

LETTER

An Improved Circuit Theory for the Analysis of Longer Co-planar Dipole Antennas

Adam Icarus IMORO[†], Student Member, Yoshihisa KANI^{††}, Nonmember,
Naoki INAGAKI[†], and Nobuyoshi KIKUMA[†], Members

SUMMARY The valid region for the application of the conventional *Improved Circuit Theory* (ICT) in the analysis of wire antennas is established. To further extend the application of ICT to the analysis of much longer antennas, Tai's trial function is used to derive new formulas for the impedance matrix. Unlike the conventional ICT trial function, Tai's trial functions lead to input impedances which are finite irrespective of antenna length. Results of the new ICT impedance formulas are comparable in accuracy with the general method of moments. Moreover, since all the elements of the new formula have been expressed in closed-form, the resulting ICT algorithm is still superior in terms of computer running time with lesser storage requirement compared to other conventional methods like method of moments. This would enhance ICT applications in CAD/CAE systems.

key words: ICT, trial function, input impedance CAD/CAE, MoM

1. Introduction

The *Improved Circuit Theory* (ICT) [1] is well known to be an accurate method for analyzing array of co-planar dipole antenna systems (antennas fed in the same plane) such as Yagi-Uda and log-periodic arrays [2]–[3]. Its accuracy in analyzing such systems is comparable to that of *method of moments* (MoM). But the key advantage of the ICT method over MoM is its computational efficiency. By expressing all the formulas in the evaluation of the input impedance in closed-form, an ICT implementation at much reduced running time has been reported [4]–[6]. This advantage of ICT has therefore been exploited in such systems like CAD/CAE [7] for antenna evaluation.

As a semi-analytical method [6], the ICT method improves the classical EMF method of analyzing multi element antennas by using a Storer two-term current function in an extended variational principle to derive the impedance matrix. This improvement is as a result of ICT addressing the inconsistency in the classical EMF method [1], [8].

Even though the efficiency of ICT has witnessed wide applications in antenna evaluations of co-planar dipole arrays including design of CAD/CAE tools [7],

the limitation of the current implementation is yet to be reported.

This letter therefore first addresses the issue of the limitation in the application of the conventional ICT in relation to the current functions used to derive the formulas for the generalized self and mutual impedances matrices in a two-term representation. It then presents much improved impedance evaluation formulas as a result of modification of the conventional current functions. This leads to an extended application of ICT, especially for the evaluation of a much larger co-planar dipole arrays. Because all the formulas can be expressed in closed-form, ICT still maintains its advantage of being an efficient method of evaluating co-planar multi element linear antennas. For simplicity the element lengths of the system under consideration are specified as being equal.

2. Limitation of Conventional ICT Method

For z -directed dipole antennas, the ICT method in conventional application uses an extended variational principle and the following Storer two-term current functions for the i^{th} element to derive the generalized input impedance.

$$f^1(z_i) = \frac{\sin k(h_i - |z_i|)}{\sin kh_i} \quad (i = 1, 2, \dots) \quad (1)$$

$$f^2(z_i) = \frac{1 - \cos k(h_i - |z_i|)}{1 - \cos kh_i}, \quad (i = 1, 2, \dots) \quad (2)$$

These functions were originally used by Storer for his variational input impedance of single isolated elements [9]. The application of these Storer trial functions in a ICT constituted a drastic improvement to the classical EMF method [1], [7], in that it removed the inconsistencies in the classical method [8].

However, for center-fed dipoles, a close examination of the current functions would show that both functions are zero at the antenna input if the lengths are 2λ or an even multiples of such lengths. For these situations, it is easily deduced from the denominators of Eqs. (1) and (2) that they are simultaneously zero, thus predicting infinite impedance at the input. This also implies that antennas with lengths around these values

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[†]The authors are with Department of Electrical and Computer Engineering, Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya-shi, 466 Japan.

^{††}The author is with Toyota Auto Body Co. Ltd, 100, Kanayama, Ichiriyama-cho, Kariya-shi, 448 Japan.

would provide very high input impedances, that is rendering the conventional ICT inapplicable for such situations.

As would be seen later, the variational method based on Tai's modification of Storer's variational theory for single isolated elements which we shall call here and thereafter as Tai's variational method, gives finite input impedances irrespective of the length of the dipole.

Tai's variational input impedance is actually an improvement of Storer trial function on which ICT is based. Tai suggested a modification to the original Storer trial function in order to extend its applications to much longer single element antennas [9], [10], although he had not also conceived that its extension to the case of multi element antennas would have led to an evolutionary method like ICT.

Even though the conventional ICT valid range covers a large number of antennas in practical applications, for antennas requiring much longer lengths for wider bandwidth operations like in some log-periodic antennas [10], we would need to resort to application of MoM to evaluate such systems, notwithstanding the fact that MoM is inherently time intensive and would therefore slow down applications to CAD/CAE systems.

To therefore extend the application of ICT to much longer dipoles, the next section considers the application of Tai's trial functions for single element to the derivation of ICT impedance for multi-element case in what we call a Tai Two-term ICT representation. For simplicity, this letter limits the derivation of the new elements of the ICT impedance to arrays of equal lengths.

3. Tai Two-Term ICT

Tai has shown that the use of the current trial function [9], [10]

$$f^2(z_i) = \frac{k(h_i - |z_i|) \cos k(h_i - |z_i|)}{kh_i \cos kh_i} \quad (3)$$

instead of Eq. (2) as used by Storer in his variational method leads to impedance formulas which are applicable for all element lengths [11], [12].

In this section we shall consider the application of Tai's trial function to the derivation of the elements of the impedance matrix of ICT.

3.1 Two-Term Representation

The ICT generalized impedance matrix is given as [1]

$$Z_{ij}^{lm} = - \int_{-h_i-h_j}^{h_i} \int_{-h_i-h_j}^{h_j} f_i^l(z_i) G_{ij}(z_i, z_j) f_j^m(z_j) dz_i dz_j \quad (4)$$

($l = 1, 2; m = 1, 2$)

where i and j represent elements of the co-planar dipole

arrays and $G_{ij}(z_i, z_j)$ is the appropriate Green's function.

The application of Tai's trial function for computing the elements of the impedance matrix in a two-term representation would involve the current distributions given in Eqs. (1) and (3).

Therefore for an array elements of the same length ($2h_i/\lambda = 2h_j/\lambda = 2h/\lambda$), Eqs. (1) and (3) have been used in a two-term representation to derive elements of the impedance matrix. Details of the final expressions for multi element antennas with identical lengths are given in the appendix. Z_{ij}^{11} is not shown because it is identical as in the conventional ICT [1].

4. Results and Discussion

In this section we compare the various admittance values using the conventional and ICT formulas based on Tai's trial function, designated as *Tai two-term ICT*, the variational method of Tai for single elements and the MoM [13]. The Computational times of the various methods are also compared.

4.1 Admittances for Various Antenna Lengths

For Hallén's parameter $\Omega = 2 \ln \frac{2h}{a} = 10, 12$ where $2h/\lambda$ is the antenna length and a/λ is the radius, we have compared the input admittances of MoM, Tai's variational method for single elements, conventional ICT or Storer Two-term ICT and Tai two-term ICT methods. It should be noted that for a single element case, the conventional ICT method is Storer's Variational method itself.

The results are shown in Figs. 1 and 2. The conventional ICT method (Storer Two-term ICT) based on Storer's variational trial functions is seen to breakdown around lengths of 1.85λ or more.

However, it is clear from the results that there is general agreement between the Tai two-term ICT and the other two methods, especially for thinner antenna lengths (see Fig. 2).

At this point it is appropriate to discuss the differences between Tai variational method for single element antennas and Tai Two-term ICT. Tai's variational method utilizes the thin wire approximation with his formulas expressed in terms of Hallén's parameter [14]. They cannot therefore be applied to multi-element case. However, Tai Two-term ICT as reported in this letter, is not constrained by this thin wire approximation. As shown in the appendix the element spacing is a factor in the computation of the self- and mutual impedances in the Tai Two-term ICT, the spacing reducing to the radius of an element for the computation of the self-term. Thus, Tai Two-term ICT only reduces to Tai's variational method when the thin approximation applies for calculating input impedances of single elements. The

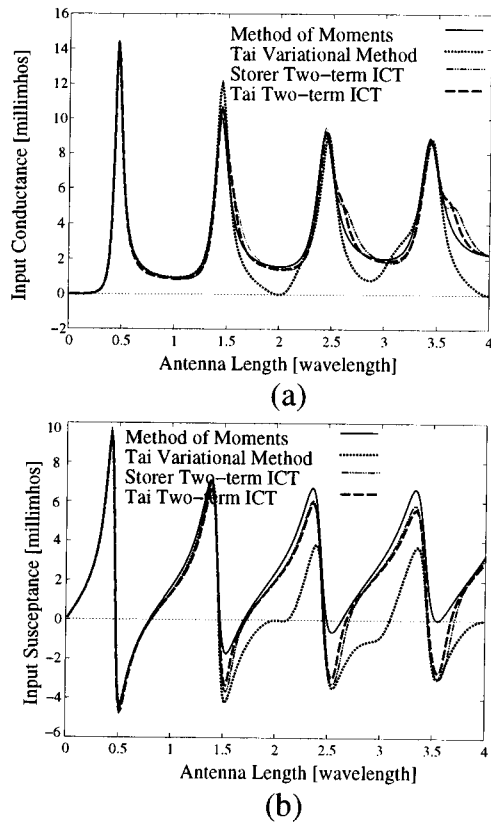


Fig. 1 Input admittance for $\Omega = 10$ (a) conductance, (b) susceptance.

two methods are therefore expected to be in greater agreement for thinner radii as suggested in Fig. 2.

4.2 Analysis of Multi-Element Antennas

For 12-element co-phasally fed linear arrays of similar lengths of half- and two-wavelengths respectively, we calculated the normalized current amplitudes, driving point impedances and their phases using MoM [13], Tai Two-term ICT and Storer Two-term ICT.

The results of the three algorithms are shown in Figs. 3 and 4 for the two different lengths respectively. They illustrate how each method computes the mutual coupling between elements in an array or multi element environment. The results of the half-wave dipole system are shown in Fig. 3 while that for the two-wavelength dipoles are shown in Fig. 4. Each computation has been done with a Hallén's parameter ($\Omega = 10$) and a load impedance of 72 ohm.

Figure 3 shows how all the three methods of computation are in good agreement. In particular, the driving point impedance of MoM with five piecewise sinusoidal modes and Tai Two-term ICT are in better agreement.

As expected, Fig. 4 shows that Storer Two-term ICT which was in good agreement with the other meth-

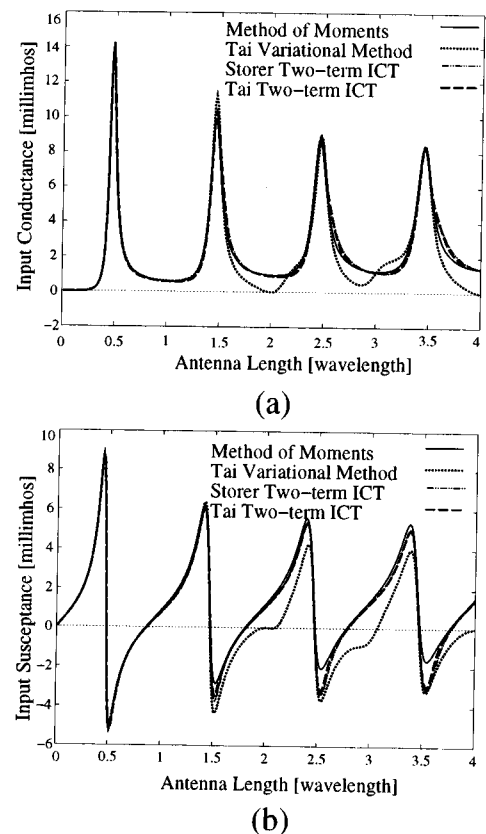


Fig. 2 Input admittance for $\Omega = 12$ (a) conductance, (b) susceptance.

ods for the half-wavelength system in the previous figure totally failing for the two-wavelength system. The extremely high driving-point impedances of Storer Two-term ICT, shown in the insert in Fig. 4(b), is an indication that for all practical purposes an open circuit exists for this case. For the same length and with 11 piecewise sinusoidal expansion modes in a Galerkin scheme, the MoM and Tai Two-term ICT are in reasonable agreement.

Thus, a change or modification of conventional trial function leads to an ICT algorithm which would be capable of analyzing much longer linear dipoles with accuracy comparable to that of more versatile MoM procedure, but with an advantage of requiring lesser CPU time and storage.

4.3 Computational Time

The main advantage of the ICT method is its relatively shorter computational time and storage compared to MoM [4]–[6]. For the analysis of the systems described in the previous section, we have compared the times of the ICT methods and MoM as shown in the Table 1 below.

The computation was carried out on an NEC PC-9801 VX personal computer with about five piecewise

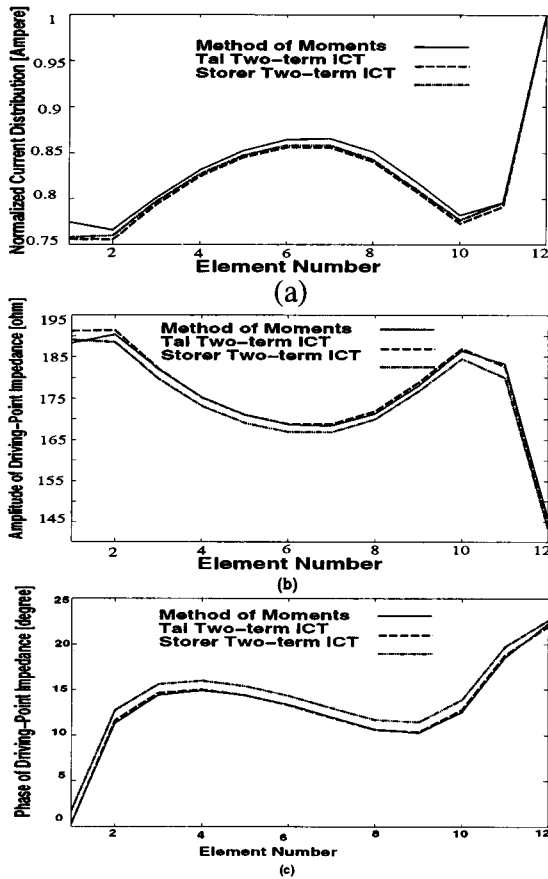


Fig. 3 Amplitude of Input (a) current, (b) impedance, (c) and phase of $\lambda/2$ identical lengths 12-element linear array with 72-ohm load for Hallén's parameter $\Omega = 10$.

Table 1 CPU Time on NEC PC-9801 VX.

Method	Time [seconds]
Tai Two-term ICT	0.11
Storer Two-term ICT	0.07
Tai Variational EMF	0.03
Method of Moments	96

sinusoidal expansion functions in a Galerkin method with the MoM [13].

It is clear from the results that the ICT algorithm with the new functions has relatively shorter computational time requirement. This is as a result of expressing all the components of the functions in closed-form.

5. Conclusion

The valid lengths of co-planar center-fed dipole antennas to which the conventional ICT method has been established. To further extend ICT to the analysis of much longer antennas, we have shown that a more judicious choice of trial functions leads to an impedance formula which expands the region of validity of ICT in the analysis of co-planar dipole antennas. As expected, the new formula gives uniformly finite results and are

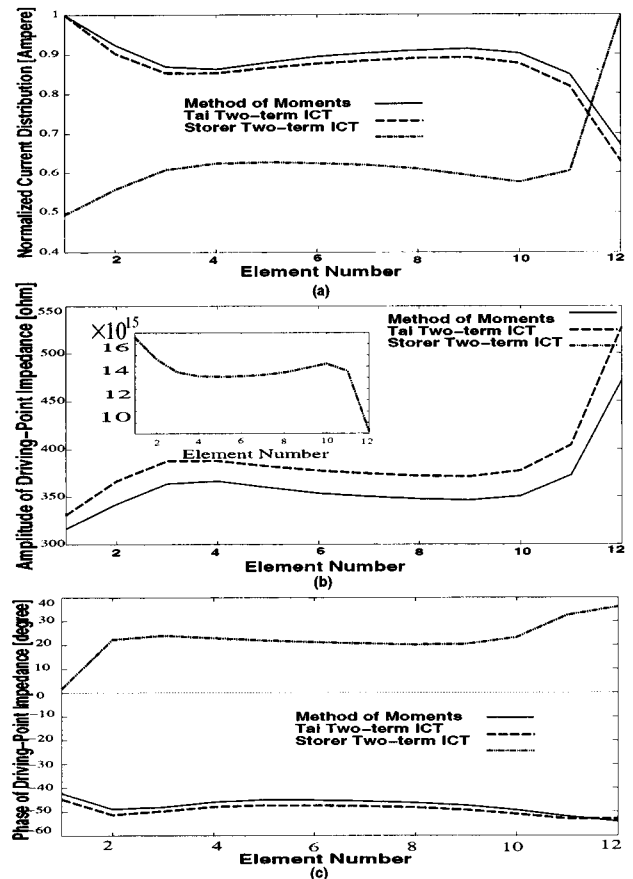


Fig. 4 Amplitude of Input (a) current, (b) impedance, (c) and phase of 2λ identical lengths 12-element linear array with 72-ohm load for Hallén's parameter $\Omega = 10$.

comparable in accuracy with conventional methods like Method of Moments. This has been confirmed by comparing the input admittances for single element cases, and then the analysis of two types of similar linear arrays of dipoles.

Because the Tai Two-term ICT formulas are in closed-form, they require relatively lesser computational time and computer storage. A comparison of the computational times shows that the ICT method still requires far lesser CPU time compared to the inherently time intensive numerical method of moments.

The results of this letter applies to arrays of equal lengths but the argument could be extended to the derivation of impedance formulas which are applicable to co-planar arrays of arbitrary lengths and configurations. This work is currently under way and the final results could be very useful in CAD/CAE antenna expert systems.

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Appendix: Generalized Impedance Matrix Formulas of Tai Two-Term ICT Representation

This appendix gives the Tai Two-term ICT generalized impedance formulas for the case of antennas of lengths $2h/\lambda = 2h_i/\lambda = 2h_j/\lambda$ where i^{th} and j^{th} element in the array are considered to be the same length. Also the inter-element spacing are defined as $D_{ij} = D \equiv kd$ where D reduces to the radius of an element in the case of single elements or the calculation of the self-impedance in a system of linear or circular arrays.

For multi-element system of the above descriptions, the generalized impedance matrix elements defined in Eq. (4), using Eqs. (1) and (3) in a Two-term representation are given in Eqs. (A-1) and (A-2) below. The definition of Z_{ij}^{11} is the same as in the conventional implementation [1] and would therefore not be given

here.

$$Z_{ij}^{12} = j60 \csc 2kh [2kh \cos 2kh C_{kd}(2kh) + 2kh \sin 2kh S_{kd}(2kh) - \cos 2kh CU1_{kd}(2kh) - \sin 2kh SU1_{kd}(2kh) + 2(1 + \cos 2kh) CU1_{kd}(kh) + 2 \sin 2kh SU1_{kd}(kh) - kh(1 + 3 \cos 2kh) C_{kd}(kh) - 3kh \sin 2kh S_{kd}(kh)] \quad (A \cdot 1)$$

$$Z_{ij}^{22} = j \frac{30}{k^2 h^2 (1 + \cos 2kh)} \left\{ [-\sin 2kh CU2_{kd}(2kh) + \cos 2kh SU2_{kd}(2kh) + [4kh \sin 2kh - \cos 2kh] CU1_{kd}(2kh) - [4kh \cos 2kh + \sin 2kh] SU1_{kd}(2kh) + [(1 - 4(kh)^2) \sin 2kh + 2kh \cos 2kh] C_{kd}(2kh) + [2kh \sin 2kh - (1 - 4(kh)^2) \cos 2kh] S_{kd}(2kh) + 2 \sin 2kh CU2_{kd}(kh) - 2 \cos 2kh SU2_{kd}(kh) + 2[1 + \cos 2kh - 4kh \sin 2kh] CU1_{kd}(kh) + 2[-kh + \sin 2kh + 4kh \cos 2kh] SU1_{kd}(kh) - 2[kh(1 + \cos 2kh) - (3(kh)^2 - 1) \sin 2kh] \cdot C_{kd}(kh) + 2[(kh)^2 + 1 - kh \sin 2kh - ((kh)^2 - 1) \cos 2kh] S_{kd}(kh) \right\} \quad (A \cdot 2)$$

where

$$CU1_D(x) = 2 \int_0^x \frac{\exp(-j\sqrt{D^2 + t^2})}{\sqrt{D^2 + t^2}} t \cos t dt \quad (A \cdot 3)$$

$$CU2_D(x) = 2 \int_0^x \frac{\exp(-j\sqrt{D^2 + t^2})}{\sqrt{D^2 + t^2}} t^2 \cos t dt \quad (A \cdot 4)$$

$$SU1_D(x) = 2 \int_0^x \frac{\exp(-j\sqrt{D^2 + t^2})}{\sqrt{D^2 + t^2}} t \sin t dt \quad (A \cdot 5)$$

$$SU2_D(x) = 2 \int_0^x \frac{\exp(-j\sqrt{D^2 + t^2})}{\sqrt{D^2 + t^2}} t^2 \sin t dt \quad (A \cdot 6)$$

Equations (A-3) to (A-6) can further be expressed in closed-form as [15]:

$$CU1_D(x) = \frac{1}{2} \left\{ -2(D + j) e^{-jD} + j(e^{-jv} + e^{-ju}) + D^2 \left[\left(\frac{e^{-jv}}{v} + \frac{e^{-ju}}{u} \right) + S_D(x) \right] \right\} \quad (A \cdot 7)$$

$$SU1_D(x) = \frac{1}{2} \left\{ jD^2 \left(\frac{e^{-jv}}{v} - \frac{e^{-ju}}{u} \right) - (e^{-jv} - e^{-ju}) - D^2 C_D(x) \right\} \quad (\text{A} \cdot 8)$$

$$CU2_D(x) = \frac{1}{2} \left\{ \left(\frac{1}{2} + j\frac{v}{2} - \left(\frac{D^2}{2v} \right)^2 + j\frac{D^4}{4v} \right) e^{-jv} - \left(\frac{1}{2} + j\frac{u}{2} - \left(\frac{D^2}{2u} \right)^2 + j\frac{D^4}{4u} \right) e^{-ju} - D^2 \left[1 + \left(\frac{D}{2} \right)^2 \right] C_D(x) \right\} \quad (\text{A} \cdot 9)$$

$$SU2_D(x) = \frac{1}{2} \left\{ \left(D - j + j\frac{D^2}{2} + \frac{D^3}{2} \right) e^{-jD} \right.$$

$$\left. - \left(\frac{v}{2} - \frac{j}{2} + j \left(\frac{D^2}{2v} \right)^2 + \frac{D^4}{4v} \right) e^{-jv} \right.$$

$$\left. - \left(\frac{u}{2} - \frac{j}{2} + j \left(\frac{D^2}{2u} \right)^2 + \frac{D^4}{4u} \right) e^{-ju} \right.$$

$$\left. - D^2 \left[1 + \left(\frac{D}{2} \right)^2 \right] S_D(x) \right\} \quad (\text{A} \cdot 10)$$

where

$$u = \sqrt{D^2 + x^2} - x, \quad v = \sqrt{D^2 + x^2} + x \quad (\text{A} \cdot 11)$$

and $C_D(x)$ and $S_D(x)$ are as defined in [1].