

# Convergence of Mobile Robots with Uniformly-Inaccurate Sensors

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## Abstract

We consider the convergence problem of autonomous mobile robots with inaccurate sensors, which may return the erroneous location of other robots. In this paper, we newly introduce a uniform error model, which is a restricted variant of the original observation-error model proposed by Cohen and Peleg [4]. The degree of an observation error is characterized by distance errors and angle errors. While the original model (non-uniform model) allows that two or more points can have different error degrees, the uniform error model assumes that the same amount of error degree is incurred to all observed points in a single observation. The main focus of our study is to reveal how much such uniformity expands the feasibility of the convergence. In the non-uniform error model, it has been shown that no algorithm can achieve the convergence if the maximum error angle is more than or equal to  $\pi/3$ . This paper shows that the convergence problem is solvable under the uniform error the maximum error angle is less than  $\pi/2$ . We also prove that there is no convergence algorithm for the maximum error angle more than or equal to  $\pi/2$  even in the uniform error model, which implies the optimality of our algorithm in the sense of angle errors.

## 1 Introduction

### 1.1 Background

In recent years, cooperations among a large number of autonomous mobile robots have received much attention. In particular, the algorithmic issues of autonomous mobile robots are actively studied in the literature of the distributed computing. In most of algorithmic studies about autonomous mobile robots, a robot is modeled as a point in a plane, and its capability is quite limited: It is usually assumed that robots are oblivious (no memory to record past situations) and anonymous (no IDs to distinguish two robots). Furthermore, they have no explicit direct means of communication. Typically, the communication between two robots is done in the implicit way that each robot observes the environment, which includes the position of other robots in terms of observer's local coordinate system. A theoretical interest of autonomous mobile robots is to reveal what kinds of coordination tasks can be accomplished by exchanging only such positional or geographic information.

Gathering and convergence problems are popular and fundamental coordination tasks for autonomous mobile robots. In short, given a set of robots with arbitrary initial locations,

gathering must make all robots meet in finite time at a point that is not predefined. The convergence problem is a weaker variant of the gathering problem. It requires the distance between any two robots converges to zero (i.e., for every  $\epsilon > 0$ , there exists a time  $t_\epsilon$  after which any two robots have a distance within  $\epsilon$ ). Both of the problems are actively studied before now, and a number of possibility/impossibility results under different assumptions are shown [10, 6, 9, 7, 8, 5, 2]. Especially, it is known that the difference of observation capability is strongly related to the solvability of those problems. The gathering problem is first discussed in [10], which proves that it is impossible to achieve gathering of two oblivious autonomous robots that have no common sense of orientation under the semi-synchronous model. This result is expanded to the general number of robots by Prencipe [8]. These impossibility results are one of reasons to make us focus on the convergence problem. The convergence problem is also considered in several papers [4, 3, 1]. Since, as we mentioned, the convergence problem is weaker than the gathering, most of those studies assume weaker models in the sense of observation capability. Recently, as such a weaker model, Cohen and Peleg introduced the robot model where each robot suffers observation errors. If a robot A observes another robot B, A may see B at the position which is slightly different from the actual location of B. More precisely, if B is located at  $(r \cos \phi, r \sin \phi)$  on A's coordinate system, an observation by A may return the coordinate  $(r(1 + \epsilon) \cos(\phi + \theta), r(1 + \epsilon) \sin(\phi + \theta))$  as the B's location (namely,  $\epsilon$  and  $\theta$  represent the error ratio about distance and direction respectively). For both of  $\epsilon$  and  $\theta$ , their absolute bounds  $\epsilon_0$  and  $\theta_0$  are assumed. They show that if the maximum angle error  $\theta_0$  can be greater than  $\pi/3$ , it is impossible to achieve to convergence, and propose a convergence algorithm for any maximum distance error  $\epsilon_0$  and maximum angle error  $\theta_0$  satisfying  $0.2 > \sqrt{2(1 - \epsilon_0)(1 - \cos \theta_0 + \epsilon_0^2)}$ .

## 1.2 Our results

This paper also considers the convergence problem under a similar inaccurate sensor model. The main focus of our study is the uniformity of observation errors: In the original model, the error ratio can be different for each robot. For example, if a robot A observes two other robots B and C, the returning coordinates of B and C can include different amounts of errors. The uniformity of observation error assumes that all coordinates returned by one observation include the same amount of error (but two distinct observations can have the different error ratio even if they are performed by a same robot.) Our interest is to answer the question how the uniformity assumption enhances the capability of robots in respect to task solvability. Interestingly, we can show that the assumption relaxes the bound on the error ratio for which the convergence task can be solved. More precisely, assuming uniform error ratio to inaccurate sensor models, we can solve the convergence problem if the maximum distance error ratio is less than one, and the maximum angle error is less than  $\pi/2$ . We present the summary of our result, which includes the comparison to the previous paper, in Table 1.

## 1.3 Organization

The following is the organization of this paper. In Section 2 we define the robot model and uniform error model. In Section 3 we present the impossibility result that robots cannot converge when maximum angle error is more than or equal to  $\pi/2$ . In Section 4 we present a convergence algorithm on the uniform error model with the maximum distance error ratio less than one and the maximum angle error less than  $\pi/2$ , and prove its correctness.

Table 1: Summary and comparison of our results

Model	Maximum distance error $\epsilon_0$	Maximum angle error $\theta_0$	Possibility of Convergence
Non-uniform error model ([4])	Any $\epsilon_0$	$\theta_0 \geq \pi/3$	<b>×</b>
	$0.2 > \sqrt{2(1 - \epsilon_0)(1 - \cos \theta_0 + \epsilon_0^2)}$		
	Other		Open
Uniform error model (This paper)	$\epsilon_0 < 1$	$\theta_0 < \pi/2$	(This paper)
	$\epsilon_0 \geq 1$	$\theta_0 < \pi/2$	Open
	Any $\epsilon_0$	$\theta_0 \geq \pi/2$	<b>×</b> (This paper)

## 2 The robot model

The robot model of this paper is an extension from that proposed by Suzuki and Yamashita [10] such that each robot suffers observation errors. The following is a concise outline of our model.

- Each robot is the point without volume that moves freely on 2-D space.
- Robots are anonymous, that is, each robot cannot be distinguished from others by ids, their physical appearances, and so on.
- Robots are oblivious. That is, they cannot remember the history of their executions.
- We only consider uniform algorithms. That is, all robots execute the same algorithm.
- Robots have no direct communication device. Each robot observes a configuration of all robots by its own local coordinate system.
- As the timing model, we adopt the *semi-synchronous model*. At each time unit, a subset of all robots (determined by the scheduler) performs movement synchronously.
- An observation of a robot is inaccurate. An observation result may include wrong locations of robots. An observation error is characterized by distance errors and angle errors. We assume the maximum distance error  $\epsilon_0$  the maximum angle error is  $\theta_0$ . All robots know the value of  $\epsilon_0$  and  $\theta_0$  as information about the accuracy of their observations. We also assume the uniformity of observation errors, which implies all observed robots necessarily have the same amount of distance/angle error in a single observation.

In the following subsections, we present the details of our model.

### 2.1 The system model

The system consists of  $n$  robots  $S = \{s_0, s_1, \dots, s_{n-1}\}$ . Each robot is modeled as a point on the two-dimensional Euclidean plane, and works based on discrete time  $0, 1, 2, \dots$ . Each robot has its own local coordinate system whose origin is the current position of the robot. A location of all other robots in an observation result is called *local coordinates*. In contrast, to define the positions of robots consistently, we introduce the global coordinate system on the plane. The coordinate of each robot on the global coordinate system are called *global coordinate*. Notice that the global coordinate system is introduced only for ease of explanations, and thus each robot

cannot be aware of them. In what follows, any coordinate is represented by two-dimensional vectors, which is described by bold-faced characters.

Each robot is either *active* or *inactive* at each time. An active robot first observes the locations of all other robots, and computes the destination from the observation result. Since each robot is oblivious and uniform, the destination is computed only from the observation result by a common algorithm. In this sense, an algorithm is formally defined as a deterministic function  $f$  that maps a set of coordinates (i.e, the positions of all robots) to a coordinate (i.e, the destination). After the computation, the robot moves toward the computed destination on the local coordinate system. We assume that it is guaranteed that any movement is necessarily completed within one time unit. That is, if a robot is active at  $t$ , its location at  $t + 1$  is the destination computed at  $t$ . The set of active robots at each time is determined by the scheduler. Throughout this paper, we assume *fair* scheduler. It ensures that each robot activated infinitely often.

Each robot observes the locations of all robots in terms of its local coordinate system. There is no assumption about the direction and unit scale of local coordinate systems. That is, each local coordinate system can have a different direction and unit distance. In this paper, it is assumed that each robot suffers *observation error*, which allows the locations of robots observed by another robot to be different from their actual locations. The detail of observation-error models is explained in the following subsection.

## 2.2 Observation error

The observation error is characterized by maximum distance error  $\epsilon_0$  and maximum angle error  $\theta_0$ . We explain the influence that these error factors give by showing an example: Consider a situation where a robot  $s_1$  observes  $s_2$ . Let  $\mathbf{V}$  be the vector representing the location of a robot  $s_2$  in terms of  $s_1$ 's coordination system, and  $\mathbf{v}$  be the vector representing the location of  $s_2$  in  $s_1$ 's observation result. (See Fig.1.) Then each error factor is explained as follows:

### Distance error

Any observed distance is affected by at most  $\pm\epsilon_0$  fraction of the actual distance. That is, the observation result satisfies  $|\mathbf{V}|(1 - \epsilon_0) < |\mathbf{v}| < |\mathbf{V}|(1 + \epsilon_0)$ .

### Angle error

Any observed angle has an additive error within  $\pm\theta_0$ . That is, letting angle formed by  $\mathbf{v}$  and  $\mathbf{V}$  be  $\theta$ ,  $\cos \theta \geq \cos \theta_0$  is satisfied.

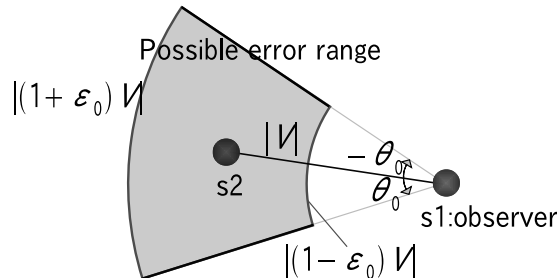


Figure 1: Observation error

### 2.3 Uniformity of Observation Error

In this subsection, we define the non-uniform error model and the uniform error model.

#### Non-uniform error model

If a robot observes other two or more robots, the observation result can involve different distance/angle errors for each observed robot. An example is shown in Fig.2.a. In this example, robot  $s_0$  observes all other robots, and the observation error occurs differently for two robots  $s_1$  and  $s_2$ : Distance error ratio  $\epsilon_1$  and angle error  $\theta_1$  is associated with robot  $s_1$  and  $\epsilon_2$  and  $\theta_2$  with robot  $s_2$ .

#### Uniform error model

In a single observation, the same observation error is associated with all observed robots (see Fig.2.b), but it is allowed that two different observations have different observation errors.

Notice that even in the uniform-error model, it is possible that two observations by one robot at different timings can have different observation errors.

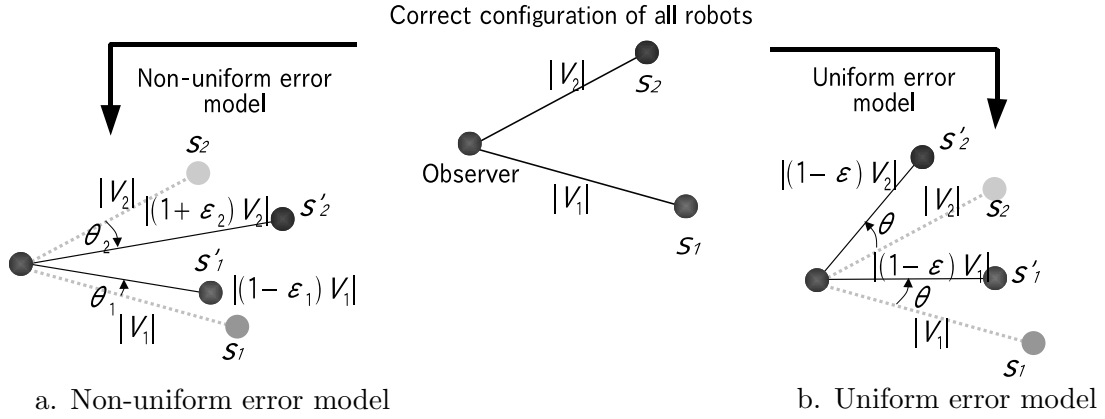


Figure 2: An example of observations in two error models

## 3 Impossibility result

This section provides the impossibility of the convergence when  $\theta_0 \geq \pi/2$ .

**Theorem 1** *For any number of robots, if  $\theta_0 \geq \pi/2$ , no algorithm can achieve the convergence.*

**Proof** Suppose for contradiction an algorithm A that achieves the convergence for  $n$  robots ( $n \geq 2$ ) and  $\theta_0 \geq \pi/2$ . We start the proof from the initial configuration where all robots are evenly located on a circle. The  $y$ -axis of their local coordinate systems are directed to the center of the circle (Figure 3). The proof idea is that to find an execution where all robots move to the outside of the circle and form a evenly-located circle again after the movement. Then, by repeating the same execution, the diameter of the circle grows infinitely, which implies the impossibility of convergence.

To construct the desired execution, we first consider the execution where only one robot is activated. Let  $s_i$  be the activated robot. If  $s_i$  moves to the outside of the circle, we obtain the desired execution by activating all robots simultaneously because, by symmetry of the

configuration, all robots symmetrically move. This implies that they form an evenly-located circle after the movement. On the other hand, If  $s_i$  moves to the inside of the circle, we consider an execution where all robots are simultaneously activated but equally suffer angle observation error  $\pi/2$ . Then, the observation result of each robot is rotated by  $\pi/2$ , and thus the destination point is also rotated by  $\pi/2$ . As a consequence, in this execution, all robots symmetrically move toward the outside of the circle.  $\square$

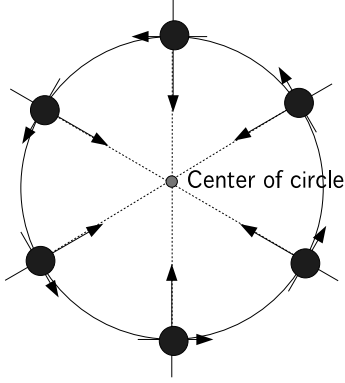


Figure 3: Initial configuration

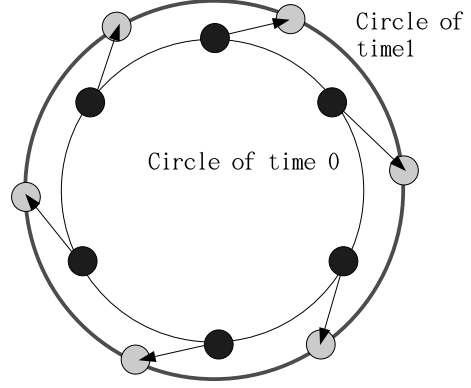


Figure 4: Symmetric outside movement by robots

## 4 Convergence algorithm

### 4.1 Outline of algorithm

In this section, we show the algorithm *Conv-SEC*, which can converge all robots under the assumption of the uniform error model with  $\theta_0 < \pi/2$  and  $\epsilon_0 < 1$ . The pseudo-code of *Conv-SEC* algorithm shown in the Figure.5. The key idea of the algorithm is to make robots move

**Code for Robot  $s_i$ :**

- 1: Observe the locations of all other robots
- 2: Compute the center of SEC from the observation result
- 3: **if**  $s_i$  is on the computed SEC **then**
- 4:     move toward the center of SEC by distance  $d \cos \theta_0 / (1 + \epsilon_0)$   
       ( $d$  is the distance between  $s_i$  and the computed center)
- 5: **endif**

Figure 5: Algorithm *Conv-SEC*

toward the center of the smallest-enclosing circle (SEC), which is the minimum-diameter circle containing all positions of robots. At each time, each active robot computes the center of SEC from the observation result (note that for any set of points, its smallest enclosing circle is uniquely determined and it can be computed in polynomial time). Then, if a robot stays on the boundary of SEC, it moves toward the center of SEC with distance  $(d \cos \theta_0 / (1 + \epsilon_0))$ , where  $d$  is the observed distance between the robot and the center. Notice that the computed center is not equal to the actual center: Because of the observation error, the robot does not move toward

the actual center. Then, long-distance movement, that is length  $d$ , may cause the robot to go out the actual SEC. (See Fig.6.) The movement with length  $d \cos \theta_0 / (1 + \epsilon_0)$  ensures robots do not go out the actual SEC. (See, Fig.7.)

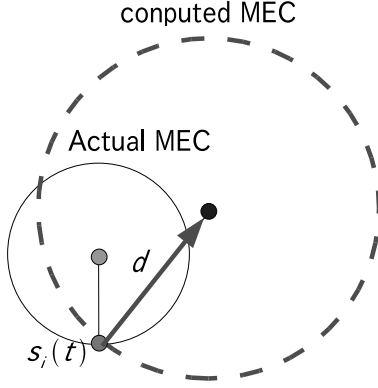


Figure 6: The movement with length  $d$

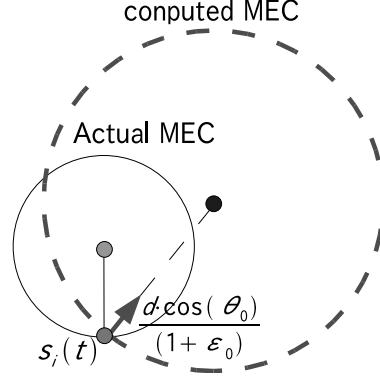


Figure 7: The movement with length  $d \cos \theta_0 / (1 + \epsilon_0)$

## 4.2 Correctness Proof

For two points  $\mathbf{p}$  and  $\mathbf{q}$ , let  $dis(\mathbf{p}, \mathbf{q})$  be the distance between them in terms of the global coordinate system. (i.e.,  $dis(\mathbf{p}, \mathbf{q}) = |\mathbf{p} - \mathbf{q}|$ .) The global coordinate of robot  $s_i$  at time  $t$  is denoted by  $\mathbf{s}_i(t)$ . The actual center of SEC at time  $t$  is denoted by  $\mathbf{C}(t)$ . Letting  $t$  be a time when a robot  $s_i$  is active,  $\mathbf{c}_i(t)$  denotes the global coordinate of the center of SEC computed by robot  $s_i$  at  $t$ . For simplicity, we also introduce the following notations:

- $d_i(t) = dis(\mathbf{c}_i(t), \mathbf{s}_i(t))$
- $D_i(t) = dis(\mathbf{C}(t), \mathbf{s}_i(t))$
- $D'_i(t) = dis(\mathbf{C}(t), \mathbf{s}_i(t+1))$

The displacement of the center of actual SEC during  $[t, t+1]$  (in terms of the global coordinate system) is denoted by  $\Delta(t)$ , i.e.,  $\Delta(t) = dis(\mathbf{C}(t), \mathbf{C}(t+1))$ . The radius of SEC at time  $t$  is denoted by  $\mathbf{R}(t)$ . (See Fig.8).

It should be noted that in the uniform error model, any observation result is a homothetic transformation of the actual robot locations, whose magnification is between  $(1 - \epsilon_0)$  and  $(1 + \epsilon_0)$ . This implies that we can obtain the following corollary.

**Corollary 1**  $(1 - \epsilon_0) D_i(t) \leq d_i(t) \leq (1 + \epsilon_0) D_i(t)$ .

By the nature of the algorithm, it is clear that the diameter of SEC is non-increasing. However, it is not so trivial to prove that the distance certainly converges to zero. The difficulty of the proof is the movement of the center of SEC, which prevents the monotonic decrease of the distance between the center and each robot. This fact implies the necessity of a little more complicated argument: The key idea of our proof is to show that any movement necessarily decreases either the diameter of SEC, or the sum of the distances between the center and robots.

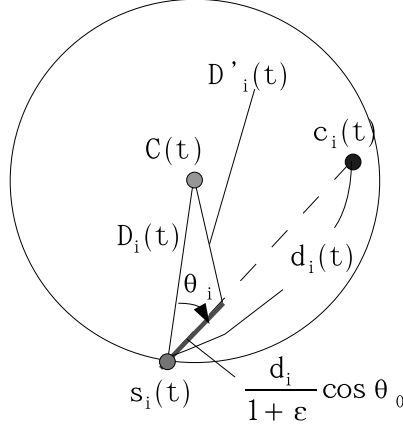


Figure 8: Notations used in the proof

**Lemma 1** *There is a constant  $\alpha$  ( $0 \leq \alpha < 1$ ) satisfying  $D'_i(t) \leq \alpha D_i(t)$  for any time  $t$  when a robot  $s_i$  is active.*

**Proof** Let  $\theta_i$  be the angle error which  $s_i$  suffers at  $t$ . Since the distance traveled by  $s_i$  is  $\frac{d_i(t)}{1+\epsilon_0} \cos \theta_0$ , by applying the law of cosine to the triangle  $\mathbf{C}(t)\mathbf{s}_i(t)\mathbf{s}_i(t+1)$ , we can obtain the following equation:

$$D_i'^2(t) = D_i^2(t) + \left( \frac{d_i(t)}{1+\epsilon_0} \right)^2 \cos^2 \theta_0 - 2D_i(t) \frac{d_i(t)}{1+\epsilon_0} \cos \theta_0 \cos \theta_i.$$

This equation can be transformed into the following inequality (the detail is shown in the appendix):

$$D_i'(t) \leq D_i(t) \sqrt{1 + \frac{(3\epsilon_0 + 1)(\epsilon_0 - 1)}{(1 + \epsilon_0)^2} \cos^2 \theta_0}.$$

Since  $0 \leq \epsilon_0 < 1$ , the term of square root is less than one, and thus the lemma holds.  $\square$

**Lemma 2**  $R(t+1) \leq \sqrt{R^2(t) - \Delta^2(t)}$ , for all times  $t \geq 0$ .

**Proof** Lemma 1 implies that the destination of any movement is necessarily inside of the actual SEC. Thus, the radius of SEC is non-increasing. In addition, it is clear that the SEC at  $t$  (denoted by  $SEC_t$ ) necessarily intersects that at  $t+1$  (denoted by  $SEC_{t+1}$ ). Thus, the following four cases are possible:

1.  $SEC_{t+1}$  and  $SEC_t$  are circumscribed to each other: All robots stay at their (unique) intersecting point, which implies  $R(t+1) = 0$ .
2.  $SEC_t$  and  $SEC_{t+1}$  are identical: We can obtain  $R(t+1) = \sqrt{R^2(t) - \Delta^2(t)} = R(t)$  because  $\Delta(t) = 0$ .
3. The boundaries of  $SEC_t$  and  $SEC_{t+1}$  have two intersecting points: Let  $\mathbf{X}_1$  and  $\mathbf{X}_2$  be the two intersecting points, and  $\mathbf{Z}$  be the intersecting point of the lines  $\mathbf{X}_1\mathbf{X}_2$  and  $\mathbf{C}(t)\mathbf{C}(t+1)$ .



Letting  $L$  be the line passing through  $\mathbf{C}(t+1)$  and orthogonal to  $\mathbf{C}(t)\mathbf{C}(t+1)$ , we define  $\mathbf{Y}_1$  and  $\mathbf{Y}_2$  as two intersecting points of  $L$  and the boundary of  $SEC_{t+1}$ . Notice that the segment  $\mathbf{Y}_1\mathbf{Y}_2$  is the diameter of  $SEC_{t+1}$ . We further divide this case into the following two sub-cases:

(a)  $dis(\mathbf{Z}, \mathbf{C}(t)) < \Delta(t)$ . (Fig. 9)

We show that this case never occurs by leading a contradiction. Suppose  $dis(\mathbf{Z}, \mathbf{C}(t)) < \Delta(t)$  for contradiction. The circle  $C$  centered at  $\mathbf{Z}$  and having diameter  $dis(\mathbf{X}_1, \mathbf{X}_2)$  contains the intersecting area of  $SEC_t$  and  $SEC_{t+1}$ , and thus it encloses all robot locations at  $t+1$ . Since  $\mathbf{Y}_1\mathbf{Y}_2$  is the diameter of  $SEC_{t+1}$  and  $dis(\mathbf{Z}, \mathbf{C}(t)) < \Delta(t)$  holds, the length of  $\mathbf{X}_1\mathbf{X}_2$  is smaller than  $R(t+1)$ . This contradicts to the fact that the  $SEC_{t+1}$  has the smallest diameter.

(b)  $dis(\mathbf{Z}, \mathbf{C}(t)) \geq \Delta(t)$ . (Fig. 10)

Let  $\mathbf{Y}'_1$  and  $\mathbf{Y}'_2$  be the two intersecting points of the line  $\mathbf{Y}_1\mathbf{Y}_2$  and the boundary of  $SEC_t$ . Then,  $R(t+1) = dis(\mathbf{Y}_1, \mathbf{C}(t)) < dis(\mathbf{Y}_1, \mathbf{C}(t+1)) = \sqrt{R^2(t) - \Delta^2(t)}$  holds.

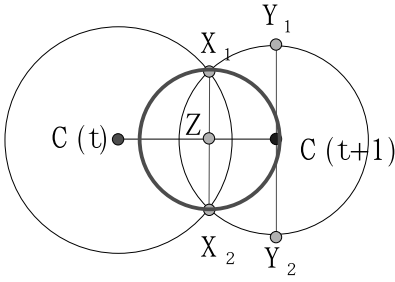


Figure 9:  $dis(\mathbf{Z}, \mathbf{C}(t)) < \Delta(t)$

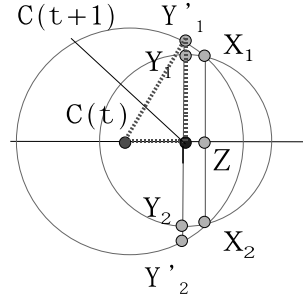


Figure 10:  $dis(\mathbf{Z}, \mathbf{C}(t)) \geq \Delta(t)$

4.  $SEC_t$  contains  $SEC_{t+1}$ : This proof is the completely same as that for the case 3(b) (Fig. 11)..

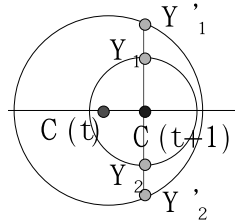


Figure 11: The SEC at time  $t+1$  exists in the SEC at time  $t$

From the above, the lemma holds for any cases.  $\square$

**Lemma 3** There exist two constants  $\beta$  and  $\gamma$  ( $0 < \beta, \gamma < 1$ ) such that either of the following holds for any time  $t$ :

- $R(t+1) \leq \beta R(t)$ .

- $\sum_i D_i(t+1) \leq \gamma \sum_i D_i(t)$ .

**Proof** Let  $S'(t)$  is a set of robots moving at time  $t$ . By the triangle inequality,  $\text{dis}(\mathbf{P}, \mathbf{C}(\mathbf{t})) + \Delta(t) \geq \text{dis}(\mathbf{P}, \mathbf{C}(\mathbf{t}+1))$  holds for any point  $\mathbf{P}$ . Combining this inequality with Lemma 1, we obtain  $\alpha D_i(t) + \Delta(t) \geq D_i(t+1)$ . It follows the inequality below:

$$\begin{aligned} & \sum_i D_i(t+1) \\ & \leq \sum_{i \notin S'(t)} (D_i(t) + \Delta(t)) + \sum_{i \in S'(t)} (\alpha D_i(t) + \Delta(t)) \\ & = \sum_{i \notin S(t)} D_i(t) + n\Delta(t) - (1-\alpha) \sum_{i \in S'(t)} D_i(t). \end{aligned}$$

Since only the robots on the boundary of SEC can move,  $\sum_{i \in S'(t)} D_i(t) = |S'(t)|R(t)$  holds for any  $t$ , and thus we have  $\sum_i D_i(t+1) \leq \sum_i D_i(t) + n\Delta(t) - (1-\alpha)|S'(t)|R(t)$ . Then we consider the following two cases:

1.  $n\Delta(t) - (1-\alpha)|S'(t)|R(t) > \frac{\alpha-1}{2n} \sum_i D_i(t)$ .

In this case, we can obtain inequality  $\Delta(t) > \frac{2(1-\alpha)|S'(t)|R(t) + \frac{(\alpha-1)}{n} \sum_i D_i(t)}{2n}$ . Moreover,  $\sum_i D_i(t) \leq nR(t)$  clearly holds. Thus, we have

$$\begin{aligned} \Delta(t) & > (1-\alpha) \frac{2|S'(t)|R(t) - R(t)}{2n} \\ & = (1-\alpha)R(t) \frac{2|S'(t)| - 1}{2n}. \end{aligned}$$

From Lemma 2, we obtain

$$\begin{aligned} R(t+1) & < \sqrt{R^2(t) - \left( (1-\alpha)R(t) \frac{2|S'(t)| - 1}{2n} \right)^2} \\ & = R(t) \sqrt{1 - \left( \frac{(2|S'(t)| - 1)(1-\alpha)}{2n} \right)^2}. \end{aligned}$$

Since  $1 \leq |S'(t)| \leq n$ ,  $R(t+1) < R(t) \sqrt{1 - \left( \frac{1-\alpha}{2n} \right)^2}$  holds. The term of the square root is smaller than one because  $0 \leq \alpha < 1$ . Thus, the lemma holds.

2.  $n\Delta(t) - (1-\alpha)|S'(t)|R(t) \leq \frac{\alpha-1}{2n} \sum_i D_i(t)$ .

Then, we can obtain

$$\sum_i D_i(t+1) \leq \sum_i D_i(t) + \frac{\alpha-1}{2n} \sum_i D_i(t) = \frac{1+\alpha}{2n} \sum_i D_i(t).$$

This implies  $\gamma = (1+\alpha)/2n$ , and thus the lemma holds.  $\square$

**Theorem 2** In the uniform error model satisfying  $\theta_0 < \pi/2$  and  $0 \leq \epsilon_0 < 1$ , algorithm Conv-SEC can converge all robots into a point.

**Proof** From Lemma 3, we can easily prove the geometric decreasing of either the radius of SEC or the sum of the distances between the center of SEC and all robots. This implies that the distance between any two robots converges to zero.  $\square$

## 5 Conclusion

In this paper, we newly introduced the notion of uniformity in observation error of autonomous mobile robots, and investigated the impact of uniformity to the solvability of the convergence problem. We showed that in the uniform error model with the maximum angle error  $\epsilon_0 \geq \pi/2$ , no algorithm can achieve the converge. In addition, assuming the semi-synchronous model and uniform observation error with  $\epsilon_0 < 1$  and  $\theta_0 < \pi/2$ , we proposed a convergence algorithm correctly working for any number of robots. This algorithm is optimal in the sense of its allowable error angle.

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## Appendix

The omitted expression transformation in Lemma 1.

$$\begin{aligned}
& D_i'^2(t) \\
= & D_i^2(t) + \left( \frac{d_i(t)}{1 + \epsilon_0} \right)^2 \cos^2 \theta_0 - 2D_i(t) \frac{d_i(t)}{1 + \epsilon_0} \cos \theta_0 \cos \theta_i \\
\leq & D_i^2(t) + \left( \frac{d_i(t)}{1 + \epsilon_0} \right)^2 \cos^2 \theta_0 - 2D_i(t) \frac{d_i(t)}{1 + \epsilon_0} \cos^2 \theta_0 \\
= & D_i^2(t) + \left( \left( \frac{d_i(t)}{1 + \epsilon_0} \right)^2 - 2D_i(t) \left( \frac{d_i(t)}{1 + \epsilon_0} \right) \right) \cos^2 \theta_0 \\
= & D_i^2(t) + \left( \left( \frac{d_i(t)}{1 + \epsilon_0} \right) - D_i(t) \right)^2 \cos^2 \theta_0 - D_i^2(t) \cos^2 \theta_0 \\
\leq & D_i^2(t) + \left( \frac{1 - \epsilon_0}{1 + \epsilon_0} D_i(t) - D_i(t) \right)^2 \cos^2 \theta_0 - D_i^2(t) \cos^2 \theta_0 \\
= & D_i^2(t) \left( 1 + \left( \frac{1 - \epsilon_0}{1 + \epsilon_0} - 1 \right)^2 \cos^2 \theta_0 - \cos^2 \theta_0 \right) \\
= & D_i^2(t) \left( 1 + \frac{3\epsilon_0^2 - 2\epsilon_0 - 1}{(1 + \epsilon_0)^2} \cos^2 \theta_0 \right) \\
= & D_i^2(t) \left( 1 + \frac{(3\epsilon_0 + 1)(\epsilon_0 - 1)}{(1 + \epsilon_0)^2} \cos^2 \theta_0 \right)
\end{aligned}$$

□