

PAPER

Matched Design Method for Concatenated Trellis-Coded Modulation

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SUMMARY A new design method, which is referred to as the matched design method, for concatenated trellis-coded modulation (TCM) is presented. Most of the conventional concatenated TCM employs TCM designed to maximize the minimum squared Euclidean free distance, d_{free}^2 . With the matched design method, we maximize $d_1^2(t)$ instead of d_{free}^2 , where $d_1^2(t)$ is the effective minimum squared Euclidean distance (MSED) when the outer code has a t -error correcting capability. The effective MSED is derived from the Euclidean/Hamming (E/H) joint weight distribution of terminated TCM. We here assume the concatenated TCM whose transmitted symbol corresponds to a symbol of outer code. The new classes of 2-dimensional (2D) and 4-dimensional (4D) codes are found by a computer search. Under the performance measures of the effective MSED or the effective multiplicity, these codes are superior to the conventional codes such as the Ungerboeck's 2D-codes when those are used as an inner code. We disclose an interesting fact that the new class of codes using rate-1/2 encoder is superior to the class of codes using rate-2/3 encoder. This fact implies that the codes using rate-1/2 encoder have two advantages: 1) better overall decoding performance and 2) less decoding complexity.

key words: trellis-coded modulation, concatenated code, weight distribution

1. Introduction

The invention of the trellis-coded modulation (TCM) by Ungerboeck [1]–[3] opened up a new field in coding theory. TCM is constructed on the basis of an expanded signal constellation (compared with an uncoded constellation at the same rate) and a binary convolutional code designed to maximize the minimum Euclidean free distance d_{free}^2 . TCM yields an asymptotic coding gain of 3–6 dB without bandwidth expansion.

In order to achieve more coding gains, Deng and Costello [4], [5] discussed a concatenation of trellis-coded M-ary phase shift keying (MPSK) and Reed-Solomon (RS) codes. In [5], they showed that the concatenation scheme with the RS code of minimum distance 3 yields an additional coding gain of 1.25–1.75 dB compared with that of non-concatenated TCM. Kasami et al. [6] showed that concatenated codes with a block MPSK code and a relatively powerful RS code can

achieve a high reliability. These results suggest that the concatenation of TCM (as an inner code) with block codes (as an outer code) has a potentiality to achieve an additional coding gain compared with that of non-concatenated TCM under the same decoding complexity.

Most of the previous concatenation schemes employ TCM designed by maximizing d_{free}^2 . We call this design method of concatenation scheme the *separate design* method. However, overall decoding performance of concatenation scheme is independent of d_{free}^2 as well as its multiplicity. This is because the error events which have the Euclidean weight* d_{free}^2 can be corrected by the outer block code. We cannot optimize overall decoding performance, under a given decoding complexity, with the separate design method.

In this paper, we present a new design method for concatenated TCM, which is referred to as the *matched design* method. We maximize $d_1^2(t)$ instead of d_{free}^2 where $d_1^2(t)$ is the effective minimum squared Euclidean distance (MSED) when the outer code has a t -error correcting capability. In other words, the matched design method enables us to construct inner-TCM matched to the t -error correcting outer code.

To determine $d_1^2(t)$, the Euclidean/Hamming (E/H) joint weight distribution plays a key role. The E/H joint weight distribution yields the number of codewords with Euclidean weight e and Hamming weight h in terminated TCM.

This paper is organized as follows. In Sect. 2, we prepare notations and definitions used throughout the paper. The effective minimum squared Euclidean weight (MSEW) and the effective multiplicity (or error coefficient) are derived based on the Euclidean/Hamming joint weight distribution. In Sect. 3, we perform an exhaustive search for finding the inner code matched to an outer code and present the results of the search. When the outer code has a t -error correcting capability, there exists superior inner code, with a larger effective MSED or with a smaller effective multiplicity, to the conventional ones (e.g. Ungerboeck's ones). We conclude in Sect. 4.

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*In this paper, the squared Euclidean weight and the squared Euclidean distance is only considered. Thus, we sometimes omit 'squared' when we refer to these quantities.

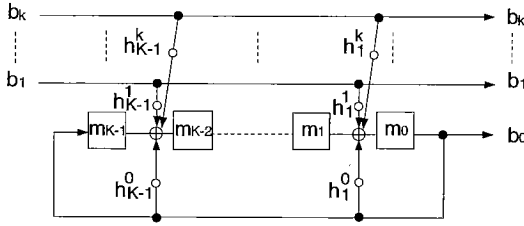


Fig. 1 Feedback encoder.

2. Preliminaries

2.1 Notations

The convolutional encoder discussed in this paper is the class of rate- $k/(k+1)$ feedback encoder as shown in Fig. 1. Let us denote the number of memories in a feedback encoder by K . Thus, the binary trellis code generated by the encoder has 2^K -states. A feedback encoder can be characterized by the parity check polynomials [1] $H^i(D) = \sum_{0 \leq j \leq K} h_j^i D^j$ ($0 \leq i \leq k$), where

$$\begin{aligned} h_0^j &= h_K^j = 0, \quad j \neq 0, \\ &= 1, \quad j = 0 \quad (K \geq 2). \end{aligned} \quad (1)$$

Hereafter, $H^i(D)$ is described in the octal form.

We define the *code complexity* N_c (i.e., the number of branches per 2D) by

$$N_c = (2^K \times 2^k)/L, \quad (2)$$

where $L = 1$ for 2D-TCM and $L = 2$ for 4D-TCM. This quantity is an appropriate measure of decoding complexity when the Viterbi algorithm is used for decoding [8].

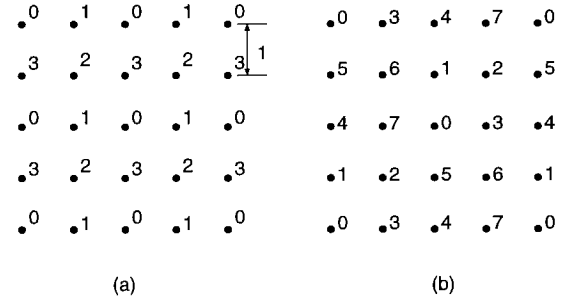
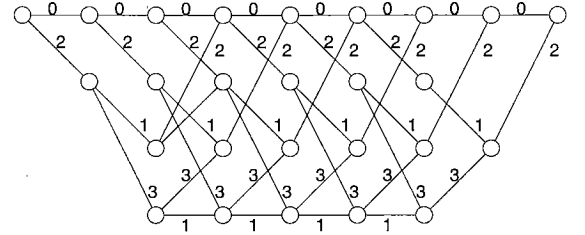
2.2 Terminated TCM

In conventional TCM, the convolutional codes are used in a non-terminated manner. Namely, no input restrictions are imposed. In this paper, we discuss a terminated version of TCM, hereafter referred to as the *terminated TCM*. Note that we use the term 'TCM' as the non-terminated code.

We now define two well known TCM [1], U_4 and U_8 , which will be discussed in the succeeding sections. The code U_4 is based on rate-1/2 feedback encoder ($H^0(D) = 5$, $H^1(D) = 2$) and the partition $Z^2/2Z^2$ (4-subconstellations). The code U_8 is based on rate-2/3 feedback encoder ($H^0(D) = 11$, $H^1(D) = 2$, $H^2(D) = 4$) and the partition $Z^2/2RZ^2$ (8-subconstellations). These partitions are shown in Fig. 2. The output from the feedback encoder is denoted by an integer L such that

$$L = b_k 2^k + b_{k-1} 2^{k-1} + \dots + b_1 2^1 + b_0 2^0. \quad (3)$$

The subconstellations corresponding to L ($0 \leq L \leq 2^{K+1}$) are shown in Fig. 3.

Fig. 2 Two partitions: (a) $Z^2/2Z^2$ (b) $Z^2/2RZ^2$.Fig. 3 Trellis diagram of terminated 4-states code $U_4^{(8)}$.

The terminated TCM of length 8 corresponding to U_4 is denoted by $U_4^{(8)}$. Figure 3 illustrates the terminated trellis diagram of 4-states Ungerboeck code $U_4^{(8)}$. Hereinafter, we shall denote the terminated TCM of length N corresponding to TCM X by $X^{(N)}$.

2.3 Euclidean/Hamming Joint Weight Distribution

We can define the E/H joint weight enumerator for the terminated TCM C by

$$J_C(x, y) = \sum_{\mathbf{V} \in C} x^{w_e(\mathbf{V})} y^{w_h(\mathbf{V})}. \quad (4)$$

where $w_e(\cdot)$ is the Euclidean weight function defined by

$$w_e(\mathbf{V}) = \sum_{0 \leq t \leq N-1} \|v_t - s_0\|^2, \quad (5)$$

$$\mathbf{V} = (v_0, v_1, \dots, v_{N-1}), \quad (6)$$

(the signal point s_0 is referred to as the origin of a signal constellation) and the function $w_h(\cdot)$ is the Hamming weight function given by

$$w_h(\mathbf{V}) = \sum_{0 \leq t \leq N-1} h(v_t), \quad (7)$$

$$h(s) = \begin{cases} 0 & s = s_0, \\ 1 & s \neq s_0. \end{cases} \quad (8)$$

The coefficient $x^e y^h$ in $J_C(x, y)$ represents the number of codewords with the Euclidean weight e and the Hamming weight h in the terminated TCM C .

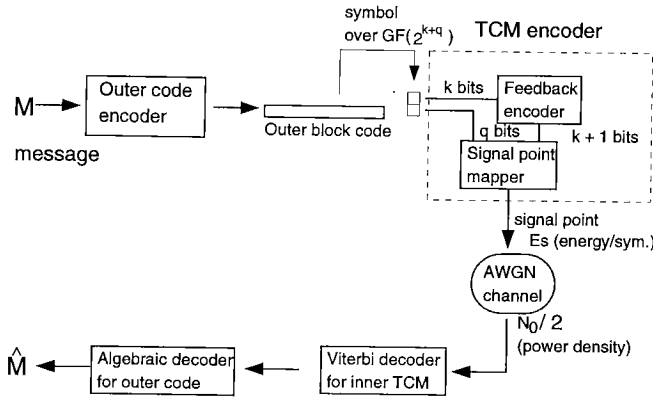


Fig. 4 Channel model.

2.4 Channel Model

Figure 4 illustrates a concatenated TCM channel with an additive white Gaussian noise (AWGN). A message M , which is equally likely transmitted, is first encoded to a block code (such as RS code) of length $N - \lceil K/(k+q) \rceil$ over $\text{GF}(2^{k+q})$ as the outer code. Each symbol consists of $k+q$ bits. The k bits in a symbol are passed to the rate- $k/(k+1)$ feedback encoder and its output bits ($k+1$ bits) choose one of 2^{k+1} subconstellations. The remaining q bits are used to select a signal point from the selected subconstellation. Each subconstellation consists of 2^q signal points. The transmitted signals are disturbed by the additive white Gaussian noise. The receiver first performs the maximum likelihood decoding for TCM with the Viterbi algorithm and the decoded sequences are then passed to the bounded distance decoder for the outer block code. The outer decoder executes an errors-only decoding. In this paper, we assume that no side information such as reliability is passed from the inner decoder to the outer decoder. The *concatenated terminated TCM* is constructed based on concatenating a terminated TCM of length N with a t -error correcting outer code. Namely, a block of an outer code is encoded to a codeword of the terminated TCM.

2.5 Effective Minimum Euclidean Weight and Effective Multiplicity

We assume that a terminated TCM of length N and t -error correcting block outer code are concatenated. When calculating the weight enumerator, we assume an infinite QAM constellation instead of a finite QAM constellation. This assumption leads the distance invariant property of the codes presented here [10]. The effective MSEW $d_1^2(t)$ is defined as the MSEW of the codewords which have Hamming weight more than t . The number of codewords with Euclidean weight $d_1^2(t)$ and Hamming weight more than t is referred to as the effective multiplicity, which is denoted as $M_1(t)$.

Euclid Weight	Hamming Weight							
	0	1	2	3	4	5	6	7
0	1							
4			32					
5					192			
6								
7								
8			32	448				
9					384	3840		3072

Fig. 5 Effective MSEW and effective multiplicity for $U_4^{(8)}$ with $t = 3$.Table 1 Effective i -th squared euclidean weight and effective i -th multiplicity for $U_4^{(8)}$.

t	1st shell		2nd shell		3rd shell	
	$d_1^2(t)$	$M_1(t)$	$d_2^2(t)$	$M_2(t)$	$d_3^2(t)$	$M_3(t)$
0	4	32	5	192	6	576
1	5	192	6	576	7	1536
2	5	192	6	576	7	1536
3	6	576	7	1536	8	3072
4	7	1536	8	3072	9	3072

As an example, we assume that $U_4^{(8)}$ is concatenated with a 3-error correcting block code, and that the transmitted block is the codeword $\mathbf{O} = (s_0, s_0, \dots, s_0)$. The received block (r_0, r_1, \dots, r_7) is decoded with the inner maximum likelihood decoder to a codeword in $U_4^{(8)}$. All the codewords of Hamming weight less than or equal to 3 are correctly decoded to \mathbf{O} with the outer decoder. In this case, the block error performance of overall decoding is dominated by the Euclidean weight distribution of the codewords of Hamming weight more than 3. Thus $d_1^2(3)$ and $M_1(3)$ should be given for evaluating the error performance of concatenated terminated TCM. We can easily compute these quantities, yielding $d_1^2(3) = 6$ and $M_1(3) = 576$ (Fig. 5).

In general, if the outer block code can correct t -errors, the block error probability for overall decoding is dominated by the Euclidean weight distribution of codewords with Hamming weight more than t . Thus we define the effective i -th smallest squared Euclidean weight $d_i^2(t)$ by the i -th smallest Euclidean weight of codewords with Hamming weight more than t and the effective i -th multiplicity $M_i(t)$ by the total number of such codewords. Namely, $d_1^2(t)$ is the MSEW, $d_2^2(t)$ is the second smaller Euclidean weight. We show $d_i^2(t)$ and $M_i(t)$ for $U_4^{(8)}$ in Table 1.

3. Searching for Inner Code Matched to Outer Code

The effective MSEW and the effective multiplicity for the concatenated terminated TCM (with t -error correcting outer code) can be obtained by the method described

in the previous sections. In this section, we perform a code search for finding the code with the largest effective MSED.

3.1 Results of Searching

In the code search, we have tried to find the *asymptotic best code*. The asymptotic best code has 1) the largest effective MSED $d_{1\max}^2(t)$ and 2) the smallest effective multiplicity at $d_{1\max}^2(t)$.

We shall refer to this performance criteria as the *asymptotic best criteria*. Under the asymptotic best criteria, we denote “the code A is superior to the code B” when 1) the code A has the larger effective MSED than that of the code B or 2) the code A and the code B has the same effective MSED but the code A has smaller effective multiplicity than that of the code B.

We perform a code search based on the following combinations:

2D-code: rate-1/2 encoder and the partition $Z^2/2Z^2$ (4-subconstellations)

2D-code: rate-2/3 encoder and the partition $Z^2/2RZ^2$ (8-subconstellations)

4D-code: rate-1/2 encoder and the partition Z^4/RZ^4 (4-subconstellations)

The 4D-TCM using the partition Z^4/RZ^4 has not been previously used, because the MSED of RZ^4 is only 2. The operator R is $\pi/4$ -rotation and $\sqrt{2}$ -scale operator.

In all the searches, the block length of 32 is used. If the block length N is sufficiently large compared with the number of states of the inner code, the block length does not influence the results of the code search.

The results are summarized in Table 2 (2D-code with rate-1/2 encoder), Table 3 (2D-code with rate-2/3 encoder) and Table 4 (4D-code with rate-1/2 encoder). As a reference, we show the parameters of the conventional 2D-codes in Table 5. The codes of 4, 8 and 16-states were found by Ungerboeck [1], 32 and 64-states by Eyuboglu and Li [9].

For comparing the performances of codes, we define the *asymptotic figure of merit of coding*, γ_a , by

$$\gamma_a = 10 \log_{10}(d_1^2(t)/2^r), \quad (9)$$

where r is the redundancy of code per 2D. For 2D-TCM and 4D-TCM, r are 1 and 0.5, respectively. Though this quantity is similar to the coding gain [9], it is defined not by taking into account of the rate loss of the outer code. The coding gain depends not only on the inner code but also on the outer code. This paper mainly deals with inner codes, thus the performance measure independent of outer codes is preferred.

We see that all the new 2D-codes (Tables 2 and 3) with more than 4-states are superior to the referenced 2D-codes (Table 5) under the asymptotic best criteria.

Table 2 Asymptotic best 2D-codes based on the partition $Z^2/2Z^2$ with rate-1/2 encoder.

t	2^K	$H^0(D)$	$H^1(D)$	$d_1^2(t)$	$M_1(t)$	N_c	γ_a
1	4	5	2	5	9.60×10^2	8	3.97
	8	13	4	6	1.79×10^3	16	4.77
	16	23	16	7	6.78×10^3	32	5.44
	32	53	26	8	1.43×10^4	64	6.02
	64	105	62	8	7.93×10^3	128	6.02
2	4	5	2	5	9.60×10^2	8	3.97
	8	13	4	6	1.79×10^3	16	4.77
	16	23	16	7	6.78×10^3	32	5.44
	32	53	26	8	6.40×10^3	64	6.02
	64	117	42	10	2.47×10^5	128	6.98
3	4	5	2	6	3.64×10^3	8	4.77
	8	11	2	6	1.66×10^3	16	4.77
	16	23	16	7	6.78×10^3	32	5.44
	32	53	26	8	6.40×10^3	64	6.02
	64	117	42	10	2.47×10^5	128	6.98
4	4	5	2	7	1.38×10^4	8	5.44
	8	13	6	8	6.40×10^4	16	6.02
	16	23	12	8	2.58×10^4	32	6.02
	32	43	10	8	5.37×10^3	64	6.02
	64	117	42	10	2.47×10^5	128	6.98
5	4	5	2	8	5.22×10^4	8	6.02
	8	11	2	8	2.33×10^4	16	6.02
	16	25	2	8	1.10×10^4	32	6.02
	32	47	10	9	1.21×10^4	64	6.53
	64	133	64	10	8.70×10^4	128	6.98
6	4	5	2	9	1.96×10^5	8	6.53
	8	13	6	10	1.07×10^6	16	6.98
	16	23	12	10	5.01×10^5	32	6.98
	32	45	16	10	1.91×10^5	64	6.98
	64	123	64	10	4.81×10^4	128	6.98

Table 3 Asymptotic best 2D-codes based on the partition $Z^2/2RZ^2$ with rate-2/3 Encoder.

t	2^K	$H^0(D)$	$H^1(D)$	$H^2(D)$	$d_1^2(t)$	$M_1(t)$	N_c	γ_a
1	4	—	—	—	—	—	—	—
	8	11	2	4	5	4.72×10^2	32	3.97
	16	23	4	16	6	1.53×10^3	64	4.77
	32	45	16	34	6	2.48×10^2	128	4.77
	64	115	52	36	7	1.03×10^3	256	5.44
2	4	—	—	—	—	—	—	—
	8	11	2	6	5	4.64×10^2	32	3.97
	16	23	4	14	6	8.64×10^2	64	4.77
	32	45	2	12	7	2.86×10^3	128	5.44
	64	111	36	74	8	1.02×10^4	256	6.02
3	4	—	—	—	—	—	—	—
	8	11	2	6	6	1.74×10^3	32	4.77
	16	21	2	6	6	8.00×10^2	64	4.77
	32	45	2	12	7	2.41×10^3	128	5.44
	64	111	36	74	8	9.80×10^3	256	6.02
4	4	—	—	—	—	—	—	—
	8	11	2	6	7	6.52×10^3	32	5.44
	16	23	6	12	8	3.37×10^4	64	6.02
	32	43	22	14	8	1.57×10^4	128	6.02
	64	103	10	30	8	3.72×10^3	256	6.02
5	4	—	—	—	—	—	—	—
	8	11	2	6	8	2.43×10^4	32	6.02
	16	21	2	6	8	1.09×10^4	64	6.02
	32	41	2	6	8	4.86×10^3	128	6.02
	64	107	10	30	8	1.24×10^3	256	6.02
6	4	—	—	—	—	—	—	—
	8	11	2	6	9	9.01×10^4	32	6.53
	16	23	6	12	10	5.90×10^5	64	6.98
	32	43	22	14	10	2.91×10^5	128	6.98
	64	105	56	24	10	1.37×10^5	256	6.98

Table 4 Asymptotic best 4D-codes based on the partition Z^4/RZ^4 with rate-1/2 encoder.

t	2^K	$H^0(D)$	$H^1(D)$	$d_1^2(t)$	$M_1(t)$	N_c	γ_a
1	4	7	2	4	4.68×10^4	4	4.51
	8	13	6	4	3.17×10^4	8	4.51
	16	21	16	4	3.17×10^4	16	4.51
	32	41	16	4	3.17×10^4	32	4.51
	64	101	16	4	3.17×10^4	64	4.51
2	4	7	2	4	1.51×10^4	4	4.51
	8	13	6	5	5.73×10^4	8	5.48
	16	37	12	6	2.75×10^6	16	6.27
	32	45	36	6	2.54×10^6	32	6.27
	64	103	56	6	2.54×10^6	64	6.27
3	4	5	2	4	7.16×10^3	4	4.51
	8	13	2	6	2.37×10^6	8	6.27
	16	37	12	6	2.17×10^5	16	6.27
	32	57	32	8	1.58×10^8	32	7.52
	64	127	72	8	1.48×10^8	64	7.52
4	4	7	2	6	2.10×10^6	4	6.27
	8	13	6	6	3.23×10^5	8	6.27
	16	23	12	7	6.79×10^6	16	6.94
	32	57	32	8	1.11×10^7	32	7.52
	64	127	72	8	1.57×10^8	64	7.52
5	4	7	2	6	1.06×10^5	4	6.27
	8	17	2	6	9.83×10^4	8	6.27
	16	23	16	8	6.69×10^7	16	7.52
	32	67	22	8	6.16×10^6	32	7.52
	64	123	32	9	1.15×10^8	64	8.03
6	4	7	2	8	9.89×10^7	4	7.52
	8	13	2	8	5.83×10^7	8	7.52
	16	21	16	9	7.12×10^8	16	8.03
	32	63	22	9	7.12×10^8	32	8.03
	64	157	52	10	6.81×10^9	64	8.49

Table 5 Reference 2D-codes (Ungerboeck's, Eyuboglu and Li's codes).

t	2^K	$H^0(D)$	$H^1(D)$	$H^2(D)$	$d_1^2(t)$	$M_1(t)$	N_c	γ_a
1	4	5	2	—	5	9.60×10^2	8	3.97
	8	11	2	4	5	4.72×10^2	32	3.97
	16	23	4	16	6	1.53×10^3	64	4.77
	32	45	16	34	6	2.48×10^2	128	4.77
	64	115	52	36	7	1.03×10^3	256	5.44
2	4	5	2	—	5	9.60×10^2	8	3.97
	8	11	2	4	5	4.72×10^2	32	3.97
	16	23	4	16	6	1.53×10^3	64	4.77
	32	45	16	34	7	3.31×10^3	128	5.44
	64	115	52	36	7	1.03×10^3	256	5.44
3	4	5	2	—	6	3.64×10^3	8	4.77
	8	11	2	4	6	1.77×10^3	32	4.77
	16	23	4	16	6	1.29×10^3	64	4.77
	32	45	16	34	7	3.31×10^3	128	5.44
	64	115	52	36	7	7.96×10^2	256	5.44
4	4	5	2	—	7	1.38×10^4	8	5.44
	8	11	2	4	7	6.65×10^3	32	5.44
	16	23	4	16	7	2.49×10^3	64	5.44
	32	45	16	34	7	3.31×10^3	128	5.44
	64	115	52	36	7	7.96×10^2	256	5.44
5	4	5	2	—	8	5.22×10^4	8	6.02
	8	11	2	4	8	2.48×10^4	32	6.02
	16	23	4	16	8	1.33×10^4	64	6.02
	32	45	16	34	8	5.84×10^3	128	6.02
	64	115	52	36	8	3.42×10^3	256	6.02
6	4	5	2	—	9	1.96×10^5	8	6.53
	8	11	2	4	9	9.21×10^4	32	6.53
	16	23	4	16	9	4.20×10^4	64	6.53
	32	45	16	34	9	1.06×10^4	128	6.53
	64	115	52	36	9	1.12×10^4	256	6.53

Table 6 E/H joint weight distribution for new 16-states 2D-code with $t = 4$ based on the partition $Z^2/2Z^2$.

Euclid Weight	Hamming Weight						
	0	1	2	3	4	5	6
0	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	128	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	1792	0	0
7	0	0	0	0	0	0	0
8	0	128	7936	0	0	0	25856
9	0	0	0	0	0	0	0
10	0	0	0	0	7168	200704	0

Table 7 E/H joint weight distribution for 16-states Ungerboeck's code with $t = 4$ based on the partition $Z^2/2RZ^2$.

Euclid Weight	Hamming Weight						
	0	1	2	3	4	5	6
0	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	240	1296	0	0
7	0	0	0	944	912	2496	0
8	0	128	0	0	1360	4896	13392
9	0	0	0	0	2624	12416	33952
10	0	0	0	0	6512	12704	48464

The comparison is performed among the codes with the same number of states and t . For example, the asymptotic best 2D-codes of 16-states with $t = 4$ are superior to the 16-states Ungerboeck's code with $t = 4$ by 0.58 dB in γ_a . We observe an interesting fact that the new 16-states code with $t = 4$ using rate-1/2 encoder has the quite different E/H JWD compared with that of the Ungerboeck's 16-states code. These weight distributions are shown in Tables 6 and 7. We compare the block error probability of the new 16-states code and the Ungerboeck's 16-states code through a simulation. The results of the simulation are shown in Fig. 6. We see that the new code has a smaller block error probability than the Ungerboeck's one with $t = 4$. Besides the code complexity of the new code ($N_c = 32$) is improved by a factor of 2, compared with the Ungerboeck's one ($N_c = 64$).

We next compare the classes of codes using rate-1/2 encoder (Table 2) and rate-2/3 encoder (Table 3). All the codes in Table 2 are superior to those in Table 3 under the asymptotic best criteria. The code complexities of the codes in Table 2 are improved by a factor of 2, compared with those in Table 3 under the same number of states. This is because the number of branches of rate-1/2 encoder is only half that of rate-2/3 encoder. For example, let us focus our attention on the asymptotic best 16-states 2D-codes with $t = 4$. Both codes, namely code using rate-1/2 encoder and rate-2/3 encoder, have $d_1(4) = 8$. However, the code using rate-1/2 encoder has $M_1(4) = 2.58 \times 10^4$, $N_c = 32$ and the code using rate-2/3 encoder, 3.37×10^4 , $N_c = 64$. It is rather sur-

prising that the codes with less code complexity are superior to those having a large code complexity. This is partly due to the fact that the MSED of the code sequences, d_{seq}^2 , is more important than the MSED of the subconstellations, d_{sub}^2 , particularly, for concatenation case.

With respect to γ_a , we see that the 4D-codes are superior to the 2D-codes under the same code complexity. However, it would be fair to point out that the effective multiplicity of the 4D-codes is very large compared with that of the 2D-codes. This is not desired in channel with low SNR channels, because the error performance largely depends on multiplicity in low SNR. For a practical use, other performance criteria (including multiplicity) instead of the asymptotic best criteria may be required when searching 4D-codes. For example, sacrificing the increasing of the code complexity, Wei [8] reduced the multiplicity (not increasing d_{free}^2). Though 4D-codes have such a large multiplicity, they have an excellent performance/complexity tradeoff compared with other codes.

4. Conclusion

The principal subject of this paper is to present the matched design method for concatenated TCM. Several new inner codes are found by a computer search. These codes are not necessarily efficient for a non-concatenated case, because the free distance d_{free}^2 of those codes are smaller than that of the conventional codes. However, for a concatenated case, these codes have a larger effective MSED or a smaller effective multiplicity than that of the conventional codes (Fig. 6). These results suggest that, in concatenated TCM, the inner code should be

chosen by taking into account of not only the signal constellation but also the minimum distance of outer code.

The matched design method for concatenated TCM scheme can be summarized as follows:

- 1) *Partitioning the signal constellation into 4 subconstellations*: Subconstellations of more than 4 (e.g., 8 or 16-subconstellations) may not give any advantage in concatenated scheme;
- 2) *Searching for the best code matched to the error-correcting capability of outer code*.

The matched design method yields the better codes compared with the separate design method. Besides, it can yield the codes where the code complexity is significantly reduced while maintaining overall decoding performance. In this paper, we have assumed the concatenated TCM that a transmitted symbol corresponds to a symbol of an outer code. Such codes are in a restricted class of concatenated TCM. Further investigations on other types of concatenated TCM are needed, such as concatenated TCM with interleaved outer codes and with unmatched inner and outer symbol.

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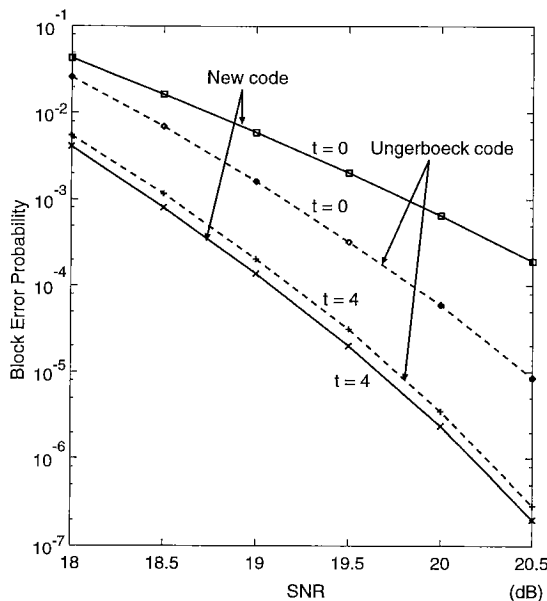


Fig. 6 Block error probabilities (16-states).

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