PAPER

An Efficient ICT Method for Analysis of Co-planar Dipole Antenna Arrays of Arbitrary Lengths

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SUMMARY A more judicious choice of trial functions to implement the Improved Circuit Theory (ICT) application to multi-element antennas is achieved. These new trial functions, based on Tai's modified variational implementation for single element antennas, leads to an ICT implementation applicable to much longer co-planar dipole arrays. The accuracy of the generalized impedance formulas is in good agreement with the method of moments. Moreover, all these generalized formulas including the radiation pattern expressions are all in closed-form. This leads to an ICT implementation which still requires much shorter CPU time and lesser computer storage compared to method of moments. Thus, for co-planar dipole arrays, the proposed implementation presents a relatively very efficient method and would therefore be found useful in applications such as CAD/CAE systems.

key words: *MoM, ICT, co-planar dipole arrays, trial function, variational principle, input impedance,. CAD/CAE applications*

1. Introduction

It is well known that to carry out an accurate analysis of a parallel coplanar dipole arrays, one first solves a system of coupled Fredholm integral equations of the first kind (or a system which is mathematically equivalent) to determine the current distribution on each dipole [1], [2]. Once the current distributions are known, all other quantities such as field patterns, and directivities can in principle be calculated [1]. One method of solution of this set of integral equations is by the *Improved Circuit Theory* (ICT) [3]. Other methods include the *Method of Moments* (MoM) [4]–[6], the "two-term theory" developed by King [7] and King et al. [8].

MoM [4]–[6] is a numerical method of solving electromagnectic boundary value problems including antennas. The MoM is very versatile, accurate but computationally intensive.

The ICT is an extension of the classical EMF for the analysis of multielement antennas. It uses an extended variational principle to derive generalized self- and mutual impedance formulas. In its conventional implementation, ICT uses a Storer Two-term representation which is equivalent to a Garlekin MoM [9] where the basis functions are chosen to be the same as the weighting functions.

Compared to MoM, ICT is a much more efficient method of analyzing co-planar dipole arrays because it uses two-term current functions and with all the formulas are expressible in closed-form [9]–[11].

It has been shown that it is possible to extend the application of ICT to much longer dipoles of similar lengths [12], [13] by use of trial functions adopted from Tai's modified implementation of Storer's variational formula for single elements [14]–[17]. But no detailed trial functions analysis has been done.

This paper does a trial function analysis which leads us to conclude that Tai's trial functions as in the similar element case [12], [13] is a more judicious choice of trial functions to implement ICT with no restriction on the antenna length. The accuracy of the generalized impedance formulas are shown to be in good agreement with the MoM. Moreover, all these generalized formulas including the radiation pattern expressions are all in closed-form. This leads to an ICT implementation which still requires much shorter CPU time and lesser computer storage compared to MoM.

Thus, for co-planar dipole arrays, the proposed implementation presents a relatively very efficient method and would therefore be found useful in applications such as CAD/CAE systems. NEC-MOM codes (NEC2, NEC3, NEC4) have been used for such applications [19] as almighty tools for linear antennas, but instead ICT is a special tool developed for co-planar structures and therefore more numerically efficient for antenna arrays in this category.

Details of the rest of the paper follows. In Sect. 2 we briefly present the main circuit equation of ICT. This is followed by Sect. 3 where we discuss how we arrive at a more judicious choice of trial functions to more accurately implement ICT, thus extending its application to longer and arbitrary dipoles. In Sect. 4 we consider the derivation of both the generalized self- and mutual impedances and the far-field pattern formulas. Validation of the new implementation using calculated results compared with other conventional methods and their CPU time statistics are carried out in Sect. 5. In Sect. 6 we discuss a hybrid implementation of the ICT algorithm consisting of the conventional and new im-

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plementation. Section 7 is the conclusion. The paper ends with the references and an appendix where more details of the main formulas are given.

2. Improved Circuit Theory

The ICT is best applied to co-planar arrays of general configuration shown in Fig. 1. For such structures, the ICT circuit equation involving the generalized impedance matrix, the voltage and current distribution is given as [3]:

$$\sum_{m=1}^{M}[Z^{lm}][I^{m}] = [V], \quad l = 1, \ldots, M. \tag{1}$$

where Z^{lm} is an M×M matrix with Z_{ij}^{lm} being the (i,j)th elements of the generalized self- or mutual impedances which are defined by [3]

$$Z_{ij}{}^{lm} = -\int_{-h_i}^{h_i}\int_{-h_j}^{h_j} f_i{}^{l}(z_i)G_{ij}(z_i,z_j)f_j{}^{m}(z_j)dz_i dz_j. \tag{2}$$

The current distribution $I_i(z_i)$ is represented as a linear combination of trial functions, each normalized to unity at the driving points and expressed as

$$[I^{m}] = \sum_{l=1}^{M} I_i^l f_i^l(z_i), \quad f_i^l(0) = 1. \tag{3}$$

$f_i^l(z_i)$ must be smooth near the driving points $z = 0$.

The EMF method applied to multielement antennas fixes the current distribution at the beginning which corresponds to the distribution in Eq. (3) for $M = 1$. This is equivalent to a first order approximation which is adequate for antenna lengths of the first resonance region or less [14]–[17].

In particular, the application of this first order current

$$f^1(z_i) = \frac{\sin k(h_i - |z_i|)}{\sin kh_i} \tag{4}$$

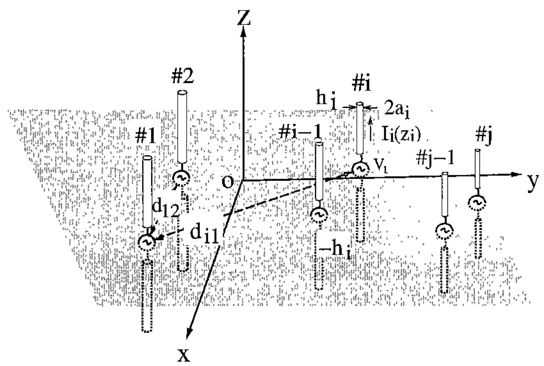


Fig. 1 Co-planar dipole arrays of arbitrary configuration.

to the analysis of co-planar arrays restricts the bandwidth of the structure to less than 2:1 [5]. To therefore improve the EMF results, the ICT uses the Storer two-term trial functions consisting of of Eq. (4) and [3], [14]–[17]

$$f^2(z_i) = \frac{1 - \cos k(h_i - |z_i|)}{1 - \cos kh_i} \tag{5}$$

to derive the elements of the generalized impedance matrix.

Storer [3], [14]–[17] was the first to use these trial functions in his variational implementation of EMF although restricted to single elements. We therefore designate the ICT implementation based on these trial functions as *Storer Two-term ICT* or SICT.

SICT is a considerable improvement of use of analytical/semi-analytical techniques to the analysis of multielement element antennas compared to EMF [3]. And to further reduce the implementation time of SICT, approximate closed-form formulas have been derived to replace a function in the SICT algorithm which requires numerical integration [9]–[11]. We designate this as *Storer Faster Two-term ICT* or SFICT.

Even though SFICT has several advantages [9]–[11], the use of Eqs. (4) and (5) in the derivation of the generalized impedance formulas means that its applications are restricted to the lengths of antennas for which the current functions are valid. For instance, their simultaneous vanishing at the input of a center-fed dipole for any particular antenna configuration would give rise to an infinite input impedance. The next section considers the choice of a more accurate trial function which would make the application of ICT more general in the evaluation of co-planar dipole arrays of arbitrary lengths.

3. Choice of Trial Functions

To apply ICT most efficiently to co-planar center-fed dipoles of the configuration of shown in Fig. 1, it is desirable for the trial functions to have the following properties:

1. they should vanish at the end of the dipole;
2. for any length of the antenna or systems of antennas, they should not simultaneously vanish or be zero at the feed point;
3. they should lead to formulas which as much as possible avoid numerical integrations for their evaluations.

But a close study of Eqs. (4) and (5) shows that these properties are only satisfied for antennas of less than two wavelengths ($kh_i < 2\pi$). For instance when $kh_i = 2\pi$ the denominators of Eqs. (4) and (5) become zero at the antenna input $z = 0$ as shown in Fig. 2

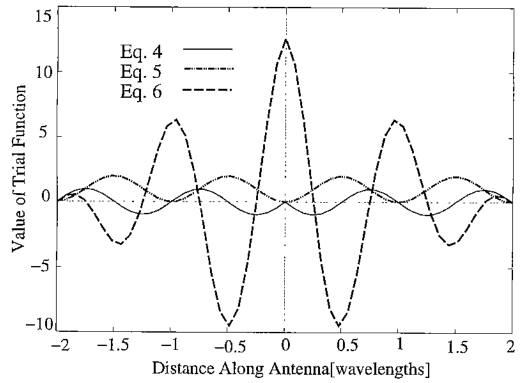


Fig. 2 Shape of trial function along dipole.

and Table 1. This situation predicts infinite impedance which is not necessarily so even though the impedance could be very high. Thus SFICT is invalid for such lengths, their even multiples and their vicinity.

3.1 Other Trial Functions

Other trial functions have been suggested as component of the first order current function (Eq. (4)) for a variational improvement of EMF method. These include[15]–[22]

$$f^2(z_i) = \frac{k(h_i - |z_i|)\cos k(h_i - |z_i|)}{kh\cos kh_i} \tag{6}$$

$$f^2(z_i) = \frac{k(h_i - |z_i|)}{kh_i} \tag{7}$$

$$f^2(z_i) = \frac{\cos\dfrac{kz_i}{2} - \cos\dfrac{kh_i}{2}}{1-\cos\dfrac{kh_i}{2}} \tag{8}$$

Equation (6) is due to Tai[16]–[18], Eq. (7) is due to Harrington[20] while Eq. (8) is found in [22]. Figure 2 and Table 1 shows the values and shapes of various trial functions along an antenna and its feed point respectively.

Even though Eqs. (7) and (8) have the possibility of extended application in second order variational formulation, they are not considered in this paper because they involve components such as

$$\int_0^x \cos\frac{t}{2}f(t)dt, \quad \int_0^x t^2 f(t)dt, \quad f(t) = \frac{e^{-j\sqrt{d^2+t^2}}}{\sqrt{d^2+t^2}} \tag{9}$$

which can only be numerically integrated, which violates the third criteria we set above.

3.2 Tai's Trial Function in ICT

From the trial functions above, we can deduce that Tai's trial function made of Eqs. (4) and (6) have the ability to give more accurate generalized self- and mutual impedance formulas. Moreover, these trial functions have been used in an ICT Two-term representation for multielement antennas of the same length with all the formulas expressed in closed-formed[12], [13]. Since they satisfy the two other conditions, they provide the the best among the trial functions which can be used to derive the generalized impedance matrix elements for an extended application of ICT.

Table 1 Values of trial functions at antenna input ($z = 0$).

kh	$\pi/2$	π	$3/2\pi$	2π	3π	4π
1. $\sin kh_i$	1	0	−1	0	0	0
2. $1-\cos kh_i$	1	2	1	0	2	0
3. $kh_i \cos kh_i$	0	$-\pi$	0	2π	-3π	4π
4. kh_i	$\pi/2$	π	$3/2\pi$	2π	3π	4π
5. $1-\cos\frac{kh_i}{2}$	$1-\frac{\sqrt{2}}{2}$	1	$1+\frac{\sqrt{2}}{2}$	2	1	0

4. New ICT Implementation

To implement ICT as defined in Eq. (1), the elements of the generalized impedance formulas are necessary. According to Eq. (3) any number of trial functions can be used to derive these formulas. But since the labor of calculations increases approximately as the square of the number of terms of Eq. (3), it is desirable to keep M small[20]. Also depending on the quality of the trial functions chosen, relatively fewer terms of Eq. (3) could be applied to achieve reasonably accurate results at lower CPU time and storage.

We saw however that, Tai modified trial function set represented by Eqs. (4) and (6) provides a combination which are not identically zero for all practical antenna lengths. This combination has been shown to provide considerably improved results when applied to arrays of identical lengths[12], [13], and it is therefore adopted to derive the generalized Two-term matrix. We shall for the rest of this paper, designate this new two-term ICT representation as *Tai Two-term ICT* or (TICT).

4.1 TICT Generalized Impedance Matrix

For the two-term ($l = 1,2,\ m = 1,2$) representation the elements of the generalized impedance matrix of TICT defined by Eqs. (4) and (6) have been used to derive $Z_{ij}^{12} = Z_{ij}^{21}$ and Z_{ij}^{22} elements of the self- and mutual generalized impedance. Details of the elements of the impedance matrix $Z_{ij}^{12} = Z_{ij}^{21}, Z_{ij}^{22}$ in TICT in the appendix. It should be noted that Z_{ij}^{11} is same as in [9], [21] and so it is not considered here.

4.2 TICT Field Patterns

The trial functions in Eqs. (4) and (6) allow as to express the far-field radiation patterns of TICT in closed-form as

$$F(\theta,\phi)=\sum_{i=1}^{N}\sum_{m=1}^{2}I_i^m g_i^m(\theta,\phi) \tag{10}$$

where

$$g^1(\theta,\phi)=\frac{\cos(kh_i\cos\theta)\cos kh_i}{\sin\theta}$$
$$g^2(\theta,\phi)=\frac{1}{\sin^3\theta}[kh_i(1-\cos 2\theta)\sin kh_i + (3+\cos 2\theta)\,[\cos kh_i-\cos(kh_i\cos\theta)]] \tag{11}$$

This involves a considerable saving in time compared with the conventional MoM method.

5. Validation of New ICT Implementation

This section discusses the validation of TICT by comparing its results with that of SFICT and MoM [4], [5] in analysis of various antenna systems. Tai single element variational implementation of EMF designated as TVAR [15] is also used in the validation process.

5.1 Input Admittances

To validate TICT for single elements, we have calculated the input admittances of various antennas for a moderate Hallen parameter ($\Omega=2\ln\frac{2h}{a}=10$, a is dipole radius). As the results show in Fig. 3, for this case there is good agreement between MoM [5], TVAR and TICT.

For moderate values of Ω and single elements, TICT and TVAR are expected to give very identical results as shown in Fig. 3. We also notice that SFICT cannot be valid at $kh_i=2\pi$. As a matter of fact, the input impedance calculated with this formula cannot be valid in the vicinity of this value [20]. We estimate that the valid length for the application of the SFICT is about $kh_i\leq 1.5\pi$, where $2h_i$ is the antenna length and $k=2\pi/\lambda$ is the free-space wave number.

5.2 Mutual Impedance Computations

Figures 4 to 6 show the mutual impedances versus spacing between various pairs of mutual coupled antennas calculated with MoM [4], TICT and SFICT.

We can see from Fig. 4 that the three methods (MoM, TICT and SFICT) produce mutual impedance results which are in good agreement because the trial functions chosen to represent the current distribution in the ICT methods are a good representation for this situation. The same conclusion can be drawn from Fig. 5 where all the results are again in very good agreement.

In Fig. 6 we notice that there is considerable disparity between the results of MoM and TICT on one hand and SFICT on the other. While there is reasonable agreement between MoM and TICT for this case,

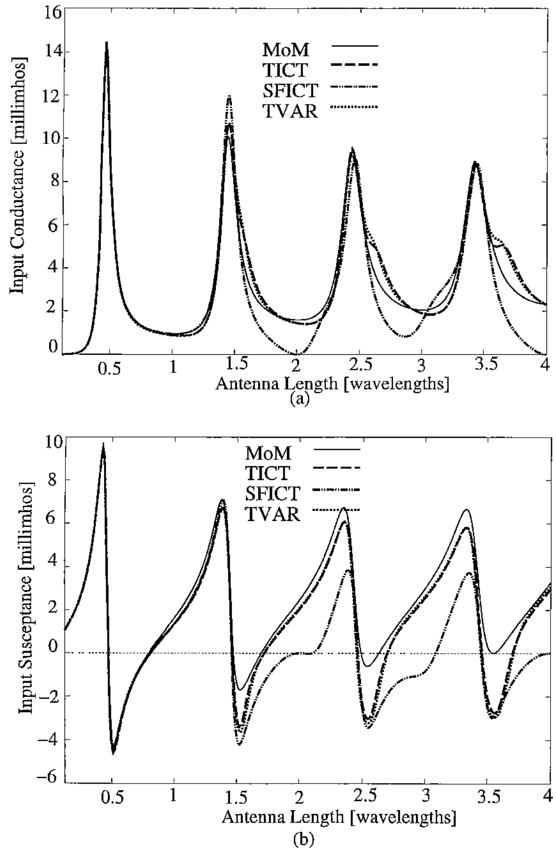


Fig. 3 Input admittance (a) conductance, (b) susceptance for $\Omega=10$.

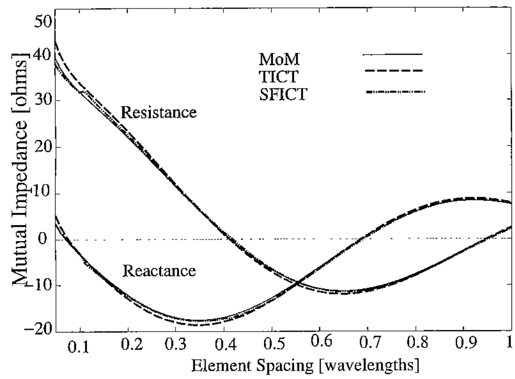


Fig. 4 Mutual impedance of two co-planar dipoles of lengths $2h_i=0.5\lambda$, $2h_j=0.25\lambda$ for $\Omega=10$.

the results of SFICT has to be shown as an insert because it is in considerable disagreement. This can be explained from the fact that the set of trial functions (Eqs. (4) and (5)) used to derive the formulas are simultaneously zero for dipole length $kh_i=2\pi$.

As a matter of fact, these trial functions can not be accurate in the vicinity of any lengths of antennas in the vicinity of $kh_i=2\pi$ (see Fig. 3). But the change introduced by the modification of the trial functions (replacing Eqs. (4) and (5) with Eqs. (4) and (6)) demonstrates that even though the first order current distribution (one current function) is accurate for mostly resonance lengths, its accuracy can be improved if a much better second order current is introduced.

The quality of the trial current functions is therefore important as shown in this case. This explains the improvement we have notice by replacing the conventional second order current component and with a qualitatively much better one in the form of a component of Tai's trial function [16], [17].

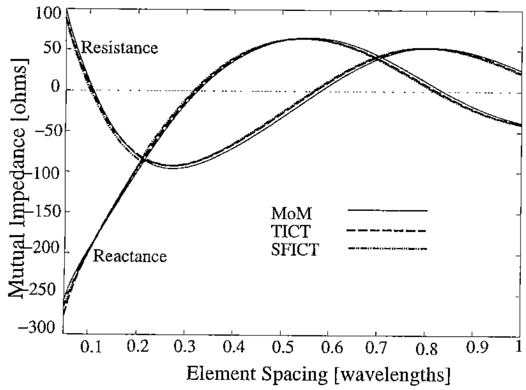
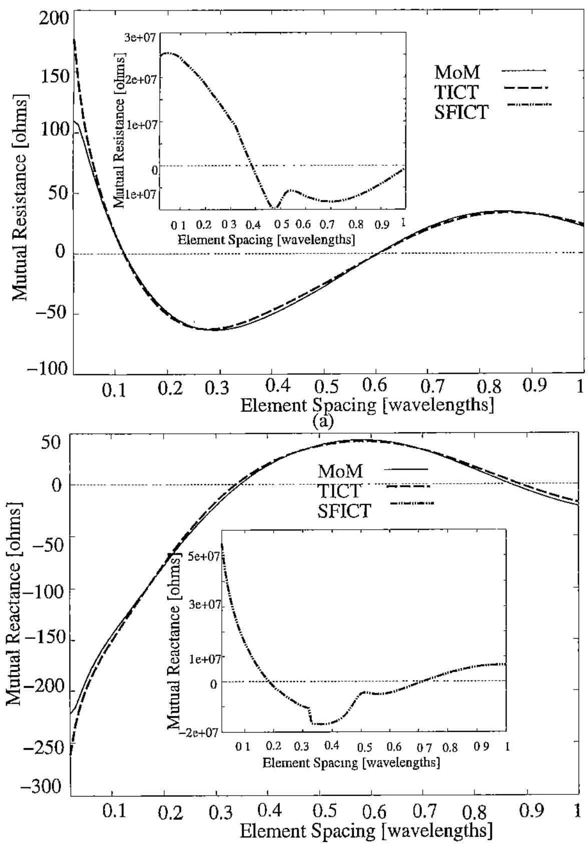


Fig. 5 Mutual impedance of two co-planar dipoles of lengths $2h_i = \lambda$, $2h_j = 0.5\lambda$, for $\Omega = 10$.



(b)

Fig. 6 Mutual impedance: (a) resistance, (b) reactance of two antennas: $2h_i = 2\lambda$, $2h_j = 0.5\lambda$, for $\Omega = 10$.

6. Analysis of a Typical Broadband Antenna

This section considers the analysis of a Log-Periodic Dipole Array (LPDA) as a typical broadband antenna, using TICT and SFICT according to the principles outlined in [5].

6.1 Geometry of LPDA

The geometrical dimensions of LPDA [5], [23]–[25], shown in Fig. 7, follow a set of pattern relating the lengths (l_n), spacing between elements (d_n), radius, spacing between centers of dipoles (s_n) and so on. The log periodic array therefore increases logarithmetrically as defined by the inverse of the geometric ratio (τ) defined as

$$\frac{1}{\tau} = \frac{l_2}{l_1} = \frac{l_{n+1}}{l_n} = \frac{x_2}{x_1} = \frac{x_{n+1}}{x_n} = \frac{d_2}{d_1} = \frac{d_{n+1}}{d_n} = \frac{s_2}{s_1} = \frac{s_{n+1}}{s_n} \tag{12}$$

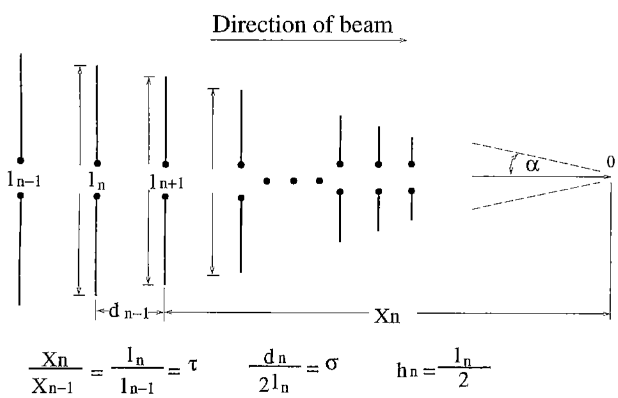
Other parameters usually associated with LPDA are the spacing factor σ and apex angle α defined as

$$\sigma = \frac{x_{n+1} - x_n}{2l_{n+1}}, \quad \alpha = 2\tan^{-1}\frac{1-\tau}{4\sigma}. \tag{13}$$

6.2 Case Analysis of LPDA

In this case analysis, we consider a LPDA which is to operate at bandwidth of 300 to 2000 MHz. This particular band would require the handling of antennas whose electrical length may be more that two wavelengths depending on the working frequency.

For a design gain of 10 dB using 28 elements, we have $\tau = 0.917$, $\sigma = 0.169$ [5]. The base current amplitudes are shown in Fig. 8 for frequencies 300 MHz, 900 MHz and 2000 MHz respectively.



(a)

Fig. 7 Geometry of a log-periodic dipole antenna.

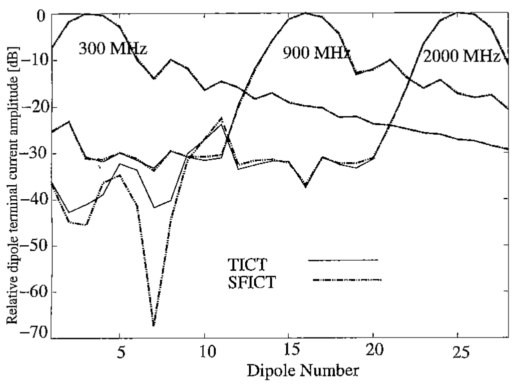


Fig. 8 Computed amplitude of dipole element base current versus element number of an optimum log-periodic dipole antenna for operation in the 300 to 2000 MHz band.

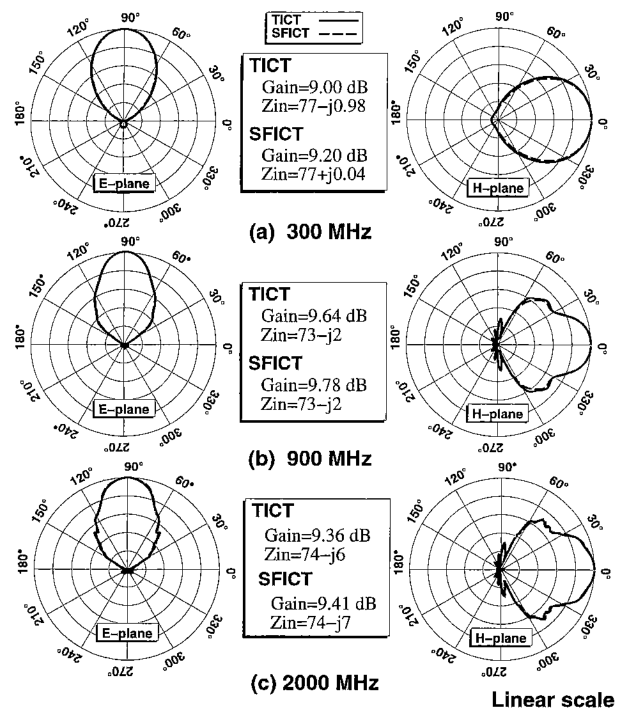


Fig. 9 Radiation pattern at three different frequencies for an optimum log-periodic dipole antenna for operation in the 300 to 2000 MHz band [5].

As we can see from the results of Fig. 8 at 300 MHz, the dipole lengths span from 0.5 to 0.05 wavelengths. For this situation, TICT and SFICT produce identical current distributions, because the trial functions are adequate. Also for a frequency of 900 MHz with the lengths spanning 1.5 to 0.14 wavelengths, we again see that the trial functions are an accurate representation of the system.

However, as we move to much longer dipoles, for the 2000 MHz band where the dipole lengths span from 3.3 to 0.3 wavelengths, we notice large discrepancies in the base currents at the elements of longer lengths.

The difference is marked at element seven which

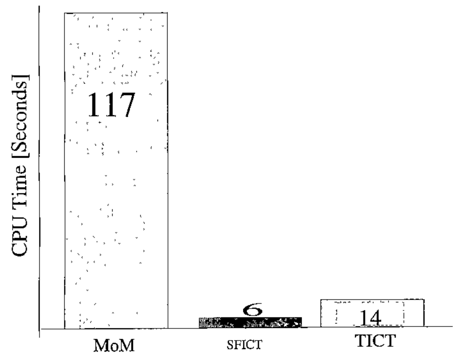


Fig. 10 CPU times of various algorithms.

is about 1.98 wavelengths at the operating frequency of 2000 MHz. However, as shown in Fig. 9 (c), this difference in current amplitude is not sufficient to drastically influence the far field pattern for this frequency.

An explanation of this unusual phenomena is that the current distribution is far greater and concentrated in the active region elements which are operating at resonance or near resonance than the rest of the elements in the structure [18].

For this situation the insignificance of the current distribution for the non-reactive region reflects the almost identical patterns. More detailed studies to accurately establish the advantage of TICT has over SFICT for these types of antennas are therefore necessary.

7. CPU Time Statistics

For two coupled dipoles of a typical form in Sect. 5.2, we have computed the CPU times required for the computation of the mutual impedances using MoM [4], TICT and SFICT methods. The results are given in Fig. 10. We used an NEC PC-9801 RA to avoid distortion on the results on a time sharing workstation.

It is clear from these results that TICT requires much shorter CPU time compared to MoM.

8. Hybrid Implementation

In this section we summarize various ICT implementations designed towards getting the most efficient ICT procedure. Our conclusion from results of the single element input admittances and the mutual coupling of two elements is that *Tai Two-term ICT* or (TICT) is the most general form of implementing the ICT algorithm. But we find out from the time statistics analysis that the computational time demanded of the new implementation is more than two and half times that required by the conventional implementation via *Storer Faster Two-term ICT*[9] or (SFICT). But SFICT is limited by length constraint of the conventional method, even though its accuracy is in good agreement where the conventional current functions are accurate. It is therefore necessary to implement the ICT method utilizing the strengths and weakness of each implementation scheme. Based on this argument, we are therefore suggesting a hybrid implementation of ICT consisting of SFICT and TICT defined as

$$\text{ICT Implementation} = \begin{cases} \text{SFICT}, & kh_i \leq 1.5\pi \\ \text{TICT}, & kh_i > 1.5\pi. \end{cases} \tag{14}$$

9. Conclusion

We have analyzed various trial functions to find the most suitable set which best represents the current distribution for the analysis of an arbitrary configured co-planar array of dipoles. The new set gives impedance and far-field formulas which are expressed in closed-form. The new implementation is called *Tai Two-term ICT* or TICT. For co-planar dipoles TICT has no restriction on the length of the antenna.

In a typical analysis, TICT has the same order of accuracy as MoM, but requires about eight times lesser CPU time than the latter. However, within the region where SFICT and TICT are valid, the former method is more than two times faster than the former. This has lead to a suggestion of a hybrid implementation of the ICT algorithm consisting of TICT and SFICT.

To demonstrate the application of TICT to broadband antennas, a widebandwidth LPDA has been analyzed. But the results of the systems has been such that we have not reached any conclusive advantages TICT has over SFICT in view of the fact in LPDAs, most of the current is concentrated in the active region. Study is underway to establish the exact advantages TICT has over SFICT in such situations.

It is important to note that the current work is coinciding with a renewed interest in variational methods in conjunction with numerical solutions of electromagnetic radiation and scattering problems [1], [26]–[28]. This is because, where these semi-analytical methods like ICT apply, they offer the antenna designer a very efficient tool for CAD/CAE systems. In these systems response time is premium. TICT would therefore be a boast for such applications.

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Appendix

Using the trial functions given in Eqs. (4) and (6) and noting that $p = h_i + h_j$, $q = h_i - h_j$, the elements Z_{ij}^{12} and Z_{ij}^{22} of the generalized impedance matrix defined in Eq. (2) are given as

$$Z_{ij}^{12} = j\frac{60}{kh_i\,[\sin kp + \sin kq]}\,[C_{p12}\cos kp + S_{p12}\sin kp + C_{q12}\cos kq + S_{q12}\sin kq] \qquad (\text{A}\cdot 1)$$

$$Z_{ij}^{22} = j\frac{30}{k^2 h_i h_j\,[\cos kp + \cos kq]}\,\{C_{p22}\cos kp + S_{p22}\sin kp + C_{q22}\cos kq + S_{q22}\sin kq\} \qquad (\text{A}\cdot 2)$$

$$C_{p12} = kpC_{kd}(kp) - CU1_{kd}(kp) + CU1_{kd}(kh_i) - kpC_{kd}(kh_i) + CU1_{kd}(kh_j) - kh_jC_{kd}(kh_j) \qquad (\text{A}\cdot 3)$$

$$S_{p12} = kpS_{kd}(kp) - SU1_{kd}(kp) + SU1_{kd}(kh_i) - kpS_{kd}(kh_i) + SU1_{kd}(kh_j) - kh_jS_{kd}(kh_j) \qquad (\text{A}\cdot 4)$$

$$C_{q12} = kqC_{kd}(kq) - CU1_{kd}(kq) + CU1_{kd}(kh_i) - kqC_{kd}(kh_i) + CU1_{kd}(kh_j) - kh_jC_{kd}(kh_j) \qquad (\text{A}\cdot 5)$$

$$S_{q12} = kqS_{kd}(kq) - SU1_{kd}(kq) + SU1_{kd}(kh_i) - kqS_{kd}(kh_i) - SU1_{kd}(kh_j) + kh_jS_{kd}(kh_j) \qquad (\text{A}\cdot 6)$$

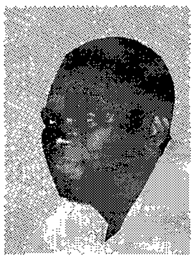
$$\begin{aligned} C_{p22} = {} & kpC_{kd}(kp) - CU1_{kd}(kp) \\ & - [1-(kp)^2]S_{kd}(kp) - 2kpSU1_{kd}(kp) \\ & + SU2_{kd}(kp) + CU1_{kd}(kh_j) - kh_jC_{kd}(kh_j) \\ & - [-1 + 2k^2h_ih_j + (kh_j)^2]S_{kd}(kh_j) \\ & + 2kpSU1_{kd}(kh_j) - SU2_{kd}(kh_j) \\ & + CU1_{kd}(kh_i) - kh_iC_{kd}(kh_i) \\ & - [-1 + 2k^2h_ih_j + (kh_i)^2]S_{kd}(kh_i) \\ & + 2kpSU1_{kd}(kh_i) - SU2_{kd}(kh_i) \qquad (\text{A}\cdot 7) \end{aligned}$$

$$\begin{aligned} S_{p22} = {} & kpS_{kd}(kp) - SU1_{kd}(kp) \\ & + [1-(kp)^2]C_{kd}(kp) + 2kpCU1_{kd}(kp) \\ & - CU2_{kd}(kp) + SU1_{kd}(kh_j) - kh_jS_{kd}(kh_j) \\ & + [-1 + 2k^2h_ih_j + (kh_j)^2]C_{kd}(kh_j) \\ & - 2kpCU1_{kd}(kh_j) + CU2_{kd}(kh_j) \\ & + SU1_{kd}(kh_i) - kh_iS_{kd}(kh_i) \\ & + [-1 + 2k^2h_ih_j + (kh_i)^2]C_{kd}(kh_i) \\ & - 2kpCU1_{kd}(kh_i) + CU2_{kd}(kh_i) \qquad (\text{A}\cdot 8) \end{aligned}$$

$$\begin{aligned} C_{q22} = {} & kqC_{kd}(kq) - CU1_{kd}(kq) \\ & - [1-(kq)^2]S_{kd}(kq) - 2kqSU1_{kd}(kq) \\ & + SU2_{kd}(kq) + CU1_{kd}(kh_j) - kh_jC_{kd}(kh_j) \\ & + [1 + 2k^2h_ih_j - (kh_j)^2]S_{kd}(kh_j) \\ & - 2kqSU1_{kd}(kh_j) - SU2_{kd}(kh_j) \\ & + CU1_{kd}(kh_i) - kh_iC_{kd}(kh_i) \\ & - [-1 - 2k^2h_ih_j + (kh_i)^2]S_{kd}(kh_i) \\ & + 2kqSU1_{kd}(kh_i) - SU2_{kd}(kh_i) \qquad (\text{A}\cdot 9) \end{aligned}$$

$$\begin{aligned} S_{q22} = {} & kqS_{kd}(kq) - SU1_{kd}(kq) \\ & + [1-(kq)^2]C_{kd}(kq) + 2kqCU1_{kd}(kq) \\ & - CU2_{kd}(kq) - SU1_{kd}(kh_j) + kh_jS_{kd}(kh_j) \\ & + [1 + 2k^2h_ih_j - (kh_j)^2]C_{kd}(kh_j) \\ & - 2kqCU1_{kd}(kh_j) - CU2_{kd}(kh_j) \\ & + SU1_{kd}(kh_i) - kh_iS_{kd}(kh_i) \\ & + [-1 - 2k^2h_ih_j + (kh_i)^2]C_{kd}(kh_i) \\ & - 2kqCU1_{kd}(kh_i) + CU2_{kd}(kh_i) \qquad (\text{A}\cdot 10) \end{aligned}$$

For an argument x the the generalized cosine and sin integrals $C_{kd}(x)$, $S_{kd}(x)$, $CU1_{kd}(x)$, $CU2_{kd}(x)$, $SU1_{kd}(x)$, and $SU2_{kd}(x)$, in Eqs. (A·3) to (A·10) are expressed in closed-form [3], [12], [13].



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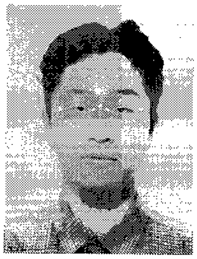

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