PAPER

2-D LMA Filters——Design of Stable Two-Dimensional Digital Filters with Arbitrary Magnitude Function——

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SUMMARY This paper proposes a technique for designing two-dimensional (2-D) digital filters approximating an arbitrary magnitude function. The technique is based on 2-D spectral factorization and rational approximation of the complex exponential function. A 2-D spectral factorization technique is used to obtain a recursively computable and stable system with nonsymmetric half-plane support from the desired 2-D magnitude function. Since the obtained system has an exponential function type transfer function and cannot be realized directly in a rational form, a class of realizable 2-D digital filters is introduced to approximate the exponential type transfer function. This class of filters referred to as two-dimensional log magnitude approximation (2-D LMA) filters can be viewed as an extension of the class of 1-D LMA filters to the 2-D case. Filter coefficients are given by the 2-D complex cepstrum coefficients, i.e., the inverse Fourier transform of the logarithm of the given magnitude function, which can be efficiently computed using 2-D FFT algorithm. Consequently, computation of the filter coefficients is straightforward and efficient. A simple stability condition for the 2-D LMA filters is given. Under this condition, the stability of the designed filter is guaranteed. Parallel implementation of the 2-D LMA filters is also discussed. Several examples are presented to demonstrate the design capability.

key words: *2-D digital filter, LMA filter, 2-D spectral factorization*

1. Introduction

In designing two-dimensional (2-D) IIR digital filters, one of difficulties of the problem is to ensure the stability of the designed filter. This is due to the fact that it is generally impossible to factor a 2-D polynomial into a product of polynomials of lower order. There have been several attempts to design stable 2-D IIR filters with an arbitrarily prescribed magnitude function[(1),(2)]. Since these techniques involve nonlinear optimization procedures, the computational complexity of the filter design tends to be high. An approach to simplifying the problem is to employ separable filters. However, it has a potential problem that separable filters can realize only the restrictive class of magnitude functions which have quadrantal symmetry[(3)]. It is known that nonsymmetric half-plane (NSHP) filters should be employed to realize a general magnitude function[(4)].

In this paper, we propose a technique for designing 2-D IIR digital filters with an arbitrarily prescribed magnitude function. The technique is based on 2-D spectral factorization and rational approximation of the complex exponential function. For a given 2-D magnitude function, a recursively stable system with nonsymmetric half-plane support is obtained using a 2-D spectral factorization technique[(4)]. However, since the obtained system has an exponential function type transfer function, it cannot be realized directly in a rational form. We introduce a class of realizable 2-D digital filters to approximate exponential function type transfer functions. We will refer such filters as 2-D log magnitude approximation (LMA) filters because this class of filters can be viewed as an extension of the class of LMA filters[(5)] to two dimensions. Unlike other filter design techniques based on the 2-D spectral factorization[(1),(7)], the 2-D LMA filter leads to a straightforward and computationally efficient design algorithm for obtaining a stable filter. Filter coefficients are given by the 2-D complex cepstrum coefficients[(8)], i.e., the inverse Fourier transform of the logarithm of the given magnitude function. The complex cepstrum may be efficiently computed using 2-D FFT algorithm. We give a simple stability condition for the 2-D LMA filters. Using this condition, we can easily guarantee the stability of the obtained filter. Moreover, we can derive an efficient network structure of 2-D LMA filters for implementation on parallel VLSI array processors.

In Sect. 2, we state a brief review of the 2-D spectral factorization algorithm. Then we introduce the class of 2-D LMA filters in Sect. 3. We also discuss stability and implementation of the 2-D LMA filters. In Sect. 4, we show several examples to demonstrate the design capability.

2. 2-D Spectral Factorization

2. 1 Two-Factor Decomposition

Let $X(z_1, z_2)$ be the z-transform of a 2-D sequence

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$\{x(m, n)\}$. Suppose that $X(z_1, z_2)$ is analytic and not zero on the unit bicircle $\Gamma^2=\{(z_1, z_2) \mid |z_1|=|z_2|=1\}$. Then $X(z_1, z_2)$ can be factored into two recursively stable factors such that

$$X(z_1, z_2)=X_{\oplus+}(z_1, z_2)\, X_{\ominus-}(z_1, z_2) \qquad (1)$$

where $X_{\oplus+}(z_1, z_2)$ and $X_{\ominus-}(z_1, z_2)$ are mix-min phase and mix-max phase, respectively[(4),(6)]. Furthermore, $X_{\oplus+}(z_1, +\infty)$ is 1-D minimum phase and $X_{\ominus-}(z_1, 0)$ is 1-D maximum phase.

The above factorization is obtained by decomposing the 2-D complex cepstrum[(8)] given by

$$\hat{x}(m, n)=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \ln[X(e^{j\omega_1}, e^{j\omega_2})]e^{j\omega_1 m}e^{j\omega_2 n} \cdot d\omega_1 d\omega_2 \qquad (2)$$

as

$$\hat{x}(m, n)=\hat{x}_{\oplus+}(m, n)+\hat{x}_{\ominus-}(m, n) \qquad (3)$$

where $\hat{x}_{\oplus+}(m, n)$ and $\hat{x}_{\ominus-}(m, n)$ are the projections of $\hat{x}(m, n)$ onto the nonsymmetric half-planes $\mathcal{R}_{\oplus+}$ and $\mathcal{R}_{\ominus-}$ shown in Fig. 1, respectively. Since regions $\mathcal{R}_{\oplus+}$ and $\mathcal{R}_{\ominus-}$ overlap at the origin, $\hat{x}(0, 0)$ should be divided up between $\hat{x}_{\oplus+}$ and $\hat{x}_{\ominus-}$ such that

$$\hat{x}(0, 0)=\hat{x}_{\oplus+}(0, 0)+\hat{x}_{\ominus-}(0, 0). \qquad (4)$$

Then $X_{\oplus+}(z_1, z_2)$ and $X_{\ominus-}(z_1, z_2)$ are given by

$$\ln[X_{\oplus+}(z_1, z_2)]=\sum_{(m,n)\in\mathcal{R}_{\oplus+}} \hat{x}_{\oplus+}(m, n)\, z_1^{-m} z_2^{-n} \qquad (5)$$

$$\ln[X_{\ominus-}(z_1, z_2)]=\sum_{(m,n)\in\mathcal{R}_{\ominus-}} \hat{x}_{\ominus-}(m, n)\, z_1^{-m} z_2^{-n}. \qquad (6)$$

2.2 Application to Filter Design

The 2-D spectral factorization technique leads to a simple approach to designing nonsymmetric half-plane filters which can approximate arbitrary magnitude functions[(4)].

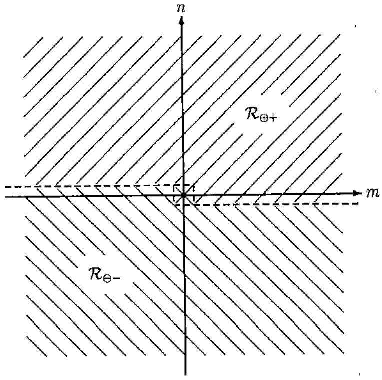


Fig. 1 Nonsymmetric half-planes $\mathcal{R}_{\oplus+}$ and $\mathcal{R}_{\ominus-}$.

Suppose that a desired 2-D magnitude function $D(\omega_1, \omega_2)$ is gven. We assume that $D(\omega_1, \omega_2)$ is continuous and that $D(\omega_1, \omega_2)=D(-\omega_1, -\omega_2)$. We also assume that there exists the inverse Fourier transform of $\ln[D(\omega_1, \omega_2)]$, i.e., the cepstrum $\hat{d}(m, n)$ given by

$$\hat{d}(m, n)=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \ln[D(\omega_1, \omega_2)]e^{j\omega_1 m}e^{j\omega_2 n} \cdot d\omega_1 d\omega_2. \qquad (7)$$

Let us consider the following transfer function:

$$F(z_1, z_2)=\sum_{(m,n)\in\mathcal{R}_{\oplus+}} w_{\oplus+}(m, n)\, \hat{d}(m, n)\, z_1^{-m} z_2^{-n} \qquad (8)$$

where

$$w_{\oplus+}(m, n)=\begin{cases} 2, & (n>0)\cup(m>0, n=0) \\ 1, & m=n=0 \\ 0, & \text{otherwise.} \end{cases} \qquad (9)$$

Since we have assumed that $D(\omega_1, \omega_2)=D(-\omega_1, -\omega_2)$ and, therefore, it holds $\hat{d}(m, n)=\hat{d}(-m, -n)$, we have

$$\mathrm{Re}[F(e^{j\omega_1}, e^{j\omega_2})]=\ln[D(\omega_1, \omega_2)]. \qquad (10)$$

Consequently, if we can realize

$$\tilde{H}(z_1, z_2)=\exp[F(z_1, z_2)], \qquad (11)$$

then $\tilde{H}(z_1, z_2)$ is mix-min phase and stable, and its magnitude response becomes

$$|\tilde{H}(e^{j\omega_1}, e^{j\omega_2})|=D(\omega_1, \omega_2). \qquad (12)$$

It is noted that the impulse response of $\tilde{H}(z_1, z_2)$ has the region of support on the nonsymmetric half-plane $\mathcal{R}_{\oplus+}$ and is in general infinite-extent sequence.

3. 2-D Log Magnitude Approximation (LMA) Filters

3.1 Realization of Exponential Function Type Transfer Functions

To obtain the stable 2-D digital filter which has the desired magnitude response, we should realize the exponential function type transfer function given by Eq. (11). However, since the complex exponential function is not a rational function, filters having the transfer function (11) cannot be realized directly. We introduce here a class of 2-D digital filters to approximate exponential transfer functions. The new class of filters is viewed as an extension of the class of log magnitude approximation (LMA) filters[(5)] to the 2-D case.

The complex exponential function $\exp(w)$ can be approximated by a rational function

$$\exp(w) \cong R_L(w) = \frac{1+\sum_{l=1}^{L} A_{L,l} w^l}{1+\sum_{l=1}^{L} A_{L,l}(-w)^l}, \quad |w| \ll W_{R_L} \tag{13}$$

where W_{R_L} is the modulus of the zero or pole of $R_L(w)$ with the smallest modulus. For example, if we choose $A_{L,l}$ as

$$A_{L,l} = \frac{1}{l!}\binom{L}{l} \Big/ \binom{2L}{l} \tag{14}$$

then Eq. (13) is the $[L/L]$ Padé approximant of $\exp(w)$ at $w=0$.

Substituting $w = F(z_1, z_2)$ into Eq. (13), we have

$$R_L[F(z_1, z_2)] = \frac{1+\sum_{l=1}^{L} A_{L,l}\{F(z_1, z_2)\}^l}{1+\sum_{l=1}^{L} A_{L,l}\{-F(z_1, z_2)\}^l}. \tag{15}$$

If $\max_{0 \leq \omega_1, \omega_2 \leq 2\pi} |F(e^{j\omega_1}, e^{j\omega_2})| \ll W_L$, then, from Eqs. (12), (13), and (15), the magnitude response of $R_L[F(z_1, z_2)]$ becomes

$$|R_L[F(e^{j\omega_1}, e^{j\omega_2})]| \cong D(\omega_1, \omega_2). \tag{16}$$

Thus the filter with the transfer function (15) gives an approximation to the desired magnitude function $D(\omega_1, \omega_2)$. We will refer to this filter as the 2-D LMA filter. In addition, as in the 1-D case[5], we will refer to the filter with the transfer function $F(z_1, z_2)$ as the basic filter. The basic filter should have a convex region of support which does not include the origin so that the 2-D LMA filter becomes recursively computable.

Since the error of the rational approximation of $\exp(w)$ affects the performance of the 2-D LMA filter crucially, it is desirable to determine the rational function in such a way that the maximum error is minimized; that is,

$$\min_{A_{L,l}} \max_{|w| \leq r} |\ln[\exp(w)] - \ln[R_L(w)]| \tag{17}$$

where

$$r = \max_{0 \leq \omega_1, \omega_2 \leq 2\pi} |F(e^{j\omega_1}, e^{j\omega_2})|. \tag{18}$$

We can easily obtain a best or near-best approximation using a conventional optimization technique such as a complex Chebyshev approximation technique[10] or modified Padé approximation technique[5].

3. 2 Structure of 2-D LMA Filters

When a desired magnitude function $D(\omega_1, \omega_2)$ is given, the transfer function of the basic filter $F(z_1, z_2)$ is obtained from Eq. (8). Since the support of $\tilde{d}(m, n)$ is generally of infinite-extent, the support of $f(m, n)$, the impulse response of the basic filter, is also of infinite-extent. In order to realize the 2-D LMA filter, we confine the support of $f(m, n)$ to some finite-extent region using a window method. Let $w_F(m, n)$ be a window function confining the support to a finite-extent region $\mathcal{R}_F$. Using this window, the transfer function of the basic filter is given by

$$F(z_1, z_2) = \sum_{(m,n) \in \mathcal{R}_{\oplus+} \cap \mathcal{R}_F} f(m, n) z_1^{-m} z_2^{-n} \tag{19}$$

where

$$f(m, n) = w_{\oplus+}(m, n)\, w_F m, n)\, \tilde{d}(m, n). \tag{20}$$

The frequency response $\mathrm{Re}[F(e^{j\omega_1}, e^{j\omega_2})]$ becomes a smoothed version of $\ln[D(\omega_1, \omega_2)]$, where the smoothing function is the Fourier transform of $w_F(m, n)$[8].

If $F(z_1, z_2)$ has a constant term $f(0, 0)$, the 2-D LMA filter $R_L[F(z_1, z_2)]$ becomes noncomputable because the constant term leads to delay-free loops. However, rewriting Eq. (19) in the form

$$F(z_1, z_2) = f(0, 0) + G(z_1, z_2) \tag{21}$$

where

$$G(z_1, z_2) = \sum_{(m,n) \in \mathcal{R}_{\oplus} \cap \mathcal{R}_F - (0,0)} f(m, n) z_1^{-m} z_2^{-n} \tag{22}$$

we can obtain a realizable transfer function of the 2-D LMA filter:

$$H(z_1, z_2) = C \cdot R_L[G(z_1, z_2)] \tag{23}$$

where C is the filter gain given by

$$C = \exp[f(0, 0)]. \tag{24}$$

It is obvious that $R_L[G(z_1, z_2)]$ is recursively computable. A network structure of the 2-D LMA filter for $L=3$ is shown in Fig. 2.

3. 3 Stability

The exponential transfer function $\exp[G(z_1, z_2)]$ is stable if the basic filter $G(z_1, z_2)$ is stable. This can be derived from the fact that the $\exp[G(z_1, z_2)]$ has no singularities and zeros in the region in which $G(z_1, z_2)$ has no singularities. We consider here the stability when the rational approximation in Eq. (13) is used. Let $H(z_1, z_2)$ of Eq. (23) be the transfer function of a 2-D LMA filter. Suppose that the basic filter is stable and has the nonsymmetric half-plane support $\mathcal{R}_{\oplus+}$. This means that $G(z_1, z_2)$ is analytic in the region

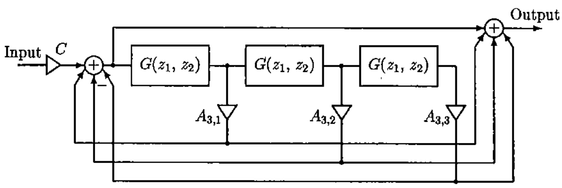


Fig. 2 Direct implementation of a 2-D LMA filter for $L=3$.

$\mathcal{D}_{\oplus+}=\{(z_1, z_2)\,|\,|z_1|=1, |z_2|\geqq 1\}\cup\{(z_1, z_2)\,|\,|z_1|\geqq 1, z_2=+\infty\}$[4]. Thus the following inequality holds for any $(z_1, z_2)\in\mathcal{D}_{\oplus+}$ from the maximum principle:

$$|G(z_1, z_2)|\leqq \max_{(z_1,z_2)\in\Gamma^2}|G(z_1, z_2)|. \tag{25}$$

Moreover, it can be shown[5] that the rational approximation $R_L(w)$ in Eq. (13), such as the Padé approximant or the best approximation of $\exp(w)$, has no poles and zeros in the region

$$|w|< W_L=\begin{cases}2.000, & L=1\\ 3.464, & L=2\\ 4.644, & L=3\end{cases} \tag{26}$$

because $W_L\leqq W_{R_L}$. If $|G(z_1, z_2)|$ is bounded such that

$$\max_{(z_1,z_2)\in\Gamma^2}|G(z_1, z_2)|< W_L \tag{27}$$

then, from Eq. (25), we have

$$|G(z_1, z_2)|< W_L, \quad (z_1, z_2)\in\mathcal{D}_{\oplus+}. \tag{28}$$

Consequently, $R_L[G(z_1, z_2)]$ has no poles and zeros in the region $\mathcal{D}_{\oplus+}$, i.e., $R_L[G(z_1, z_2)]$ is mix-min phase and $R_L[G(z_1, +\infty)]$ is 1-D minimum phase. Therefore, the 2-D LMA filter is stable. In other words, for the given transfer function of the basic filter $G(z_1, z_2)$, if we choose the order of the rational approximation L such that

$$r=\max_{0\leqq\omega_1,\omega_2\leqq 2\pi}|G(e^{j\omega_1}, e^{j\omega_2})|< W_L \tag{29}$$

then the 2-D LMA filter $R_L[G(z_1, z_2)]$ is stable. In addition, the same result can be derived for the other nonsymmetric half-plane support or quarter-plane support basic filters.

3.4 Parallel Implementation of 2-D LMA Filters

In Fig. 2 we have shown that the 2-D LMA filter with the rational approximation of order L consists of L basic filters. When an $M\times N$ NSHP FIR filter is used as the basic filter, i.e., the basic filter has the region of support $\{(m, n)\,|\,|m|\leqq M, |n|\leqq N\}\cap\mathcal{R}_{\oplus+}-(0, 0)$, an output sample from the 2-D LMA filter requires $L(2MN+M+N+1)+1$ multiplications and $L(2MN+M+N+1)$ additions. Thus the number of multiplications and additions per output point is roughly L times that of the $M\times N$ NSHP FIR filter. However we can reduce the processing time per output point as follows.

Let us consider computing the value of an output sample at (m, n) in Fig. 2. From the definition of $G(z_1, z_2)$ in Eq. (22), it is easily seen that computing the output sample of each basic filter at (m, n) does not require output samples of the other basic filters at (m, n). This means that the computations in each basic filter are performed simultaneously. Therefore, network structure shown in Fig. 2 is inherently suited for parallel implementation. To derive a more efficient network structure of 2-D LMA filters for implementation on parallel VLSI array processors, we rewrite Eq. (13) in the form

$$R_L(w)=\frac{P(w)}{P(-w)}=\frac{1+O(w)/E(w)}{1-O(w)/E(w)} \tag{30}$$

where

$$P(w)=1+\sum_{l=1}^{L}A_{L,l}w^l \tag{31}$$

$$O(w)=\{P(w)-P(-w)\}/2 \tag{32}$$

$$E(w)=\{P(w)+P(-w)\}/2. \tag{33}$$

Using the fact that there exists a continued-fraction expansion of $O(w)/E(w)$ in the form

$$\frac{O(w)}{E(w)}=\frac{1|}{|(wB_{L,1})^{-1}}+\frac{1|}{|(wB_{L,2})^{-1}}+\cdots+\frac{1|}{|(wB_{L,L})^{-1}} \tag{34}$$

we obtain a network structure of $R_L[G(z_1, z_2)]$ shown in Fig. 3(a)[11]. When $R_L(w)$ is the $[L/L]$ Padé approximant of $\exp(w)$, the coefficient $B_{L,l}$ is given by

$$B_{L,l}=1/2(2l-1). \tag{35}$$

If we use a configuration as shown in Fig. 3(b) to realize the network of Fig. 3(a), then the network has the following features:

(a) Regularity: Each processor has the same structure with local interconnections except that the first processor has extra two adders.

(b) Temporal locality: Since the basic filter $G(z_1, z_2)$ does not require the input sample at (m, n) to compute its output sample at (m, n), there exists one

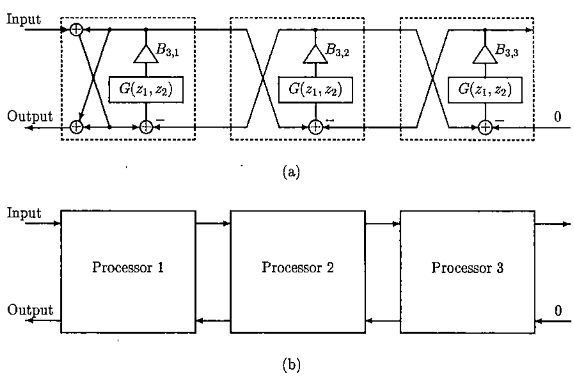


Fig. 3 Parallel implementation of a 2-D LMA filter for $L=3$.
(a) A structure of 2-D LMA filter based on continued-fraction expansion.
(b) A systolized version.

unit-time delay at each interconnection.
(c) Synchrony: The data can be computed synchronously with a global clock.

As a result, this network can be viewed as a systolic array[13]. If we ignore the processing time for two adders and the communication time between processors, the speed-up factor[13] is L, where L is the number of processors.

4. Design Examples

In this section, we present examples of the 2-D LMA filter design algorithm applied to the approximation of general magnitude functions. The design algorithm is summarized as follows:
(1) For a given magnitude function $D(\omega_1, \omega_2)$, find the cepstrum $\hat{d}(m, n)$ by inverse Fourier transforming $\ln[D(\omega_1, \omega_2)]$.
(2) Using an appropriate window function $w_F(m, n)$, obtain the basic filter coefficients $f(m, n)$ given by Eq. (20).
(3) Determine L, the order of the rational approximation of the complex exponential function, such that Eq. (29) holds. Then find $A_{L,l}$ using the modified Padé approximate[5].

In the following examples, we used 64×64-point DFT to compute the Fourier transform.

Example 1: *Lowpass filter*—Consider a circularly symmetric magnitude function given by

$$20\log_{10}[D(\omega_1, \omega_2)]=\begin{cases} 0\text{ dB}, & \sqrt{\omega_1^2+\omega_2^2}\leqq\frac{1}{2}\pi \\ -30\text{ dB}, & \text{otherwise.}\end{cases}$$

Since this magnitude function yields an infinite-extent sequence $\hat{d}(m, n)$, we confine the support of $f(m, n)$ by applying a circularly symmetric Kaiser Window[8],[9]

$$w_F(m, n)=\begin{cases}\dfrac{I_0[\alpha\sqrt{1-(m^2+n^2)/N^2}]}{I_0(\alpha)}, & \sqrt{m^2+n^2}\leqq N \\ 0, & \text{otherwise}\end{cases} \tag{36}$$

where $I_0(x)$ is the modified Bessel function of the first kind of order zero, and α is a parameter. Then the basic filter becomes the NSHP FIR filter with the region of support $\{(m, n)|0<\sqrt{m^2+n^2}\leqq N\}\cap\mathcal{R}_{\oplus+}$. For the case $N=10$ and $\alpha=6.0$, the maximum value of the magnitude response of the basic filter became $r=3.43$. From Eq. (29) we let $L=3$ to obtain a stable filter. Figure 4 shows the log magnitude response of the designed filter with $L=3$, $N=10$, and $\alpha=6.0$.

Example 2: *Bandpass fan filter*—Let the following bandpass fan specification be the desired magnitude response:

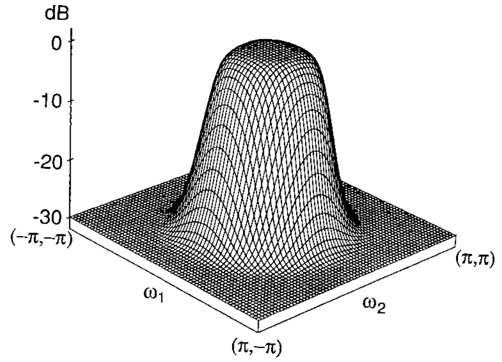


Fig. 4 Example 1. Log magnitude response of the designed filter with $L=3$, $N=10$, and $\alpha=6.0$.

$$20\log_{10}[D(\omega_1, \omega_2)]=\begin{cases}0\text{dB}, & \frac{1}{4}\pi\leqq\theta\leqq\frac{1}{2}\pi \text{ or } -\frac{3}{4}\pi\leqq\theta\leqq-\frac{1}{2}\pi, \\ & \text{and } \frac{1}{4}\pi\leqq|\omega_2|\leqq\frac{3}{4}\pi \\ -30\text{dB}, & \text{otherwise.}\end{cases}$$

where $\theta=\arctan(\omega_2/\omega_1)$. We used here the separable Kaiser window given by

$$w_F(m, n)=\begin{cases}\dfrac{I_0[\alpha\sqrt{1-(m/N)^2}]I_0[\alpha\sqrt{1-(n/N)^2}]}{I_0^2(\alpha)}, \\ \qquad |m|\leqq N,\quad |n|\leqq N \\ 0, \quad \text{otherwise}\end{cases} \tag{37}$$

with $N=12$ and $\alpha=6.0$ to confine the support of $f(m, n)$.. The obtained basic filter was 12×12 NSHP FIR filter with $r=3.56$. We, therefore, let $L=3$ from Eq. (29). The log magnitude response of the obtained filter is shown in Fig. 5.

Example 3: *General magnitude function*—In this example, we show the result of the approximation of an arbitrary magnitude function. Let the function shown in Fig. 6(a) be the desired log magnitude response. We generated this magnitude function from estimated power spectral data taken from a 128×128 face image. Using the rectangular window

$$w_F(m, n)=\begin{cases}1, & |m|\leqq N,\quad |n|\leqq N \\ 0, & \text{otherwise}\end{cases} \tag{38}$$

with $N=8$, we obtained the 8×8 NSHP basic filter with $r=2.51$. Although $L=2$ is sufficient to stabilize the designed 2-D LMA filter, we let $L=3$ to reduce the approximation error. Figure 6(b) shows the log magnitude response of the designed filter. Note that

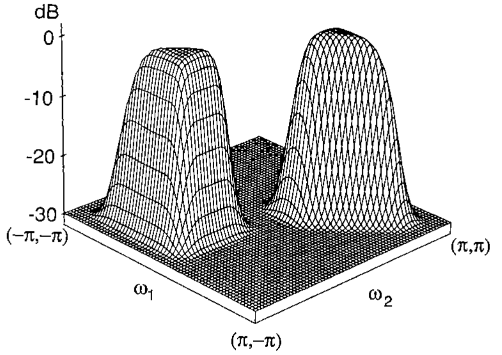
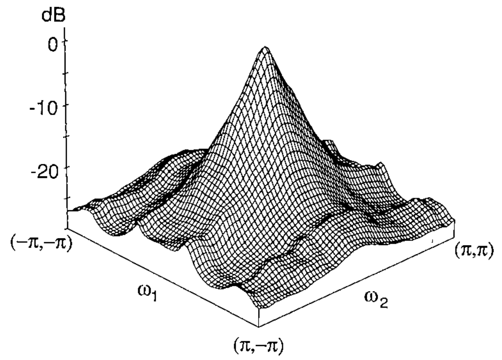
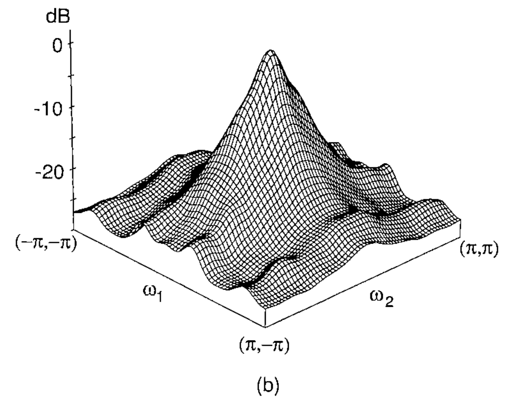


Fig. 5 Example 2. Log magnitude response of the designed filter with $L=3$, $N=12$, and $\alpha=6.0$.



(a)



(b)

Fig. 6 Example 3.
(a) Desired log magnitude function.
(b) Log magnitude response of the designed filter with $L=3$, $N=8$.

the root mean square (rms) log magnitude error defined by

$$e_{rms}=\left\{\frac{1}{K^2}\sum_{0\le k,l<K}\left|20\log_{10}|H(e^{j\omega_{1k}}, e^{j\omega_{2l}})|-20\log_{10}[D(\omega_{1k}, \omega_{2l})]\right|^2\right\}^{1/2} \quad (39)$$

where K is the DFT size, is $e_{rms}=0.16$ dB. In addition, the obtained filter can be used to implement a realizable Wiener filter[12].

5. Conclusion

We have developed a technique for designing stable 2-D digital filters which can approximate arbitrary magnitude functions. We introduced a class of filters referred to as 2-D LMA filters to realize recur sively computable and stable systems drived from the 2-D spectral factorization problem. The design procedure is straightforward and simple. Filter coefficients are given by the 2-D complex cepstrum coefficients which can be efficiently computed using 2-D FFT algorithm. We presented a stability condition under which we can easily guarantee the stability of the designed filters. We also discussed parallel implementation of the 2-D LMA filters.

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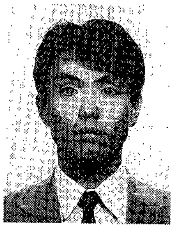

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