	89	89	82
b_i	77	77	77	73
p_i	116.154 (96.902)	116.154 (96.902)	115.532 (97.788)	112.445 (90.259)
y_i	22.105 (37.717)	22.105 (37.717)	21.233 (34.608)	32.826 (58.887)

Table 2.3 Uniform Demand and Linear Function

a_i	0.1	0.3	0.5	0.7
$\Pi_M(p)$	2531.42	2352.36	2176.38	2003.38
$\pi_i(p_i, y_i)$	513.03	481.51	450.56	414.12
Entire expected profits	3557.49 (4303.71)	3315.38 (3999.12)	3077.51 (3700.00)	2832.62 (3407.00)
$w_1(=w_2)$	87	87	87	87
$b_1(=b_2)$	75	75	75	74
$p_1(=p_2)$	110.31 (87.08)	110.97 (88.46)	111.69 (89.96)	112.55 (91.56)
$y_1(=y_2)$	23.51 (40.26)	24.55 (41.75)	25.59 (43.20)	26.05 (44.57)

2.6 Conclusion

In this study, we first show the sufficient condition that unique Nash equilibrium retail prices exist and they are greater than wholesale prices. We then give the equations whose solutions are those retail prices. In numerical examples we compute these equilibrium prices, optimal wholesale and buy-back prices for the supplier and supply chain optimal retailers' prices, and discuss properties on these values. As mentioned in above, this model can be considered as a benchmark for other competition studies where new decision variables and coordination contracts can be introduced. In next chapter, we will focus on multiple competing retailers on retail price and safety-stock where this new decision variable is extremely important in the increase or decrease of customization degree.

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Chapter 3

Competition in a Decentralized Supply Chain under Price and Safety Stock Sensitive Stochastic Demand and Buyback Contract

3.1 Introduction

In this chapter, we study a model of competitive newsvendor problem between a single supplier and multiple retailers under simultaneous price and safety stock competition and under buy-back contract. In this model, we compute Nash equilibrium prices, safety stocks, optimal wholesale, optimal supplier and retailers' profits, numerically. In addition, we discuss the effect of chain and distribution parameters on optimal decision variables and expected profits. This chapter is organized as follows: First, we introduce the literature related to this study, its objective, and the application of this model in industrial management. Second, we present our contract model and we discuss the sufficient conditions on the existence and the uniqueness of the Nash solution. Third, we investigate the model with exponential distribution function and with linear demand function. Forth, we present the supply chain optimization and its solution. Finally, we present numerical results and discuss the behavior of Nash equilibrium solution with the competition and distribution parameters. The results will be compared to that of non-competitive case and chain optimization.

3.2 Literature and Objective

The majority of problems in supply chain management can be translated into mathematical models, which are solved based on the setting of the chain parameters. As a simple model, we find the newsvendor problem, in which a single supplier sells his products to a single retailer. This elementary problem was studied intensively in several publications such as in Cachon and Netessine , Zhao and Atkins, and Solyali and Sural [1-3]. Although, the differences between these studies are the decision criteria and the nature of the demand function, the objective is relatively the same. In first stage, the supplier and the retailer must take actions to optimize their profits. In second stage, their objective is to optimize the total supply chain. Finally, the problem is entirely solved. The actions can be taken with coordination and/or by setting a contract between the supplier and the retailer as reported by Xue *et al.* and Fugate *et al.* [4, 5]. However, there are not only one retailer and one supplier in the market. This reality leads to the study of newsvendor problems in multi-retailers and/or multi-suppliers supply chains as in Soares *et al.* [6]. In another term, a competition between the different actors of the chain cannot be avoided. For example, the retailers compete in the market to attract the maximum number of consumers. This competition was studied in some types of contracts, such as the contract models between multiple retailers with stochastic demand and under various variables of decision.

As discussed in chapter 1, Bernstein and Federgruen [7] have studied the competition between multiple retailers in the case of price dependent demand, where retailers compete on retail prices. Zhao and Atkins [2] have developed, through a contraction mapping approach, the sufficient conditions of existence and uniqueness of Nash equilibrium in simultaneous price and safety-stock dependent demand function. Safety-stock is extremely important in the increase or decrease of customization degree since in case where a retailer faces out of stock, his consumer can move to another retailer. In this model, Zhao and Atkins [2] has not introduced coordination contract between the supplier and the retailers which exist in real market and also they did not deeply analyzed the effect of price and safety stock competition.

Keeping in mind the model of Bernstein and Federgruen where buyback contract was used, we will introduce this coordination contract in the model of Zhao and Atkins and we will discuss the new condition of existence and uniqueness of Nash solution. Buy back contract is a good example of appropriate contracts, which redistributes the risk of overstocking. Furthermore, the effect of price and safety stock competition factors will be simulated numerically and compared to the case of only price competition (chapter 1) and non-competitive model. Concerning the application of this model in management industry, it can be used in all perishable products industries that are needed by consumer every day. In this application, the retailer orders a limited quantity depending on its random customization. This retailer can lose some of his consumers who can move to another retailer to by the same product in case of out of stock. From this interpretation, it is necessary for each retailer to introduce a safety stock to keep his consumers. However, this safety stock increases the total inventory and can result in decrease of profits and increase of costs related inventory.

For this model, we compute Nash equilibrium prices, optimal wholesale, and optimal buyback rates for the supplier's and the retailer's profits, and supply chain optimal retailers' prices, numerically. We also discuss properties on a relationship between these values and the demand distribution. We present our contract model and we discuss the sufficient conditions on the existence and the uniqueness of the Nash solution. We investigate the model with exponential distribution function and with linear demand function. We present the supply chain optimization and its solution. Finally, we present numerical results and discuss the behavior of Nash equilibrium solution with the competition and distribution parameters. The results will be compared to that of non-competitive case and chain optimization.

3.3 Competing Retailers' Model

The model of competing retailers for one supplier (S) and N retailers $(R_i)_{1\leq i\leq N}$ is shown in Fig. 3.1. The supplier incurs retailer R_i a wholesale price w_i for each product. The supplier produces products at a constant production cost rate c_i including transportation cost to R_i. We assume that the supplier has ample capacity to satisfy any retailer demand. A buyback rate $-\infty < v_i < +\infty$ for unsold items is used in the supply chain. To avoid trivial setting, the model parameters are chosen as $v_i < c_i < w_i$ for $1\leq i \leq N$. Each R_i fixes his selling price p_i and safety stock y_i before ordering his quantity from the supplier. The demand function is expressed as $L_i(\vec{p}) + \varepsilon_i$, where $\vec{p} = (p_1, p_2, \ldots, p_N)$, $L_i(\vec{p})$, and ε_i denote the retailer price vector, the deterministic part of demand, and the stochastic part of demand, respectively. The deterministic part of demand decreases with retail price p_i and increases with other retailers' prices p_j , which gives $\partial L_i(\vec{p})/\partial p_i < 0$ and $\partial L_i(\vec{p})/\partial p_j > 0$ for $(i \neq j)_{1\leq i,j\leq N}$. The stochastic parts of the demand ε_i are assumed to be mutually independent for $1 \leq i \leq N$ with a Cumulative Density Function (cdf) F_{ε_i} and a Probability Distribution Function (pdf) f_{ε_i}. The total inventory level of R_i is expressed as $Y_i = L_i(\vec{p}) + y_i$. A price-independent coefficient γ_{ji}, called *spill rate*, is used to characterize lost sales of retailer j in regards to retailer i. The total demand of R_i can be expressed as $D_i(y_{-i}) = L_i(\vec{p}) + D_i^s(y_{-i})$, where $y_{-i} = (y_1, y_2, \ldots, y_{i-1}, \; y_{i+1}, \ldots, y_N)$ denotes the safety stocks vector without y_i and $D_i^s(y_{-i}) = \varepsilon_i + \sum_{j\neq i}^{N} \gamma_{ji}(\varepsilon_j - y_j)^+$ is the effective stochastic component of the demand of R_i with $(a)^+ = max\,(a, 0)$. The cdf and pdf of $D_i^s(y_{-i})$ are given by $F_{D_i^s(y_{-i})}$ and $f_{D_i^s(y_{-i})}$, respectively. They are calculated from cdf and pdf of ε_i. A failure rate of a stochastic variable X is defined by $r_X = f_X/(1 - F_X)$.

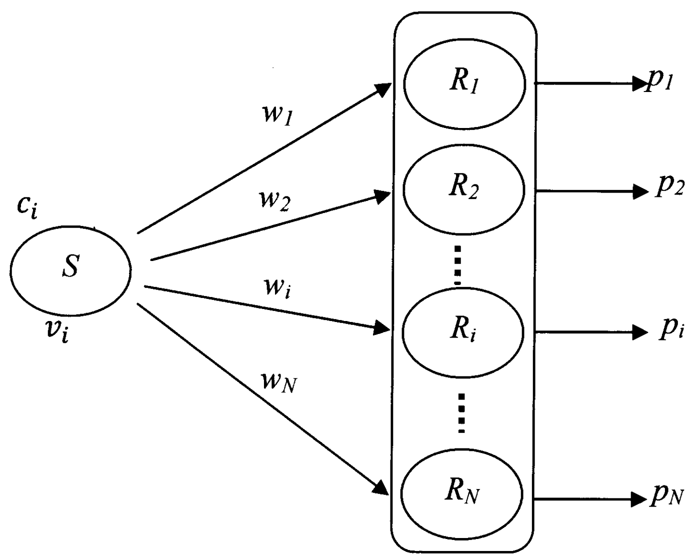

Fig. 3.1 Competing retailers model

As mentioned in the introduction, this model studies multiple retailers' competition in a decentralized supply chain, where the demand depends simultaneously on price and safety- stock. The safety stock competition factor is given by the *spill rate* γ_{ji}, defined in the stochastic part of the demand. The price competition factor is given in the deterministic part of the demand. In this study, we use a linear form of demand with symmetric price competition factor θ. It is expressed with $L_i(\vec{p}) = a - bp_i + \sum_{j\neq i}^{N} \theta(p_j - p_i)$ for $1 \leq i \leq N$, with $a > 0$ and $b > 0$. This form of demand is largely used in supply chain management literature. It goes back to Shubik and Levitan [8]. Then, it is used in many models such as: in Dixit [9], Banker *et al.* [10], Tsay and Agrawal [11], and Boyaci and Ray [12]. In practice, Shubik and Levitan reported that this linear form of price competition is used in automobile market in the United States. The profit function of R_i is expressed as $\pi_i(\vec{p}, \vec{y}) = p_i E[min(Y_i, D_i(y_{-i}))] + v_i E\left[(Y_i - D_i(y_{-i}))^+\right] - w_i Y_i$. Using $(Y_i - D_i(y_{-i}))^+ = max\left((Y_i - D_i(y_{-i})), 0\right) = Y_i - min(Y_i, D_i(y_{-i}))$, it can be rewritten as:

$$\pi_i(\vec{p}, \vec{y}) = \pi_i^d(\vec{p}) + (v_i - w_i)y_i + (p_i - v_i)E[min(y_i, D_i^s(y_{-i}))], \tag{3.1}$$

where $\pi_i^d(\vec{p}) = (p_i - w_i)L_i(\vec{p})$ denotes the deterministic part of the profit function. This model is studied under the assumption that $(p_i, y_i) \in \{w_i \leq p_i \leq p_i^{max}, 0 \leq y_i \leq y_i^{max}\}$, where p_i^{max} and y_i^{max} are chosen arbitrarily large. The supplier profit function is given by

$$\Pi_s = \sum_{i=1}^{N}(w_i - c_i)\, L_i(\vec{p}) + \sum_{i=1}^{N}(w_i - c_i - v_i)y_i + v_i E[min(y_i, D_i^s(y_{-i}))].$$

3.4 Nash Equilibrium Conditions

In this model, the supplier is a Stackelberg leader who decides the wholesale price for each retailer. The retailers compete each other on retail prices and safety stocks to maximize their own profits. To derive the conditions of existence and uniqueness of Nash solution, we apply *theorem* 1 of reference [2], where the difference between our model and that of this reference is the adoption of buyback cost in the chain. In [2], Zhao and Atkins proved that the quasi-concavity of retailer's profit function $\pi_i(\vec{p}, \vec{y})$ in p_i and y_i requires two conditions (A) and (B), given by

(A) $\partial^2 \pi_i^d(\vec{p})/\partial p_i^2 < 0$ and $\partial^3 \pi_i^d(\vec{p})/\partial p_i^3 \leq 0$,

(B) ϵ_i has an increasing failure rate (IFR) distribution for $1 \leq i \leq N$

Then, the quasi-concavity of retailer's profit function $\pi_i(\vec{p}, \vec{y})$ in p_i and y_i proves the existence of Nash solution. The conditions (A) and (B) are independent of buyback cost. Thus, they are used to prove the existence of Nash solution in our model. In addition, Zhao and Atkins reported that the best solution for the profit function $\pi_i(\vec{p}, \vec{y})$ is given by (3.2) – (3.3):

$$\partial \pi_i^d(\vec{p})/\partial p_i + E[min(y_i, D_i^s(y_{-i}))] = 0, \tag{3.2}$$

$$-w_i + p_i E\left[min\left(y_i, D_i^s(y_{-i})\right)\right] = 0. \tag{3.3}$$

By introducing buyback cost v_i in the chain, only equation (3.3) is changed and becomes

$$(v_i - w_i) + (p_i - v_i)E\left[min\left(y_i, D_i^s(y_{-i})\right)\right] = 0. \tag{3.4}$$

About the uniqueness of the Nash solution, Zhao and Atkins decomposed the demand on three components: deterministic, demand depends only on retail price, and demand depends only on safety stock. From the conditions of uniqueness of Nash solution in each part, they developed strong conditions of uniqueness for the entire problem, as given by

$$-\partial^2\pi_i^d(\vec{p})/\partial p_i^2 > \sum_{j\neq i}^N \partial^2\pi_i^d(\vec{p})/\partial p_i\partial p_j \text{ and } \sum_{j\neq i}^N \gamma_{ji} + 1/(w_i r_{D_i^s(y_{-i})}(y_i)) < 1$$

The first condition is valid for our model because it is independent of buy-back cost. However, the second condition is changed by introducing buy-back parameter and given by:

$$\sum_{j\neq i}^N \gamma_{ji} + 1/((w_i - v_i) r_{D_i^s(y_{-i})}(y_i)) < 1.$$

With the demand function described above, the deterministic part of R_i profit function is expressed as $\pi_i^d(\vec{p}) = (p_i - w_i)(a - bp_i + \sum_{j\neq i}^N \theta(p_j - p_i))$. The first and second derivatives of $\pi_i^d(\vec{p})$ on p_i are $\partial\pi_i^d(\vec{p})/\partial p_i = a + (w_i - 2p_i)(b + (N-1)\theta) + \theta\sum_{j\neq i}^N p_j$ and $\partial^2\pi_i^d(\vec{p})/\partial p_i^2 = -2(b + (N-1)\theta) < 0$. This result satisfies the condition (A). The demand function is linear and symmetric on price competition. Then, the first condition of Nash solution's uniqueness is satisfied if $2b + (N-1)\theta > 1$. In addition, to satisfy condition (B), the pdf of ε_i must have an IFR. This property is satisfied only by some kind of pdf such as exponential and uniform distributions. In this study, we restrain our study to exponential distribution because it facilities theoretical analysis more than in case of uniform one. In addition, our essential objective is studying the effect of chain parameters on decision variables and expected profits, which is insensitive to the distribution function of stochastic variables. Then, for $1 \leq i \leq N$, the cdf and pdf of ε_i are expressed for all $x \geq 0$ by $F_{\varepsilon_i}(x) = 1 - e^{-\lambda_i x}$ and $f_{\varepsilon_i}(x) = \lambda_i e^{-\lambda_i x}$, respectively. Now, we have to explicit the cdf and pdf of effective stochastic component of R_i demand function $D_i^s(y_{-i})$.

At this step, we have to avoid progressive difficulties when the number of retailers exceeds two because the number of stochastic variables will be equal or greater than two. Then, we restrain our theoretical and numerical analysis to the case of two retailers, and we believe that this condition does not limit our fundamental understandings of this model. We start by evaluating the cdf of $D_1^s(y_2)$ using the following equation:

$$P_r(D_1^s(y_2) \leq x) = \int_0^{y_2 + x/\gamma_{21}} P_r\left(D_1^s(y_2) \leq x \,|\, \epsilon_2 = u\right) f_{\varepsilon_2}(u)du.$$

The right hand side can be written as

$$\lambda_2 \int_0^{y_2} P_r(\epsilon_1 \leq x)\, e^{-\lambda_2 u}du + \lambda_2 \int_{y_2}^{y_2 + x/\gamma_{21}} P_r(\epsilon_1 + \gamma_{21}(\epsilon_2 - y_2) \leq \epsilon_2 = u)\, e^{-\lambda_2 u}du$$

After calculation, for $i = 1,2$, the cdf and pdf of $D_1^s(y_2)$ and $D_2^s(y_1)$ are given by:

$$F_{D_1^s(y_2)}(x) = 1 - e^{-\lambda_1 x} - \frac{\gamma_{21}\lambda_2 e^{-\lambda_2 y_2}}{\lambda_2 - \gamma_{21}\lambda_1}(e^{-\lambda_1 x} - e^{-\frac{\lambda_2}{\gamma_{21}}x}),$$

$$f_{D_1^s(y_2)}(x) = \lambda_1 e^{-\lambda_1 x} - \frac{\gamma_{21}\lambda_2 e^{-\lambda_2 y_2}}{\lambda_2 - \gamma_{21}\lambda_1}(\frac{\lambda_2}{\gamma_{21}} e^{-\frac{\lambda_2}{\gamma_{21}}x} - \lambda_1 e^{-\lambda_1 x}),$$

$$F_{D_2^s(y_1)}(x) = 1 - e^{-\lambda_2 x} - \frac{\gamma_{12}\lambda_2 e^{-\lambda_1 y_1}}{\lambda_1 - \gamma_{12}\lambda_2}(e^{-\lambda_2 x} - e^{-\frac{\lambda_1}{\gamma_{12}}x}), \text{ and}$$

$$f_{D_2^s(y_1)}(x) = \lambda_2 e^{-\lambda_2 x} - \frac{\gamma_{12}\lambda_2 e^{-\lambda_1 y_1}}{\lambda_1 - \gamma_{12}\lambda_2}(\frac{\lambda_1}{\gamma_{12}} e^{-\frac{\lambda_1}{\gamma_{12}}x} - \lambda_2 e^{-\lambda_2 x}),$$

From equation (3.3), we obtain for $i = 1,2$:

$$p_i = \frac{w_i - v_i F_{D_i^s(y_{-i})}(y_i)}{1 - F_{D_i^s(y_{-i})}(y_i)} \tag{3.5}$$

Then, the retail prices can be expressed from (3.5). For $i = 1,2$ we have :

$$\begin{cases} p_1 = \dfrac{w_1 - v_1(1 - e^{-\lambda_1 y_1} - \frac{\gamma_{21}\lambda_2 e^{-\lambda_2 y_2}}{\lambda_2 - \gamma_{21}\lambda_1}(e^{-\lambda_1 y_1} - e^{-\frac{\lambda_2}{\gamma_{21}}y_1}))}{e^{-\lambda_1 y_1} + \frac{\gamma_{21}\lambda_2 e^{-\lambda_2 y_2}}{\lambda_2 - \gamma_{21}\lambda_1}(e^{-\lambda_1 y_1} - e^{-\frac{\lambda_2}{\gamma_{21}}y_1})} = h_1(y_1, y_2) \\ p_2 = \dfrac{w_2 - v_2(1 - e^{-\lambda_2 y_2} - \frac{\gamma_{12}\lambda_2 e^{-\lambda_1 y_1}}{\lambda_1 - \gamma_{12}\lambda_2}(e^{-\lambda_2 y_2} - e^{-\frac{\lambda_1}{\gamma_{12}}y_2}))}{e^{-\lambda_2 y_2} + \frac{\gamma_{12}\lambda_2 e^{-\lambda_1 y_1}}{\lambda_1 - \gamma_{12}\lambda_2}(e^{-\lambda_2 y_2} - e^{-\frac{\lambda_1}{\gamma_{12}}y_2})} = h_2(y_1, y_2) \end{cases}.$$

The equation (3.5) shows the relationship between retail prices and safety stocks in case of Nash equilibrium. From that, the decision variables can be simplified to only safety stock variables. To explicit the solution of the problem we have to evaluate the equation (3.2). The second term of (3.2) can be expressed as $E[min(y_i, D_i^s(y_{-i}))] = y_i - E\left[(y_i, -D_i^s(y_{-i}))^+\right] = E_i(y_i, y_{-i})$, with $E\left[(y_i, -D_i^s(y_{-i}))^+\right] = y_i F_{D_i^s(y_{-i})}(y_i) - \int_0^{y_i} f_{D_i^s(y_{-i})}(u)\, du$. Then, we obtain

$$\begin{cases} E_1(y_1, y_2) = \dfrac{(1 - e^{-\lambda_1 y_1})}{\lambda_1} - \dfrac{\gamma_{21}\lambda_2 e^{-\lambda_2 y_2}}{\lambda_2 - \gamma_{21}\lambda_1}\left(\dfrac{\gamma_{21}(1 - e^{-\frac{\lambda_2}{\gamma_{21}}y_1})}{\lambda_2} + \dfrac{(1 - e^{-\lambda_1 y_1})}{\lambda_1}\right) \\ E_2(y_1, y_2) = \dfrac{(1 - e^{-\lambda_2 y_2})}{\lambda_2} - \dfrac{\gamma_{12}\lambda_2 e^{-\lambda_1 y_1}}{\lambda_1 - \gamma_{12}\lambda_2}\left(\dfrac{\gamma_{12}(1 - e^{-\frac{\lambda_1}{\gamma_{12}}y_2})}{\lambda_1} + \dfrac{(1 - e^{-\lambda_2 y_2})}{\lambda_2}\right) \end{cases}.$$

The left hand side of equation (3.2) depends on demand function. The equations (3.2) and (3.3) can be rewritten in a non-linear system of equations $g_1(y_1, y_2) = 0$ and $g_2(y_1, y_2) = 0$, with:

$$\begin{cases} g_1(y_1, y_2) = a + (\theta + b)w_1 - 2(\theta + b)h_1(y_1, y_2) + \theta h_2(y_1, y_2) + E_1(y_1, y_2) = 0 \\ g_2(y_1, y_2) = a + (\theta + b)w_2 - 2(\theta + b)h_2(y_1, y_2) + \theta h_1(y_1, y_2) + E_2(y_1, y_2) = 0 \end{cases}$$

The Newton method is used to solve this non-linear system of equation, simultaneously. We set the initial value $Y^0 = (y_1^0, y_2^0)^T$ and we make an iterative computation $Y^k = \left(y_1^k, y_2^k\right)^T$ to solve $g_1(y_1, y_2) = 0$ and $g_2(y_1, y_2) = 0$, according to the following solution:

$$Y^{k+1} = Y^k - \begin{bmatrix} \frac{\partial g_1(y_2^k)}{\partial y_1} & \frac{\partial g_1(y_1^k)}{\partial y_2} \\ \frac{\partial g_j(y_j^k)}{\partial y_1} & \frac{\partial g_2(y_2^k)}{\partial y_2} \end{bmatrix} \begin{pmatrix} g_1(y_1^k) \\ g_1(y_2^k) \end{pmatrix}.$$

The computation starts to search the wholesale price which maximizes the supplier profit function by incrementing its value from $c_i + 1$. After that, the Nash safety stocks solution is obtained using the Newton method as described before. The Nash retail prices will be calculated according to equation (3.4). Finally, the retailers' profit functions are given by

$$\begin{cases} \pi_1(\vec{p},\vec{y}) = (p_1 - w_1)L_1(\vec{p}) + (v_1 - w_1)y_1 + (p_1 - v_1)E_1(y_1, y_2) \\ \pi_2(\vec{p},\vec{y}) = (p_2 - w_2)L_2(\vec{p}) + (v_2 - w_2)y_2 + (p_2 - v_2)E_i(y_1, y_2) \end{cases}.$$

Three parameters in the model have a particular importance. These parameters are the spill rate, the price competition factor, and the distribution parameter. It is important to note that in the case of zero spill rates, the competition is restrained to retail prices. In the case of zero price competition factors, we obtain safety stock competition model. However, if the two parameters are equal to zero, we obtain a non-competitive model with only price sensitive demand. Thus, it will be important to compare our results to that non-competitive case. Furthermore, the effect of the model parameters on the wholesale price, the safety stock, the retail prices, the total inventory, the retailer's profit functions, the supplier profit function, and the total profit function will be compared with the solution of supply chain optimization, studied in next section.

3.5 Supply Chain Optimization

Supply chain optimization is the setting of processes and tools to ensure the optimal operation of manufacturing and distribution in a supply chain. For example, this can be translated to the setting of the optimal prices and safety stocks to maximize the total profit of the chain. For example, in our case, to search the optimal solution, we consider the total profit function for two retailers and one supplier as given by

$$\Pi_T(\vec{p},\vec{y}) = \sum_{i=1}^{2}((p_i - c_i)L_i(\vec{p}) - c_i y_i + p_i E_i(y_1, y_2)) \tag{3.6}$$

The optimal solution can be obtained by differentiating equation (3.6) on the four independent variables (retail prices and safety stocks) and set the system to zero. Then, we obtain

$$\begin{cases} \frac{\partial \Pi_T(\vec{p},\vec{y})}{\partial p_1} = a - 2(\theta + b)p_1 + 2\theta p_2 + (\theta + b)c_1 - \theta c_2 + E_1(y_1, y_2) = 0 \\ \frac{\partial \Pi_T(\vec{p},\vec{y})}{\partial p_2} = a - 2(\theta + b)p_2 + 2\theta p_2 + (\theta + b)c_2 - \theta c_1 + E_2(y_1, y_2) = 0 \\ \frac{\partial \Pi_T(\vec{p},\vec{y})}{\partial y_1} = -c_1 + p_1 \frac{\partial E_1(y_1,y_2)}{\partial y_1} + p_2 \frac{\partial E_2(y_1,y_2)}{\partial y_1} = 0 \\ \frac{\partial \Pi_T(\vec{p},\vec{y})}{\partial y_2} = -c_2 + p_{i2} \frac{\partial E_2(y_i,y_j)}{\partial y_2} + p_1 \frac{\partial E_1(y_1,y_2)}{\partial y_2} = 0 \end{cases} \tag{3.7}$$

The system of equations (3.7) is independent of wholesale prices and buyback rates. It is a nonlinear system. Thus, Newton method is used to solve it. The solution will be compared with that of Nash equilibrium in the next section.

3.6 Numerical Results

Our computation is restrained to symmetric parameters and we define for $i, j = 1, 2$, the spill rate $\gamma = \gamma_{ij}$, the distribution parameter $\lambda = \lambda_i$. The numerical values of chain and distribution parameters, used for simulation are: $a = 300$, $b = 1$, $\theta = 0.5$, $\gamma = 0.2$, $\lambda = 1$, $v_i = 45$, and $c_i = 60$. The program is coded by C and the computations are done using Fujitsu C compiler on PC. As explained in section 3.4, the strategy of the simulations is based on the maximization of the supplier profit function to set the wholesale price. In Tables 3.1, 3.2, and 3.3, we present the numerical results for different values of the spill rate γ, the price competition factor θ, and the distribution parameter λ. We report the optimal·supplier profit function, the two retailer's profit functions, the entire profit function, the wholesale prices which maximize the supplier profit function, the two retail prices, the two retail safety stocks, and the two retail total demand functions. As a first result, we find that the optimal wholesale prices to maximize the supplier profit function are not affected by the various of γ, λ, and θ. The prices and the safety stocks for the two retailers are the same due to the symmetric value of the chain parameters.

3.6.1 Behavior of the Nash solution with the chain parameters

3.6.1.1 Non-completive model

The case of non-competitive model is obtained by setting the spill rate and the price competition factor to zero. The results are given in the second column of Table 3.1. This elementary newsvendor problem was studied by Petruzzi and Dada [13]. The retail prices, the safety stocks, the profit function of retailers are higher than that found in case of competition. However, the supplier profit function, the total profit function in case of Nash, and the total inventories of the retailers are less than in the case of competition.

3.6.1.2 Effect of the spill rate γ

First, in absence of safety stock competition ($\gamma = 0$), the effective stochastic component of the demand is restrained to ε_i and the model is only under price competition and price sensitive-demand. In this case, we find our results published in [14]. In addition, the retail prices are high and the total inventory is low. Increasing the spill rate γ increases the stochastic part of the demand function and consequently increases the total demand function. As a consequence, the retail prices and the retailers' profit functions decrease. However, the supplier profit function increases but it cannot compensate the decrease of the retailers' profit functions, thus the total profit function decreases.

3.6.1.3 Effect of price competition factor θ

As in last paragraph, we discuss the model in absence of price competition ($\theta = 0$). In this case, the results are near that of the case of non-competitive model except the safety stock. This is explained by the low value of the spill rate (= 0.2) which depends directly the safety stock. In addition, increasing the price competition factor θ affects the Nash solution due its existence in the left hand side of equation (3.2). In reference [2], Proposition 2, it was proved that in the case of linear demand function, the total demand increases with θ in contrast to the safety stocks. Our results are in accordance with this proposition. Thus, as the intensity of price competition increases, the increase of the deterministic part of the total demand function exceeds the decrease of the safety stocks. Therefore, the retailers increase their total demand and keep lower their safety stocks, which results in decrease of their selling prices. In addition, the supplier's profit increases and compensates the dropping of retailers' profits, which results in increase of total profit function.

Table 3.1 Effect of spill factor γ

Spill factor	Non-competition	0.0	0.1	0.2	0.3
Supplier profit function	14446.03	17324.14	17334.70	17342.93	17349.06
Retailers profit functions	3610.38	3465.48	3459.58	3448.80	3433.22
Entire expected profits (EEP-Nash)	21666.80	24255.08	24253.86	24240.52	24215.51
Wholesale price	180	180	180	180	180
Retail prices	240.15	228.11	228.11	228.11	228.08
Safety stocks	0.368	0.304	0.376	0.430	0.469
Total Inventories	60.21	72.20	72.26	72.32	72.38

Table 3.2 Effect of retail price competition factor θ

Price competition factor	0	0.5	0.7	1.5
Supplier profit function	14464.76	17342.93	18195.21	20628.74
Retailers profit functions	3592.34	3448.79	3350.73	2930.91
Entire expected profits (EEP-Nash)	21649.45	24240.52	24896.67	26490.56
Wholesale price	180	180	180	180
Retail prices	240.15	228.11	224.54	214.35
Safety stocks	0.492	0.430	0.411	0.351
Total Inventories	60.43	72.32	75.87	86.004

3.6.1.4 Effect of the distribution parameter λ

The distribution parameter λ characterizes the distribution function rate in the stochastic part of the demand function. Increasing λ has a weak impact on retail prices, supplier profit function, total demand function, and total profit; however drops the safety stock.

Table 3.3 Effect of distribution parameter λ

Distribution parameter	1	2	3
Supplier profit function	17342.93	17311.44	17300.96
Retailers profit functions	3448.79	3452.39	3453.59
Entire expected profits (EEP-Nash)	24240.52	24216.23	24208.15
Wholesale price	180	180	180
Retail prices	228.11	228.05	228.03
Safety stocks	0.430	0.215	0.143
Total Inventories	72.32	72.16	72.11

3.6.2 Comparison between the Nash solution and the optimal one

The results of the optimal solution, analyzed in section 3.4, are given in Tables 3.4, 3.5, and 3.6. In the case of supply chain optimization, the retail prices are not affected considerably by the increase of γ in contrast to the safety stocks which increase with it. This can be explained by the correlation between the spill rate γ and the stochastic part of the demand function. The ratio (EEP-Nash)/(EEP-optimal) is nearly constant because the impact of retail prices on the total profit function is higher than the impact of the safety stocks and in the two cases of Nash solution and optimal one. The factor θ does not affect the optimal solution due to the symmetric values of the retail prices. However, the comparison between the Nash solution and the optimal one shows a large difference on the retail prices and the safety stocks. The retails prices in the optimal solution are lower than in the case of Nash solution in contrast to the safety stocks. This result affects considerably the total demand functions which are high in the case of optimal solution due essentially to the effect of the retail prices. The ratio (EEP-Nash)/(EEP-optimal) increases with increase of the competition factor θ. This can be explained by the effect of the strong price competition which leads retailers to reduce their safety stocks and increase their profits. The impact of the distribution rate λ on the safety stocks is considerable in contrast to the retail prices which are nearly constant. This can be explained by the same effect of the spill rate γ, however in this case the safety stocks decrease dramatically. The ratio (EEP-Nash)/(EEP-optimal) is nearly constant because the impact of retail prices on the total profit function is higher than the impact of the safety stocks and in the two cases of Nash solution and optimal one the retail prices are not considerably affected by the various of λ.

Table 3.4 Effect of spill factor γ

Spill rate	Non-competition	0.0	0.1	0.2	0.3
Entire expected profits(EEP-optimal)	28908.39	28908.39	28898.30	28883.52	28863.46
(EEP-Nash)/ (EEP-optimal)	0.749	0.839	0.839	0.839	0.839
Retail prices	180.33	180.33	180.33	180.32	180.32
Safety stocks	1.10	1.10	1.15	1.25	1.39
Total Inventories	120.76	120.76	120.82	120.92	121.07

Table 3.5 Effect of retail price competition factor θ

Price competition factor	0.0	0.5	0.7	1.5
Entire expected profits(EEP-optimal)	28883.52	28883.52	28883.52	28883.52
(EEP-Nash)/ (EEP-optimal)	0.749	0.839	0.862	0.917
Retail prices	180.32	180.32	180.32	180.32
Safety stocks	1.24	1.24	1.24	1.24
Total Inventories	120.92	120.92	120.92	120.92

Table 3.6 Effect of distribution parameter λ

Distribution parameter	1	2	3
Entire expected profits(EEP-optimal)	28883.52	28841.27	28827.79
(EEP-Nash)/ (EEP-optimal)	0.840	0.840	0.840
Retail prices	180.32	180.16	180.11
Safety stocks	1.24	0.62	0.41
Total Inventories	120.92	120.46	120.30

3.7 Conclusion

In this study, the condition of Nash equilibrium solution are presented for a buyback contract model for one supplier and multiple retailers, where the demand is stochastic and depends on price and safety stock. The performances of the Nash solution are discussed numerically for various price and safety stock competition factors. They are also compared with that of the optimization solution and the case of non-competitive model. The Nash solution is computed based on the maximization of the supplier profit function and the use of the Newton method to solve the non-linear equations. This solution is found to depend strongly on the price competition factor, the spill rate, and the distribution parameter. In addition, the ratio of entire profit function of the Nash solution and the optimal one is found to increase with price completion factor; however, it is nearly constant when the spill rate and the distribution parameters are varied. This problem can be extended by introducing new decision parameters such as the lead-time.

In chapter 2 and 3, we have focused our study to retailers' competition. In next chapter, we move to suppliers' competition under sale rebate contract in a decentralized supply chain with price sensitive demand.

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Chapter 4

Competing Suppliers under Sale-Rebate Contract and Price sensitive Demand in a Decentralized Supply Chain

4.1 Introduction

The current chapter focuses on the study of a competition between two independent suppliers who sell their products to a common retailer in a decentralized supply chain, under sales-rebate contract, and wherein the demand is sensitive to retail price. This chapter is organized as follows: First, we introduce the literature related to this study, its objective, and the application of this model in industrial management. Next, we formulate the model. Then, we explore the condition of existence and uniqueness of the retailer's optimal solution and also that that of suppliers. Finally, we present the numerical results and their discussion.

4.2 Literature and Objective

Competition in supply chain management has been reported rarely in the literature of economics, where different contracts and scenarios have been studied. Most of these publications have focused only on competition between retailers who order their products from a single supplier and compete on different types of decision variables, such as retail price, lead-time, order quantity, time service,... etc. The contracts that have been studied in supply chain management are the wholesale price contracts, the buyback contracts, the revenue–sharing contracts, the quantity–flexibility contracts, the quantity–discount contracts, and the sale-rebate contracts (see Cachon [1]). The common conclusion, from the study of these contracts, is the favor that the supplier achieves than the competing retailers. In actual globalized and competitive market, however, the retailer has the possibility to provide his products from different suppliers, in order to maximize his profits and to compensate the spoiled choices of the consumers. For that, the impact of competition between suppliers should be studied with the same importance as in retailers' case.

Actually, few studies were focused on this subject. The only recent study that discusses the competition between two suppliers under different contractual forms is reported by Cachon and Kök [2]. In this work, the authors have studied three types of contracts: wholesale-price contract, quantity-discount contract, and two-part-tariff contract. However, they did not study the sale-rebate contract and its impact on the different chain performances. For that, the current chapter focuses on the study of a competition between two independent suppliers who sell their products to a common retailer in a decentralized supply chain, under sales-rebate contract, and wherein the demand is sensitive to retail price. In addition, retailer's inventory related operational costs are included based on economic order quantity (EOQ) model Hax *et al.* [3].

As described by Taylor [4], two forms of sale–rebate contracts exist. The first one is called linear, in which the supplier offers channel rebate to the retailer for any product sold. This form of sale rebate contract was used by Nissan Company and the market of hardware [4].The second form is more restrictive, in which the channel rebate is conditioned by the setting of a target. This last type of contract was studied in several publications [4-5]. Its main objective is to incite the retailer to make effort to order a quantity more than the target. It is used in the fields of hardware, software, and auto industries [4]. In personal computer hardware industry and during the last decade, Compaq, Hewlett-Packard (HP), and IBM have introduced sale rebate contracts based on the volume of sales to their consumers and increase of channel rebate between 3% and 6% [4]. Channel rebates are also important in the software industry. Microsoft and Novel have offered channel rebates between 3% and 5.5% [6]. Furthermore, Lotus and Symantec have also used channel rebates [7-8]. In automobile field, sale-rebate contracts were characterized to be more incentive and have included 13 auto industrials through more than 188 models [9]. This form of sale rebate contract will be the essence of this chapter. The conditions of existence and uniqueness of the retailer's optimal solution and that of the suppliers are characterized, however, due to the non-linearity of the inventory costs, it was difficult to obtain a close theoretical solution form. The optimal demand rates and wholesale prices of the model are calculated numerically. The profit functions of the retailers and suppliers are evaluated and the total Nash profit is compared to that of the integrated system. Furthermore, the impact of the inventory related costs is investigated numerically.

4.3 Model Formulation

A schematic illustration of our model is given in Fig. 4.1. It consists of a common retailer who buys two products from two competing suppliers $(S_i)_{1 \le i \le N}$). Each supplier announces his payment scheme by offering his whole-sale price w_i, and his channel rebate u_i (i.e., the amount paid by the supplier to the retailer for each sold unit beyond a target t_i. The supplier S_i has ample capacity to satisfy any retailer demand and produces products at a constant production cost rate c_i. To avoid trivial setting, it is assumed that $0 < c_i < w_i < p_i, u_i \geq 0$, and $t_i \geq 0$. The demand rate vector $d = (d_i, d_j)$ depends on retail price for the pair of products. This study is restrained to the linear form of demand, which satisfies for all $i, j \in [1, 2]$, $\partial d_i(p)/\partial p_i < 0$ and $\partial d_i(p)/\partial p_j \geq 0$. It can be expressed as $d_i(p_i, p_j) = a_i - \alpha_i p_i + \gamma_{ij} p_j$ where a_i, α_i, and γ_{ij} ($\min(\alpha_i, \alpha_j) > \gamma_{ij}$) are the base market potential from the supplier i, the sensitivity of the demand to the product i, and the sensitivity of the demand to product j, respectively. This linear form of demand was largely used in management literature [10-15]. For simplicity, the inverse form of the demand is used as $p_i(d_i, d_j) = \theta_i - \beta_i d_i + \gamma_j d_j$ where $\theta_i = (\alpha_j a_i + \gamma_{ij} a_j)/(\alpha_j \alpha_i - \gamma_{ij}\gamma_{ji})$, $\beta_i = \alpha_j/(\alpha_j \alpha_i - \gamma_{ij}\gamma_{ji})$, $\gamma_i = \gamma_{ij}/(\alpha_j \alpha_i - \gamma_{ij}\gamma_{ji})$, and $\beta_i > \gamma_j > 0$ for all $i, j \in [1, 2]$ (Cachon and Kök [2]). Note that the demand rates d_i and d_j will be different when $\beta_i = \beta_j = \beta$, $\gamma_i = \gamma_j = \gamma$, and $\theta_i \neq \theta_j$. As the retailer will profit from the sale-rebate contract by ordering large quantity, he will face an increase in the inventory related operational costs. In this model, the inventory costs that exist in the economic order quantity (EOQ) model is adopted (Hax and Candea [3]). In such case, the retailer's inventory related operational costs are given by $G_i(d_i) = K_i d_i^{\lambda}$ where $K_i = \sqrt{2k_i h_i} \geq 0$, k_i, h_i, and $\lambda = 0.5$ denote the economics of scale, the cost per order quantity, the holding cost, and a coefficient, respectively. Let $R_i(d_1, d_2) = p_i(d_1, d_2)d_i$ be the revenue of the retailer from the selling of product i without considering the sale-rebate contract. The total retailer profit function is expressed as

$$\pi(d_1, d_2) = \sum_{i=1}^{2}\left[(\theta_i - \beta_i d_i - \gamma_{3-i} d_{3-i} - w_i)d_i - K_i d_i^{\lambda} + u_i \max((d_i - t_i), 0)\right]. \quad (4.1)$$

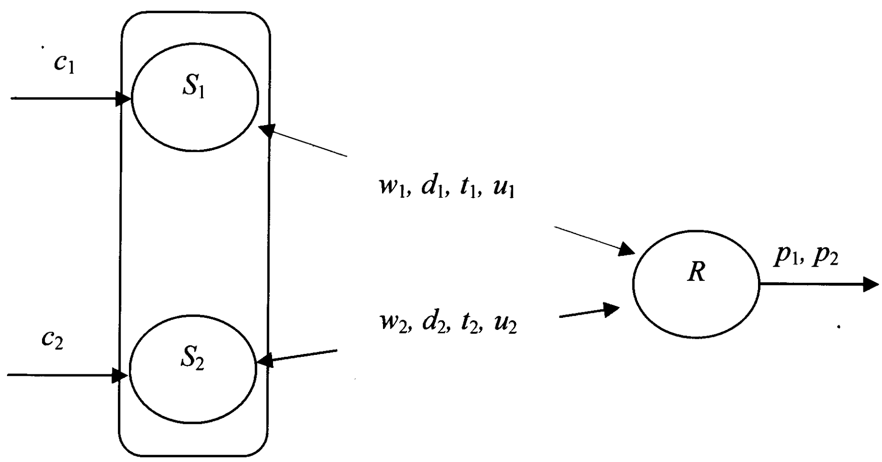

Fig. 4.1 Model of a supply chain consisting of one retailer and two suppliers

This profit function can take four different forms based on the position of the demand rate from the two sides of the target. If a unique optimal solution exists, it will be localized in one of the four regions limited by the targets or at the boundaries. The position of the solution depends on the chain parameters. In the case where the two demand rates are less or equal to the targets given by the suppliers, the problem is equivalent to the wholesale contract. For $i \in [1, 2]$, the profit function of the supplier i is given by

$$\Pi_i(d_1, d_2, w_i) = (w_i - c_i)d_i - u_i \max((d_i - t_i), 0). \quad (4.2)$$

In the next section, the conditions of existence and uniqueness of the optimal solution are discussed by considering the concavity of the profit function of the retailer.

4.4 Retailer's Optimal Decision

Let $S_i(d_1, d_2)$ denote the first order derivative of the total retailer profit function with respect to d_i. For $i, j \in [1, 2]$, it is expressed as

$$S_i(d_1, d_2) = \theta_i - 2\beta_i d_i - w_i - \lambda K_i d_i^{\lambda-1} + u_i 1(d_i > t_i) - (\gamma_i + \gamma_j)d_i, \tag{4.3}$$

where $1(A)$ denotes the indicator function which takes 1 if A is satisfied and 0 otherwise. Equation (4.3) depends on the on the sale-rebate u_i, wholesale price w_i, and independent of the targets t_i . Then, the position of the solution cannot be known from the two sides of the targets or at the boundaries. This random situation makes the problem difficult. In addition, the second order derivative of the retailer profit function is given by

$$\partial^2\pi_i(d_1, d_2)/\partial d_i^2 = -2\beta_i + \lambda(1-\lambda)K_i d_i^{\lambda-2}. \tag{4.4}$$

It is worth to note that if $K_i > 0$, $\lim_{x\to 0^+} S_i(d_1, d_2) = \infty$. For that, it is optimal for the retailer and the system to carry both products. The condition of concavity of the profit function for the retailer is given by Lemma 1.

Lemma 1 For a given whole-sale price vector (w_i, w_j), $i, j \in [1, 2]$, the retailer profit function is strictly concave on $\{(d_i, d_j), d_i d_j > 0\}$ under the following conditions:

$$\beta_i = \beta_j = \beta, \gamma_i = \gamma_j = \gamma, \text{ and} \tag{4.5}$$

$$2R_i(d_1, d_2)/G_i(d_i) > [\beta_i/(\beta_i\beta_j - \gamma_i\gamma_j)]\, p_i/d_i. \tag{4.6}$$

Proof The retailer profit function depends on both d_i and d_j. Then, it is strictly concave if its Hessian is a negative definite matrix, which can be satisfied by the two following conditions:

(A) $\partial^2\pi/\partial d_i^2 < 0$ for $i \in [1, 2]$,

(B) $|\partial^2\pi/\partial d_i^2| > \partial^2\pi/\partial d_i \partial d_j$ for $i, j \in [1, 2]$ (i.e., the Hessian is strictly diagonally dominant).

First, $\partial^2\pi/\partial d_i^2$ can be translated to $2p_i d_i/G_i(d_i) > \lambda(1-\lambda)\beta_i^{-1} p_i/d_i$, which holds under (4.6) since $\beta(\beta^2 - \gamma^2) > \beta^{-1}$, and $\lambda^{-1}(1-\lambda)^{-1} = 4$. The condition given by (4.6) means that two times of the ratio between the revenue of the retailer without any contract and the inventory-related costs must be greater than the absolute value of the own price elasticity. This condition was developed by Bernstein and Federgruen [11] for decentralized retailers and then used by Cachon and Kök [2].

Second, as $\beta > \gamma$ in price equation, $\beta(\beta^2 - \gamma^2) > (2\beta - 2\gamma)^{-1}$ and $p_i d_i / G_i(d_i) > \lambda(1 - \lambda)(2\beta - 2\gamma)^{-1} p_i / d_i$ holds under Eq. (4.6).

The satisfaction of conditions given by (4.5) and (4.6) guarantees the existence of a uniqueness of the optimal demand rate (d_i^*, d_j^*)that maximizes the profit function of the retailer. However, this solution depends on the wholesale price. In addition, under positive economics of scale $K_i > 0$, the system of equation $S_i(d_1, d_2) = 0$ for $i \in [1, 2]$ that gives $(d_i^*, d_j^*)$is non-linear, which requires a numerical resolution.

4.5 Competing Suppliers' Optimal Decision

In this section, a competition between two suppliers, who offer sale-rebate to a common retailer, is studied. It is worth to note that the optimal demand rate that the retailer searches for depends on the wholesale prices. Differentiating the profit function Π_i with respect of w_i gives

$$\partial \Pi_i(w_1, w_2)/\partial w_i = (w_i - c_i - u_i)\, \partial d_i(w_1, w_2)/\partial w_i + d_i(w_1, w_2). \tag{4.7}$$

Its second order derivative gives

$$\partial^2 \Pi_i(w_1, w_2)/\partial w_i^2 = (w_i - c_i - u_i)\, \partial^2 d_i(w_1, w_2)/\partial w_i^2 + 2\, \partial d_i(w_1, w_2)/\partial w_i. \tag{4.8}$$

The concavity of profit function of the supplier i depends on the sign of the first and second orders derivative of the demand rate d_i of product i on the wholesale price w_i. Normally with increasing w_i, the demand rates d_i decreases and d_j increases simultaneously. This statement will be proved in the next part of this study and the conditions of concavity of the supplier profit function will be discussed.

Lemma 2 Under the conditions given by (4.5), (4.6), and symmetric optimal demand rates solution $d^* = d_i^* = d_j^*$, there exists a Nash (w_i^*, w_j^*) that satisfies for $i \in [1, 2]$

$$d_i^* + (w_i - c_i - u_i)\, \partial d_i^*/\partial w_i = 0. \tag{4.9}$$

Proof Under the conditions given by (4.5) and (4.6), the unique (d_i^*, d_j^*) exists and satisfies $S_i(d_i^*, d_j^*) = S_j(d_i^*, d_j^*) = 0$, The first order derivative of $S_i(d_i^*, d_j^*)$and $S_j(d_i^*, d_j^*)$on the wholesale price w_i gives the following system of equations

$$\begin{cases} \frac{\partial S_i(d_i, d_j^*)}{\partial w_i} = 1 + (-2\beta_i + \lambda(1-\lambda)K_i d_i^{*(\lambda-2)}\, \partial d_i^*/\partial w_i - (\gamma_i + \gamma_j)\, \partial d_j^*/\partial w_i = 0 \\ \frac{\partial S_j(d_i^*, d_i)}{\partial w_i} = 0 + (-2\beta_j + \lambda(1-\lambda)K_j d_j^{*(\lambda-2)}\, \partial d_j^*/\partial w_i - (\gamma_i + \gamma_j)\, \partial d_i^*/\partial w_i = 0 \end{cases}.$$

Let $A_i = 2\beta_i - \lambda(1-\lambda)K_i d_i^{*(\lambda-2)} > 0$, $A_j = 2\beta_j - \lambda(1-\lambda)K_j d_j^{*(\lambda-2)} > 0$, and $B = \gamma_i + \gamma_j$. The impact of w_i on the two demand rates is given by

$$\begin{pmatrix} \partial d_i^*/\partial w_i \\ \partial d_j^*/\partial w_i \end{pmatrix} = \frac{1}{A_iA_j - B^2}\begin{pmatrix} -A_j \\ B \end{pmatrix}. \tag{4.10}$$

Note that $A_iA_j - B^2 > 0$ because the Hessian of retailer's profit function is strictly diagonally dominant. The demand functions d_i^* and d_j^* decreases and increases with the wholesale price w_i, respectively. Then, the sign of $\partial^2 d_i^*/\partial w_i^2$ must be evaluated in order to evaluate the sign of $\partial^2 \Pi_i(w_1, w_2)/\partial w_i^2$. Using equation (4.10), $\partial B/\partial w_i$, $\partial A_j/\partial w_i = (\partial A_j/\partial d_j^*)B(A_iA_j - B^2)^{-1}$, and $\partial A_i/\partial w_i = -(\partial A_i/\partial d_i^*)A_j(A_iA_j - B^2)^{-1}$. Therefore,$\partial^2 d_i^*/\partial w_i^2 = (A_iA_j - B^2)^{-3}[(\partial A_j/\partial d_j^*)B^3 - (\partial A_i/\partial d_i^*)A_j^3]$. As $A_iA_j - B^2 > 0$, the sign of $\partial^2 d_i^*/\partial w_i^2$depends on the sign of$(\partial A_j/\partial d_j^*)B^3 - (\partial A_i/\partial d_i^*)A_j^3$, which is negative under symmetric optimal demand rates solution $d^* = d_i^* = d_j^*$.

When the economics of scale are zero ($K_i > 0$), the optimal demand and Nash wholesale price are

$$d_i^* = \frac{2\beta_j(\theta_i + u_i - w_i) + (\gamma_i + \gamma_j)(\theta_j + u_i - w_j)}{4\beta_i\beta_j - (\gamma_i + \gamma_j)^2}, \tag{4.11}$$

$$w_i^* = \frac{4\beta_i\beta_j(\theta_i + c_i + 2u_i) - 2\beta_i(\gamma_i + \gamma_j)(\theta_j + u_i)}{4\beta_i\beta_j - (\gamma_i + \gamma_j)^2}. \tag{4.12}$$

The condition of symmetry of the optimal demand rates limits the regions in which the optimal solution of the retailer is located, to only two regions or at the boundaries. These two regions are delimited by the targets and differ on the setting of the sale-rebate rate. Then, the retailer and suppliers profit functions will be discussed depending on the value of this parameter. When $u_i = u_j = u$, the problem is restrained to a wholesale contract and the target has no meaning. However; if $u > 0$, the setting of the target will not affect the optimal demand rate or the wholesale price. It affects only the profit functions of the different actors of the chain. For the retailer, its profit function decreases linearly with increasing the target and will be limited by its maximum at a zero target and its minimum when the target is equal to the optimal demand rate solution. In addition, in contrast to the retailer, the supplier profit function increases linearly with decreasing its target. Its minimum will be obtained at a target equal to zero; however, its maximum will be achieved at a target equal to the optimal demand rate. In contrast to the supplier, the retailer seems to achieve more profits when $u > 0$; however, this intuitive result will be not guaranteed and depends on the impact of the value of u on the optimal wholesale price and optimal demand rate.

4.6 Numerical Results

This section presents the numerical results of the different equilibrium solutions under symmetric parameters ($\beta_i = \beta_j = \beta, \gamma_i = \gamma_j = \gamma, \theta_i = \theta_j = \theta$, $K_i = K_j = K$, $u_i = u_j = u$, $c_i = c_j = c$, $t_i = t_j =$

t) and especially with a positive economic of scale $K > 0$. The following combination of parameters is used for the simulation ($\beta = 2, \gamma = 0.75, \theta = 100, K = \{1, 2, 3\}, u = \{0, 5\}, c = 10$) a . To obtain the Nash solution, a non-linear system of equations is formed from the first order derivatives of the profit functions of the suppliers and the retailer. This system is given by

$$\begin{cases} g_1(d, w) = \theta + u - w - 2(\beta + \gamma)d - \lambda K d^{\lambda-1} = 0 \\ g_2(d, w) = \left(\left(\lambda(1-\lambda)Kd^{\lambda-2} - 2\beta\right)^2 - 4\gamma^2\right)d + (w - c - u)\left(\lambda(1-\lambda)Kd^{\lambda-2} - 2\beta\right) = 0 \end{cases} \quad (4.13)$$

This system of equations is solved using the Newton method as expressed by

$$\begin{pmatrix} d^{k+1} \\ w^{k+1} \end{pmatrix} = \begin{pmatrix} d^{k} \\ w^{k} \end{pmatrix} - \begin{pmatrix} \partial g_1(d^k)/\partial d & \partial g_1(d^k)/\partial w \\ \partial g_2(w^k)/\partial d & \partial g_2(w^k)/\partial w \end{pmatrix}^{-1} \begin{pmatrix} g_1(d^k) \\ g_2(w^k) \end{pmatrix}. \quad (4.14)$$

In this numerical study, the solutions of the wholesale price contract and the sale-rebate contract will be presented, compared, and their effect on the profits of the retailer, the suppliers, and the integrated system in presence of inventory related costs, will be discussed. For the optimization problem, the total profit function of the integrated system is obtained by excluding the endogenous parameters of the chain and it is given by

$$\pi_I = 2((\theta - c - (\beta + \gamma)d)d - Kd^{\lambda}) = 0. \quad (4.15)$$

The optimal solution of the profit function of the integrated system is obtained by solving its first order derivative, as given by

$$\partial \pi_I / \partial d = 2(\theta - c - 2(\beta + \gamma)d - \lambda K d^{\lambda-1}) = 0. \quad (4.16)$$

The different optimal results for the Nash equilibrium and for the integrated system are summarized in Table 1. In this simulation, the target takes two different values ($t = 0$ or $t = d$). For $0 < t < d$, the profit function of the retailer decrease linearly with the target. However, the profit of the supplier increases linearly in such target range. The case, in which the optimal demand rate is high than the target, is not studied here. For the wholesale contract, the profit functions of the retailer and the suppliers are the same. However, in the sale-rebate contract, they increase and decrease with varying the target between $t = 0$ and $t = d$, respectively. The Nash wholesale price in the sale-rebate contract is higher than of that in the wholesale contact and the difference between them is equal to the rebate rate ($u = 5$). However, the optimal demand rate and retail price are unchanged. This can be explained by the leadership of the supplier to take decision in the chain. Although the competition is between the suppliers, the retailer did not benefit from it in the sale rebate contract. It seems here that the decision variables depend on the leadership decision and not on the competition. The total profit of the chain under Nash equilibrium is independent of the contracts. It is explained by the linear change of profits with the target between $t = 0$ and $t = d$ (what is gained by a retailer is lost by two suppliers together to make a compensation). The demand rate in the optimization problem (integrated system) is high than that of the Nash equilibrium, in contrast to the retail price. This can be explained by the remove of the leadership decision in the optimization problem, where the wholesale price is an endogenous parameter. The integrated system achieves around 30 % profit more than the Nash profit. Furthermore, increasing the economics of scale drops the different performances of the chain in the two forms of contract. Finally, using sophisticated contract is proven to increase the profits of the system, in accordance with the results of Cachon and Kök [2]. However, using sale rebate contract increases the profit of the supplier, in contrast to the same result of Cachon and Kök [2].

Table 4.1 Summary of Nash and optimal numerical results for the different parameters.

		Wholesale contract (u = 0)	K = 1	K = 2	K = 3	Sale-rebate contract (u = 5)	K = 1	K = 2	K = 3
Nash wholesale price (w)			44.499	44.383	44.267		49.499	49.383	49.267
Optimal demand rate (d)			10.062	10.055	10.047		10.062	10.055	10.047
Optimal retail price (p)			72.328	72.349	72.370		72.328	72.349	72.370
Target t = 0	Retailer profit function		553.709	549.699	545.695		553.709	549.699	545.695
	supplier profit function		347.147	345.718	344.289		347.147	345.718	344.289
Target t = d	Retailer profit function		553.709	549.699	545.695		453.085	449.151	445.223
	supplier profit function		347.147	345.718	344.289		397.459	395.992	394.525
Nash profit of the chain			1248.002	1241.136	1234.273		1248.002	1241.136	1234.273
Optimal demand for integrated system			16.341	16.319	16.296		16.341	16.319	16.296
Optimal price for integrated system			55.062	55.124	55.186		55.062	55.124	55.186
Profit function of integrated system			1766.950	1758.452	1749.959		1766.950	1758.452	1749.959
Nash profit / integrated profit			0.706	0.706	0.705		0.706	0.706	0.705

4.7 Conclusion

In summary, the existence and uniqueness of optimal demand rate for the retailer in a target sale-rebate based decentralized supply chain was found to be conditioned by the symmetric of the chain parameters β_i and γ_i for $i \in [1, 2]$. The existence of the optimal wholesale price solution was found to be conditioned by the symmetric of the optimal demand rate solution. Further, the setting of the target affects only the profit functions of the supplier and the retailer and does not affect the optimal wholesale and demand rate. As important result, the optimal wholesale price; in the case of sale rebate contract; increases with approximately a difference equals to the channel rebate rate. The total profit of the chain under Nash equilibrium is independent of the contracts.

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Chapter 5

Impact of Lead-time Decision in a Decentralized Supply Chain under Price and Lead-time Sensitive Demand

5.1 Introduction

This chapter analyses the impact of lead-time decision in a decentralized supply chain, where the retailer demand is sensitive to price and lead-time. First, we introduce the literature related to this study, its objective, and the application of this model in industrial management. Second, we formulate the models in the three scenarios. Third, we describe the modelling of lead-time using exponential distribution. Then, the power distribution of lead-time will be modelled and the different scenarios are studied and compared. Furthermore, the impact of own price and lead-time sensitivity demand factors on the performances of the chain are discussed theoretically and numerically.

5.2 Literature and Objective

In actual globalized and competitive market, the consumer benefits from the variety of choices. Therefore, considering the selling price as a unique competition factor in a supply chain became insufficient. For that, the market actors have been investigating new competition criteria based on consumers' attention. Sterling et al. [1] and Ballou et al. [2] reported that the rapidity and the regularity of delivery time have a particular importance in the customer service. Such delivery time is related to the so called "lead-time" factor. Generally, lead-time depends on the efficiency and the capacity of the selling system.

For example, So [3] reported that a retailer needs to provide sufficient capacity and guarantees the efficiency of his delivery system to achieve desired lead-time performances. This competition factor is widely discussed in supply chain management literature. It started with Yano [4-6], Li [7], Hopp and Spearman [8], Li and Lee [9], Lederer and Li [10], Palaka et al. [11], So and Song [12], Song et al. [13], Cachon and Harker [14], Boyaci and Ray [15]. Recently, Liu et al. [16] have studied pricing and lead-time decisions in a two level decentralized supply chain consisting of one supplier and one retailer, in which the supplier decides the lead-time and faces related costs. Furthermore, Pekgun et al. [17] have compared centralized and decentralized supply chains under price and lead-time sensitive demand.

In most cases of these studies, the supplier is a lead-time decision maker. However, the consumer places his order to the retailer and gets information about the time of delivery. The retailer is more suitable to determine the lead-time in the practice. Thus, the scenario in which the retailer is a lead-time decision maker should be considered and studied. However, in literature related to industrial management, this scenario has not been deeply studied. For that reason, we will focus in the current work on effect of lead-time decision on the performances of the supply chain management. Furthermore, we will compare the optimal decision variables and expected profits in this scenario with that when the supplier is a lead-time decision maker.

In this chapter, we study the impact of centralized and decentralized lead-time decision in a two-level supply chain management, consisting of one supplier, one retailer, and wherein the demand is sensitive to both retail price and lead-time. The decentralized chain is based on a leader-follower model. However, in the centralized chain a single decision maker is considered. The lead-time is defined, in this work, as the interval of time separating the moment of placing an order by a consumer to the moment of receiving that order, including the time of intermediary process between the retailer and the supplier. When a consumer places an order to the retailer, a promised time to receive this order will be announced. Such time is defined as the promised delivery lead-time (PDL), which is also expressed in literature as *quoted lead time* or *planned lead time*.

However, this PDL can be smaller or greater than the exact interval of time to deliver the order of the consumer. Such period of time is defined as the realized delivery lead-time (RDL) or as referred in the literature to the *response time* or *cycle time*. The RDL is a stochastic variable and may deviate from the PDL due to many reasons such as high demands. As consequence, the actor of the chain who decides the lead-time faces holding and tardiness costs incurred by the difference between PDL and RDL. Three different scenarios based on lead-time decision are studied and compared. In the first scenario, the retailer is a leader and the supplier is a follower. The retailer decides the PDL and the retail price to be quoted to the consumer, however, the supplier determines the wholesale price. The second scenario describes a supply chain in which the supplier is a leader and the retailer is a follower. The supplier determines the PDL and the wholesale price, however, the retailer quotes the retail price. The lead-time decision in these two first scenarios is decentralized. The third scenario is a centralized problem, where a single lead-time decision maker is considered. In addition, another problem is faced when choosing the using the distribution function of lead-time.

In general, the exponential distribution function is the commonly used distribution in management literature; however, with it, we cannot obtain a close form solution and some assumptions must be satisfied to compute the solution numerically. To overcome these limitations, the power distribution of lead-time is used under some conditions to imitate the properties of exponential distribution. Under such distribution, the optimal decision variables and expected profits are characterized and compared in the three scenarios. Further, the effect of own price and lead-time sensitive demand are studied numerically. The results of this study can be used in several industrial management applications such as internet retailing, online selling transaction or e-retailing, post services...etc. Mike Eskew, the chairman and chief executive officer of UPS, explains: "*Globalization has raised the competitive stakes, forcing companies to compete on more than just product features and price. Companies can achieve competitive differentiation based on how well they deliver the right product to the right place at the right time* [18]". This citation deals with the importance of shot lead-time in posting services applications. In internet retailing, most of websites in this field announce their delivery lead-time and try to minimize it as possible in order not only to satisfy their consumers but also to increase to battle their competitors.

5.3 Formulation of the Models

Three different seniors are studied to determine the optimal decision variables and expected profits in a two level supply chain, consisting of one supplier and one retailer. In the first scenario, the retailer decides the lead-time; however, this decision is taken by the supplier in the second scenario and centralized in the third one. The supplier produces products at a constant production cost rate (c), including the transportation cost to the retailer or to the consumer. The supplier has ample capacity to satisfy any received demand. The retailer faces an administrative cost per unit (c_r). The actor of the chain who decides the lead-time faces lead-time costs incurred by the difference between the PDL and the RDL. If the RDL is less than the PDL, the product is kept in stock and a holding cost (h) per unit per unit time is introduced; however, he faces a tardiness cost (b) per unit per unit time, when the RDL exceeds the PDL. We assume a demand rate λ dependent cumulative distribution function (cdf) R_λ and a probability distribution function (pdf) r_λ for lead-time. The lead-time costs are defined as in [16] and [22], where they are expressed, for a given λ, by

$$C(l, R_\lambda) = h\int_0^l (l-t) r_\lambda(t)dt + b\int_l^\infty (t-l) r_\lambda(t)dt, \tag{5.1}$$

where the demand function λ is deterministic and linear in retail price and lead-time. It is expressed as

$$\lambda(p, l) = \lambda_0 - \alpha p - \beta l, \tag{5.2}$$

with λ_0, α, β are the base market potential, own price sensitivity demand factor, and own lead-time sensitivity demand factor, respectively. We define the standard waiting cost $c_w = \frac{\beta}{\alpha}$ per unit of PDL and the maximum retail price $p^{max} = \frac{\lambda_0}{\alpha}$. The demand function is similar to that reported by Boyaci and Ray [15], Pekgun el al. [17], Tsay and Agrawal [19], and Balasubramanian and Bhardwaj [20].

In the first scenario, to maximize their profits, the supplier decides his wholesale price w_{d1}; however, the retailer decides his lead-time l_{d1} and retail price p_{d1}. The optimization problem of the supplier is given by $max_{w_{d1}} \pi_{s1}(w_{d1}, p_{d1}(w_{d1}), l_{d1}(w_{d1})) = (w_{d1} - c)\lambda_{d1}(p_{d1}(w_{d1}), l_{d1}(w_{d1}))$, where

$p_{d1}(w_{d1})$ and $l_{d1}(w_{d1})$ are the optimal solutions for following retailer's optimization problem. The index $d1$ refers to decentralized chain in scenario1. For a given w_{d1}, the optimization problem of the retailer is expressed as

$max_{p_{d1},l_{d1}} \pi_{r1}(p_{d1}, l_{d1}) = \left(p_{d1} - w_{d1} - c_r - C(l_{d1}, R_{\lambda_{d1}})\right)\lambda_{d1}(p_{d1}(w_{d1}), l_{d1}(w_{d1}))$. The supplier decides his wholesale price w_{d2} and lead-time l_{d2}. His problem is given by

$$max_{w_{d2}} \pi_{s2}(w_{d2}, l_{d2}) = \left(w_{d2} - c - C(l_{d2}, R_{\lambda_{d2}})\right)\lambda_{d2}(p_{d2}(w_{d2}), l_{d2}(w_{d2})),$$

where the index d_2 refers to the decentralized chain in scenario 2. The optimization of the retailer is given for given w_{d2}, by $max_{p_{d2},l_{d2}} \pi_{r2}(p_{d2}, l_{d2}) = (p_{d2} - w_{d2} - c_r)\lambda_{d2}(p_{d2}, l_{d2})$.

In the third scenario, one of the chain' actors is a decision maker and the other one is a follower. The wholesale price is excluded from the optimization problem, as an internal variable. The total profit function of the centralized chain is given by $\pi_c(\lambda_c(p,l)) = \left(p^{max} - \frac{\lambda_c}{\alpha} - c_w l_c - c - c_r - C(l_c, R_{\lambda_c})\right)\lambda_c(p,l)$, where the index c refers to the centralized chain. In all these scenarios, we assume that the right hand-sides of the optimization problems are positive.

5.4 Exponential Distribution

In M/M/1 system, the service times are independent and identically exponentially distributed. As reported by Boyaci and Ray [15], the exponential distribution gives an important approximation of waiting times. Its cdf and pdf of lead-time are given by $R_{\lambda_{d1}}(t) = 1 - e^{-(\gamma-\lambda_{d1})t}$ and $r_{\lambda_{d1}}(t) = (\gamma - \lambda_{d1})e^{-(\gamma-\lambda_{d1})t}$ for $0 \leq t \leq \infty$, respectively, where γ is the mean service rate. Here, only the first scenario will be studied under the exponential distribution of lead-time. The other scenarios were studied by Liu et al. [16]. For a given wholesale price w_{d1}, the retailer profit function depends on three dependent parameters; the price, the lead-time, and the demand. To solve this technical problem, the retail price is expressed as a function of lead-time and demand function. Then, the optimal lead-time solution will be obtained for a given demand. Using Eq. (5.2), we obtain

$$p_{d1} = p^{max} - \frac{\lambda_{d1}}{\alpha} - c_w l_{d1}. \tag{5.3}$$

The optimization problem of the retailer can be rewritten by

$$max_{\lambda_{d1},l_{d1}} \pi_{r1}(\lambda_{d1}, l_{d1}) = \left(p^{max} - \frac{\lambda_{d1}}{\alpha} - c_w l_{d1} - w_{d1} - c_r - C(l_{d1}, R_{\lambda_{d1}})\right)\lambda_{d1}.$$

Lemma 5.1 For a given wholesale price w_{d1} and demand λ_{d1}, there is a unique optimal lead-time $l^*_{d1}(\lambda_{d1})$, which depends on λ_{d1} and expressed by $l^*_{d1}(\lambda_{d1}) = R^{-1}_{\lambda_{d1}}\left(\frac{b-c_w}{b+h}\right)$, where $R^{-1}_{\lambda_{d1}}$ is the inverse of the distribution function $R_{\lambda_{d1}}$.

Proof By differentiating $\pi_{r1}(\lambda_{d1}, l_{d1})$ on the lead-time l_{d1}, we obtain $\frac{\partial \pi_{r1}(\lambda_{d1}, l_{d1})}{\partial l_{d1}} = \left(-c_w - \frac{\partial c(l_{d1}, R_{\lambda_{d1}})}{\partial l_{d1}}\right)\lambda_{d1}$ and $\frac{\partial^2 \pi_{r1}(\lambda_{d1}, l_{d1})}{\partial {l_{d1}}^2} = -\lambda_{d1}\frac{\partial^2 c(l_{d1}, R_{\lambda_{d1}})}{\partial {l_{d1}}^2}$, with $\frac{\partial c(l_{d1}, R_{\lambda_{d1}})}{\partial l_{d1}} = -b + (b+h)R_{\lambda}(l_{d1})$ and $\frac{\partial^2 c(l_{d1}, R_{\lambda_{d1}})}{\partial {l_{d1}}^2} = (b+h)r_{\lambda_{d1}}(l_{d1}) > 0$ for all l_{d1}. Thus, the retailer profit function is strictly concave on l_{d1} and the unique optimal lead-time $l^*_{d1}(\lambda_{d1})$ is given by $l^*_{d1}(\lambda_{d1}) = R^{-1}_{\lambda_{d1}}\left(\frac{b-c_w}{b+h}\right)$.

The ratio $\left(\frac{b-c_w}{b+h}\right)$ reflects the cost parameter [22]. The optimal lead-time is dependent on retail price through λ_{d1}. Note that if $b - c_w \le 0$, the optimal lead-time is zero. Using the exponential distribution, the optimal lead-time can be expressed if $b > c_w$ as $l^*_{d1}(\lambda_{d1}) = \frac{d}{\gamma - \lambda_{d1}}$, where $d = -ln\left(\frac{h+c_w}{b+h}\right)$. Substituting $l^*_{d1}(\lambda_{d1})$ in (1), gives $(l_{d1}, R_{\lambda_{d1}}) = \frac{hd+c_w}{\gamma-\lambda_{d1}}$. Then, the retailer profit function is given by $\pi_{r1}(\lambda_{d1}) = \left(p^{max} - \frac{\lambda_{d1}}{\alpha} - \frac{(c_w+h)d+c_w}{\gamma-\lambda_{d1}} - w_{d1} - c_r\right)\lambda_{d1}$.

Lemma 5.2 For a given wholesale price w_{d1}, there are unique optimal demand λ^*_{d1} and retail price p^*_{d1}. The unique optimal demand is given by $\lambda^*_{d1} = \gamma - \phi^*$, with $\phi^*$is the solution of the cube equation $\phi^{*3} + A\phi^{*2} + B\phi^* + C = 0$, with $A \equiv \frac{\alpha(p^{max} - w_{d1} - c_r)}{2} - \gamma$, $B \equiv 0$, and $C \equiv -\frac{\alpha((c_w+h)d+c_w)\gamma}{2}$.

Proof By differentiating the retailer profit function on the demand function λ_{d1} we obtain $\frac{\partial \pi_{r1}(\lambda_{d1})}{\partial \lambda_{d1}} = p^{max} - \frac{2\lambda_{d1}}{\alpha} - ((c_w+h)d + c_w)\left(\frac{1}{\gamma-\lambda_{d1}} + \frac{\lambda_{d1}}{(\gamma-\lambda_{d1})^2}\right) - w_{d1} - c_r$ and $\frac{\partial^2 \pi_r(\lambda_{d1})}{\partial {\lambda_{d1}}^2} = -\frac{2}{\alpha} - ((c_w+h)d + c_w)\left(\frac{1}{(\gamma-\lambda_{d1})^2} + \frac{\lambda_{d1}+\gamma}{(\gamma-\lambda_{d1})^3}\right) < 0$ for all λ_{d1}. Thus, the retailer profit function is strictly concave on λ_{d1} and the unique optimal demand function is given by $\lambda^*_{d1} = \gamma - \phi^*$, with $\phi^*$is the solution of the third order equation $\phi^{*3} + A\phi^{*2} + B\phi^* + C = 0$, with $A \equiv \frac{\alpha(p^{max} - w_{d1} - c_r)}{2} - \gamma$, $B \equiv 0$, and $C \equiv -\frac{\alpha((c_w+h)d+c_w)\gamma}{2}$.

The cube equation on λ^*_{d1} has three possible solutions. However, the retailer profit function is concave in λ^*_{d1}, which guarantees the existence of unique positive solution. This solution cannot be obtained analytically or numerically because it depends on wholesale price w_{d1}, which is an unknown decision variable. Liu et al. [16] discussed nearly the same cube equation, where the constants depend on chain and distribution parameters only and can be solved numerically using the formulas reported in Spiegel and Liu [21]. They introduced some approximations to model the lead-time as function of demand. In this study, a new approach based on the so called power distribution is used to solve the problem analytically. Such a distribution has, under some conditions, the properties of exponential distribution of lead-time in M/M/1 system. Its advantages will be discussed in detail in the next section.

5.5 Power Distribution

As mentioned in last section, the problem cannot be solved under exponential distribution. Then, a new approach based on the power distribution function is used. This distribution function of lead-time was introduced by Zhengping et al. in [22]. It is a parametric function which models a wide variety of distributions such as uniform and triangular distributions. In addition, we will show that the power distribution is more suitable in the context of modelling lead-time in a general environment industrial management. It has the same properties as the exponential distribution for specific parameters. The advantages of this distribution will be discussed after its definition. The cdf and the pdf of lead-time are expressed by $R_{\lambda_{d1}}(t) = \left(\frac{t}{\rho\lambda_{d1}}\right)^{\varpi}$ and $r_{\lambda_{d1}}(t) = \frac{\varpi t^{\varpi-1}}{(\rho\lambda_{d1})^{\varpi}}$ for $0 \leq t \leq \rho\lambda_{d1}$, respectively, where $\varpi > 0$ and $\rho > 0$ are the shape and the scale parameters, respectively. The interval $\rho\lambda_{d1} = T$ represents the longest possible lead-time for a job in the system, when the demand rate is λ_{d1}. The properties of power distribution function of lead-times are summarized in the following points:

- The service mean rate and the demand are $1/\rho$ and λ_{d1}, respectively. They are analogous to γ and λ_{d1}, respectively, in the exponential distribution.
- Infinite lead-time is not allowed as in practice.
- It can be used in different situation by varying the shape parameter ϖ. As shown in Fig. 5.1, the pdf of lead-time under various ϖ has different behaviors. For $\varpi = 0$ or $\varpi = \infty$, the optimal lead-time is deterministic and equal to 0. For $0 < \varpi < 1$, the pdf drops with increasing lead-time as in M/M/1 system. In this case, short lead-times have high probability, indicating a rapid delivery of the order. The cases where $\varpi = 1$ and 2 correspond to the uniform and the triangular distributions, respectively. For $\varpi > 1$, the pdf increases with lead-time, which indicates that long lead-time has high probability, in contrast to exponential distribution. The order that the supplier receives from the retailer tends to stay long in the system, which is consistent with common practice where deliveries normally take place near end of promised lead-time (PDL) or even beyond in some cases.

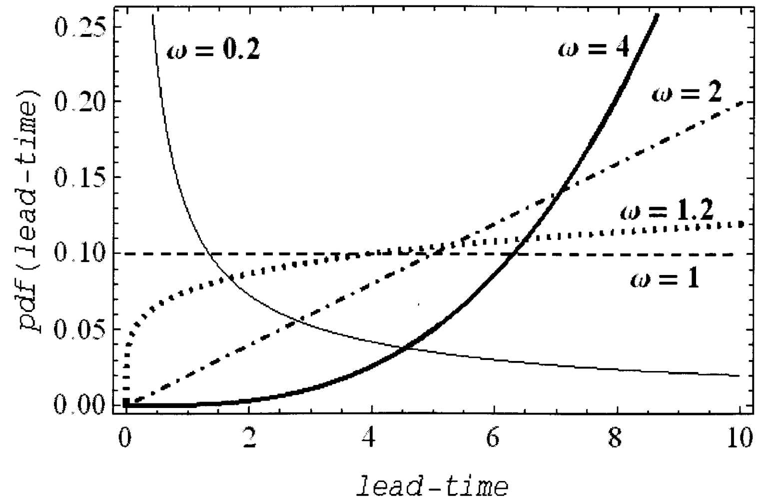


Fig. 5.1 Probability distribution function of lead-time for various shape parameters ϖ.

5.5.1 Retailer decides the lead-time

Using results of lemma 1 and the expression of power distribution function, the optimal lead-time can be expressed as $l_{d1}^{*}(\lambda_{d1}) = \rho\lambda_{d1}\tau$, where $\tau = \sqrt[\varpi]{\frac{b-c_w}{b+h}}$. The lead-time costs can be expressed by $(\lambda_{d1}) = \eta\rho\lambda_{d1}$, where $\eta = \frac{\tau^{\varpi+1}h+b(\tau^{\varpi+1}+\varpi-\tau-\tau\varpi)}{\varpi+1}$. Then, the retailer profit function can be rewritten as $\pi_{r1}(\lambda_{d1}) = \left(p^{max} - \frac{\lambda_{d1}}{\alpha} - c_w\rho\lambda_{d1}\tau - w_{d1} - c_r - \eta\rho\lambda_{d1}\right)\lambda_{d1}$.

Lemma 5.3 For given wholesale price w_{d1}, there are unique optimal demand function λ^{*} and retail price p^{*}. They are expressed as $\lambda_{d1}^{*} = \frac{\alpha(p^{max}-w_{d1}-c_r)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}$ and $p_{d1}^{*} = p^{max} - \frac{\lambda_{d1}^{*}}{\alpha}(1+\alpha c_w\tau\rho)$.

Proof By differentiating the retailer profit function on the demand λ_{d1}, we obtain $\frac{\partial\pi_{r1}(\lambda_{d1})}{\partial\lambda} = p^{max} - w_{d1} - c_r - \frac{2\lambda_{d1}}{\alpha} - 2c_w\rho\lambda_{d1}\tau - 2\eta\rho\lambda_{d1}$ and $\frac{\delta^2\pi_{r1}(\lambda_{d1})}{\delta\lambda_{d1}{}^2} = -\frac{2}{\alpha} - 2c_w\rho\tau - 2\eta\rho < 0$. Thus, the retailer profit function is strictly concave on λ_{d1} and the unique optimal demand function is given by $\lambda_{d1}^{*} = \frac{\alpha(p^{max}-w_{d1}-c_r)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}$. Substituting l_{d1}^{*} and λ_{d1}^{*} in Eq. (5.3), the unique optimal retail price is given by $p_{d1}^{*} = p^{max} - \frac{\lambda_{d1}^{*}}{\alpha}(1+\beta\tau\rho)$.

Inserting the optimal demand in the supplier profit function, we obtain

$$\pi_{s1}(w_{d1}) = (w_{d1} - c)\frac{\alpha(p^{max}-w_{d1}-c_r)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}.$$

Lemma 5.4 There is a unique optimal wholesale price w_{d1}^*, given by

$$w_{d1}^* = \frac{p^{max} - c_r + c}{2}. \tag{5.4}$$

Proof By differentiating the supplier profit function on the wholesale price w_{d1}, we obtain $\frac{\partial \pi_{s1}(w_{d1})}{\partial w_{d1}} = \frac{-\alpha(2w_{d1} - p^{max} + c_r - c)}{2(1+\alpha c_w \rho\tau + \alpha\eta\rho)}$ and $\frac{\partial^2 \pi_{s1}(w_{d1})}{\partial {w_{d1}}^2} = \frac{-\alpha}{1+\alpha c_w \rho\tau + \alpha\eta\rho} < 0$. Thus, the supplier profit function is strictly concave on w_{d1} and the unique optimal wholesale price is given by $w_{d1}^* = \frac{p^{max} - c_r + c}{2}$.

Then, the optimal decision variables and expected profits are expressed by

$$l_{d1}^* = \frac{\rho\tau\alpha(p^{max} - c - c_r)}{4(1+\beta\rho\tau+\alpha\eta\rho)}, \tag{5.5}$$

$$\lambda_{d1}^* = \frac{\alpha(p^{max} - c - c_r)}{4(1+\beta\rho\tau+\alpha\eta\rho)}, \tag{5.6}$$

$$p_{d1}^* = p^{max} - \frac{\lambda_{d1}^*}{\alpha}(1 + \beta\tau\rho), \tag{5.7}$$

$$\pi_{s1}^* = \frac{\alpha(p^{max} - c - c_r)^2}{8(1+\beta\rho\tau+\alpha\eta\rho)} = \frac{\lambda_{d1}^*}{2}(p^{max} - c - c_r) \tag{5.8}$$

$$\pi_{r1}^* = \frac{\alpha(p^{max} - c - c_r)^2}{16(1+\beta\rho\tau+\alpha\eta\rho)} = \frac{\lambda_{d1}^*}{4}(p^{max} - c - c_r). \tag{5.9}$$

The sum of the supplier and retailer profits is given by

$$\pi_{d1}^* = \pi_{r1}^* + \pi_{s1}^* = \frac{3\lambda_{d1}^*}{4}(p^{max} - c - c_r) \tag{5.10}$$

5.5.2 Supplier decides the lead-time

Using lemma 1 in Liu et al. [16]; for given w_{d2} and lead-time l_{d2}, the optimal retail price is given by $p_{d2}^* = \frac{p^{max} + c_r - c_w l_{d2} + w_{d2}}{2}$. Substituting p_{d2}^* in Eq. (5.2) and expressing the wholesale price as function of demand, we obtain $w_{d2}(\lambda_{d2}) = p^{max} - c_w l_{d2} - 2\lambda_{d2}/\alpha - c_r$. Using the same methodology as in §4.1; for a given demand function λ_{d2}, there is a unique optimal lead-time $l_{d2}^*(\lambda_{d2})$, which depends on λ_{d2} and given by $l_{d2}^*(\lambda_{d2}) = \rho\lambda_{d2}\tau$. Then, the supplier profit function is rewritten as $\pi_{s2}(\lambda_{d2}) = \left(p^{max} - c_r - c - \lambda_{d2}(c_w\rho\tau + 2/\alpha + \eta\rho)\right)\lambda_{d2}$.

Lemma 5.5 Under power distribution, there is a unique optimal demand function λ_{d2}^* expressed by

$$\lambda_{d2}^* = \frac{\alpha(p^{max} - c - c_r)}{2(2+\beta\rho\tau+\alpha\eta\rho)}, \tag{5.11}$$

Proof By differentiating the supplier profit function on the demand λ_{d2}, we obtain $\frac{\partial \pi_{s2}(\lambda_{d2})}{\partial \lambda_{d2}} = p^{max} - c - c_r - 2\lambda_{d2}(c_w\rho\tau + 2/\alpha + \eta\rho)$ and $\frac{\partial^2 \pi_{2s}(\lambda_{d2})}{\partial {\lambda_{d2}}^2} = -2(c_w\rho\tau + 2/\alpha + \eta\rho) < 0$. Thus, the supplier profit function is strictly concave on λ_{d2} and the unique optimal demand function is given by $\lambda_{d2}^* = \frac{\alpha(p^{max} - c - c_r)}{2(2+\beta\rho\tau+\alpha\eta\rho)}$.

Then, the unique optimal lead-time is expressed by

$$l_{d2}^{*} = \frac{\rho\tau\alpha(p^{max}-c-c_r)}{2(2+\beta\rho\tau+\alpha\eta\rho)}. \tag{5.12}$$

Substituting l_{d2}^{*} and λ_{d2}^{*} in w_{d2}^{*}, the unique optimal wholesale price can be given by

$$w_{d2}^{*} = p^{max} - c_r - \frac{(p^{max}-c-c_r)(2+\beta\rho\tau)}{2(2+\beta\rho\tau+\alpha\eta\rho)}. \tag{5.13}$$

From that, we obtain the unique optimal retail price as

$$p_{d2}^{*} = p^{max} - \frac{\lambda_{d2}^{*}}{\alpha}(\beta\rho\tau + 1) \tag{5.14}$$

Finally, the retailer and supplier profit functions are given, respectively by

$$\pi_{r2}^{*} = \frac{\lambda_{d2}^{*2}}{\alpha} = \frac{\alpha(p^{max}-c-c_r)^2}{4(2+\beta\rho\tau+\alpha\eta\rho)^2} \tag{5.15}$$

$$\pi_{s2}^{*} = (2+\beta\rho\tau+\alpha\eta\rho)\frac{\lambda_{d2}^{*2}}{\alpha} = (2+\beta\rho\tau+\alpha\eta\rho)\pi_{r2} \tag{5.16}$$

The sum of the supplier and retailer profits is given by

$$\pi_{d2}^{*} = \pi_{r2}^{*} + \pi_{s2}^{*} = \frac{\alpha(3+\beta\rho\tau+\alpha\eta\rho)(p^{max}-c-c_r)^2}{4(2+\beta\rho\tau+\alpha\eta\rho)^2} \tag{2.17}$$

5.5.3 Centralized scenario

In this case, one of the chain' actors is a decision maker and the other one is a follower. The wholesale price is excluded from the optimization problem because it is an endogenous variable. This scenario is considered as a benchmark to be compared with decentralized scenarios. As calculated in § 5.5.1, the optimal lead-time and lead-time cost are given, respectively, by $l^{*}(\lambda) = \rho\lambda\tau$ and $C(\lambda) = \eta\rho\lambda$. The profit function of the centralized chain is given by $\pi(\lambda) = \left(p^{max} - \frac{\lambda}{\alpha} - c_w\rho\lambda\tau - c - c_r - \eta\rho\lambda\right)\lambda$. Using the same methodology as in lemma 5.2, we obtain the optimal demand function, optimal price, optimal lead-time, and expected profit as

$$\lambda^{*} = \frac{\alpha(p^{max}-c-c_r)}{2(1+\beta\rho\tau+\alpha\eta\rho)} = 2\lambda_{d1}^{*}, \tag{5.18}$$

$$p^{*} = p^{max} - \frac{\lambda^{*}}{\alpha}(1+\beta\tau\rho) \tag{5.19}$$

$$l^{*}(\lambda) = \rho\tau\frac{\alpha(p^{max}-c-c_r)}{2(1+\beta\rho\tau+\alpha\eta\rho)}, \tag{5.20}$$

$$\pi^{*} = \frac{\lambda^{*2}}{\alpha}(1+\beta\rho\tau+\alpha\eta\rho) = \frac{\alpha(p^{max}-c-c_r)^2}{4(1+\beta\rho\tau+\alpha\eta\rho)} \tag{5.21}$$

5.5.4 Comparison between the scenarios

In this sub-section, the optimal decision variables and expected profits are compared in the three scenarios. As reported in [16], an inefficiency of lead-time decision factor is used to evaluate the gap between the centralized and decentralized solutions. This factor is expressed by

$$q_{\pi_{di}} = 1 - \frac{\pi_{di}^*}{\pi^*} \text{ for } i = \{1,2\} \tag{5.22}$$

5.5.4.1 Comparison between scenario1 and centralized one

From results of §5.5.1 and §5.5.3, the optimal demand function and the optimal lead-time in the centralized chain are the double of that in the first scenario, where the retailer decides the lead-time, that is $\lambda^* = 2\lambda_{d1}^*$ and $l^* = 2l_{d1}^*$. However, the optimal retail price in scenario 1 is higher than in that of centralized chain, that is $p_{d1}^* = \frac{p^{max}+p^*}{2} > p^*$. Concerning the comparison of total profits, we have $\pi_{d1}^* = \frac{3\pi^*}{4}$. Therefore, an inefficiency of lead-time decision, when the retailer decides the lead-time, is independent of chain and distribution parameters and equal to $q_{\pi_{d1}} = 0.25$.

5.5.4.2 Comparison between scenario 2 and centralized one

From results of §5.5.2 and §5.5.3, the optimal demand function, the optimal lead-time, and the optima retail price in the centralized chain are higher than that in the case of the second scenario, where the supplier decides the lead-time which satisfies this equation $\frac{\lambda_{d2}^*}{\lambda^*} = \frac{1+\beta\rho\tau+\alpha\eta\rho}{2+\beta\rho\tau+\alpha\eta\rho} = \frac{l_{d2}^*}{l^*} = \frac{p^{max}-p_{d2}^*}{p^{max}-p^*} < 1$. Concerning the comparison of total profits, we have $0.75 < \frac{\pi_{d2}^*}{\pi^*} = \frac{3+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2}{4+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2} < 1$, which results in an inefficiency of lead-time decision of $q_{\pi_{d2}} = \frac{1}{4+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2} < 0.25$. This result depends on chain and distribution parameters; however, it is often less than that in the case when the retailer decides the lead-time.

5.5.4.3 Comparison between scenario 1 and 2

From results of §5.4.2 and §5.4.3, we have $\frac{\lambda_{d2}^*}{\lambda_{d1}^*} = \frac{l_{d2}^*}{l_{d1}^*} = \frac{2(1+\beta\rho\tau+\alpha\eta\rho)}{2+\beta\rho\tau+\alpha\eta\rho} > 1$. From that, the retailer orders more quantity when the supplier decides the lead-time. It can be explained by the non responsibility of the retailer to compensate the waiting cost for the consumer. As consequence to high demand, long lead-time is required to complete the job. Concerning the retail prices, we have $\frac{p_{d2}^*-p^{max}}{p_{d1}^*-p^{max}} = \frac{\lambda_{d2}^*}{\lambda_{d1}^*} > 1$, which gives $p_{d2}^* < p_{d1}^*$. From Eq. (5.9) and (5.15), we have $\frac{\pi_{r2}^*}{\pi_{r1}^*} = \frac{4(1+\beta\rho\tau+\alpha\eta\rho)}{(2+\beta\rho\tau+\alpha\eta\rho)^2} < 1$. This means that the retailer achieves more profits when he decides lead-time. The same result is found for the supplier, where $\frac{\pi_{s2}^*}{\pi_{s1}^*} = \frac{2(1+\beta\rho\tau+\alpha\eta\rho)(2+\beta\rho\tau+\alpha\eta\rho)}{(2+\beta\rho\tau+\alpha\eta\rho)^2} > 1$. Therefore, the chain's actor who decides the lead-time achieves more profits. Concerning the total profits in the two scenarios, we have $\frac{\pi_{d2}^*}{\pi_{d1}^*} = \frac{4(3+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2)}{3(4+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2)} > 1$.

5.5.5 Numerical results

In this section, we compute numerically the optimal decision variables and expected profits under power distribution function. Two different values of shape parameter will be used. The first one is set to $\varpi = 0.2$, where the pdf of lead-time drops with increasing lead-time. This case imitates the properties of the exponential distribution. However, the second value is set to $\varpi = 1.2$, where the pdf of lead-time increases with increasing lead-time. For all our simulations, the numerical values of chain parameters are set as $\lambda_0 = 100$, $\alpha = 1$ if β is varied, $\beta = 1$ if α is varied, $b = 2$, $h = 0.3$, $c_r = 5$, $c = 20$, $\varpi \epsilon \{0.2, 1.2\}$, and $\rho = 10$. The decision variables and expected profits that can be discussed theoretically will not be presented in the numerical results.

5.5.5.1 Effect of own price sensitivity α

First, we have $\frac{\partial\tau}{\partial\alpha} = \frac{\beta}{\alpha^2\varpi(b+h)}\left(\frac{b-\frac{\beta}{\alpha}}{b+h}\right)^{\frac{1-\varpi}{\varpi}} > 0$, $\frac{\partial(\alpha(p^{max}-c-c_r))}{\partial\alpha} = -c - c_r < 0$, $\frac{\partial\eta}{\partial\tau} = \frac{1}{\varpi+1}[(\varpi + 1)(b+h)\tau^{\varpi} - b(\varpi+1)] = -\frac{\beta}{\alpha} < 0$, and $\frac{\partial(1+\beta\rho\tau+\alpha\eta\rho)}{\partial\alpha} = \beta\rho\frac{\partial\tau}{\partial\alpha} + \eta\rho + \alpha\rho\frac{\partial\eta}{\partial\tau}\frac{\partial\tau}{\partial\alpha} = \eta\rho > 0$. From that, it is easy to see in the first and the second scenarios that $\frac{\partial\lambda_{di}^*}{\partial\alpha} < 0$, $\frac{\partial\pi_{ri}^*}{\partial\alpha} < 0$, and $\frac{\partial\pi_{si}^*}{\partial\alpha} < 0$ for $i = 1, 2$. Then, increasing the own price sensitivity demand factor αdecreases the demand function and the expected profits. The sensitivity of the other decisions variables to α will be discussed numerically. In all figures, the solid, the dashed and the dot-dashed lines correspond to the first, second, and third scenarios, respectively.

- **Case of shape parameter ϖ = 0.2**

For $\varpi = 0.2$, the pdf decreases with increasing the lead-time as in M/M/1 system. As shown in Fig. 5 (a), at low values of α, the wholesale price in the second scenario is more high than that in the first one. However, it converges in the two cases to the product cost rate c for $\alpha_c = \frac{\lambda_0}{c+c_r}$. In addition, as plotted in Fig. 5.2 (b-left), the retail price decreases with increasing α. Its sensitivity to lead-time decision is weak. For such α_c, the optimal price is equal to $c + c_r$ and the others decision variables are zero. Furthermore, the important result in this sub-section concerns the lead-time, which is found to be a non-monotone function with α. As shown in Fig. 5.2(c-left), it reaches its maximum for $\alpha = \alpha_{lmax}$, which depends on the setting of chain and distribution parameters. It is worth to note that the non-monotony of lead-time disappears for high tardiness cost b. Further, it is easy to see numerically that in contrast to the other parameters, α_{lmax} right shifts with increasing own lead-time sensitive demand factor β. As conclusion to this sub-section, an infinity lead-time is not allowed in the chain, which is in accordance with the practice.

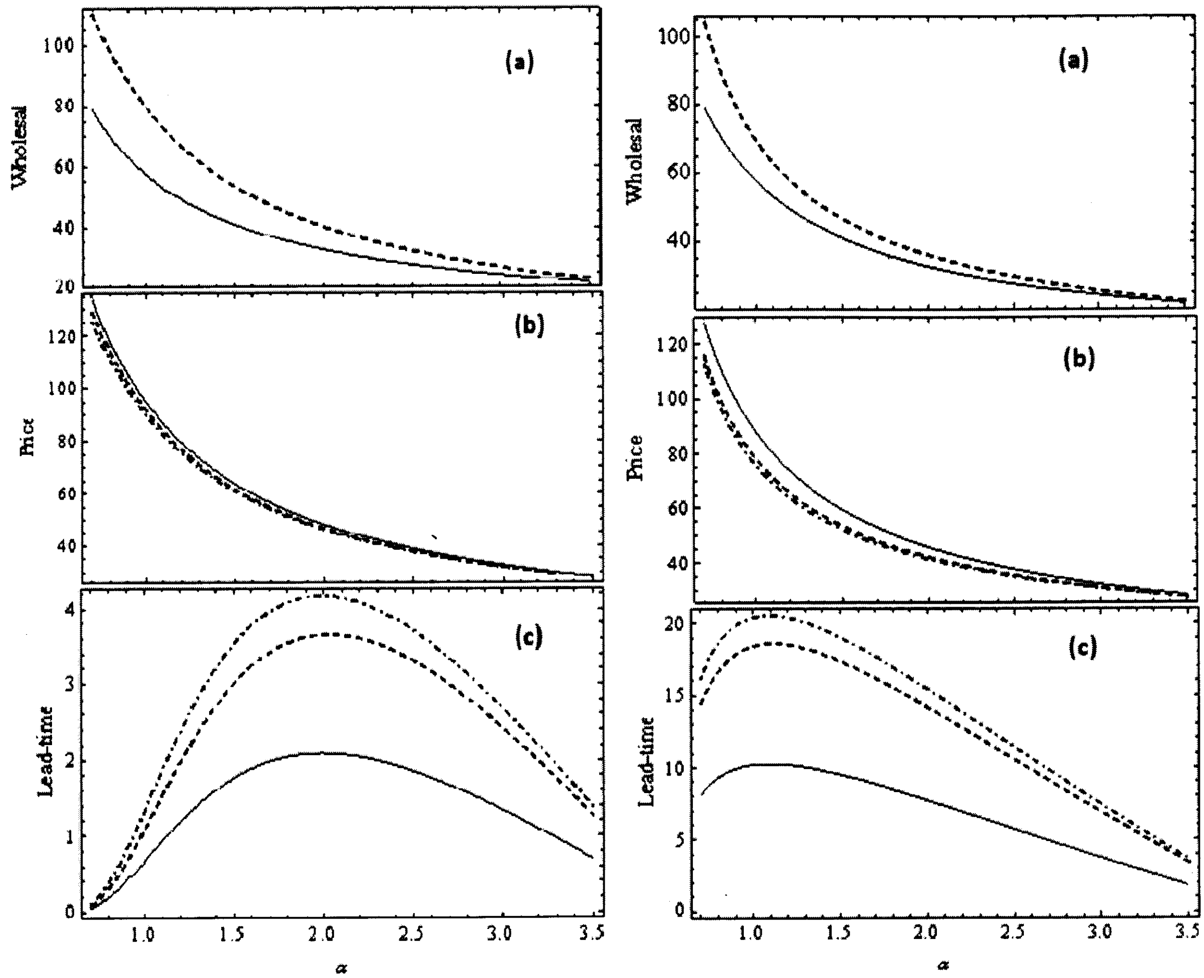


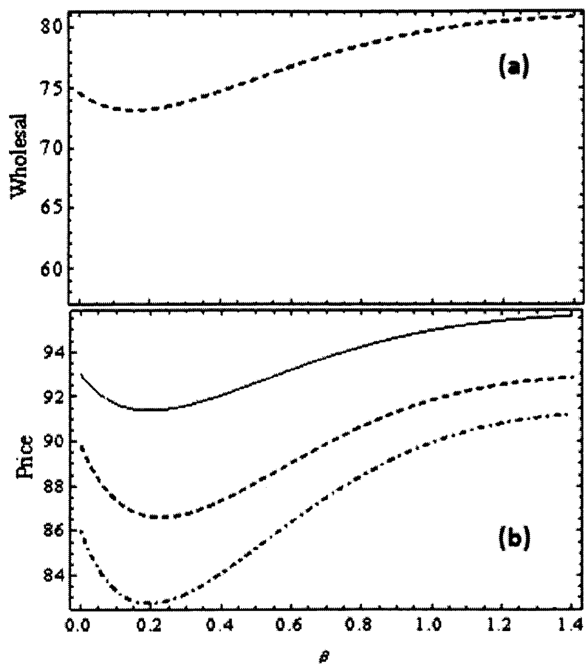
Fig. 5.2 Own price sensitive demand dependence of wholesale, retail price, and lead-time for a shape parameter $\varpi = 0.2$ (left) and $\varpi = 1.2$ (right).

- **Case of shape parameter** $\varpi = 1.2$

For $\varpi = 1.2$, the pdf decreases with increasing the lead-time. This means that long lead-times have high probabilities. This case is in contrast to the behavior of the pdf in M/M/1 system. The dependence of the wholesale, the retail price, and the lead-time to the own price sensitivity demand factor α is plotted in Fig. 5.2 (a-right), (b-right), and (c-right), respectively. At low values of , the gap between the retail price in the centralized and decentralized decisions are sharp more than in the case where $\varpi = 0.2$. In addition, α_{lmax} shifts to low values of α with increasing the shape parameter.

5.5.5.2 Effect of own lead-time sensitivity β

First, we have $\frac{\partial \tau}{\partial \beta} = -\frac{1}{\alpha\varpi(b+h)}\left(\frac{b-\frac{\beta}{\alpha}}{b+h}\right)^{\frac{1-\varpi}{\varpi}} < 0$, $\frac{\partial \eta}{\partial \tau} = \frac{1}{\varpi+1}[(\varpi+1)(b+h)\tau^{\varpi} - b(\varpi+1)] = -\frac{\beta}{\alpha} < 0$, and $\frac{\partial(1+\beta\rho\tau+\alpha\eta\rho)}{\partial \beta} = \rho\tau + \beta\rho\frac{\partial \tau}{\partial \beta} + \alpha\rho\frac{\partial \eta}{\partial \tau}\frac{\partial \tau}{\partial \beta} = \rho\tau > 0$. Then, it is easy to see, in the first and the second scenarios, that $\frac{\partial l_{di}^*}{\partial \beta} < 0\ \frac{\partial \lambda_{di}^*}{\partial \beta} < 0$, $\frac{\partial \pi_{ri}^*}{\partial \beta} < 0$, and $\frac{\partial \pi_{si}^*}{\partial \beta} < 0$ for $i = 1, 2$. Then, increasing the own lead-time sensitivity demand factor β drops the lead-time, the demand function, and the expected profits. For the wholesale and retail prices, we will discuss their behavior with β, numerically. In all figures, the solid, the dashed and the dot-dashed lines correspond to the first, second, and third scenarios, respectively.



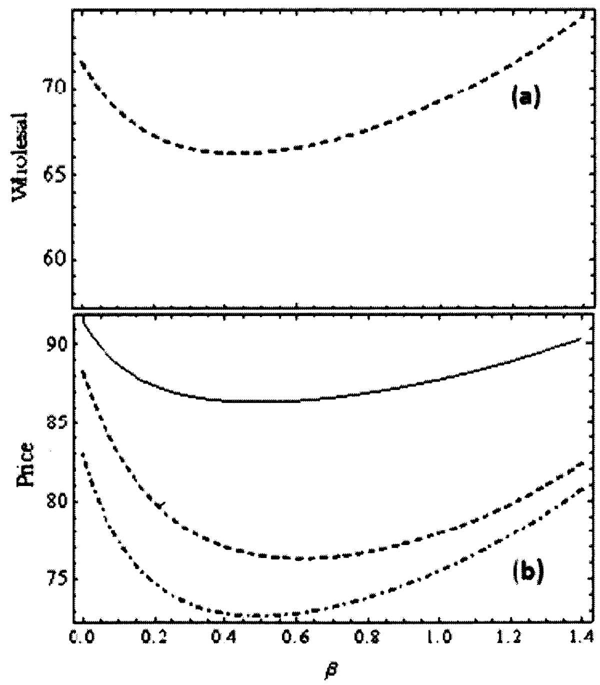


Fig. 5.3 Own lead-time sensitive demand dependence of wholesale and retail price for a shape parameter $\varpi = 0.2$ (left) and $\varpi = 0.2$ (right).

- **Case of shape parameter** $\varpi = 0.2$

The wholesale price in the first scenario is independent of own lead-time sensitivity demand factor β. However, as shown in Fig. 5.3 (a-left), it is not monotone in the second scenario and limited by a minimum wholesale price value w_{min}. Further, as given in Fig. 5.3 (b-left), the retail price in the three scenarios is also not monotone and limited by a minimum price p_{min}. The value of own lead-time sensitivity demand factor $\beta = \beta_{min}$ that corresponds to w_{min} and p_{min} is not the same and it depends on the chain and distribution parameters. This value depends strongly on the tardiness cost b and the own price sensitivity demand factor α. It is worth to note that the non-monotony of wholesale and retail price disappears for high tardiness cost b. In addition, in contrast to the other parameters, β_{min} shifts to high values with increasing α. As important results of this sub-section, the wholesale and retail prices are limited with minimum values, under them the supplier and retailer cannot sell their products.

- **Case of shape parameter ϖ = 1.2**

In contrast to the wholesale and retail prices, increasing the shape parameter increases the lead-time for all values of β. The optimal demand and profits start with high values for high shape parameter, however they drop rapidly with increasing β more than in the case of low ϖ. Furthermore, as shown in Fig. 5.3 (a-right) and (b-right), β_{min} shifts to high values with increasing ϖ.

5.6 Conclusion

As summary, in a supply chain consisting of one supplier and one retailer and wherein the demand is sensitive to retail price and lead-time, three different scenarios based on lead-time decision are studied and compared theoretically and numerically. The important results are:

- Under the exponential distribution of lead-time, it is difficult to obtain a close form solution due to the non-linearity of lead-time and waiting costs with the demand function.

To overcome this problem, we have used a specific form of power distribution, where, under some conditions, it has the properties of exponential distribution. The important results are:

- Under the power distribution function, the chain's actor who decides the lead-time is found to achieve more profits than the other one, independently of chain and distribution parameters. Furthermore, when the retailer decides the lead-time, an inefficiency of lead-time decision is found to be constant (= 0. 25). However, when the supplier decides it, the inefficiency of lead-time decision is less than 0.25 and depends strongly on chain and distribution parameters. Numerically, we find that the consumers are sensitive to the own price sensitive demand factor, where infinity lead-time is not allowed. Further, the retailer is sensitive to the own lead-time sensitive demand factor, where he cannot decrease his retail price under a minimum value. The two limits of lead-time and retail price are sensitive essentially to the tardiness cost.

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Chapter 6

Conclusions

In this thesis, we focused on the modeling and analysis of competition in a decentralized supply chain with retail price sensitive demand. Some models were inspired from previous recent publication with the addition of new parameters and/or numerical analysis. Three competition based works were completely done with an introduction to a new model based on lead-time in a single echelon supply chain. The results of these studies are summarized in the following points:

In chapter 2, we analyze the properties of Nash equilibrium retail prices in contracting model in a supply chain consisting of one supplier, multiple competing retailers, and wherein the demand stochastic and sensitive to retail price. As summary, optimal whole sale prices and buybacks determined by the supplier give more profits to the supplier than retailers. In the symmetric cost cases, the optimal retail prices of two retailers become the same. Compared to supply chain optimization, the retail prices are higher and the quantities of orders are smaller in the individual optimal case. It is because under the chain optimization more amounts of demand are satisfied by decreasing retail prices and increasing order quantities, whereas in the individual optimal case the supplier wants to obtain its own profit, which leads to higher wholesale prices and as a result retail prices become higher. In the anti-symmetric cost case, the optimal wholesale price to the retailer with the smaller production cost is smaller than that to another retailer, which leads to more profits for the former retailer. The reason is that the retailer with small wholesale price sets the less retail price and more quantities of order, which implies that more amounts of demand occur in total and the supplier can sell more products to customers. In particular, with Logit demand function the demand depends on the retail prices more intensively, and the wholesale prices, retail prices and the order quantities change more. In both cases the entire expected profits in the individual optimal cases is about 80 to 85 % of that under supply chain optimization. When the chain consists of one supplier and one retailer, it is shown in Song et al. (2008) that the fraction is 3/4(in linear case) or 2/e = 0.736 (in Logit case). The competition among retailers makes retail prices lower, which makes the fraction higher. Furthermore, it was found that as the variance increases in uniform distribution, retail prices are higher, and profits of the supplier and retailers decrease. This is because when the variance increases, the quantity of order must be increased to apply the fluctuation of demand, whereas the retail price must be also increased to obtain profits of retailers. When the distribution parameter increases, the optimal wholesale prices and buyback prices for the supplier are almost the same. It means that the optimal wholesale and buyback prices for the supplier are robust in the variance of the demand distribution.

In chapter 3, the condition of Nash equilibrium solution are presented for a buyback contract model for one supplier and multiple retailers, where the demand is stochastic and depends on price and safety stock. The performances of Nash solution are discussed numerically for various price and safety stock competition factors. They are also compared with that of the optimization solution and the case of non-competitive model. The Nash solution is computed based on maximization of supplier profit function and use of Newton method to solve non-linear equations. This solution is found to depend strongly on price competition factor (θ), the spill rate (γ), and the distribution parameter (λ). In addition, the ratio of entire profit function of Nash solution and optimal one is found to increase with price competition factor; however, it is nearly constant when the spill rate and the distribution parameters are varied. In the case of supply chain optimization, the retail prices are not affected considerably by the increase of γ in contrast to the safety stocks which increase with it. This can be explained by the correlation between γ and the stochastic part of the demand function. The ratio (Nash profit)/(Optimal profit) is nearly constant because the impact of retail prices on the total profit function is higher than the impact of the safety stocks and in the two cases of Nash solution and optimal one. The factor θ does not affect the optimal solution due to the symmetric values of the retail prices. However, the comparison between the Nash solution and the optimal one shows a large difference on the retail prices and the safety stocks. The retails prices in the optimal solution are lower than in the case of Nash solution in contrast to the safety stocks. This result affects considerably the total demand functions which are high in the case of optimal solution due essentially to the effect of the retail prices. The ratio (Nash profit)/(Optimal profit) increases with increase of θ. This can be explained by the effect of the strong price competition which leads retailers to reduce their safety stocks and increase their profits. The impact of (λ) on the safety stocks is considerable in contrast to the retail prices which are nearly constant. This can be explained by the same effect of γ, however in this case the safety stocks decrease dramatically. The ratio (Nash profit)/(Optimal profit) is nearly constant because the impact of retail prices on the total profit function is higher than the impact of the safety stocks and in the two cases of Nash solution and optimal one the retail prices are not considerably affected by the various of λ.

Chapter 4 has studied a competition between two independent suppliers who sell their products to a common retailer in a decentralized supply chain, under sales-rebate contract, and wherein the demand is sensitive to retail price. As concluded remarks, the wholesale contract, the profit functions of the retailer and the suppliers are the same. However, in the sale-rebate contract, they increase and decrease with varying the target between target = 0 and target = optimal demand, respectively. The Nash wholesale price in the sale-rebate contract is higher than of that in the wholesale contact and the difference between them is equal to the rebate rate. However, the optimal demand rate and retail price are unchanged. This can be explained by the leadership of the supplier to take decision in the chain. Although the competition is between the suppliers, the retailer did not benefit from it in the sale rebate contract. It seems here that the decision variables depend on the leadership decision and not on the competition. The total profit of the chain under Nash equilibrium is independent of the contracts. It is explained by the linear change of profits with the target between target = 0 and target = optimal demand (what is gained by a retailer is lost by two suppliers together to make compensation). The demand rate in the optimization problem (integrated system) is high than that of the Nash equilibrium, in contrast to the retail price. This can be explained by the remove of the leadership decision in the optimization problem, where the wholesale price is an endogenous parameter. The integrated system achieves around 30 % profit more than the Nash profit. Furthermore, increasing the economics of scale drops the different performances of the chain in the two forms of contract.

In Chapter 5, three different scenarios based lead-time decision were studied in a supply chain. The two first scenarios are decentralized and lead-time decision is made by the retailer and the supplier in the first scenario and in the second one, respectively. The supply chain in the third scenario is centralized. The optimal decision variables and expected profits were evaluated and compared each others in decentralized case and to that in centralized one. As concluded remarks, the chain's actor who decides the lead-time is found to achieve more profits than the other one, independently of chain and distribution parameters. Furthermore, when the retailer decides the lead-time, an inefficiency of lead-time decision is found to be constant (= 0.25). However, when the supplier decides it, the inefficiency of lead-time decision is less than 0.25 and depends strongly on chain and distribution parameters. Numerically, we found that the consumers are sensitive to the own price sensitive demand factor, where infinity lead-time is not allowed. Further, the retailer is sensitive to the own lead-time sensitive demand factor, where he cannot decrease his retail price under a minimum value. The two limits of lead-time and retail price are sensitive essentially to the tardiness cost.

As concluding remarks of this thesis, we have:

- In case of retailers' competition, the existence and uniqueness of Nash solution is conditioned with specific conditions related to the context of the model and there is no global condition that can be used. The unique Nash equilibrium retail prices are greater than wholesale prices; however, they depend strongly on competition factors setting, essentially related to retail price.
- In case of suppliers' competition, the problem is more complicated and the existence of Nash solution is restrained by the symmetry of both chain parameters and optimal demand rates.
- The chain's member who decides the lead-time is found to achieve more profits than the other one, independently of chain and distribution parameters. Furthermore, an inefficiency of lead-time

decision is found to be less than 0.25 when the supplier decides lead-time in contrast to the case of retailer

The study of competition in retail supply chain is very important in actual globalized and opened market. Its objective is optimizing the profits of chain members and setting the conditions of existence and uniqueness of Nash solution. However, in SCM there are many contexts in which it can be modelled owing to the existence of various decision variables and coordination contracts. For that, we propose to continue the analysis of competition in new models based on new decision variables such as lead-time, service after delivery… etc.

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