

A Method for Evaluating Minimum Free Chernov Distance of Trellis-Codes for Discrete Memoryless Channel

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SUMMARY In this paper, we present a method for evaluating the minimum free Chernov distance of trellis-codes for a discrete memoryless channels (DMC). In order to design an efficient trellis-code for the DMC, we need to evaluate the minimum free Chernov distance of the target code. However, the lack of the additive property of the Chernov distance prevents a conventional branch-and-bound search for evaluating the minimum distance. To overcome the difficulty, we present a lower bound on the Chernov distance with an additive property. The lower bound plays a key role in the minimum distance evaluation algorithm presented here. By using the proposed algorithm, we have derived the minimum free Chernov distance of some binary linear convolutional codes over Z-channel.

key words: Chernov distance, discrete memoryless channel, trellis-code, minimum free distance

1. Introduction

Trellis-codes (including convolutional codes) are mainly constructed by computer code search. In order to obtain good trellis-codes, we need fast computers and an efficient method for evaluating the minimum free distance of a target code. The latter is more important because an inefficient method, such as exhaustive search, cannot be used for code search even with a today's fast computer. An efficient evaluation method must yield a tight lower bound on the minimum free distance and be efficiently executed.

The simplest target codes are binary linear convolutional codes for a binary symmetric channel. In this case, we only need to find an error event with the minimum Hamming weight for evaluating the minimum free Hamming distance. The branch-and-bound technique can be used to find such error events. Some sophisticated methods have been proposed to improve the simple branch-and-bound technique [4].

Unfortunately, these evaluation methods for binary convolutional codes cannot be applied for construction of non-binary trellis-codes for the additive white Gaussian channel, such as trellis-coded modulation. How-

ever, the breakthrough was given by Ungerboeck in 1970's. His set partitioning method allows us to derive a lower bound on the minimum free Euclidean distance of a target code. The fact that the trellis-coded modulation could not appear before the invention of the set partitioning also shows the importance of an efficient evaluation method.

We can now evaluate trellis-codes even for a partial response channel [3], however, some important channels remain unsolved. One of such channels is a *discrete memoryless channel* involving asymmetric channels.

There exist practically important channels with asymmetric nature. For example, in photon communications, an asymmetric channel such as the Z-channel is used as a simple channel model. The Z-channel is a channel model such that the transmitted symbol 0 is always received correctly, though the transmitted symbol 1 might be incorrectly received. Some works on construction and decoding scheme of the codes for Z-channel have been presented [2].

We need a tight bound on the error probability for evaluating the performance of a trellis-code over the DMC. One of the tight bounds is the Chernov bound [1], which is applicable for any DMC. From the Chernov bound, the Chernov distance is naturally defined. The minimum free Chernov distance for a trellis-code can be defined in such a way that the minimum free Hamming distance is defined. The minimum free Chernov distance is the most essential parameter which characterizes the error performance of a trellis-code over the DMC.

Unfortunately, we encounter a problem when dealing with the Chernov distance. The problem is that the Chernov distance has no additive property in general. The additive property means that the distance between two n -tuples consisting of channel alphabet is given by the sum of the symbol-wise distance of the two n -tuples. In order to evaluate the minimum Chernov distance of a trellis-code efficiently, this property is necessary. To overcome the difficulty, we derive a lower bound on the Chernov distance which has a form of "constant (a real number) multiple of the Hamming distance." Due to the additive property of the Hamming distance, the lower bound is also additive and we can exploit the branch-and-bound search. In this paper, we present an effi-

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cient method for evaluating the minimum free Chernov distance of a trellis-code for the DMC using such an additive lower bound.

The paper is organized as follows: In Sect. 2, we give some fundamental definitions such as the Chernov distance. In Sect. 3, an evaluation algorithm is proposed. In Sect. 4, we present a summary as a conclusion.

2. Preliminaries

2.1 Chernov Distance

Let us denote the set of input alphabet and output alphabet by \mathcal{X} and \mathcal{Y} , respectively. A discrete memoryless channel can be specified by a stochastic matrix $\pi = \{\pi_{ij}\}$,

$$\pi_{ij} = p(y_j|x_i), \quad x_i \in \mathcal{X}, \quad y_j \in \mathcal{Y}. \quad (1)$$

Suppose that a sender transmits a message m ($0 \leq m \leq 2^k - 1$) to a receiver. The message m is encoded to a codeword $\mathbf{x}_m \in C$. The code C is a set of n -tuples of elements in \mathcal{X} with the cardinality 2^k . The codeword is sent to the receiver via the DMC defined by π . The pairwise error probability, corresponding to an event that the receiver fail to decode, is given by

$$P_e(m \rightarrow m') = P\{\mathbf{y} \in \Lambda_{mm'} | \mathbf{x}_m\}, \quad (2)$$

for $C = \{\mathbf{x}_m, \mathbf{x}_{m'}\}$. The set $\Lambda_{mm'}$ is the decision region of $\mathbf{x}_{m'}$ defined by

$$\Lambda_{mm'} = \left\{ \mathbf{y} : \frac{p(\mathbf{y}|\mathbf{x}_{m'})}{p(\mathbf{y}|\mathbf{x}_m)} \geq 1, \mathbf{y} \in \mathcal{Y}^n \right\}. \quad (3)$$

In order to bound $P_e(m \rightarrow m')$ for any code C , we prepare the indicator function f defined by

$$f(\mathbf{y}) = \left(\frac{p(\mathbf{y}|\mathbf{x}_{m'})}{p(\mathbf{y}|\mathbf{x}_m)} \right)^\rho, \quad (4)$$

where ρ is a non-negative real number. The indicator function f satisfies the following inequalities:

$$f(\mathbf{y}) \geq 1, \mathbf{y} \in \Lambda_{mm'} \quad (5)$$

$$f(\mathbf{y}) \geq 0, \mathbf{y} \notin \Lambda_{mm'}, \quad (6)$$

and we have an upper bound on P_e :

$$P_e(m \rightarrow m') \leq \sum_{\mathbf{y} \in \mathcal{Y}^n} f(\mathbf{y}) p(\mathbf{y}|\mathbf{x}_m). \quad (7)$$

By using the memoryless property, the right hand side of Eq. (7) can be transformed into the following form:

$$\begin{aligned} & \sum_{\mathbf{y} \in \mathcal{Y}^n} f(\mathbf{y}) p(\mathbf{y}|\mathbf{x}_m) \\ &= \sum_{\mathbf{y} \in \mathcal{Y}^n} p(\mathbf{y}|\mathbf{x}_{m'})^\rho p(\mathbf{y}|\mathbf{x}_m)^{1-\rho} \\ &= \sum_{\mathbf{y} \in \mathcal{Y}^n} \prod_{0 \leq i \leq n-1} p(y_i|x_{m'i})^\rho p(y_i|x_{mi})^{1-\rho} \\ &= \prod_{0 \leq i \leq n-1} \sum_{y \in \mathcal{Y}} p(y|x_{m'i})^\rho p(y|x_{mi})^{1-\rho}. \end{aligned}$$

We now define the ρ -distance between \mathbf{x}_m and $\mathbf{x}_{m'}$ by

$$\begin{aligned} d^{(\rho)}(\mathbf{x}_m, \mathbf{x}_{m'}) &= -\ln \prod_{0 \leq i \leq n-1} \sum_{y \in \mathcal{Y}} p(y|x_{m'i})^\rho p(y|x_{mi})^{1-\rho} \\ &= \sum_{0 \leq i \leq n-1} g^{(\rho)}(x_{mi}, x_{m'i}), \end{aligned} \quad (8)$$

where $g^{(\rho)}(x, x')$ is given by

$$g^{(\rho)}(x, x') = -\ln \sum_{y \in \mathcal{Y}} p(y|x')^\rho p(y|x)^{1-\rho}. \quad (9)$$

From the above definition, we immediately have the following upper bound on the error probability:

$$P_e(m \rightarrow m') \leq e^{-d^{(\rho)}(\mathbf{x}_m, \mathbf{x}_{m'})}. \quad (10)$$

Since $g^{(\rho)}(x, x) = 0$, the ρ -distance can be rewritten into the following form:

$$d^{(\rho)}(\mathbf{x}_m, \mathbf{x}_{m'}) = \sum_{x \neq x' \in \mathcal{X}} N_{x \rightarrow x'}(\mathbf{x}_m, \mathbf{x}_{m'}) g^{(\rho)}(x, x'), \quad (11)$$

where $N_{x \rightarrow x'}(\mathbf{x}_m, \mathbf{x}_{m'})$ is the number of indices satisfying $x_{mi} = x, x_{m'i} = x'$. For given \mathbf{x}_m and $\mathbf{x}_{m'}$, we can choose the parameter ρ such that $d^{(\rho)}$ is maximized. The optimized ρ gives the tightest bound on the error probability. The optimized ρ -distance ($\rho \geq 0$) is referred to as the *Chernov distance*, which is denoted by d_c :

$$d_c(\mathbf{x}_m, \mathbf{x}_{m'}) = \max_{\rho} d^{(\rho)}(\mathbf{x}_m, \mathbf{x}_{m'}). \quad (12)$$

From Eq. (11), we can observe that the composition of \mathbf{x}_m and $\mathbf{x}_{m'}$ affects the Chernov distance. This shows the non-additive property of the Chernov distance.

2.2 Upper Bound on Bit Error Probability of Binary Linear Convolutional Code

Suppose that the sender uses a trellis-code instead of a block code. In this case, the error event probability for a trellis-code is given by the Chernov distance between the correct path and the error event path. The minimum free Chernov distance for a trellis-code can be defined in a similar manner as the minimum free Hamming distance. We denote the minimum free Chernov distance by d_c^{free} .

The minimum free Chernov distance is one of the dominant parameters for the error performance of trellis-codes over the DMC. We here discuss the case of binary convolutional codes to show the importance of the minimum free Chernov distance. When binary convolutional codes are exploited, we can obtain an upper bound on the bit error probability. Let us consider the binary linear convolutional code of rate k/n and S -state. We denote the set of error events of length l by

E_l and the set of code sequences of length l by W_l . The upper bound is given by

$$P_b \leq \sum_{l=1}^{\infty} \sum_{\mathbf{w} \in W_l} p(\mathbf{w}) \sum_{\mathbf{e} \in E_l} w_i(\mathbf{e}) e^{-d_c(\mathbf{w}, \mathbf{e})} \quad (13)$$

$$= \sum_{l=1}^{\infty} \sum_{\mathbf{w} \in W_l} \sum_{\mathbf{e} \in E_l} \frac{w_i(\mathbf{e})}{2^{kl} S} e^{-d_c(\mathbf{w}, \mathbf{e})}, \quad (14)$$

where $p(\mathbf{w})$ is the probability of the occurrence \mathbf{w} . The value $w_i(\mathbf{e})$ is the Hamming weight of an input sequence \mathbf{t} such that $\mathbf{e} = \phi(\mathbf{t})$, where $\phi(\mathbf{t})$ is the code sequence corresponding to \mathbf{t} . In the above discussion, we assume that all the code sequences are equiprobable. From the above bound, we have an approximation of bit error probability:

$$P_b \cong N_{d_c^{free}} e^{-d_c^{free}}, \quad (15)$$

where

$$N_{d_c^{free}} = \sum_{l=1}^{\infty} \sum_{\mathbf{w} \in W_l} \sum_{\mathbf{e} \in E_l} [d_c(\mathbf{w}, \mathbf{e}) = d_c^{free}] \frac{w_i(\mathbf{e})}{2^{kl} S}, \quad (16)$$

and the notation $[A = B]$ means that $[A = B] = 0$ for $A \neq B$, $[A = B] = 1$ for $A = B$. We call $N_{d_c^{free}}$ the first error coefficient.

Example 1: Figure 1 shows the bit error probability of 4-state binary convolutional code over Z-channel, obtained from the approximation in Eq. (15) and by the simulation. In the Fig. 1, G_i 's are the generator polynomials (in octal form) which specify the encoder [5]. The values d_c^{free} and $N_{d_c^{free}}$ can be computed by the algorithms described later. \square

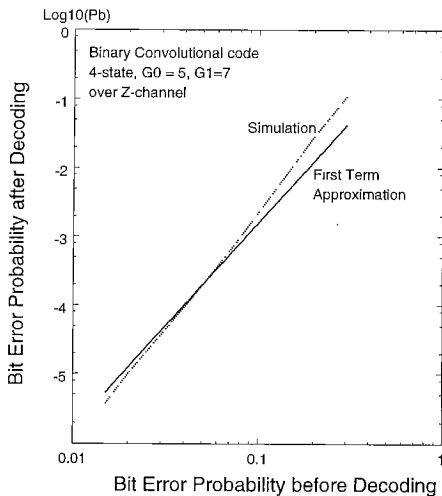


Fig. 1 Bit error probability of 4-state binary convolutional code over Z-channel.

3. Evaluation of Minimum Free Chernov Distance

3.1 Additive Lower Bound on Chernov Distance

In order to use branch-and-bound search, it is preferable to derive a lower bound of the Chernov distance with the additive property. We here derive such a lower bound, which is useful for evaluating the minimum free Chernov distance.

The sum of $N_{x \rightarrow x'}(\mathbf{x}_m, \mathbf{x}_{m'})$ for all the pairs of alphabet in \mathcal{X} is equal to the Hamming distance between \mathbf{x}_m and $\mathbf{x}_{m'}$, denoted by $d_h(\mathbf{x}_m, \mathbf{x}_{m'})$:

$$\sum_{x \neq x' \in \mathcal{X}} N_{x \rightarrow x'}(\mathbf{x}_m, \mathbf{x}_{m'}) = d_h(\mathbf{x}_m, \mathbf{x}_{m'}) \quad (17)$$

Thus, $d^{(\rho)}$ is represented by

$$d^{(\rho)}(\mathbf{x}_m, \mathbf{x}_{m'}) = d_h(\mathbf{x}_m, \mathbf{x}_{m'}) \times \sum_{x \neq x' \in \mathcal{X}} q(\mathbf{x}_m, \mathbf{x}_{m'}) g^{(\rho)}(x, x'), \quad (18)$$

where

$$q(\mathbf{x}_m, \mathbf{x}_{m'}) = \frac{N_{x \rightarrow x'}(\mathbf{x}_m, \mathbf{x}_{m'})}{d_h(\mathbf{x}_m, \mathbf{x}_{m'})}, \quad (19)$$

and

$$\sum_{x \neq x' \in \mathcal{X}} q(\mathbf{x}_m, \mathbf{x}_{m'}) = 1. \quad (20)$$

From Eq. (18), we have a lower bound on the Chernov distance:

$$d_c(\mathbf{x}_m, \mathbf{x}_{m'}) \geq \alpha d_h(\mathbf{x}_m, \mathbf{x}_{m'}), \quad (21)$$

where the tightest parameter α for the lower bound is obtained by solving

$$\alpha = \min_{q_{x,x'}} \max_{\rho} \sum_{x \neq x' \in \mathcal{X}} q_{x,x'} g^{(\rho)}(x, x') \quad (22)$$

under the conditions that $\sum_{x \neq x' \in \mathcal{X}} q_{x,x'} = 1$, $q_{x,x'} \geq 0$. Since the parameter α is a constant value and the Hamming distance has the additive property, the lower bound also has the additive property.

3.2 Lower Bound for Binary Memoryless Channel

Let us consider the case $\mathcal{X} = \mathcal{Y} = \{0, 1\}$. We denote $p(0|1) = p_a$ and $p(1|0) = p_b$ for simplicity. From the definition of $g^{(\rho)}(i, j)$, we have $g^{(\rho)}(0, 0) = g^{(\rho)}(1, 1) = 0$ and

$$g^{(\rho)}(0, 1) = -\ln \left((1 - p_b)^{1-\rho} p_a^\rho + p_b^{1-\rho} (1 - p_a)^\rho \right),$$

$$g^{(\rho)}(1, 0) = -\ln \left(p_a^{1-\rho} (1 - p_b)^\rho + (1 - p_a)^{1-\rho} p_b^\rho \right).$$

Thus, the parameter α for the lower bound is given by

$$\alpha = \min_{q_{0,1}} \max_{\rho} \left(q_{0,1} g^{(\rho)}(0, 1) + q_{1,0} g^{(\rho)}(1, 0) \right), \quad (23)$$

where $q_{0,1} + q_{1,0} = 1$. For a given $p_a > 0$ and $p_b > 0$, we can obtain the parameter α by setting $q_{0,1} = q_{1,0} = 1/2$ and $\rho = 1/2$. We thus have

$$\alpha = -\ln \left\{ \sqrt{p_a(1-p_b)} + \sqrt{p_b(1-p_a)} \right\}. \quad (24)$$

When $p_b = 0$, we set $q_{0,1} = q_{1,0} = 1/2, \rho = 0$ and have the optimized value:

$$\alpha = -1/2 \ln p_a. \quad (25)$$

Similarly, when $p_a = 0$, we have

$$\alpha = -1/2 \ln p_b. \quad (26)$$

Since these values are the special cases of Eq. (24), we can yield the parameter for the binary memoryless channel (BMC):

$$\alpha = -\ln \left\{ \sqrt{p_a(1-p_b)} + \sqrt{p_b(1-p_a)} \right\}, \quad (27)$$

where $0 \leq p_a, p_b \leq 1$.

Example 2: One of the most important BMC is the Z-channel. The Z-channel is the channel with the $p_a > 0$ and $p_b = 0$. From the condition $p_b = 0$, we have

$$g^{(\rho)}(0, 1) = -\rho \ln p_a, \quad (28)$$

$$g^{(\rho)}(1, 0) = -(1-\rho) \ln p_a, \quad (29)$$

and

$$d_c^{(\rho)}(\mathbf{x}_m, \mathbf{x}_{m'}) = (-\ln p_a) \{ N_{0 \rightarrow 1}(\mathbf{x}_m, \mathbf{x}_{m'}) \rho + N_{1 \rightarrow 0}(\mathbf{x}_m, \mathbf{x}_{m'}) (1-\rho) \}. \quad (30)$$

If $N_{0 \rightarrow 1} > N_{1 \rightarrow 0}$, then the Chernov distance is given by

$$d_c(\mathbf{x}_m, \mathbf{x}_{m'}) = \infty \quad (31)$$

for $\rho = \infty$. Otherwise ($N_{0 \rightarrow 1} \leq N_{1 \rightarrow 0}$), the Chernov distance is given by

$$d_c(\mathbf{x}_m, \mathbf{x}_{m'}) = (-\ln p_a) N_{1 \rightarrow 0}(\mathbf{x}_m, \mathbf{x}_{m'}) \quad (32)$$

for $\rho = 0$.

For the Z-channel, the parameter α is equal to $(-\ln p_a)/2$, we thus have

$$d_c(\mathbf{x}_m, \mathbf{x}_{m'}) \geq \frac{(-\ln p_a)}{2} d_h(\mathbf{x}_m, \mathbf{x}_{m'}). \quad (33)$$

□

3.3 Lower Bound for Hamming Distance

Let C be the set of code sequences of a trellis-code. The Hamming distance enumerator of C around \mathbf{a} is given by

$$HDE(\mathbf{a}) = \sum_{\mathbf{c} \in C} Z^{d_h(\mathbf{c}, \mathbf{a})}. \quad (34)$$

If the equality $HDE(\mathbf{a}) = HDE(\mathbf{b})$ holds for any $\mathbf{a}, \mathbf{b} \in C$, we say that C has the distance invariant property with respect to the Hamming distance.

In the previous subsection, we obtained a lower bound on the Chernov distance. The bound is based on the Hamming distance between two codewords. For a binary linear code, we can easily deal with the Hamming distance in a conventional way. However, when considering a trellis-code defined over $|\mathcal{X}|$ -ary alphabet, we need a lower bound on the Hamming distance having the distance invariant property. This is because the target code does not have the distance invariant property in general. The distance invariant property is necessary for efficient evaluation on the minimum free Chernov distance. For example, the algorithm described later heavily depends on the distance invariant property.

The following lower bounding technique was presented in [3] and it is used for constructing trellis-codes for a partial response channel. The bound has the distance invariant property and it exists when a trellis-code has the *linear trellis structure*.

We here discuss a time-invariant trellis-code for simplicity. The extension to the time-variant case is straightforward.

Let S be the set of states in the encoder of a target trellis-code and $l(\sigma_{from}, \sigma_{to}), \sigma_{from}, \sigma_{to} \in S$, be the output symbols (in \mathcal{X}) corresponding to the state transition from σ_{from} to σ_{to} . The branch set B is defined by

$$B = \{l(\sigma_{from}, \sigma_{to}) : \sigma_{from}, \sigma_{to} \in S\}. \quad (35)$$

The branch set is considered as the set of branches in a state transition. Let us assume that each branch in B has a *branch label* i , where the branch label is a binary s -tuple. If the set of all branch label sequences defined by a trellis forms a binary linear code, we say that the trellis has the linear trellis structure.

A trellis-code defined by the image of a binary linear code with a mapping $\phi : F_2^n \rightarrow \mathcal{X}$ has the linear trellis structure. The class of trellis-codes with the linear trellis structure includes practically important trellis-codes such as trellis-coded modulation and trellis-codes for a partial response channel.

The branch weight with branch label i is defined by

$$L(i) = \min_{\{j,k | j \oplus k = i\}} d_h(V_j, V_k), \quad (36)$$

where V_j and V_k are the output n -tuples corresponding to branches with the branch label j and k , respectively. The operator \oplus is the addition over GF(2). The path weight $L(v)$ is defined by

$$L(v) = \sum_i L(v_i), \quad v = v_0 | v_1 | \cdots | v_m, \quad (37)$$

where v is the concatenation of branch labels. For the paths a and b with branch label sequences v_a and v_b , we have the lower bound of the Hamming distance between them [3]:

$$d_h(a, b) \geq L(v_a \oplus v_b). \quad (38)$$

By using the lower bound, we can evaluate the minimum free Chernov distance of trellis-codes with linear trellis structure other than binary linear convolutional code.

3.4 Branch-and-Bound Search Algorithm

In this section, we focus on the binary linear convolutional codes for the BMC. By using the lower bound on Hamming distance presented in the previous subsection, the extension to the non-binary trellis-codes is straightforward.

Let us now review a simple branch-and-bound search method for evaluating the minimum free Hamming distance of binary linear convolutional codes. We need to prepare the following notations to present the recursive search algorithm. The zero-state of a target convolutional code is denoted by σ_0 , the set of states reached from a state σ by S_σ , and the Hamming weight of outputs generated at the state transition $\sigma_{from}, \sigma_{to}$ by $W_h(\sigma_{from}, \sigma_{to})$. The evaluation algorithm is given as follows:

Algorithm 0 (Evaluation of d_h^{free})

Step 1 Set $d_{min} := \infty$, and $w := 0$.

Step 2 Call eval (σ_0, σ, w) for each $\sigma (\neq \sigma_0) \in S_{\sigma_0}$.

Step 3 Output d_{min} and quit the algorithm.

Sub-routine eval ($\sigma_{from}, \sigma_{to}, w$)

Step 1 If $w \geq d_{min}$ then return.

Step 2 Set $w := w + W_h(\sigma_{from}, \sigma_{to})$.

Step 3 If $\sigma_{to} = \sigma_0$ and $d_{min} > w$ then set $d_{min} := w$.

Step 4 If $\sigma_{to} = \sigma_0$ then return.

Step 5 Call eval (σ_{to}, σ, w) for each $\sigma \in S_{\sigma_{to}}$.

Step 6 Return.

The algorithm can be regarded as a search algorithm on the code tree, which tries to find the error event with the minimum Hamming weight. The conditions $w \geq d_{min}$ and $\sigma = \sigma_0$ in the sub-routine are used for cutting a

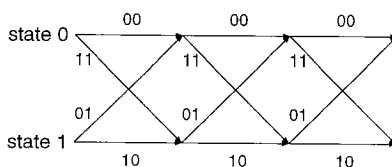


Fig. 2 2-state Trellis-Code.

branch of the tree and it makes the search space finite. The validity of the condition $w \geq d_{min}$ is guaranteed by the additive property of the Hamming distance. In other words, when $w \geq d_{min}$ is satisfied for a code path x , we can exclude the future paths of x from the search space because the future paths of x cannot yield the Hamming weight less than d_{min} .

Example 3: Let us consider the 2-state binary convolutional code shown in Fig. 2. Applying Algorithm 0 to the code, we have $d_h^{free} = 3$. The evaluation process is presented in Fig. 3. In Fig. 3, the branch cut caused by the condition $\sigma = \sigma_0$ is denoted by “0state-cut” and $w \geq d_{min}$ by “dmin-cut.” □

In order to evaluate the minimum free Chernov distance of a binary linear convolutional code, we are required to solve the two more problems, i.e., the lack of additive property and the distance invariance.

The first problem, the lack of additive property, can be solved by using the additive lower bound presented here. The second is that linear codes are not distance invariant with respect to the Chernov distance in general. This prevents the use of the conventional search algorithm (such as Algorithm 0) because the conventional algorithm heavily depends on the distance invariance property of the target code.

The following Algorithm 1 for evaluating the minimum free Chernov distance can solve the above problems.

Algorithm 1 (Evaluation of d_c^{free})

Step 1 Set $d_{min} := \infty$, and $w := 0$, and u to be the empty string. Compute α for a given target channel.

Step 2 Call eval (σ_0, σ, w, u) for each $\sigma (\neq \sigma_0) \in S_{\sigma_0}$.

Step 3 Output d_{min} and quit the algorithm.

Sub-routine eval ($\sigma_{from}, \sigma_{to}, w, u$)

Step 1 If $\alpha w \geq d_{min}$ then return.

Step 2 Set $u := u | l(\sigma_{from}, \sigma_{to})$, where $|$ is the concatenation operator.

Step 3 Set $w := w + W_h(\sigma_{from}, \sigma_{to})$.

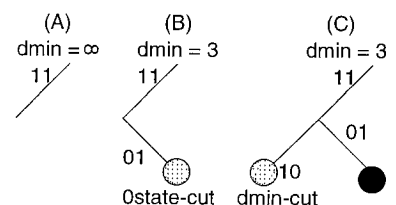


Fig. 3 Evaluation Process of Algorithm 0.

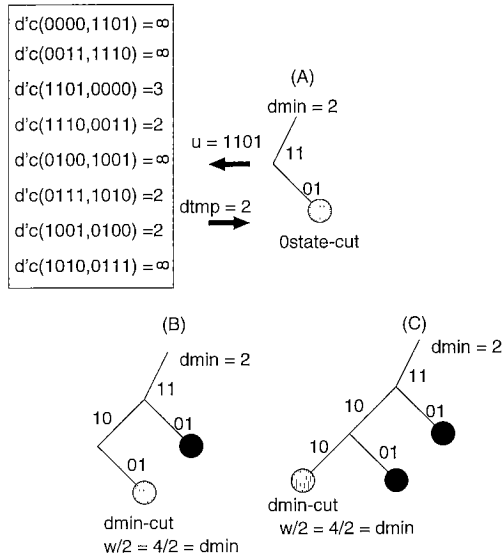


Fig. 4 Evaluation Process of Algorithm 1.

Step 4 If $\sigma_{to} = \sigma_0$ and $d_{min} > d_{tmp}(u)$ then set $d_{min} := d_{tmp}(u)$.

The value $d_{tmp}(u)$ is defined by

$$d_{tmp}(u) = \min_{a \in W_{|u|}} \{d_c(a, a \oplus u)\}, \quad (39)$$

where $|u|$ denotes the length of u .

Step 5 If $\sigma_{to} = \sigma_0$ then return.

Step 6 Call eval $(\sigma_{to}, \sigma, w, u)$ for each $\sigma \in S_{\sigma_{to}}$.

Step 7 Return.

In Step 1 of the sub-routine, the additive lower bound of the Chernov distance is used to discard the paths which will give the Chernov distance larger than d_{min} . In Step 4 of the sub-routine, all the possible pair of code sequences are generated for a given u . The process is somewhat time consuming, but it is necessary when the target code is not distance invariant with respect to the Chernov distance.

Example 4: For the 2-state code defined in Fig.2, the evaluation process performed by Algorithm 1 is shown in Fig.4. The Z-channel with error probability p_a is assumed. In Fig.4, the notation $d'_c(x, y) = d_c(x, y)/(-\ln p_a)$ is used for simplicity. By Algorithm 1, we have $d'_c{}^{free} = d'_c{}^{free}/(-\ln p_a) = 2$. \square

Example 5: We here present the minimum free Chernov distance of rate 1/2 binary convolutional codes[5] over Z-channel in Table 1. These are evaluated by Algorithm 1. The convolutional codes assumed here are generated by a feedforward convolutional encoder with M memories. A slightly modified version of Algorithm 1 can be used for computing $N_{d'_c{}^{free}}$. \square

Table 1 Minimum Free Chernov Distance of Binary Convolutional Codes (rate 1/2) over Z-channel.

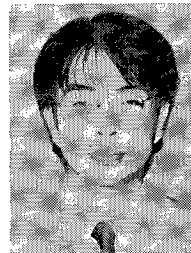
| M | G_0 | G_1 | $d'_c{}^{free}/(-\ln p_a)$ | $N_{d'_c{}^{free}}$ |
|-----|-------|-------|----------------------------|---------------------|
| 2 | 5 | 7 | 3 | 1.56 |
| 3 | 15 | 17 | 3 | 0.63 |
| 4 | 23 | 35 | 4 | 4.38 |
| 5 | 53 | 75 | 4 | 0.54 |
| 6 | 133 | 171 | 5 | 8.90 |

4. Conclusion

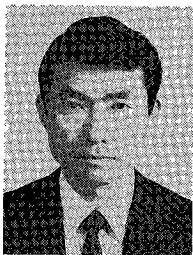
In this paper, we have presented a method for evaluating the minimum free Chernov distance of the trellis-codes for the DMC. The method is based on the additive lower bound on the Chernov distance which enables us to utilize the branch-and-bound technique. The proposed evaluation method would be used as a basic tool for constructing the trellis-codes for the DMC, including asymmetric channels.

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