Effect of an infinite plane wall on the motion of a spherical Brownian particle

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(Received 11 June 1981; accepted 12 November 1981)

The motion of a spherical Brownian particle in an incompressible fluid bounded by an infinite plane wall is studied on the basis of the linearized Landau–Lifshitz equations for the fluctuating hydrodynamics. The asymptotic forms for large time t of the autocorrelation function $\Phi_{ij}(t)$ for the random force acting on the particle and the autocorrelation function $\Psi_{ij}(t)$ for its velocity are discussed for the case that the distance l between the wall and the particle is finite but much larger than the particle radius a. It is shown that Φ_{ii} and Ψ_{ii} fall off as $t^{-3/2}$ for $a^2/\nu \ll t \ll l^2/\nu$, but for $t \gg l^2/\nu$ they fall as $t^{-5/2}$ or $t^{-7/2}$, accordingly, as the direction i is parallel or perpendicular to the wall, where ν is the kinematic shear viscosity of the fluid.

I. INTRODUCTION

In recent years, the motion of a Brownian particle has been studied extensively and successfully from an approach based on the Landau–Lifshitz equations[1] for the fluctuating fluid.[2–9] This approach is convenient, especially for calculating explicitly the correlations of fluctuating quantities. From this approach, it can be shown, for example, that the autocorrelation function Ψ_{ii} of the velocity $\mathbf{U}$ of a spherical particle of radius a, in an unbounded incompressible fluid, falls as $t^{-3/2}$ for large t, which is in good agreement with the numerical simulation by Alder and Wainwright.[10]

In the calculation of such correlations, crucial information is provided by solving hydrodynamic equations. In hydrodynamics, the existence of a wall, e.g., a container wall, is known to have important effects on the motion of a particle, and the effects have been studied extensively (see Happel and Brenner[11] for references).

In these studies, however, the thermal agitation in the fluid has been neglected and the motion of the particle is usually assumed to be steady. And little is known about the wall effect on the unsteady motion of the Brownian particle, which is driven by the thermal agitation. The purpose of this paper is to study such a wall effect on the basis of Landau–Lifshitz equations. We confine ourselves to the simplest case, in which a sphere is immersed in an incompressible fluid bounded by a single infinite plane wall, and the distance between the wall and particle is much larger than the particle radius.

As is explained in Sec. II, both the autocorrelation function Φ_{ij} for the random force acting on the particle, and the autocorrelation function Ψ_{ij} for its velocity, can be expressed in terms of the friction coefficient D_{ij} for a translational motion of the particle. In Sec. III, we consider the wall effect on D_{ij}. In Sec. IV, the asymptotic behaviors of Φ_{ij} and Ψ_{ij} for large times are discussed.

II. BASIC EQUATIONS AND FLUCTUATIONS

Let us consider a particle moving in an incompressible fluctuating fluid bounded by a wall of an arbitrary shape. It is assumed that the fluid motion is described by the linearized stochastic Landau–Lifshitz equations[1]

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \operatorname{div}(\mathbf{P} + \mathbf{S}) , \tag{1a}$$

$$\operatorname{div} \mathbf{v} = 0 , \tag{1b}$$

with

$$P_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) , \tag{2}$$

where $\mathbf{v}$ is the fluid velocity, p the pressure, ρ the uniform fluid density, μ the shear viscosity, and S_{ij} the random stress tensor, which has the following stochastic properties:

$$\langle S_{ij}(\mathbf{x}, t)\rangle = 0 \tag{3}$$

$$\langle S_{ij}(\mathbf{x}, t) S_{kl}(\mathbf{x}', t')\rangle = 2k_B T\mu \times (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\delta(\mathbf{x} - \mathbf{x}')\delta(t - t') , \tag{4}$$

where the square brackets denote the average over an equilibrium ensemble.

We confine ourselves to the case in which the particle and the wall are solid surfaces, to which the fluid adheres. The boundary conditions to be satisfied are then

$$\mathbf{v} = \begin{cases} \mathbf{U} + \boldsymbol{\Omega} \times (\mathbf{x} - \mathbf{x}_p) , & \text{on } P , \\ 0 , & \text{on } W , \end{cases} \tag{5}$$

$$\langle \mathbf{v}\rangle \to 0 , \quad \text{at infinity} ,$$

where $\mathbf{U}$ is the translational velocity of the particle, Ω the rotational one around $\mathbf{x}_p$, and P refers to the surface of the particle, and W refers to the wall. If W completely bounds P, then $\langle \mathbf{v}\rangle$ need not vanish at infinity

Let us write

$$\mathbf{v} = \bar{\mathbf{v}} + \tilde{\mathbf{v}}, \quad p = \bar{p} + \tilde{p}, \quad \mathbf{P} = \bar{\mathbf{P}} + \tilde{\mathbf{P}} , \tag{6}$$

where $(\bar{\mathbf{v}}, \bar{p})$ is the field satisfying

$$\rho \frac{\partial \bar{\mathbf{v}}}{\partial t} = \operatorname{div} \bar{\mathbf{P}} , \tag{7a}$$

$$\operatorname{div} \bar{\mathbf{v}} = 0 , \tag{7b}$$

$$\bar{\mathbf{v}} = \begin{cases} \mathbf{U} + \boldsymbol{\Omega} \times (\mathbf{x} - \mathbf{x}_p) , & \text{on } P , \\ 0 , & \text{on } W, \end{cases}$$

$$\bar{\mathbf{v}} \to 0, \quad \text{at infinity.} \tag{7c}$$

Then $(\tilde{\mathbf{v}}, \tilde{p})$ satisfies

$$\rho \frac{\partial \tilde{\mathbf{v}}}{\partial t} = \operatorname{div}(\tilde{\mathbf{P}} + \mathbf{S}) , \tag{8a}$$

$$\operatorname{div} \tilde{\mathbf{v}} = 0 , \tag{8b}$$

$$\begin{cases} \tilde{\mathbf{v}} = 0, & \text{on } P \text{ and } W \\ \langle \tilde{\mathbf{v}} \rangle \to 0, & \text{at infinity} . \end{cases} \tag{8c}$$

It is convenient to introduce the Fourier-time transformation defined by

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} \, dt . \tag{9}$$

Hereafter, for ease of writing, the symbol ^ will be omitted at will. There should arise no confusion.

The force F exerted on the particle by the fluid is given by

$$\mathbf{F}(\omega) = \overline{\mathbf{F}}(\omega) + \tilde{\mathbf{F}}(\omega) , \tag{10}$$

where

$$\overline{\mathbf{F}} = \int_P \overline{\mathbf{P}} \cdot \mathbf{n} \, dS, \quad \tilde{\mathbf{F}} = \int_P (\tilde{\mathbf{P}} + \mathbf{S}) \cdot \mathbf{n} \, ds , \tag{11}$$

in which $\mathbf{n}$ is the unit vector normal to the surface P, pointing out of the particle, and the integral is over P. The force $\overline{\mathbf{F}}$ due to the deterministic field $(\overline{\mathbf{v}}, \overline{p})$ can be expressed, as is well known, as

$$\overline{\mathbf{F}}(\omega) = -[\mathbf{D}(\omega) \cdot \mathbf{U}(\omega) + \mathbf{C}(\omega) \cdot \mathbf{\Omega}(\omega)], \tag{12}$$

with suitable friction coefficient matrices $\mathbf{D}(\omega)$ and $\mathbf{C}(\omega)$.

It is not difficult to verify from the above relations the fluctuation–dissipation theorem

$$\Phi_{ij}(t) \equiv \frac{\pi}{k_B T} \langle F_i(0) F_j(t) \rangle = \int_{-\infty}^{\infty} \operatorname{Re}[D_{ij}(\omega)] e^{-i\omega t} d\omega . \tag{13}$$

It is well known that Eq. (13) holds in an unbounded fluid, i.e., when the wall is absent or at infinity.[4] It should be noted that Eq. (13) also holds when there is a boundary wall of arbitrary shape at a finite distance.

The velocity U of the particle obeys

$$M \frac{d\mathbf{U}}{dt} = -\int_{-\infty}^{t} [\mathbf{D}(t-s) \cdot \mathbf{U}(s) + \mathbf{C}(t-s) \cdot \mathbf{\Omega}(s)] ds + \tilde{\mathbf{F}}(t) + \mathbf{F}^{\mathrm{ext}}(t) , \tag{14}$$

where M is the mass of the particle and $\mathbf{F}^{\mathrm{ext}}$ an external force. In particular, if $\mathbf{C} = 0$ and $D_{ij}(\omega) = D_i(\omega)\delta_{ij}$, then

$$\Psi_{ij}(t) \equiv \langle U_i(0) U_j(t) \rangle = \Psi_i(t) \delta_{ij} , \tag{15}$$

where

$$\Psi_j(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \left[\frac{1}{-i\omega M + D_j(\omega)} + \text{c.c.} \right] = \frac{1}{2\pi i} \int_{-\infty}^{0} dz \, e^{z|t|} \left\{ \frac{1}{zM + D_j[i(z - i0)]} + \frac{1}{zM + D_j[i(z + i0)]} \right\}, \tag{16}$$

here, c.c. denotes complex conjugates (cf. Ref. 8).

Let us introduce Green's functions G_{ij} and P_j, satisfying

$$\rho \frac{\partial}{\partial t} G_{ij}(\mathbf{x}, t; \mathbf{x}', t') + \frac{\partial}{\partial x_i} P_j(\mathbf{x}, t; \mathbf{x}', t') - \mu \Delta G_{ij}(\mathbf{x}, t; \mathbf{x}', t') = \delta_{ij} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') , \tag{17a}$$

$$\frac{\partial}{\partial x_i} G_{ij}(\mathbf{x}, t; \mathbf{x}', t') = 0 , \tag{17b}$$

$$G_{ij}(\mathbf{x}, t; \mathbf{x}', t') = 0 , \quad \text{for } t < t' , \tag{17c}$$

and the boundary condition

$$G_{ij}(\mathbf{x}, t; \mathbf{x}', t') = 0 , \quad \text{for } \mathbf{x} \in W \text{ or } P . \tag{18}$$

Then it can be shown (Jones[12]) that

$$Q_{ij}(\mathbf{x}, \mathbf{x}'; \omega) = 2k_B T \operatorname{Re}[G_{ij}(\mathbf{x}, \mathbf{x}'; \omega)] , \tag{19}$$

where Q_{ij} is the Fourier-time transform [see Eq. (9)] of Q_{ij} defined by

$$Q_{ij}(\mathbf{x}, \mathbf{x}'; t) = \langle v_i(\mathbf{x}, 0) v_j(\mathbf{x}', t) \rangle . \tag{20}$$

III. THE FRICTION COEFFICIENT D FOR A SPHERICAL PARTICLE IN A SEMI-INFINITE FLUID

Let us consider a simple case when the wall is an infinite plane and the particle is a sphere of radius a. We take the origin of the coordinate system to be at the instantaneous position of the center of the sphere, and the x_3 axis perpendicular to the plane wall. The wall is represented by $x_3 = -l$. We assume $l \gg a$.

The friction coefficient $\mathbf{D}$ can be found by solving Eq. (7), where without loss of generality we may put $\mathbf{\Omega} = 0$ to find $\mathbf{D}$. Then Eq. (7) may be written in the frequency representation as

$$-i\omega\rho\mathbf{v} = -\nabla p + \mu \Delta \mathbf{v} , \tag{21a}$$

$$\operatorname{div} \mathbf{v} = 0 , \tag{21b}$$

$$\mathbf{v} = 0, \quad \text{on } W (x_3 = -l) , \tag{22a}$$

$$\mathbf{v} = \mathbf{U}(\omega) , \quad \text{on } P(|\mathbf{x}| = a) , \tag{22b}$$

$$\mathbf{v} \to 0 , \quad \text{at infinity} . \tag{22c}$$

(In this section the symbol ¯ is omitted.)

Because of the geometrical symmetry and the linearity of Eqs. (21) and (22), D_{ij} for any i and j can be known, if the drags of the following two cases are known: Case (A) U is perpendicular to the wall, and case (B) U is parallel to the wall. Wakiya[13] has studied case (B), and his result can be used. Thus, we need solve here only case (A), where Eq. (22b) may be written as

$$v_i(\omega) = \delta_{i3} U(\omega), \quad \text{on } P(|\mathbf{x}| = a) . \tag{22b'}$$

The general solution of Eq. (21) may be written in the form[14]

$$\mathbf{v} = \operatorname{grad} \operatorname{div}(\chi - \phi) - k^2 \chi , \tag{23a}$$

$$p = \nu k^2 \operatorname{div} \phi , \tag{23b}$$

where χ, ϕ satisfy

$$(\Delta - k^2) \chi = 0 , \tag{24a}$$

$$\Delta \phi = 0 , \tag{24b}$$

and $k^2 = -i\omega/\nu$, $\nu = \mu/\rho$.

In order to find the solution satisfying the boundary conditions (22a), (22b′), and (22c), we use the method of reflections[15] and try a solution of the form

$$\mathbf{v}=\sum_{n=0}^{\infty}\mathbf{v}^{(n)},\quad p=\sum_{n=0}^{\infty}p^{(n)}, \tag{25}$$

each term of which ($\mathbf{v}^{(n)}$, $p^{(n)}$), respectively, satisfies the equations of motion [Eq. (21)].

The individual fields are to be determined successively by application of the following boundary conditions:

$$\mathbf{v}^{(0)}=\mathbf{U},\quad \text{on } P(|\mathbf{x}|=a), \tag{26a}$$

$$\mathbf{v}^{(1)}=-\mathbf{v}^{(0)},\quad \text{on } x_3=-l, \tag{26b}$$

$$\mathbf{v}^{(2)}=-\mathbf{v}^{(1)},\quad \text{on } P(|\mathbf{x}|=a),\quad \text{etc.}, \tag{26c}$$

and, in addition, for $n=0,1,2,3,\ldots$,

$$\mathbf{v}^{(n)}\to 0,\quad \text{at infinity}. \tag{26d}$$

The drag $\mathbf{F}(\omega)$, due to $(\mathbf{v},p)$ exerted on the particle, is known to be given by[15]

$$\mathbf{F}=\mathbf{F}^{(0)}+\mathbf{F}^{(2)}+\mathbf{F}^{(4)}+\cdots, \tag{27}$$

where $\mathbf{F}^{(n)}$ is the contribution from the nth field ($\mathbf{v}^{(n)}$, $p^{(n)}$).

A. The field $\mathbf{v}^{(0)}$ and the drag $\mathbf{F}^{(0)}$

The velocity field $\mathbf{v}^{(0)}$ is equal to the one due to a motion of the particle in an unbounded fluid,[16] and $\mathbf{v}^{(0)}$, $\mathbf{F}^{(0)}$ are given by

$$\frac{\mathbf{v}^{(0)}}{U(\omega)}=\operatorname{grad}\operatorname{div}\left[\left(A\frac{e^{-kr}}{r}-B\frac{1}{r}\right)\mathbf{e}_3\right]-k^2A\frac{e^{-kr}}{r}\mathbf{e}_3, \tag{28}$$

$$\frac{\mathbf{F}^{(0)}}{\zeta U(\omega)}=-(1+ak+\tfrac{1}{3}a^2k^2)\mathbf{e}_3, \tag{29}$$

$$A=-\frac{3}{2}\frac{a}{k^2}e^{ak},\quad B=(1+ak+\tfrac{1}{3}a^2k^2)e^{-ak}A, \tag{30}$$

where $\zeta=6\pi a\mu$ and $\mathbf{e}_3$ is the unit vector in the x_3 direction.

B. The field $\mathbf{v}^{(1)}$ and the drag $\mathbf{F}^{(2)}$

In order to find the solution ($\mathbf{v}^{(1)}$, $p^{(1)}$) satisfying Eqs. (21), (26b), and (26d), we try the solution in the form of Eq. (23), with χ and ϕ replaced by the following $\chi^{(1)}$ and $\phi^{(1)}$, respectively:

$$\chi^{(1)}=\frac{U\mathbf{e}_3}{2\pi}\iint_{-\infty}^{\infty}A^{(1)}\exp[i(\alpha x+\beta y)-\lambda(z+2l)]\frac{d\alpha d\beta}{\lambda}, \tag{31a}$$

$$\phi^{(1)}=\frac{U\mathbf{e}_3}{2\pi}\iint_{-\infty}^{\infty}B^{(1)}\exp[i(\alpha x+\beta y)-\kappa(z+2l)]\frac{d\alpha d\beta}{\kappa}, \tag{31b}$$

with

$$\kappa=(\alpha^2+\beta^2)^{1/2},\quad \lambda=(\kappa^2+k^2)^{1/2}, \tag{31c}$$

(cf. Wakiya[13]), where $A^{(1)}$ and $B^{(1)}$ are constants to be determined by applying the boundary condition (26b).

After some calculations, it is found that

$$A^{(1)}=\frac{1}{\lambda-\kappa}[(\lambda+\kappa)A-2\lambda e^{(\lambda-\kappa)l}B], \tag{32a}$$

$$B^{(1)}=\frac{1}{\lambda-\kappa}[2\kappa e^{(\kappa-\lambda)l}A-(\kappa+\lambda)B]. \tag{32b}$$

From Eqs. (23a), (31), and (32), for $|\mathbf{x}|/a\sim 1$ and $\epsilon\equiv a/l\ll 1$,

$$\frac{v_i^{(1)}}{U}=\delta_{i3}k^3(a_\perp A+b_\perp B)+O(\epsilon^2), \tag{33}$$

where

$$a_\perp=\int_0^\infty s^3(s+\sqrt{s^2+1})\left[\frac{s+\sqrt{s^2+1}}{\sqrt{s^2+1}}\exp(-2kl\sqrt{s^2+1})-2\exp[-kl(s+\sqrt{s^2+1})]\right]ds, \tag{34a}$$

$$b_\perp=\int_0^\infty s^2(s+\sqrt{s^2+1})\{(s+\sqrt{s^2+1})\exp(-2kls)-2s\exp[-kl(s+\sqrt{s^2+1})]\}ds. \tag{34b}$$

The drag $\mathbf{F}^{(2)}$ can be found without explicitly solving the field ($\mathbf{v}^{(2)}$, $p^{(2)}$), for we may apply a generalized formula of Faxen's theorem to the case of the unsteady motion.[17,18] By the application we have

$$\frac{\mathbf{F}^{(2)}}{\zeta U}=k^3(1+ak+\tfrac{1}{3}a^2k^2)(a_\perp A+b_\perp B)\mathbf{e}_3+O(\epsilon^3). \tag{35}$$

The contribution from $\mathbf{F}^{(4)}+\mathbf{F}^{(6)}+\cdots$ is shown to be $\zeta U\times O(\epsilon^2)$.

C. The friction coefficient $D_{ij}(\omega)$

From Eqs. (29), (30), and (35), the friction coefficient $D_\perp$ (defined by $F=-D_\perp U$) for case A (the velocity U is perpendicular to the wall) is given by

$$\frac{D_\perp}{\zeta}=1+\epsilon[1+\tfrac{3}{2}(a_\perp+b_\perp)]lk+O(\epsilon^2). \tag{36}$$

The coefficient $D_\parallel$ (defined by $F=-D_\parallel U$) for case B (the velocity U is parallel to the wall) is found from Wakiya's paper[13] to be

$$\frac{D_\parallel}{\zeta}=1+\epsilon[1+\tfrac{3}{2}(a_\parallel+b_\parallel)]lk+O(\epsilon^2), \tag{37}$$

where

$$a_\parallel=\int_0^\infty\left\{\frac{s}{\sqrt{s^2+1}}[1+\tfrac{1}{2}s(s+\sqrt{s^2+1})^2(2\sqrt{s^2+1}-s)]\exp(-2lk\sqrt{s^2+1})-s^2\sqrt{s^2+1}(s+\sqrt{s^2+1})\exp[-lk(s+\sqrt{s^2+1})]\right\}ds, \tag{38a}$$

$$b_\parallel=\frac{1}{2}\int_0^\infty\{s^2(s+\sqrt{s^2+1})^2\exp(-2lks)-s^2\sqrt{s^2+1}(s+\sqrt{s^2+1})\exp[-lk(s+\sqrt{s^2+1})]\}ds. \tag{38b}$$

For $ak\ll 1$, $\xi\equiv kl\gg 1$, i.e., $1/l\ll k\ll 1/a$,

$$a_\perp+b_\perp=O(\xi^{-3}), \tag{39a}$$

$$a_\parallel+b_\parallel=O(\xi^{-3}). \tag{39b}$$

For $\xi\ll 1$[19]

$$a_\perp + b_\perp = \tfrac{3}{4}\xi^{-1} - \tfrac{2}{3} + \tfrac{1}{4}\xi + \tfrac{1}{15}\xi^4 + O(\xi^5 \log \xi) , \tag{40a}$$

$$a_\parallel + b_\parallel = \tfrac{3}{8}\xi^{-1} - \tfrac{2}{3} + \tfrac{3}{4}\xi - \tfrac{2}{3}\xi^2 + O(\xi^3 \log \xi) . \tag{40b}$$

For general i and j

$$D_{ij}(\omega) = D(\omega)\delta_{ij} , \quad D(\omega) = \begin{cases} D_\parallel(\omega), & \text{if } i = 1 \text{ or } 2 , \\ D_\perp(\omega), & \text{if } i = 3 . \end{cases} \tag{41}$$

D. The Green's function for $|\mathbf{x}-\mathbf{x}'| \ll l$

Taking the limit $a \to 0$, with aU fixed in the expression $\mathbf{v}$, we can find Green's function G_{ij} for the present case of an infinite plane wall.

Let us write

$$G_{ij}(\mathbf{x}, \mathbf{x}'; \omega) = G^0_{ij}(\mathbf{x}, \mathbf{x}'; \omega) + G^w_{ij}(\mathbf{x}, \mathbf{x}'; \omega) , \tag{42}$$

where

$$G^0_{ij}(\mathbf{x}, \mathbf{x}'; \omega) = \frac{1}{4\pi\mu k^2 r^3}\left\{\left[(\Gamma + \tfrac{2}{3}k^2 r^2)\delta_{ij} - 3\Gamma\frac{r_i r_j}{r^2}\right] e^{-kr} - \left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right)\right\} , \tag{43}$$

$(\mathbf{r} = \mathbf{x} - \mathbf{x}', \quad r = |\mathbf{r}|)$,

$\Gamma = 1 + kr + \frac{1}{3}k^2 r^2$,

and G^w_{ij} represents the wall effect.

For $r/l \ll 1$, from Wakiya's paper[13]

$$G^w_{ij} = -\frac{k}{4\pi\mu}(a_\parallel + b_\parallel)\delta_{ij} + O\left(\frac{r}{l}\right) , \quad \text{for } j = 1 \text{ or } 2 , \tag{44a}$$

and from Eqs. (30) and (33)

$$G^w_{i3} = -\frac{k}{4\pi\mu}(a_\perp + b_\perp)\delta_{i3} + O\left(\frac{r}{l}\right) . \tag{44b}$$

IV. ASYMPTOTIC BEHAVIORS OF THE AUTOCORRELATION FUNCTIONS $\Phi_{ij}(t)$ AND $\Psi_{ij}(t)$ FOR LARGE t

Let us write

$$\Phi_{11}(t) = \Phi_{22}(t) = \Phi^0(t) + \Phi^w_\parallel(t) , \tag{45a}$$

$$\Phi_{33}(t) = \Phi^0(t) + \Phi^w_\perp(t) , \tag{45b}$$

where

$$\frac{\Phi^0(t)}{\pi\zeta} = 2\delta(t) - \frac{a}{2\sqrt{\pi\nu}}t^{-3/2} , \tag{46}$$

and $\Phi^w_\parallel$, $\Phi^w_\perp$ represent the wall effects. The consideration of the geometrical symmetry yields $D_{ij}(t) = \Phi_{ij}(t) = \Psi_{ij}(t) = 0$ for $i \neq j$.

From Eqs. (36), (37), and (39), for $\sigma \equiv t/(l^2/\nu) \ll 1$ and $\sigma' \equiv t/(a^2/\nu) \gg 1$, i.e., for $a^2/\nu \ll t \ll l^2/\nu$

$$\frac{\Phi^w_\parallel(t)}{\pi\zeta} = \epsilon\frac{\nu}{l^2}O(\sigma^{1/2}), \quad \frac{\Phi^w_\perp(t)}{\pi\zeta} = \epsilon\frac{\nu}{l^2}O(\sigma^{1/2}) , \tag{47}$$

which yield

$$\frac{\Phi_{11}(t)}{\pi\zeta} = \frac{\Phi_{22}(t)}{\pi\zeta} = -\frac{a}{2\sqrt{\pi\nu}}t^{-3/2} + \epsilon\frac{\nu}{l^2}O(\sigma^{1/2}) , \tag{48a}$$

$$\frac{\Phi_{33}(t)}{\pi\zeta} = -\frac{a}{2\sqrt{\pi\nu}}t^{-3/2} + \epsilon\frac{\nu}{l^2}O(\sigma^{1/2}) . \tag{48b}$$

From Eqs. (36), (37), and (40), for $\sigma \gg 1$, i.e., $t \gg l^2/\nu$

$$\frac{\Phi^w_\parallel(t)}{\pi\zeta} = \frac{3}{2}\epsilon\frac{\nu}{l^2}\left[\frac{1}{3\sqrt{\pi}}\sigma^{-3/2} - \frac{1}{2\sqrt{\pi}}\sigma^{-5/2} + O(\sigma^{-3})\right] , \tag{49a}$$

$$\frac{\Phi^w_\perp(t)}{\pi\zeta} = \frac{3}{2}\epsilon\frac{\nu}{l^2}\left[\frac{1}{3\sqrt{\pi}}\sigma^{-3/2} - \frac{1}{8\sqrt{\pi}}\sigma^{-7/2} + O(\sigma^{-4})\right] , \tag{49b}$$

which yield, for $t \gg l^2/\nu$,

$$\frac{\Phi_{11}(t)}{\pi\zeta} = \frac{\Phi_{22}(t)}{\pi\zeta} = \epsilon\frac{\nu}{l^2}\left[-\frac{3}{4\sqrt{\pi}}\left(\frac{l^2}{\nu}\right)^{5/2}t^{-5/2} + O(\sigma^{-3})\right], \tag{50a}$$

$$\frac{\Phi_{33}(t)}{\pi\zeta} = \epsilon\frac{\nu}{l^2}\left[-\frac{3}{16\sqrt{\pi}}\left(\frac{l^2}{\nu}\right)^{7/2}t^{-7/2} + O(\sigma^{-4})\right] . \tag{50b}$$

For the autocorrelation function Ψ_{ij}, for the particle velocity, from Eqs. (15), (16), (36), (37), and (39), we have for $a^2/\nu \ll t \ll l^2/\nu$

$$\Psi_{11}(t) = \Psi_{22}(t) = \frac{k_B T}{\zeta}\left[\frac{a}{2\sqrt{\pi\nu}}t^{-3/2} + \frac{\nu}{a^2}O[(\sigma')^{-5/2}] + \epsilon\frac{\nu}{l^2}O(\sigma^{1/2})\right], \tag{51a}$$

$$\Psi_{33}(t) = \frac{k_B T}{\zeta}\left[\frac{a}{2\sqrt{\pi\nu}}t^{-3/2} + \frac{\nu}{a^2}O[(\sigma')^{-5/2}] + \epsilon\frac{\nu}{l^2}O(\sigma^{1/2})\right]. \tag{51b}$$

From Eqs. (15), (16), (37), and (40), for $t \gg l^2/\nu$,

$$\Psi_{11}(t) = \Psi_{22}(t) = \frac{k_B T}{\zeta}\epsilon\frac{\nu}{l^2}\left[\frac{3}{4\sqrt{\pi}}\left(\frac{l^2}{\nu}\right)^{5/2}t^{-5/2} + O(\sigma^{-3})\right] , \tag{52a}$$

$$\Psi_{33}(t) = \frac{k_B T}{\zeta}\epsilon\frac{\nu}{l^2}\left[\frac{3}{16\sqrt{\pi}}\left(\frac{l^2}{\nu}\right)^{7/2}t^{-7/2} + O(\sigma^{-4})\right] . \tag{52b}$$

In Eqs. (48) and (51), the leading terms ($t^{-3/2}$ terms) are just the same as those for a spherical particle in an unbounded fluid. Thus, for $t \ll l^2/\nu$, where l^2/ν is the characteristic time of the vorticity diffusion, over the distance l between the particle and the wall, the wall has little effect on Φ_{ij} and Ψ_{ij}. But, for $t \gg l^2/\nu$, the existence of the wall essentially affects the long time behavior of Φ_{ij} and Ψ_{ij}, as is clear from Eqs. (50) and (52).

It is hoped that the long-time tails as discussed in this paper will be verified experimentally or by numerical simulations.

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$$a_{\parallel}=-\tfrac{1}{4}\,\xi^{-3}+\tfrac{5}{16}\,\xi^{-1}-\tfrac{2}{3}-\tfrac{1}{8}\xi\log\xi+(\tfrac{71}{96}-\tfrac{1}{8}\gamma)\xi-\tfrac{7}{15}\xi^{2}+O(\xi^{3}\log\xi)$$

$$b_{\parallel}=\tfrac{1}{4}\,\xi^{-3}+\tfrac{1}{16}\,\xi^{-1}+\tfrac{1}{8}\,\xi\log\xi+(\tfrac{1}{96}+\tfrac{1}{8}\gamma)\xi-\tfrac{1}{5}\xi^{2}+O(\xi^{2}\log\xi).$$

where γ is Euler's constant. In Wakiya's paper, the $\xi\log\xi$ and $\gamma\xi$ terms are dropped.