# MIMO Detect-and-Forward Relay scheme using Symbol LLR Threshold

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*Abstract* — **It is proposed here a way to improve the BER in a relay channel using maximum likelihood detection (MLD) over flat Rayleigh fading channels in a wireless Multiple Input-Multiple Output (MIMO) antenna system. The proposed relaying scheme uses the detect-and-forward protocol and consists of a new threshold mechanism based on symbol LLR at the relay. The relay determines whether each symbol can be relayed or not based on the error probability obtained from the received symbol LLR. The proposed relaying scheme over the conventional detect-and-forward presents the great improvement on Bit Error Rate (BER) for BPSK and QPSK modulations.**

## I. INTRODUCTION

A relay channel means that the destination receives information directly from the source as well as the intermediate relay node located between the source and the destination, as can be seen on Fig.1. This kind of system has a growing interest as a mean to obtain spatial diversity in wireless communications over fading channels, because it allows to enhance the system coverage and to increase the system capacity. The detect-and-forward protocol has simple implementation and easy understanding as characteristics, and this makes it one of the most used protocols of relaying. But it also has a well-known disadvantage of error propagation [1]. To try to minimize this problem, it was studied a MIMO relay system using detect-and-forward protocol over a flat Rayleigh fading channel, where the Maximum Likelihood Detection (MLD) [2] is used at the relay. At the destination, the symbol LLR combining technique developed by [3] is used. The basic threshold idea developed in [4] is expanded here, where it is proposed a new symbol LLR based threshold scheme at relay. The relay will decide if the signal is reliable or not, and choose whether the information will be sent to destination or not according to the reliability information. Also, it is implemented at the destination a selective combining scheme, where the destination will decide if the signal that comes from the relay is valid or not. The objective of the proposed new scheme is to improve the BER at destination. The result of this new scheme is also compared with the hypothetical case when the destination knows the S-R link information [5] [6]. We are also going to compare the developed system with the soft-forward scheme [7]. The modulation scheme used as base is QPSK in a $2 \times 2$ antennas for each link, but it can be easily expanded for other 2-D constellation schemes in any $N \times N$ antennas system.

It will be also shown the results for MIMO $2 \times 2$ BPSK and SISO $1 \times 1$ BPSK cases.

## II. MODEL SYSTEM

We consider the model presented in Fig. 1, and assume that the relay has knowledge of the S-R channel information, i.e. it knows the flat Rayleigh channel matrix $\boldsymbol{H}_{SR}$ and the AWGN variance $\sigma_{SR}^2$. The destination knows the flat Rayleigh channel matrixes $\boldsymbol{H}_{RD}$, $\boldsymbol{H}_{SD}$ and the AWGN variances $\sigma_{RD}^2, \sigma_{SD}^2$. The relay will detect the receive signal with MLD and retransmit the signal to the destination (detect-and-forward protocol). At the destination, it is used symbol LLR to combine the signals from source and from relay.
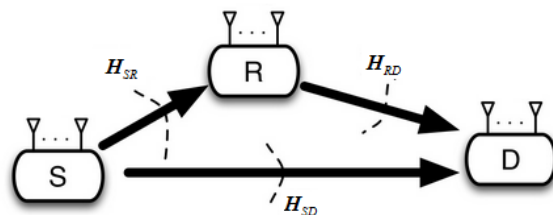


Figure 1.   System block diagram for a MIMO ( $N \times N$ for each relay link) cooperative relay system

## III. CONVENTIONAL DETECT-AND-FORWARD RELAYING

Detect-and-Forward is a simple technique that is largely used in relay systems. Using as a base the scheme presented in [1] and considering a MIMO system with $N \times N$ antennas for each link as seen in Fig.1, the signal sent by source to the relay can be calculated as

$$\boldsymbol{y}_{SR} = \boldsymbol{H}_{SR}\boldsymbol{s} + \boldsymbol{n}_{SR} \qquad (1)$$

where $\boldsymbol{y}_{SR}$ is the received signal $N \times 1$ vector at the relay, $\boldsymbol{H}_{SR}$ is the Rayleigh $N \times N$ channel matrix in which each element follows the complex Gaussian random variable with mean 0, $\boldsymbol{s}$ is the original QPSK signal $N \times 1$ vector transmitted and $\boldsymbol{n}_{SR}$ is the AWGN $N \times 1$ vector of mean 0 and variance

$\sigma_{SR}^2$. The signal vector $\hat{s}$ is then detected at the relay as a hard-symbol using the MLD criterion

$$\hat{s} = \arg\min_{\tilde{s}=(\pm 1 \pm j)} \left\| \boldsymbol{y}_{SR} - \boldsymbol{H}_{SR}\tilde{s} \right\|^2 \tag{2}$$

The signal detected at relay is then sent to destination

$$\boldsymbol{y}_{RD} = \boldsymbol{H}_{RD}\hat{s} + \boldsymbol{n}_{RD} \tag{3}$$

The signal sent directly from the source is also received at destination

$$\boldsymbol{y}_{SD} = \boldsymbol{H}_{SD}\boldsymbol{s} + \boldsymbol{n}_{SD} \tag{4}$$

where $\boldsymbol{H}_{SR}$, $\boldsymbol{H}_{RD}$ and $\boldsymbol{H}_{SD}$ represent the Rayleigh $N \times N$ matrixes independent of each other with the same statistical characteristics. Also, $\boldsymbol{n}_{SR}$, $\boldsymbol{n}_{RD}$ and $\boldsymbol{n}_{SD}$ represent $N \times 1$ AWGN vectors independent of each other with the same statistical characteristics. At the destination, both the signals that come from relay and directly from the source are combined using symbol LLR combining technique [3].

## IV. CONVENTIONAL SOFT - FORWARD RELAYING

Soft-Forward is a technique that is also largely used in relay system. It presents a better overall result than the conventional detect-and-forward, but it also increases the system complexity. The receive signal can be detected at the relay and forwarded as the soft-symbol generated by using symbol LLR. The symbol LLRs can be defined as

$$
\begin{aligned}
\lambda_{l_1}^{(n)} &\triangleq \log_e \left[ \frac{P\left(s_{l_1}^{(n)} \mid \boldsymbol{y}\Delta y\right)}{P\left(s_0^{(n)} \mid \boldsymbol{y}\Delta y\right)} \right] \\
&= \log_e \left[ \frac{P\left(s_{l_1}^{(1)}, \boldsymbol{y}\right)\Delta y / \{p(\boldsymbol{y})\Delta y\}}{P\left(s_0^{(1)} \mid \boldsymbol{y}\right)\Delta y / \{p(\boldsymbol{y})\Delta y\}} \right] \\
&= \log_e \left[ \frac{\sum_{l_2=1}^{4} p\left(\boldsymbol{y} \mid \{s_{l_1}^{(1)}, s_{l_2}^{(2)}\}\right)}{\sum_{l_2=1}^{4} p\left(\boldsymbol{y} \mid \{s_0^{(1)}, s_{l_2}^{(2)}\}\right)} \right]
\end{aligned} \tag{5}
$$

where $\lambda_{l_1}^{(n)}$ defines the symbol LLR, $l_1 = 1, 2, 3, 4$ denotes the symbol number and $n$ means which transmit antenna is used in a MIMO system, i.e., the stream number of spatial multiplex. Also $P\left(s_{l_1}^{(n)} \mid \boldsymbol{y}\Delta y\right)$ means the posteriori probability of transmit symbol being $s_{l_1}^{(n)}$ when the receive vector $\boldsymbol{y}$ falls in the small sphere region $\boldsymbol{y}\Delta y$. The transition probability density $p\left(\boldsymbol{y} \mid \{s_{l_1}^{(1)}, s_{l_2}^{(2)}\}\right)$ can be also defined as

$$p\left(\boldsymbol{y} \mid \{s_{l_1}^{(1)}, s_{l_2}^{(2)}\}\right) = \left(\frac{1}{2\pi\sigma^2}\right)^2 \exp\left(-\frac{\|\boldsymbol{y} - \boldsymbol{Hs}\|^2}{2\sigma^2}\right) \tag{6}$$

where $\boldsymbol{s} = \{s_{l_1}^{(1)}, s_{l_2}^{(2)}\}$.

Using the symbol LLR values, we can define the expected symbol value (soft-symbol) in $1 \times 1$ case using the following formula

$$E\{s \mid y\} = \frac{s_1 e^{\lambda_1} + s_2 e^{\lambda_2} + s_3 e^{\lambda_3} + s_4 e^{\lambda_4}}{e^{\lambda_1} + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}} \tag{7}$$

The power allocation when using soft-forward at relay is the same as detect-and-forward, but the expected symbol value is smaller than 1, i.e., the amplitude in detect-and-forward. The soft-symbol is then sent to the destination where it is combined with the signal that comes directly from the source using symbol LLR combining.

## V. SYMBOL LLR THRESHOLD

Using the concept of threshold developed in [4], it is proposed here a new threshold scheme based on symbol LLR for QPSK modulation. The threshold condition for $1 \times 1$ case is given by

$$\begin{cases} P(x = s_1 \mid y) > 1 - 3\alpha \\ P(x = s_2, s_3 \text{ or } s_4 \mid y) \le \alpha \end{cases} \xrightarrow[\text{highly reliable.}]{P(x=s_1|y)\text{ is}} \hat{s} = s_1 \tag{8}$$

where $s_i$, $i = 1, \cdots, 4$ can assume 4 values, i.e., 4 different possibilities of a QPSK signal point and $\alpha$, $0 < \alpha \ll 1$, means the threshold value. So, only if one of the 4 symbol probabilities is larger than the value $(1 - 3\alpha)$, then the signal with the highest reliability $s_1$, for this example, is normally retransmitted. In other words, the probability $\alpha$ of other signal $s_2, s_3$ or $s_4$ is estimated to be quite low compared with $s_1$. It is assumed that the relay always sends the relayed signal if the above conditions are satisfied, otherwise it will send nothing. The probability $P(x = s_i \mid y)$ in (8) is calculated using symbol LLR's $\lambda_1 \sim \lambda_4$ for QPSK signaling

$$P(x = s_i \mid y) = \frac{e^{\lambda_i}}{e^{\lambda_1} + e^{\lambda_2} + e^{\lambda_3} + e^{\lambda_4}} \tag{9}$$

The value of the symbols LLR's can be calculated as defined in (5). The perfect threshold value can vary according to the signal-noise ratio (SNR) between S-D link, as we can see in Fig.2. Based on simulations realized, we determine that a good threshold value for high SNR values is $\alpha \approx 0.01$.
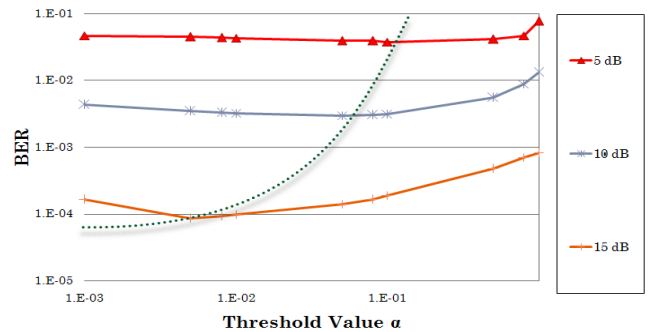


Figure 2. BER versus the threshold value $\alpha$ for many values of SNR's

The green dotted line shows us how the optimum value of the threshold $\alpha$ varies according to the SNR. This new threshold scheme can also be expanded for other 2-D constellation schemes, using the same principle.

## VI. SELECTIVE COMBINING SCHEME AT DESTINATION

Depending on the condition of threshold in (8) being satisfied or not, the relay will send to the destination a hard-symbol (detect-and-forward) or will send nothing. So, we propose here a new selective combining scheme at destination, based on the destination trying to find out when a hard-symbol was sent by the relay or not. The condition can be resumed by as follows. If the magnitude of the signal that comes from the relay for all the antennas is smaller than the maximum possible estimated AWGN value, we consider that the condition in (8) was not satisfied, so the signal sent by the relay is predicted as 0. Otherwise, the destination will accept the signal that receives normally

$$
\begin{cases}
\left| y_{RD}^n \right| \le \max \left| n_{RD}^n \right| \longrightarrow & \hat{s} = 0 \\
\\
\text{Otherwise} \longrightarrow & \hat{s} = s_i
\end{cases}
\quad (10)
$$

where $y_{RD}^n$ is the receive signal from transmit antenna $n$ of the relay-destination link and $n_{RD}^n$ is the AWGN value. Note that, $\max \left| n_{RD}^n \right|$ is the estimation of the maximum AWGN value based on the fact that the destination knows the AWGN variance $\sigma_{RD}^2$ and the fact that the $\left| n_{RD}^n \right|$ follows a Rayleigh distribution. The maximum AWGN value can be estimated as 99% of the Rayleigh distribution's PDF, and can be calculated since the variance $\sigma_{RD}^2$ is known. Also, $n$ represents each of the antenna in the MIMO $N \times N$ system. The result obtained by this selective combining scheme is the result of the following logic. At a large SNR, the max value of $\left| n_{RD}^n \right|$ is a really small value, so in the case when the relay sent nothing to the destination, the signal received at destination will have a really small value (only noise was received). So, the destination accepts that probably no signal was sent by the relay, otherwise the relay considers probably a normal hard-symbol was sent.

This new selective combining scheme provides us a very close result to the perfect result that it is expected when the destination has the knowledge of when exactly the threshold condition is satisfied or not. Fig. 4 shows us the comparison between the selective combining scheme proposed and the perfect result assuming that the destination knows exactly when the threshold condition is satisfied or not.

## VII. KNOWN S-R LINK AT DESTINATION

There is one specific scenario where the destination knows the S-R link information, i.e., the destination also knows the Rayleigh gains in $\boldsymbol{H}_{SR}$ and the AWGN variance $\sigma_{SR}^2$. In this case, we can consider the S-R link and the R-D link as one equivalent link S-R-D [5]. So, the probability expressed in (6), can be calculated as

$$
p\left( y \mid \left\{ s_{l_1}^{(1)}, s_{l_2}^{(2)} \right\} \right) = \left( \frac{1}{2\pi\sigma_{SRD}^2} \right)^2 \exp\left( -\frac{\left\| \boldsymbol{y} - \boldsymbol{H}_{SRD}\boldsymbol{s} \right\|^2}{2\sigma_{SRD}^2} \right) \quad (11)
$$

where $\sigma_{SRD}^2$ can be calculated as [5]:

$$
\sigma_{SRD}^2 = \frac{\sum_{j=1}^{2} \left| h_{RD}^{ij} \right|^2}{\gamma_{SRD}^{(i)}} \sigma^2 = \frac{\sum_{j=1}^{2} \left| h_{RD}^{ij} \right|^2}{\min\left( \gamma_{SR}^{(i)}, \gamma_{RD}^{(i)} \right)} \sigma^2 \quad (12)
$$

where $\gamma_{SR}$ and $\gamma_{RD}$ represent the instantaneous SNR values of the information received from the S-R link and from the R-D link respectively, and $i$ represents the respective transmit antenna. In the situation where the destination knows the S-R link information, the BER will present the improvement compared to the conventional way without S-R knowledge.

## VIII. SIMULATION RESULTS

The simulation conditions are given in Table I, II and III. In Fig.3, we have the results of three simulation curves. First, it was simulated a conventional detect-and-forward relay scheme with MLD. The second result shows the improvement for large SNR when we use the proposed symbol LLR threshold at the relay. The last result represents the proposed selective combining receiving scheme at destination. The modulation used is BPSK for a SISO ($1 \times 1$) system. We can notice a gain of almost 11 dB with the proposed scheme at BER $=10^{-5}$.

TABLE I. SIMULATION CONDITIONS FOR FIGURE 3

| Modulation scheme | BPSK |
|---|---|
| Number of relays | 1 |
| Number of antennas | SISO $1 \times 1$ for each relay link |
| Wireless channel | Quasi-static Rayleigh fading channel |
| Relay scheme | Detect-and-Forward (MLD) |
| Channel estimation | Known channel (R-D and S-D) at D |
| Combining scheme | Symbol LLR combining at D |
| Threshold value | $\alpha = 0.01$ |

In Fig.4, we have the results of three simulation curves. The meaning of the curves is the same of Fig.3 but this time we use a MIMO $2 \times 2$ for each link. We can notice a gain of almost 5 dB with the proposed scheme at BER $=10^{-5}$.

In Fig.5, we have the results of three simulation curves. The meaning of the curves is the same as Fig.3 and Fig.4, but this time we use a MIMO $2 \times 2$ for each link with QPSK modulation. We can notice a gain of 8 dB with the proposed scheme at BER $=10^{-5}$.

In Fig.6, we have the results of three simulation curves. First we present the result for the proposed symbol LLR threshold at the relay. The second result occurs when we apply
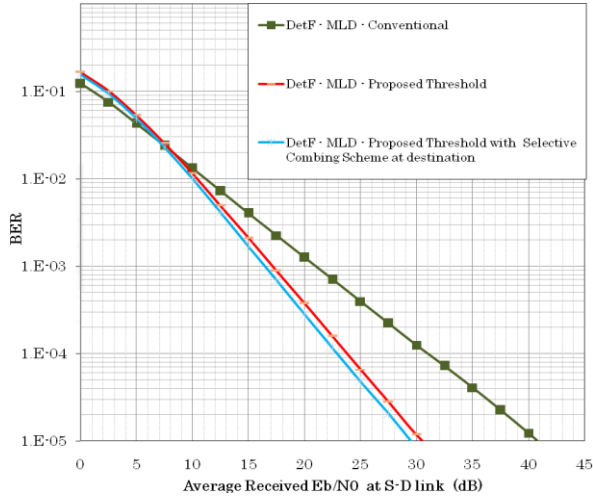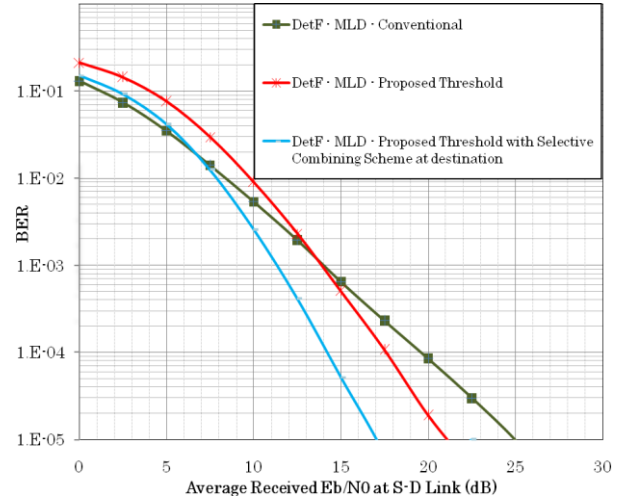
Figure 3.   Proposed scheme simulation for BPSK $1 \times 1$

TABLE II.        SIMULATION CONDITIONS FOR FIGURE 4

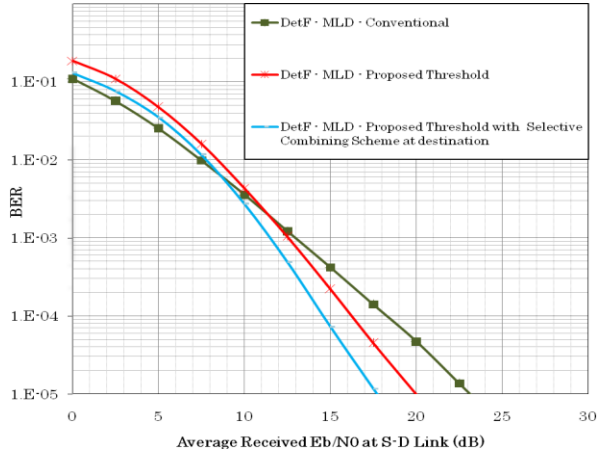| Modulation scheme | BPSK |
|---|---|
| Number of relays | 1 |
| Number of antennas | MIMO $2 \times 2$ for each relay link |
| Wireless channel | Quasi-static Rayleigh fading channel |
| Relay scheme | Detect-and-Forward (MLD) |
| Channel estimation | Known channel (R-D and S-D) at D |
| Combining scheme | Symbol LLR combining at D |
| Threshold value | $\alpha = 0.01$ |



Figure 4.   Proposed scheme simulation for BPSK $2 \times 2$

TABLE III.        SIMULATION CONDITIONS FOR FIGURES 5, 6 AND 7

| Modulation scheme | QPSK |
|---|---|
| Number of relays | 1 |
| Number of antennas | MIMO $2 \times 2$ for each relay link |
| Wireless channel | Quasi-static Rayleigh fading channel |
| Relay scheme | Detect-and-Forward (MLD) |
| Channel estimation | Known channel (R-D and S-D) at D |
| Combining scheme | Symbol LLR combining at D |
| Threshold value | $\alpha = 0.01$ |



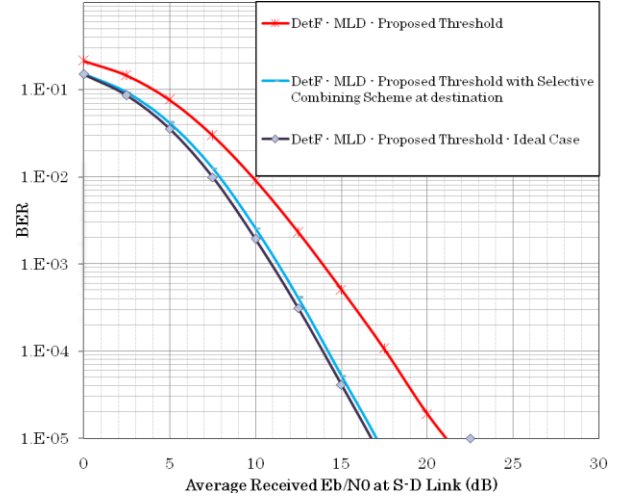Figure 5.   Proposed scheme simulation for QPSK $2 \times 2$



Figure 6.   Proposed selective combining scheme at destination compared to ideal case

the proposed selective combining scheme at destination. The last result represents the ideal case, i.e., the destination knows exactly when the threshold condition was satisfied or not. We can see that the proposed scheme at destination is almost equal to the ideal case result.

In Fig.7, we have the results of four simulation curves. First it was simulated a conventional detect-and-forward relay scheme. The second result represents the situation when the destination has information about the S-R link, i.e. the destination knows the matrix $\boldsymbol{H}_{SR}$ and the variance of $\sigma_{SR}^2$. The third simulation shows the improvement for large SNR when we use the proposed symbol LLR threshold at the relay. The last result occurs when we apply the proposed selective combining receiving scheme at destination. We can conclude that the proposed scheme is better even if compared to the hypothetical situation when destination knows the S-R link information [5].

In Fig.8, we have the results of four simulation curves. First it was simulated a conventional soft-forward relay scheme. The

second result represents the situation when the destination has information about the S-R link for soft-forward. The third result shows the improvement for large SNR when we use the proposed symbol LLR threshold at the relay. The last result occurs when we apply the proposed selective combining receiving scheme at destination. We can conclude that the proposed scheme is better even if compared to the situation when destination knows the S-R link information in a more complex system like soft-forward.
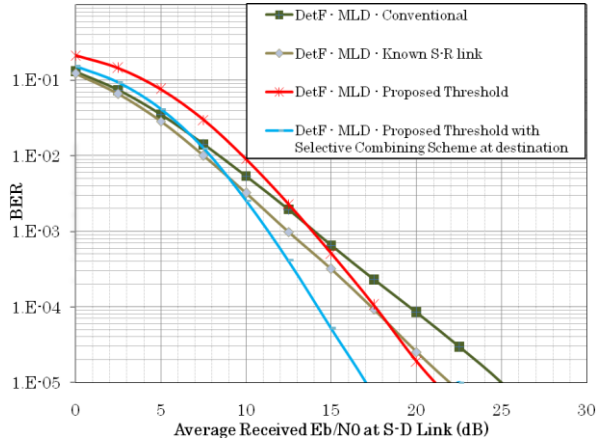


Figure 7. Proposed scheme simulation compared to detect-and-forward when destination knows the S-R link information

TABLE IV. SIMULATION CONDITIONS FOR FIGURE 8

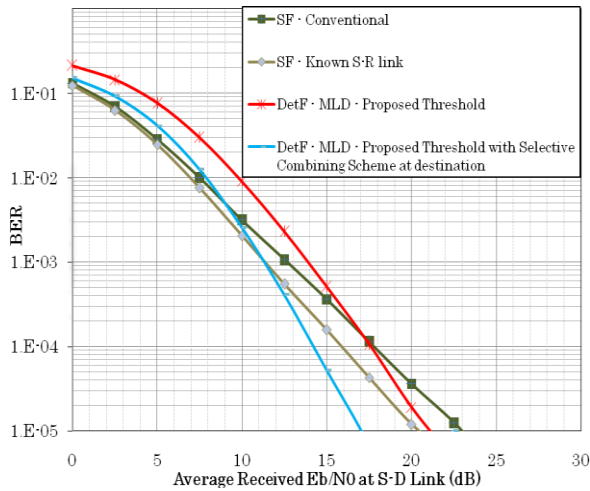| Modulation scheme | QPSK |
|---|---|
| Number of relays | 1 |
| Number of antennas | MIMO $2 \times 2$ for each relay link |
| Wireless channel | Quasi-static Rayleigh fading channel |
| Relay scheme | Soft-Forward |
| Channel estimation | Known channel (R-D and S-D) at D |
| Combining scheme | Symbol LLR combining at D |
| Threshold value | $\alpha = 0.01$ |



Figure 8. Proposed scheme simulation compared to soft-forward system when destination knows the S-R link information

## IX. CONCLUSION

The result obtained for a detect-and-forward relaying scheme based on reliability threshold using symbol LLR shows the 11dB improvement at BER=$10^{-5}$ for a BPSK $1 \times 1$ system. For a BPSK $2 \times 2$ system, we have the improvement of 5dB. For QPSK $2 \times 2$, the improvement is around 8 dB. Also, when we compare it to more complex schemes, like soft-forward, we have improvement of 5 dB. The results obtained are much better than the original detect-and-forward relaying scheme result. With the simple implementation of these proposed 2 schemes, the gain is better even when compared to the hypothetical situation where the destination has the S-R link information. The basic modulation scheme used is QPSK in a $2 \times 2$ system, but this model can be expanded for other 2-D constellations and any MIMO $N \times N$ system. The results presented for BPSK have a similar behavior to the QPSK presented. How to determine the threshold value of $\alpha$ analytically will be the future study.

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