

PAPER

A Use of Current Continuity Condition in GTD-MM Hybrid Technique

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SUMMARY A current continuity equation is proposed as the *additional* equation for the GTD-MM hybrid technique formulation to acquire the uniqueness of the solution which were nonexistent in the conventional formulation with the matching-point equation. The current continuity equation, which ensures the current continuity and satisfies the boundary condition, can directly be written down through equating the MM-region current to the GTD-region current at the regions boundary. It is proved that the current continuity equation is equivalent to the matching-point equation of special case when the matching-point located very close to the boundary, which were able to give the best solution in the conventional formulation with the matching-point equation as explained by Burnside et al. The validity of the new equation is confirmed through the numerical results.

1. Introduction

Recent treatments of electromagnetic radiation and scattering problems followed two major paths. One approach sought to elucidate more accurately the diffraction effects of edges and curved surfaces by generalization of classical optics, leading to the geometrical theory of diffraction (GTD)^{(1),(2)} and the physical theory of diffraction (PTD)⁽³⁾. The other approach applied the method of moments (MM)⁽⁴⁾ to electromagnetic scattering problems by use of the linear spaces and orthogonal projection principles. The principal shortcoming of GTD or PTD is that their solutions exist only for objects with limited shapes. The MM technique, on the other hand, can be applied to bodies with complex shapes, but it becomes intractable for bodies much beyond resonance length because of too large number of unknowns. To extend their application through the incorporation of the advantages of each technique, two hybrid techniques, MM-GTD⁽⁵⁾ and GTD-MM⁽⁶⁾ hybrid technique, were proposed both in 1975.

The GTD-MM hybrid technique introduced by Burnside et al.⁽⁶⁾ and later discussed by Sahalos and Thiele⁽⁷⁾ is a useful mathematical tool for diffraction problems to which GTD can not be applied by itself without the help of MM owing to the difficulty in modeling the structure to that of a canonical problem.

In theory, GTD-MM hybrid technique enables one to compute the diffraction coefficient by a numerical procedure, and accordingly extends the application of GTD to some new problems which could not have been solved previously. However, there remains several unsolved problems in the conventional formulation of the GTD-MM hybrid technique. One of these is how to construct the *additional* equations in addition to the MM equations. Figure 1 depicts an arbitrarily shaped, perfectly conducting three-dimensional scatterer with a discontinuity in curvature at C which causes diffraction. In GTD-MM hybrid technique, the total scatterer surface is divided into the near discontinuity region (MM-region) which is hatched in Fig. 1 and the far discontinuity region (GTD-region). The *additional* equations are required so that the system of equations is determined or overdetermined since the diffraction coefficients for the diffracted fields in the GTD-region are unknowns as well as the current expansion coefficients for the MM-region. In the conventional formulation, the magnetic field integral equation (MFIE) satisfied at one or any necessary number of points empirically selected in the GTD-region has been used as the necessary *additional* equation. The selected point and the associated equation are called

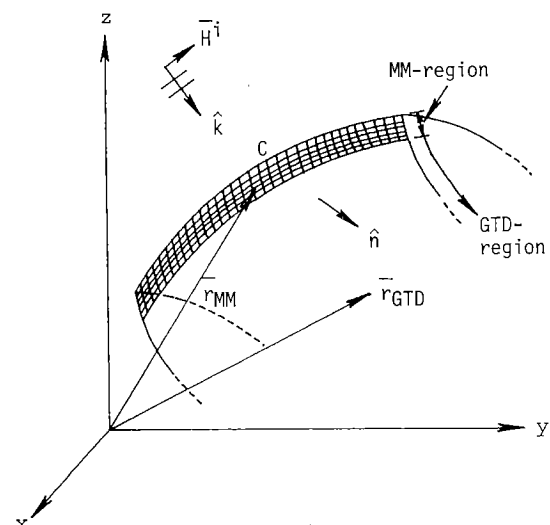


Fig. 1 Three dimensional surface with TE plane wave incidence divided into MM-region and GTD-region.

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“matching-point” and “matching-point equation”, respectively. Such way of obtaining the *additional* equations is not an ideal one. The solution of the GTD-MM hybrid technique has not been unique because the matching-point location on which the solution depends is not uniquely determined. In the paper by Burnside et al.⁽⁶⁾ it has been empirically found that the best solution is obtained when the matching-point is selected very close to the boundary between the GTD-and the MM-region, although the reason has not been explained. This paper sheds light on this problem, and proposes an equation named “current continuity equation” for the GTD-MM formulation instead of the matching-point equation.

Section 2 is concerned with the original formulation of GTD-MM hybrid technique, in which the basic equations are rederived for later convenience and the fundamental underlying problems including the indefinite matching-point location are explained. Section 3 is devoted to introduce current continuity equation, which is derived from the physical requirement of current continuity at the MM-region to GTD-region boundary. The relation of this new equation to the matching-point equation is investigated in Sect. 4. We find that the current continuity equation is equivalent to the special case of the matching-point equation when the matching-point is located at the GTD-region to MM-region boundary. This fact implies that the current continuity equation can give the best solution which would be obtained by the conventional GTD-MM formulation with the use of matching-point equation. In Sect. 5, some numerical results for a two-dimensional wedge are given to show the validity of our new formulation.

2. Original Formulation of GTD-MM Hybrid Technique

According to the method of moments, the surface current in the MM-region is expressed in terms of unknowns α_m and the prescribed basis functions \mathbf{j}_m as

$$\mathbf{J}^{\text{MM}}(\mathbf{r}) = \sum_{m=1}^N \alpha_m \mathbf{j}_m(\mathbf{r}) \quad (1)$$

where \mathbf{r} is the position vector for a point on the scatterer surface S and m extends over the dimension of the MM-region functional space assumed here to be N . In the GTD-region, the surface current is assumed to be represented in the GTD current form as⁽⁶⁾

$$\mathbf{J}^{\text{GTD}} = \mathbf{J}^i + \mathbf{J}^r + \mathbf{J}^d \quad (2)$$

where \mathbf{J}^i , \mathbf{J}^r and \mathbf{J}^d are the currents associated with the incident, reflected, and diffracted fields, respectively. Furthermore, the diffracted current \mathbf{J}^d is represented by N_d sets of yet unknown diffraction coefficients $D^{(n)}$, and diffraction functions $\mathbf{F}_n(\mathbf{r})$ determined according to the asymptotic behavior of the current near the

discontinuity. Thus

$$\mathbf{J}^d(\mathbf{r}) = \sum_{n=1}^{N_d} D^{(n)} \mathbf{F}_n(\mathbf{r}) \quad (3)$$

The magnetic field integral equation (MFIE) over surface S is employed to derive the basic equations used in the GTD-MM hybrid technique, which is given by

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{H}^i(\mathbf{r}) \hat{\mathbf{n}} \times \iint_S \mathbf{J}(\mathbf{r}') \times \nabla G(\mathbf{r}, \mathbf{r}') dS' \quad (4)$$

where \mathbf{J} is the surface current, \mathbf{H}^i is the incident magnetic field, $G(\mathbf{r}, \mathbf{r}')$ is the Green's function, and $\hat{\mathbf{n}}$ is the outward unit normal to the surface.

For an arbitrary sample point \mathbf{r}_{MM} in the MM-region, we have from Eq. (4)

$$\begin{aligned} \mathbf{J}^{\text{MM}}(\mathbf{r}_{\text{MM}}) &= \hat{\mathbf{n}} \times \mathbf{H}^i(\mathbf{r}_{\text{MM}}) \\ &\quad - \hat{\mathbf{n}} \times \iint_{S_{\text{MM}}} \mathbf{J}^{\text{MM}}(\mathbf{r}') \times \nabla G(\mathbf{r}_{\text{MM}}, \mathbf{r}') dS' \\ &\quad - \hat{\mathbf{n}} \times \iint_{S_{\text{GTD}}} \mathbf{J}^{\text{GTD}}(\mathbf{r}') \times \nabla G(\mathbf{r}_{\text{MM}}, \mathbf{r}') dS' \end{aligned} \quad (5)$$

where the integration surface S in the right hand side of Eq. (4) has been divided into the MM-region S_{MM} and the GTD-region S_{GTD} . Substituting Eqs. (1), (2) and (3) into Eq. (5), we obtain the basic equation of the GTD-MM hybrid technique as follows.

$$\begin{aligned} &\sum_{m=1}^N (\mathbf{j}_m + \hat{\mathbf{n}} \times \iint_{S_{\text{MM}}} \mathbf{j}_m \times \nabla G dS') \alpha_m \\ &\quad + \sum_{n=1}^{N_d} D^{(n)} \hat{\mathbf{n}} \times \iint_{S_{\text{GTD}}} \mathbf{F}_n \times \nabla G dS' \\ &= \hat{\mathbf{n}} \times \mathbf{H}^i - \hat{\mathbf{n}} \times \iint_{S_{\text{GTD}}} (\mathbf{J}^i + \mathbf{J}^r) \times \nabla G dS' \end{aligned} \quad (6)$$

We may take N sample points \mathbf{r}_{MM} in the MM-region which has been assumed to be N dimensional functional space with respect to the surface current. Then, we get N linear simultaneous equations from Eq. (6). However, there are $N + N_d$ unknowns included in Eq. (6), and the N_d *additional* equations have to be constructed by another way, which is the main object for discussion hereafter.

As seen above, there are three problems which have to be addressed in GTD-MM hybrid technique, i.e., the choice of MM-current expansion functions, the determination of the diffraction functions, and how to construct the necessary *additional* equations. It is desirable to incorporate the edge condition in the choice of MM-region current expansion function as in the usual MM⁽⁸⁾, but the wedge with the TE-plane wave incidence as treated here is characterized by the smoothly changing surface current and does not require the incorporation of the edge condition to

accelerate the convergence. As for the *additional* equations, Burnside et al. proposed a procedure to obtain them by setting N_d equations similar to Eq.(6) at N_d points r_{GTD} in the GTD-region whose locations were determined empirically. These points are called the “matching-point” and thus obtained equations are called “matching-point equation”. In association with the second problem, an improved power series of three terms for the wedge diffracted current was given by Sahalos et al. in Ref.(7).

3. Current Continuity Equation

Let us begin by inspecting the physical meaning of the matching-point equation used in Ref.(6). The boundary condition in the MM-region has been satisfied in Eq.(6), and that in the GTD-region is automatically satisfied for points far from the discontinuity because of the asymptotic behavior of the fields contained in the form of the currents expressed by Eq.(2). For points in the GTD-region not far from the discontinuity, it is expected that the boundary condition is partially satisfied by the matching-point equation. However, the matching-point equation does not ensure the current continuity of GTD-region to MM-region at the regions boundary.

Here we require the current continuity across the MM-and the GTD-region boundary, and propose to utilize this relationship as the necessary *additional* equations instead of the matching-point equations. This continuity condition can be written down in a simple mathematical equation, which is here referred to as the current continuity equation, as

$$\lim_{r \rightarrow r_c^+} \mathbf{J}^{GTD}(\mathbf{r}) = \lim_{r \rightarrow r_c^-} \mathbf{J}^{MM}(\mathbf{r}) \quad (7)$$

where r_c^+ and r_c^- are position vectors adjacent to the boundary in the GTD- and the MM-region, respectively. The two-dimensional 90° wedge depicted in Fig. 2 has been discussed in Ref.(6), where the diffraction function has been taken as a function of the form $e^{-jk\rho}/\sqrt{\rho}$ and the MM-region current has been expanded in the pulse functions, then the current continuity equation reduces simply to

$$\left. \begin{aligned} \alpha(x_c) &= J_x^i(x_c) + J_x^r(x_c) + D_x \frac{e^{-jkx_c}}{\sqrt{x_c}} \\ \alpha(y_c) &= J_y^i(y_c) + J_y^r(y_c) + D_y \frac{e^{-jky_c}}{\sqrt{y_c}} \end{aligned} \right\} \quad (8)$$

where x_c and y_c are the regions boundary positions on the x -wall and y -wall as shown in Fig. 2. $\alpha(x_c)$ and $\alpha(y_c)$ are the complex coefficients associated with the MM-current pulse segments at x_c and y_c . Comparing Eq.(8) with the matching-point equation (see Ref.(6)), one finds that it is extremely concise in mathematics and very easy for computation. The form of *additional* equation influences barely the computation

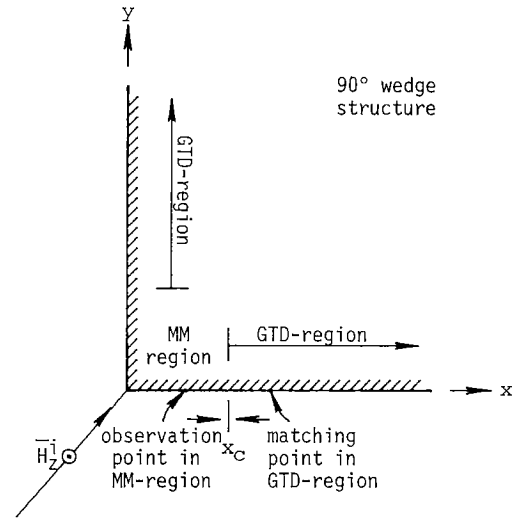


Fig. 2 A two-dimensional wedge diffraction problem.

time of GTD-MM as its number N_d is generally much fewer than the MM segments number N , but it influences substantially the computation accuracy. It will be seen in the next section that the current continuity equation can give the best solution which would be obtained by the matching-point equation.

A special case where a careful treatment is necessary exists. It is when the wave is grazing incidence on the discontinuity. Generally, more kinds of functions and more unknown constants have to be used to represent the GTD-region current precisely⁽⁷⁾, so that more *additional* equations become necessary. One can get such equations by satisfying the current continuity condition at several MM-current segments most near the regions boundary. For wedge diffraction problem with TE-plane wave grazing incidence, another method is here proposed to construct the further necessary *additional* equations using the current continuity at regions boundary. A power function is chosen as the expansion function of MM-region current, and the current continuity condition is imposed up to derivatives. That gives the following equations:

$$J_l^{MM}(L) = \sum_{m=1}^N a_m L^{m-1} \quad (9)$$

$$\left. \begin{aligned} \lim_{L \rightarrow L_c+0} J_l^{GTD}(L) &= \lim_{L \rightarrow L_c-0} J_l^{MM}(L) \\ \lim_{L \rightarrow L_c+0} \frac{dJ_l^{GTD}(L)}{dL} &= \lim_{L \rightarrow L_c-0} \frac{dJ_l^{MM}(L)}{dL} \\ &\vdots \end{aligned} \right\} \quad (10)$$

where l denotes tangential to the wedge wall, L the distance from wedge edge and L_c the distance of the regions boundary from the wedge edge.

4. Relation to Matching-Point Equation

For brevity, a two-dimensional 90° wedge as

depicted in Fig. 2 is used to clarify the relation of the current continuity equation to the matching-point equation. Because of the symmetrical nature of the two walls, only the analysis on the x -wall is described here. For the two-dimensional problem the MFIE of Eq. (4) reduces to

$$J_x(x) = - \left[H_z^i + \hat{z} \cdot \nabla \times \left\{ \hat{x} \int_{x\text{-wall}} J_x(x') \frac{H_0^{(2)}(k|x-x'|)}{4j} dx' + \hat{y} \int_{y\text{-wall}} J_y(y') \frac{H_0^{(2)}(k\sqrt{x^2+y'^2})}{4j} dy' \right\} \right] \quad (11)$$

and Eq. (6) reduces to

$$\sum_{m=1}^{N_x+N_y} l_{mn} \alpha_m + D_x L_n \left(\frac{e^{-jkx}}{\sqrt{x}} \right) + D_y L_n \left(\frac{e^{-jky}}{\sqrt{y}} \right) = g_n, \quad n=1, 2, \dots, N_x+N_y \quad (12)$$

in which N_x and N_y are the numbers of the x -wall and the y -wall current segments in the MM-region, respectively. For details of l_{mn} , g_n , $L_n(\ast)$ see Ref. (6). The matching-point equation on the x -wall was derived by Burnside et al. as follows.

$$\sum_{m=1}^{N_x+N_y} l_{mn} \alpha_m + \frac{1}{2} D_x \frac{e^{-jkx}}{\sqrt{x}} + D_y L_n \left(\frac{e^{-jky}}{\sqrt{y}} \right) = -H_z^i(x, 0) - \frac{J_x^i + J_x^r}{2} - L_n(J_y^i + J_y^r) \quad (13)$$

We shall investigate how Eq. (12) and Eq. (13) behave when the observation point x approaches the regions boundary x_c from the MM side and the GTD side, respectively.

From Eq. (12), as $x \rightarrow x_c - 0$ in MM-region, we have

$$\left. \begin{aligned} \sum_{m=1}^{N_x+N_y} l_{mn} \alpha_m &= \sum_{m=1}^{N_y} l_{mn} \alpha_m + \frac{1}{2} \alpha(x_c - 0) \\ L_n \left(\frac{e^{-jkx}}{\sqrt{x}} \right) &= L_n(J_x^i + J_x^r) = 0 \end{aligned} \right\} \quad (14)$$

where the factor $1/2$ multiplied to $\alpha(x_c - 0)$ is a result of the fact that l_{mn} becomes $1/2$ if $m = n$. Then Eq. (12) is simplified to

$$\sum_{m=1}^{N_y} l_{mn} \alpha_m + \frac{1}{2} \alpha(x_c - 0) = -H_z^i(x_c - 0, 0) - L_n(J_y^i + J_y^r) - D_y L_n \left(\frac{e^{-jky}}{\sqrt{y}} \right) \quad (15)$$

On the other hand, as $x \rightarrow x_c + 0$ from GTD side Eq. (13) reduces to

$$\sum_{m=1}^{N_y} l_{mn} \alpha_m = -H_z^i(x_c + 0, 0) - \frac{1}{2} (J_x^i + J_x^r)_{x=x_c+0} - \frac{1}{2} D_x \frac{e^{-jkx_c}}{\sqrt{x_c}} - L_n(J_y^i + J_y^r)$$

$$-D_y L_n \left(\frac{e^{-jky}}{\sqrt{y}} \right) \quad (16)$$

since,

$$\sum_{m=1}^{N_x+N_y} l_{mn} \alpha_m = \sum_{m=1}^{N_y} l_{mn} \alpha_m \quad (17)$$

After subtracting Eq. (15) from Eq. (16), we finally obtain

$$\alpha(x_c) = J_x^i(x_c) + J_x^r(x_c) + D_x \frac{e^{-jkx_c}}{\sqrt{x_c}} \quad (18)$$

Obviously, it is just the current continuity equation (8) given in the previous section. Thus, we conclude that the current continuity equation can be derived from the matching-point equation by letting the matching-point approach the regions boundary. This is a very meaningful result because it explains why the best solution in the conventional GTD-MM hybrid technique is obtained when the matching-point location is determined in this way as described by Burnside et al. in Ref. (6). The matching-point equation satisfies both the boundary condition and the current continuity condition at the matching-point and at the regions boundary, respectively, if the matching-point is located adjacent to the regions boundary.

5. Numerical Results

To show the validity of our new equation, several computed results for the 90° and 30° two-dimensional wedges with TE plane wave incidence are given in Fig. 3 and Fig. 4 in comparison with the exact solution based on the canonical problem⁽⁹⁾. We have, in the computation, utilized three diffraction functions, \sqrt{x} to the order of $+1$, 0 and -1 , to express the GTD-region current, and a power series of Eq. (9) to expand the MM-region current. The theoretical solutions are drawn by lines and the numerical results by symbol marks. Figure 3 shows the magnitude of currents along the x -wall when wave is incident at near grazing angles. In Fig. 4 the computed diffraction coefficients are shown. The theoretical diffraction coefficients are obtained by solving the linear simultaneous equations derived by equating the GTD approximate surface current to the canonical problem surface current at some observation points. The theoretical solution shown in Fig. 4 is the *least square* value of letting the observation points along the x -wall extend in the range $0.2\lambda - 10\lambda$. In the GTD-MM computation, the MM-regions extension in both walls have been taken as 0.8λ . The superscripts on the diffraction coefficients in Fig. 4 are associated with the order of n for the $(\sqrt{x})^n$ component. Note that the most dominant component of the three computed coefficients is always $D_x^{(-1)}$ except for the grazing-incident case when $D_x^{(0)}$ becomes dominant. The numerical results present-

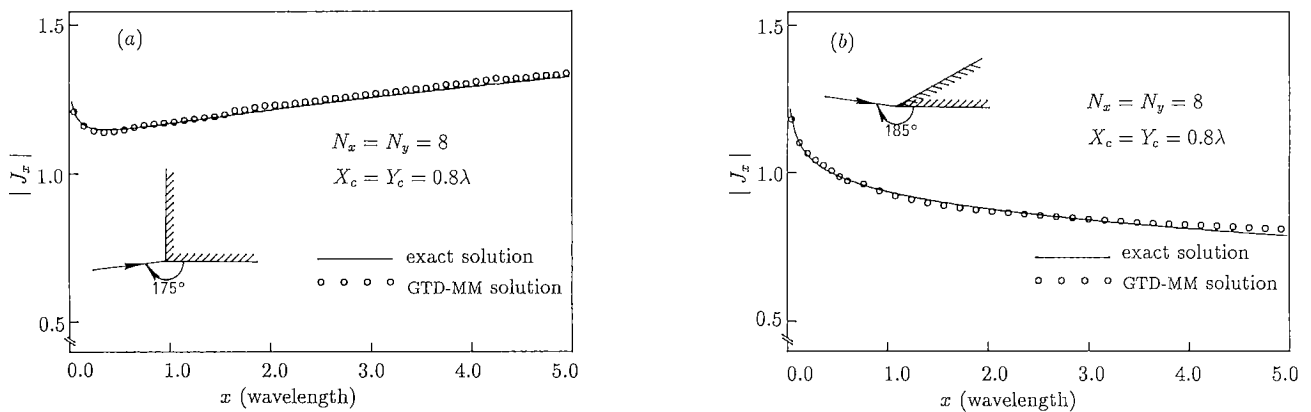


Fig. 3 Magnitude of x -wall surface current.
 (a) 90° wedge, 175° incident angle
 (b) 30° wedge, 185° incident angle

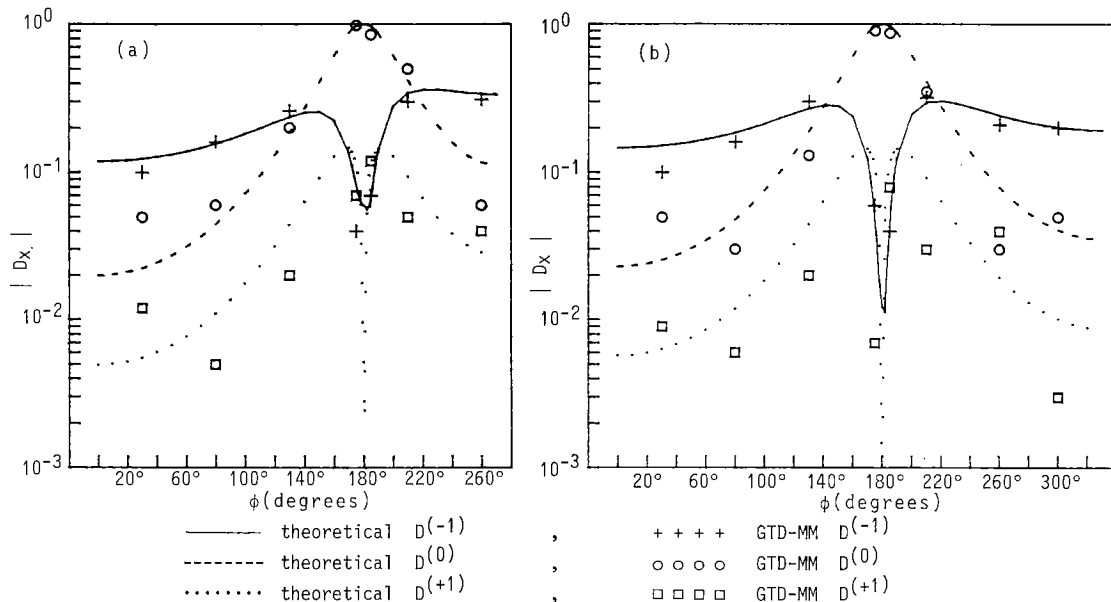


Fig. 4 Numerical diffraction coefficients of two-dimensional wedge compared with theoretical solutions.
 (a) 90° wedge
 (b) 30° wedge

ed here are in good agreement with the theoretical solution⁽⁹⁾.

6. Conclusion

A current continuity equation is proposed as the *additional* equation in the GTD-MM hybrid technique to acquire the uniqueness of the solution which were nonexistent in the conventional formulation with the matching-point equation. The current continuity equation, which ensures the current continuity and satisfies the boundary condition, can directly be written down through equating the MM-region current to the GTD-region current at the regions boundary. It is proved

that the current continuity equation is equivalent to the matching-point equation of special case when the matching-point located very close to the regions boundary, which were able to give the best solution in the conventional formulation with the matching-point equation as explained by Burnside et al⁽⁶⁾. The validity of the new equation is confirmed through the numerical results. The treatment for a two-dimensional wedge diffraction problem in this paper may be extended to three-dimensional diffraction problems.

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