A Co-dependent Value-based Mechanism for the Internet Advertisement Auction

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Abstract. Advertisements on the webpage provide good opportunity to get new customers. In recent years, a lot of webpages providing a search service have advertisements, which are related with searched word by user. A basic structure of the Internet advertisement is that the service providers decide order of placement of many advertisements and advertising fees by auctions when advertisers offer their promotions. Generalized Second Price Auction (GSP) mechanism is most efficient auction mechanism of the advertisement auction. Some searching companies, such as Google and Yahoo, employ GSP mechanism basically. There are many researches on GSP in order to analyze and clarify its feature and advantages. However, these researches assume that traded advertisements are mutually independent. It means that each advertisement does not influence other advertisements. Also these researches do not consider a value of advertisement, which means some criterions of a name value of a company, an effectiveness and an importance, that is dependently each other. This paper proposes a new advertisement auction mechanism based on GSP with considering the co-dependent value of advertisement. We analyze the auctioneer's profit in comparison between normal GSP, normal VCG (Vickrey-Clarke-Groves Mechanism) and our proposed mechanism.

1 Introduction

Agent-based electronic commerces are parts of promising techniques to enhance effectiveness and performance of trading. In this paper, we give an analysis of agent-based advertisement auction, which is displayed on a webpage. Internet advertisement auction is one of important income source for some search engines such as Yahoo! and Google[1][2]. Some searching companies have a advertising space on his/her webpages and allocates it for some advertisers based on an advertising fee. As same as items trading in the Internet auctions, a displayed advertisement on web page is also based on the auction, called the Internet advertisement auction. When users search for some words on the search engine, an advertisement related with the searched keywords is displayed with result of search[3]. The order of advertisements to be displayed is determined based on bid value in an auction. Advertisers can set up the interval and period to display the advertisement as a time slot. The advertising fee is determined based on the Generalized Second Price Auction, which is known higher revenues than the Generalized Vickrey Auction[4]. A winner in the auction gets a space to display their advertisement and the web page owner allocates time and position in the web page to show the advertisement. There are a lot of contributions about GSP(Generalized Second Price Auction) researches in electronic commerce research. In this auction, bidding and winner determination are conducted multiple time. Advertiser advertiser can change his/her bid value because the auction is continued with repetition. When advertisers try to bid in an auction, they bid on their strategy. However, GSP has an envy free equilibrium and webpage owner providing advertisement space can get larger benefit compared with VCG (Vickrey-Clark-Groves) Mechanism.

Generally, possibility of click is high order of display. This means that the advertisement fee of top-displayed advertisement is more expensive than lower advertisements. Google earned about 5.2 million USD by this advertisement system in 2008.

In previous research, the value of advertisement is assumed as independent with each other. Otherwise, some of their researches do not refer the value of the advertisement. However, each advertisement has a certain value for users. It means some criterions of a name value of a company, an effectiveness, an importance and an attribution, that is dependently each other. When same or similar item is soled in two e-commerce sites, the price on the advertisement is different from another one. If a buyer considers the price is important attribute to choose item, the advertisement selling items at low price has more value for the buyer. For example, a shop A gives an advertisement to sell an item for \$100. When a shop B gives the advertisement to sell the same item for a shop \$80, its value of the advertisement is higher than shop A's value if the condition of item and other situations between shop A and B. In this paper, we focus on such situation and simulate the revenue of advertisers. Also, we analyze a result of simulation of Internet advertisement auction with relationship between value of each advertisement. After the simulation, we reformulate our proposed model and mechanism based on the preliminary simulation. Concretely, we discuss about dynamical environment. It is more realistic situation of the advertisement market on the Internet.

The rest of this paper consists of the following four parts. In Section 2, we show preliminaries on several terms and concepts of auctions. In Section 3, we propose our value-based GSP and describe a preliminary experiment in some conditions. And we reformulate our proposed model and mechanism for applying a dynamic environment based on the preliminary simulation. Also we discuss some applications of the proposed model, especially about the GrobalAd system. Finally, we present our concluding remarks and future work.

2 Preliminaries

In this section, we describe a generally advertisement auction model. Suppose that there are n advertisers and k slots. A slot is a place of advertisement on a webpage. Let c_i be a click-through-count (CTC) of the advertisement placed on the slot i. CTC is the number of clicks of the advertisement per an unit time. We assume following rule for each c_i:

$$c_{i-1} \geq c_i, \text{ for } 2 \leq i \leq k.$$

This rule means that CTC of the slot i is lower than the slot $i-1$ for $2 \leq i \leq k$.

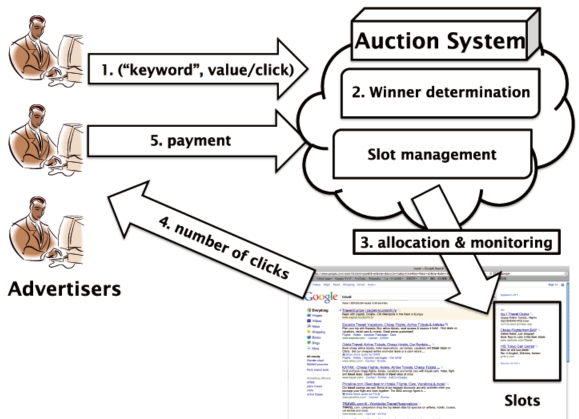

Fig. 1.

When an advertiser j bids a pair of a keyword and value per click to use a slot as $\{$"keyword", $b_j\}$, a payment of the advertiser, who was allocated a slot i, is defined by $b_j \cdot c_i$. Figure 1 shows the generally advertisement auction model. In this model, there exists an advertisement auction system which decides some winners of the auction and management of slots. First of all, each advertiser bids some pairs of a keyword and value per click for advertisement slot to the auction system. After that, the auction system decides some winners of the auction and allocates the advertisements to the slots based on the bidding values. Also the auction system announces CTC to the winners, and the winner pays decided payment to the auction system.

The auction system employs some auction mechanisms for a winner determination. The auction mechanism is a rule of allocation and decision of payment. Generally, the auction mechanism describes the auction system. We introduce some typical auction mechanisms for the Internet advertisement auction. We assume every following auction satisfies Nash equilibrium. The Nash equilibrium shows that a strategy $S_A^\star$ is a best strategy for agent A if every agent without agent A chooses an optimal strategy $S^\star$.

Vickrey auction mechanism Vickrey auction is an auction protocol which deals single item as same as second price sealed bid auction[4]. In this protocol, every advertiser bids own value for the auction system, which their bids do not be opened. A winner of the auction is the highest valued bidder, and he/she pays a second highest value of the auction. The Vickrey auction has week dominant strategy in which every advertiser bids own truth value. It is well known that the English and Dutch auction has also the same week dominant strategy[5].

Vickrey-Clark-Groves (VCG) mechanism VCG mechanism is generalized from Vickrey auction, which has dominant strategy as truthful bidding[8]. Each advertiser j bids own value per click for auctioneer. The auction system allocates a slot for the advertiser by descending order of bids. Suppose $\tilde{A}$ is a set of winners of the auction and $\tilde{A}^{-j}$ is a set of winners of the auction which eliminates the advertiser j, we define a payment per click p_j of the winner as follows,

$$p_j = \sum_{k \in \tilde{A}^{-j}} b_k - \sum_{k \in \tilde{A}} b_k - b_j.$$

VCG mechanism satisfies incentive compatibility and Pareto efficiency. The Incentive compatibility (Strategyproofness) means that each advertiser choice an optimal strategy without influence of other advertisers. The Pareto efficiency means a total utilities of each advertiser and the auction system[4].

We show an example, suppose that there are two slots and three advertisers. Advertiser 1, 2 and 3 bids $\$300$, $\$200$ and $\$100$ per click, respectively. In this case, the auction system allocates slot 1 and 2 to advertiser 1 and 2, and advertiser 1 and 2 pays $\$100$ and $\$100$ per click, respectively. Also, the auction system's gain is $(\$100 + \$100) = \$200$.

GFP (Generalized First Price Auction) mechanism This protocol is nearly single item first price auction, that is an advertiser who is a winner of the auction pays own value. The GFP protocol has dominant strategy for each advertiser. This protocol gives a highest utility for an advertiser when bids a lowest value he/she is able to win[5] . We consider an auction of one slot and two advertiser. If advertiser 1 and 2 bids $\$300$ and $\$200$, respectively, then the advertiser 1 get the slot. However, if the advertiser 1 bids $\$201$, then also the winner. Therefore, GFP protocol has an incentive that every advertiser try to decrease own value. This means that the more increasing a number of advertisers, the more decreasing the auction system's gain.

GSP (Generalized Second Price Auction) protocol GSP protocol is an auction protocol which is natural extended form second price auction[9]. The auction system sorts all bided values by descending order, and allocates slot i to i-th highest valued advertiser for all slots. The advertiser who is allocated slot i pays b_{i+1} per click for the auction system.

It is known that GSP protocol does not satisfies incentive compatibility[10]. Therefore, the truthful bidding is not dominant strategy in GSP. On the other hands, GSP converges on Locally Envy Free equilibrium[6]. The auction is Locally Envy Free equilibrium, if an advertiser who gets a slot i does not increase own utility neither getting a slot $i-1$ nor getting a slot $i+1$[11]. Hence, the slot i is an optimal position which maximizes the advertisers' utility.

We consider the same example in VCG. Suppose that there are three advertisers and two slots, and advertiser 1, 2 and 3 bids $\$300$, $\$200$ and $\$100$ per click. In this case, the advertiser 1 and 2 gets the slot 1 and 2, and pays $\$200$ and $\$100$ per click, respectively. The gain of auction system is $\$200 + \$100 = \$300$. Therefore, the GSP protocol is better than VCG mechanism in the advertisement auction, since the auction systme gains $\$200$ on the VCG mechanism. Note that if there is one slot, then the result of auction is the same on both GSP protocol and VCG mechanism.

Their mechanisms are employed not only the advertisement auction, but also the general Internet auction. Next we discuss an auction system of the Google Adwords.

2.1 Google Adwords

Google Adwords is an auction protocol similar to GSP protocol. Google Adwords employs CTR (Click-Through-Rate) and QS (Quality-Score). CTR is a ratio of click denoted by

$$\text{CTR} = \frac{\text{Click-through-count of an advertisement}}{\text{Number of page view of an advertisement}}$$

Quality score is decided by Google from CTR and relationship between text of the advertisement and searching keyword. Also, Google sets a minimum bidding value. An allocation of slots are based on descending order of multiplying the value by quality score, called $evaluation\ score$. It means that if high quality score has a possible to get a good position of slot by cheap payment. Google requires all advertisements positioned on upper slots must have a certain quality score level. Let q be a quality score of an advertiser allocated on a slot i, and b_i $(b_1 > b_2 > \cdots > b_i > \cdots > b_k)$ be a evaluation score. A payment p per click is denoted by

$$p = \frac{b_{i+1}}{q} + 1$$

It is known that CTR is proportional to order of slots. N. Brooks[7] say that there is a strong correlation between CTR and order of slots. The report also shows the ratios of CTR when a first ordered CTR is 100%. in this result, a second ordered is 77.4%, and third is 66.6%. However, Google suggests there is an exception. For example, some famous companies positioned lower slots has larger CTR than some upper positioned companies, since the famous companies get many click-through-counts even lower position.

On the other hand, Sponsored search which is derived by Yahoo! Search Marketing has technique similar to Google Adwords, but, there is a difference that order of slots is descending order of only bidding value.

We consider the same example in VCG. Suppose that there are three advertisers and two slots, and advertiser 1, 2 and 3 bids $\$300$, $\$200$ and $\$100$ per click. Also advertiser $1, 2$ and 3's quality score is $2, 1.5$ and 1, respectively. In this case, the evaluate scores are 600, 300 and 100, respectively. The advertiser 1 and 2 gets the slot 1 and 2, and pays $\$300/2 = \$150$ and $\$100/1.5 = \$66$ per click. The gain of auctioneer is $\$150 + \$66 = \$216$.

2.2 Our Advertisement Auction Model

Above mechanisms do not consider some value aside from advertising fee. It means a criterion of name value, effectiveness and importance for users. This criterion is not independent with each other, since users compare two or more advertisements for increasing them utilities. Thus each advertisement has a co-dependent value and it is expressed by linear to be evaluated.

Let Ad ($|Ad| \leq k$) be a set of advertisements which are now placed on the site, $Co(j) \subseteq \{1, ..., n\} \cap Ad$ be a set of co-dependent advertisements with advertisement j. When a value of company j's advertisement changes A^j_{after} from A^j_{before}, co-dependent value of other advertisement ℓ with company j is shown B^ℓ_{before} and B^ℓ_{after}. B^ℓ_{before} is changed a value effected by all advertisement in $Co(\ell)$ to B^ℓ_{after}. That is

$$B^\ell_{after} = B^\ell_{before} + \alpha \sum_{j \in Co(\ell)} (A^j_{after} - A^j_{before}) \quad (1)$$

The condition of the above equation is given as $0 \leq \alpha \leq 1$. $\alpha = 0$ shows independent with each advertisement. Quality score in the GSP auction protocol used in Google Adwords is placed a value in which we have defined above definition in the simulation. Figure 2 is an example of the model of our proposed mechanism. When a user clicks the link of the advertisement, its value is increased. Relatively, other advertisement's value becomes going down. When low-ranked advertisement is clicked by users, the advertisement is regarded as valuable comparing with high-ranked advertisement. In the Figure 2, we assume all of advertisers participate to bid for the first time. After bidding, the winners are determined be the auction. Then, each advertisement is displayed at the website as (A). Users click the advantages and the value of each advertisement changes based on number of click as (B). After one period passes, advertisers bid at second round auction to keep their advertisement in the website. We also assume all advertiser bid same price comparing with first round auction. The order of advertisement is changed based on both bid price and advertisement's value. In this case, although advertisement 1's value decreased in (B), position of advertisement 1 is kept at top because bid price is very high as (C). Because advertisement 4's value is quite high in (B), the rank of advertisement in (C) becomes second although bid price is the lowest in other three advertisers.

Next, we show some preliminary experiments for evaluation of our proposed model, and for finding new conditions or characteristics.

3 Preliminary Experiment

3.1 Condition

We set 3-10 slots to be put advertisements and 10-50 advertisers (companies to join in the auction) who bid to get a space for their advertisement. The lowest bid price in the auction is set \$10 and advertiser's bid value is defined a uniform distribution between

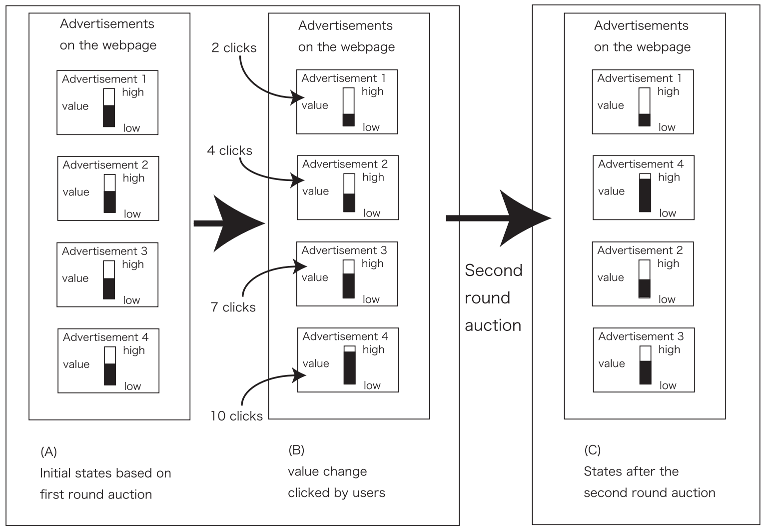

Fig. 2. Concept of the proposed mechanism

\$10 and \$100. Initial value of each advertisement is defined on a uniform distribution between 0.2 and 2.2. Number of clicking by end-user is assumed on a uniform distribution between 1 and 100 in a time slot.

3.2 Procedure of Trade

We now simulate our proposing mechanism by using following procedure. The procedure gives in above condition.

1. Decide a number of slots for advertisements.
2. Decide a number of clicks for each slot in a period.
3. Decide each advertiser's bid value and advertisement value.
4. Allocate each slot in descending order according to a valuation that is multiplied by a bid price and a value of advertisement.
5. Calculate each advertiser's payment and benefit.
6. Change a value of advertisement of a certain advertiser.
7. Compute a new value of advertisement based on equation (1).
8. Conduct step 4) and 5) based on the new value of advertisement and bid price.

We run this procedure at 100 thousands times.

3.3 Results

Table 1 shows result of simulation in which value of advertisement is changed. There are 20 advertiser advertisers and value of a certain advertisement is reduced and it effects other values of advertisement. When number of slot is changed from 4 to 10 and a value of one advertisement is reduced, 54,000 auctions make whole profit in the market increase in 100,000 trial. Average of the increased profit is $21.38. We discuss result of simulation from Table 1.

Table 1. A value of one advertisement is reduced.

Number of slot	Increase (%)	Decrease (%)	Average of increased /decreased profit
4	54.1	45.9	$22.38
6	55.3	44.7	$25.92
8	55.7	44.3	$27.56
10	56.3	43.7	$30.22

1. Averages of profit is normally increased and the profit increases when number of slots increases.
2. Possibility of profit increase is increased when number of slot increases.

This feature is apparent because the curve in Table 1 is monotonic increase.

As same as the above, Table 2 shows the case where 20 advertisers join in the auction and value of one advertiser's advertisement is increased. The number of slot is changed from 4 to 10 in each trial. We discuss result of simulation from Table 2.

Table 2. A value of one advertisement is increased.

Number of slot	Increase (%)	Decrease (%)	Average of increased /decreased profit
4	43.8	56.2	-$32.85
6	43.0	57.0	-$36.63
8	42.2	57.8	-$38.81
10	42.0	58.0	-$40.31

1. Averages of profit is normally decreased and the profit decreases when number of slots increases.
2. Possibility of profit increase is decreased when number of slot increases.

This feature is also apparent because the curve in Table 1 is monotonic decrease.

Table 3 is a result where number of slot is fixed as 5 and one advertiser changes value of his/her advertisement. The number of advertiser is changed 10 to 50 in each trial. Rate of increase/decrease of value of advertisement is assumed by uniform distribution. The result shows a comparison of profits between non-affective and affective.

Table 3. Number of slot is fixed as 5.

Number of advertisers	Increase (%)	Decrease (%)	Average of increased /decreased profit
10	49.8	50.2	-$2.81
20	49.1	50.9	-$4.87
30	48.7	51.3	-$5.17
40	48.3	51.7	-$6.27
50	48.1	51.9	-$6.85

1. Averages of profit is normally decreased and the profit decreases when number of slots increases.
2. Possibility of profit increase is decreased when number of slot increases.

Average of profit is negative because possibility that the profit decreases is large. From above simulation and analysis, we find out the following features. First, total profit of webpage owner reduces when each advertisement has co-dependence between its value. Second, when the size of auction becomes large, average of profit is decreased.

3.4 Comparison to VCG

Table 4 shows the result of simulation when the number of advertisers is 20 and number of slots are changed from 4 to 10 in each trial. When number of advertiser increases, our proposed GSP mechanism makes large profit comparing with general VCG mechanism.

Table 4. A value of one advertisement is increased.

Number of slot	Increase (%)	Decrease (%)
4	80.3	19.7
6	83.6	16.4
8	83.7	16.3
10	83.8	16.2

Table 5 shows the result of simulation when the number of slot is fixed as 5 in comparison between our proposed GSP and general VCG mechanism. Our proposed GSP makes larger profit compared with normal VCG with monotonic increase when the number of advertisers increases. When number of advertisers is not many, the increase rate is high. After number of advertisers is 30, increase rate becomes less and it seems to become convergence.

To analyze more special case, we try to run a simulation when the number of slots are fixed as 3. We find out the following two features from the simulation. First, when number of advertiser increases, our GSP provides larger profit than VCG. Second, rate of increase becomes small when number of advertisers decreases.

Table 5. Number of slot is fixed as 5.

Number of advertisers	Increase (%)	Decrease (%)
5	51.6	48.4
10	62.3	37.7
20	82.9	17.1
30	89.5	10.5
40	92.8	7.2
50	94.5	5.5

4 Discussion

The preliminary experiment is static environment, however, the real is dynamic environment. The co-dependent evaluation, which is in our proposed model, should be decided dynamically. Figure 3 shows an image of dynamic changing of co-dependent values. In this figure, there are two advertisements, they influence each other. In dynamic environment, there is a deadline of co-dependent value decision. This deadline shows a time in which a final co-dependent value is decided. White circle and gray circle shows two advertisements' co-dependent value. In this figure, their initial states are fixed on time 0. When the white circle is going down, then this phenomenon influence the gray circle. Each co-dependent value iterates this phenomena until deadline. This image also shows in left part of figure 3.

In the dynamic environment, there are many advertisements which influence each other. Hence we should reformulate our proposed model and create efficient mechanism for a dynamic environment.

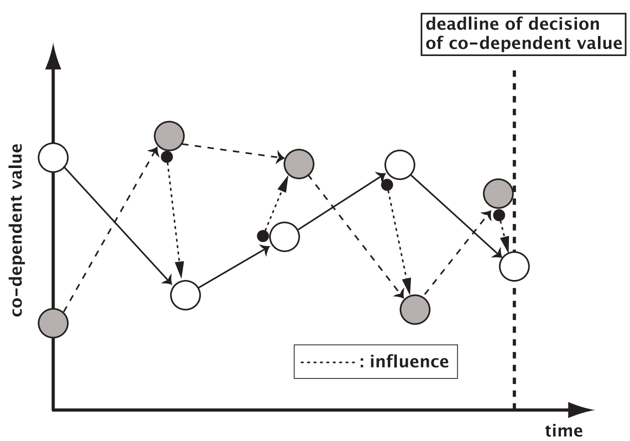

Fig. 3. Image of dynamically decision of co-dependent value

4.1 Reformulation

In this section, we reformulate our proposed model and an auction mechanism used by the model. Now we redefine some terms and formulas.

Suppose that there are n advertisers and k slots in the advertisement auction. Let Ad ($|Ad| \leq k$) be a set of advertisements which are now placed on the site and $Co(j) \subseteq Ad$ be a set of co-dependent advertisements with advertisement j. Also let C_j^t be a co-dependent value of j at time t. C_j^t is computed by the following:

$$C_j^t = C_j^{t-1} + \sum_{k \in Co(j)} \beta_k^t (C_k^t - C_k^{t-1}), \tag{2}$$

where β^t is a condition parameter. Also C_j^t's range is $(0, 1]$. We define β^t as a function on a co-dependent vector $\boldsymbol{C}^{t-1}$. Hence,

$$\beta_j^t := \beta_j^{t-1} + f(\boldsymbol{C}^{t-1}).$$

The auction system is able to compute the formulation 2 by using a simultaneous equation.

Next we reformulate an auction mechanism for the dynamic auction model. The auction system allocates each slot by descending order of a function of pair of the co-dependent value and advertiser's bids $g_j(C_j^t, b_j)$. Note that since an advertiser j's bidding value b_j does not change among the auction, the advertisement's position of slots is decided by only co-dependent value. If a ordered sequence of bid value is $b_1 > b_2 > \cdots > b_j > b_{j+1} > \cdots b_n$, a payment per click p_j^t is denoted by

$$p_j^t = \frac{b_{j+1}}{C_j^t} + 1.$$

4.2 GlobalAd

In this section, we explain an application of our advertisement auction. In the online advertisement, it is important to calculate the number of clicks of the link of advertisement shown on the webpage. The number of successful trade after clicking an advertisement is also sometimes considered to know the quality of the advertisement. On the other hands, actual newspapers are not clicked by the subscribers, and business providers (who apply to publish their advertisement) may know the reputation of their advertisement from trading history using survey. Generally, because of the strong limitation of that, it is easier to make a formalization to determine winners in actual advertisement auction than the online advertisement auction, except for the constraints regarding space, multiple pages, position, and size.

There are some web-based advertisement application systems to be used in actual newspaper. None of them provide the procurement, winner-determination, and bidding mechanisms. The GlobalAd is a useful system to apply an advertisement that is used internationally. The GlobalAd is developed with a technology that will help a user to book its advertisement in any newspaper all over the world. Figure 4 shows the architecture of GlobalAd. First, a user will be able to select its preferences viz. its country, state, city, newspaper and date on which it wants to publish its advertisement in the selected newspaper. Upon selected preferences like size of the advertisement and newspaper, a bill is generated. If the user agrees to the amount in the bill, it is asked to upload its material on the database. After uploading the material, the user has to make payment via its credit/debit card or its bank account over a secured payment gateway of the Paypal. On making payment, a receipt is generated for the user. Also, an email of user preferences is sent to the advertisement agency on making the payment. On the basis of the preferences and the material uploaded by the user on the database, the advertisement agency books the advertisement in the newspaper selected by the user in its preferences. On the date selected by the user in its preferences, it can view the copy of the published advertisement in the newspaper by clicking a link on the Global Ad website. In this way, the Global Ad website will help a user book its advertisement in any newspaper of its choice all the world.

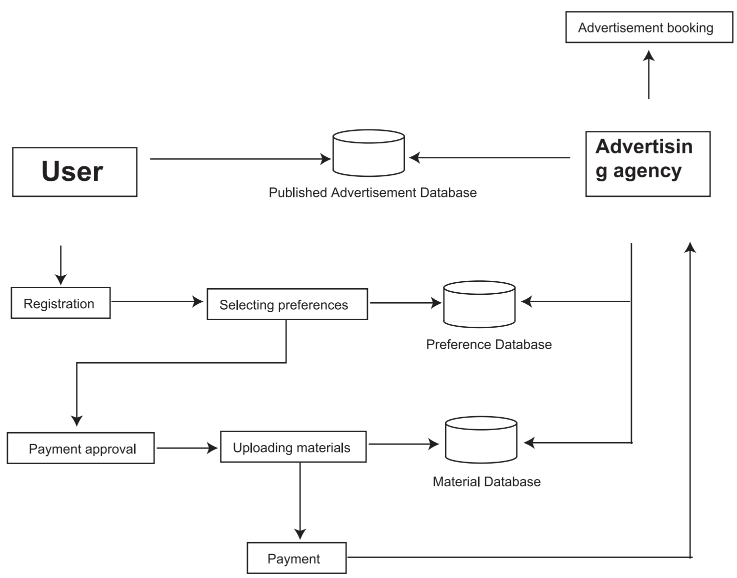

Fig. 4. System Architecture of GlobalAd

The GlobalAd currently provides the simple function explained above, however the combinations of users preferences are complicated; users have some preferences including advertisement's size, position, page, order, and cost. If users !Gome preferences are overlapped, it is rational to determine winners in the economics viewpoints. Namely, the auction type becomes applied multiple auctions or combinatorial auctions. Figure 5 shows the process using the auction-based winner determination in the GlobalAd. Using the second price auction, it is easy to determine winners, that becomes social surplus is maximum. The formalization is shown as follows. We show a multiple auction formalization. Suppose that b_{ij} is a bid value of advertiser i for a frame j. Also let N be a set of advertisers and F be a set of frames. Then,

$$
\text{(AP)}\left|\begin{array}{ll}\text{maximize} & \sum_{i\in N}\sum_{j\in F} b_{ij}x_{ij} \\ \text{subject to} & \sum_{j\in F} x_{ij} \leq 1,\ \forall i \in N \\ & \sum_{i\in N} x_{ij} = 1,\ \forall j \in F \\ & x_{ij} \in \{0,1\},\ \forall i \in N,\ j \in F\end{array}\right.
$$

x_{ij} is a binary decision variable. If x_{ij} takes 1, then the system allocates a frame j to an advertiser i. Let $\boldsymbol{X}^*$ be an optimal solution of the problem (AP). The optimal solution $\boldsymbol{X}^*$ shows the winners. The problem (AP) is one of general allocation problems, it is easy to compute by using some integer programming solver such as CPLEX[12] or Gurobi[13]. There are some paid softwares, however, the problem (AP) is also able to solvable by using graph theoretical technique written by [14] and [15]. Our auction system employs a graph theory based algorithm.

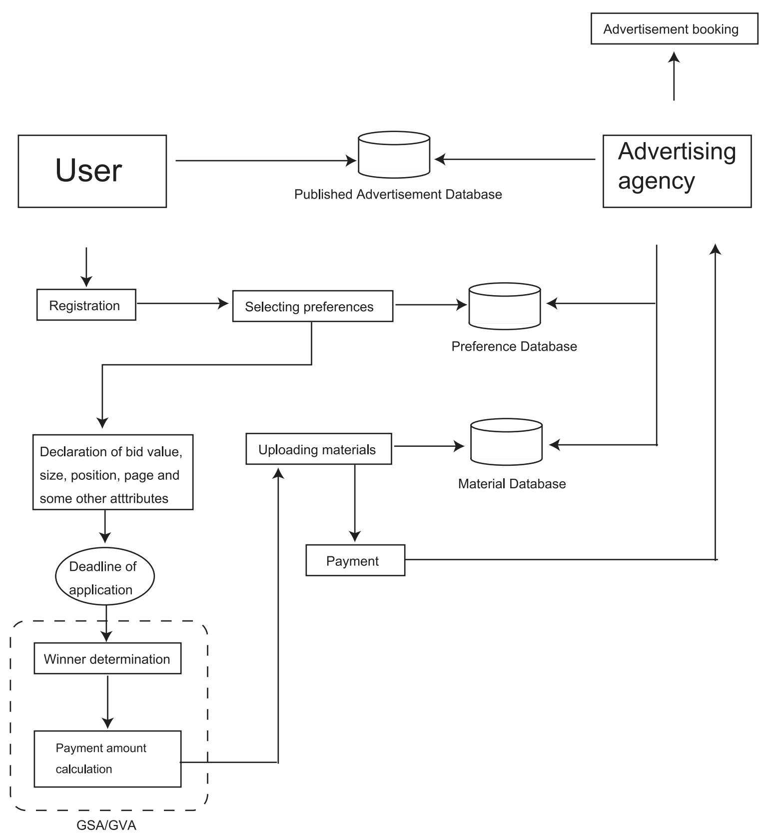

Fig. 5. Process of Auction-based Winner Determination

5 Conclusion

This paper proposed a co-dependent value-based GSP mechanism in the Internet advertisement auctions. For analysis of the mechanism, we ran a preliminary simulation based on multi-agents. Our analysis showed that total profit changes in different auctions mechanism GSP, VCG, and our proposed mechanism. From the analysis, auctioneer changes the auction protocol based on his/her estimate profit. Our auction protocol had an advantage where the website provides more useful advertisement for users, because the order of allocation is based on both price and value. Also we reformulated our proposed model and mechanism for dynamic environment. The co-dependent value is changing dynamically among a few times. Our model showed this phenomena. And we introduced the GrobalAd system as an application of our advertisement mechanism. The GrobalAd system is already implemented as an advertisement allocation system for some paper medias. Also we discussed possible application of our advertisement auction mechanism for the system.

Our future work includes the analysis of profit and expected utility for agents in the mixed type of normal GSP, VCG, and our protocol. Also future work is evaluation of a dynamic model by some simulation.

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