

# Optimal Design Method of Delta-Sigma Modulator for Digital Audio Amplifier

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**Abstract**—In recent years, low-power digital audio (Class-D) amplifiers have been widely used in portable digital devices such as smart phones and tablet PCs. The  $\Delta\Sigma$  modulator is a key component of the Class-D amplifier, and higher-order  $\Delta\Sigma$  modulators provide the advantages of better performance and lower cost. For higher-order  $\Delta\Sigma$  modulator design, it is imperative to determine the optimal design parameters for realizing the maximum SNR while maintaining system stability, which requires the expertise of an experienced designer. To overcome this problem, we propose a new fully automatic optimal design method based on simulated annealing (SA). Compared with the conventional steepest descent optimal design method, this method provides better performance and higher system stability.

**Keywords**-  $\Delta\Sigma$  modulator, Class-D amplifier, Audio, Simulated Annealing

## I. INTRODUCTION

In recent years, low-power high-efficiency amplifiers, called Class-D amplifiers, have been widely used as audio output units of high-performance portable devices such as smart phones and tablet PCs.

Although pulse-width modulation (PWM) is widely used as the modulation method for Class-D amplifiers, it is well known that distortion will occur when input audio signals are converted into PWM signals. Therefore, a novel Class-D amplifier using  $\Delta\Sigma$  modulation instead of PWM was proposed in previous studies [1],[2].  $\Delta\Sigma$  modulators have a noise-shaping property that causes quantization noise in the signal band to move to a higher frequency range; consequently, a high signal-to-noise ratio (SNR) is achieved. As compared to PWM,  $\Delta\Sigma$  modulation for Class D amplifiers provides the advantages of low cost, high power efficiency, and high performance.

High-order  $\Delta\Sigma$  modulators provide much higher performance and SNR than low-order modulators. However, third- or higher-order  $\Delta\Sigma$  modulators may become unstable. The design of high-order  $\Delta\Sigma$  modulators is often complicated because the initial values must be set by an experienced designer; moreover, solutions to the local minimum problem are required for an optimized algorithm.

In this paper, we propose a new design algorithm for high-order  $\Delta\Sigma$  modulators on the basis of expanded simulated annealing (SA) [3],[4], which yields solutions to abovementioned problems.

## II. $\Delta\Sigma$ MODULATOR IN CLASS-D AMPLIFIER

Figure 1 shows the block diagram of a Class-D amplifier with a  $\Delta\Sigma$  modulator. Input digital signals are converted to 1-bit signal streams by the  $\Delta\Sigma$  modulator. Then, a 1-bit signal is amplified by the switching circuit, and the output signal is obtained through a low-pass filter and fed to a speaker. High performance can be realized by increasing the accuracy of the  $\Delta\Sigma$  modulator used in the class-D amplifier.

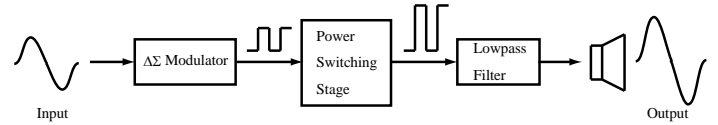


Figure 1. Block diagram of class-D amplifier

Figure 2 shows the block diagram of a third-order  $\Delta\Sigma$  modulator. Higher-order  $\Delta\Sigma$  modulators can be stabilized by assigning a weight factor to each integrator feedback path; the modulator performance is highly dependent on the weight factors. Therefore, we proposed a linear analysis technique for a high-order  $\Delta\Sigma$  modulator by considering its quantizer as a combination of white noise and linear gain in a previous study [2]. Thus, by using power integration, we developed a design algorithm for a stable high-performance modulator. However, the steepest descent method is used for this algorithm; therefore, the initial values must be set by an experienced designer.

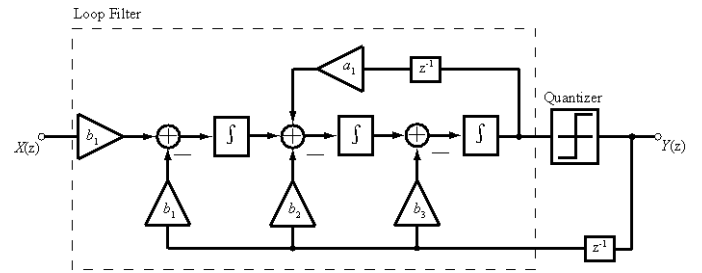


Figure 2. Third-order  $\Delta\Sigma$  modulator

## III. OPTIMAL DESIGN ALGORITHM

Figure 3 shows the proposed optimization algorithm. In [2], when the input sinusoidal amplitude is  $A$ ,  $P_{\text{design}}$  and  $P_{e0}$  are the desired value and the current minimum value of power integration, respectively. Then, the extended objective function  $F(c_i, p)$  expressed by equation (1) is minimized by multi-point search type SA, which is superior to conventional SA for the local minimum problem.

$$F(c_i, p) = -\text{SNR}(c_i) + p\{g(c_i)\}^2 \quad (1)$$

Here,  $c_i$  is a set of parameters,  $p$  is a penalty coefficient greater than 1,  $\text{SNR}(c_i)$  is a function obtained by power integration, and  $g(c_i)$  is a penalty function given by

$$g(c_i) = \varepsilon - |P_{e0} - P_{\text{design}}| \quad (2)$$

where  $\varepsilon$  is a small constant that denotes the error tolerance level.

The major advantage of multi-point search type SA over conventional SA is a move judging process. Whenever the calculated points become worse points, we regard new search points as the generated current point and worse point on the basis of equation (3).  $\Delta F$  in equation (3) denotes the difference between a value at the current point and a value at the generated point, while  $T_k$  denotes a temperature coefficient.

$$A(\Delta F, T_k) = \begin{cases} 1 & (\Delta F < 0) \\ \exp\left(-\frac{\Delta F}{T_k}\right) & (\Delta F \geq 0) \end{cases} \quad (3)$$

#### IV. SIMULATION RESULTS

##### A. Validity of Multi-Point Search Type SA

Equation (4) expresses an evaluation function for validating the design based on multi-point search type SA.

$$F(x) = 418.98 \times n + \sum_{i=1}^n \{-x_i \sin \sqrt{|x_i|}\} \quad (4)$$

Figure 4 shows the distribution of convergent points of evaluation functions  $F(x)$  when  $n = 2$ . Because the optimal minimum value of  $F(x)$  is 0, multi-point search type SA yields better results than conventional SA for points suffering from the local minimum problem.

##### B. Design Example of $\Delta\Sigma$ Modulator

Figure 5 shows the computer simulation results for the frequency spectrum of the designed third-order  $\Delta\Sigma$  modulator with  $OSR = 64$  and  $A/\Delta|_{\text{design}} = 0.7$ . Table 1 lists the results of the proposed modulator design algorithm and those of the algorithm using the steepest descent method in [2].

Compared to the conventional method, the proposed method enables us to design a more stable modulator, as shown in Figure 2. In addition, the proposed method yields a higher SNR than the conventional method [2], as shown in Table I. Thus, it is possible to design a  $\Delta\Sigma$  modulator with high SNR and high stability. Moreover, we note that higher-order  $\Delta\Sigma$  modulators have more coefficients, and their design is complicated. Therefore, the proposed method based on the multi-point search type SA algorithm solves the local minimum problem more effectively.

#### V. CONCLUSION

We proposed an optimal method for designing  $\Delta\Sigma$  modulators for class-D amplifiers on the basis of a multi-point search type SA algorithm. In order to verify the validity of the proposed method, we compared the evaluation function values of the proposed algorithm with those of the conventional SA algorithm, and we designed a third-order modulator based on the proposed algorithm. The proposed design method enabled us to realize a high-order  $\Delta\Sigma$  modulator for Class-D amplifiers, with high SNR and high stability. Moreover, the design parameters were not required to be set by an experienced designer, and they did not suffer from the local minimum problem.

TABLE I. DESIGN RESULTS

	$r$	$\theta$	$\phi$	SNR
Steepest Descent Method in [2]	0.885	0.340	0.0380	92.19
Multi-Point Search Type SA	0.872	0.339	0.0378	92.06

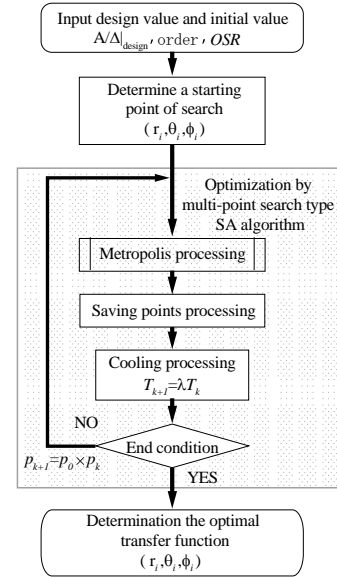


Figure 3. Multipoint search type SA algorithm

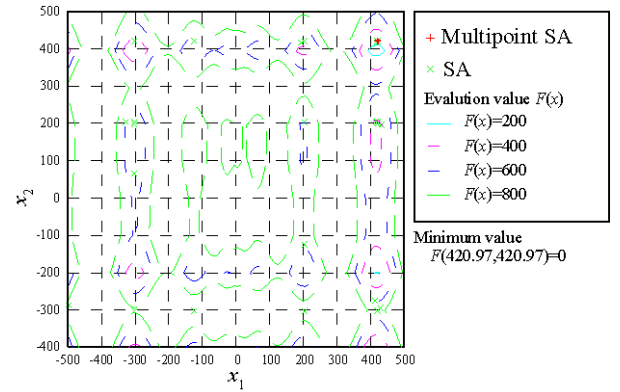


Figure 4. Distribution of convergent points

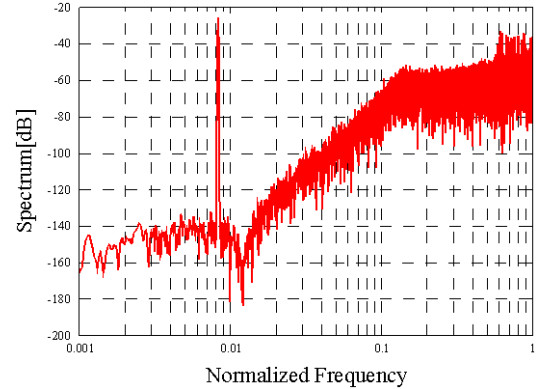


Figure 5. Example of design results

#### REFERENCES

- [1] Kyoungsik Kang, Jeogjin Roh, Youngkil Choi, Hyungdong Roh, Hyunsuk Nam, Songjun Lee: "Class-D Audio Amplifier Using 1-Bit Fourth-Order Delta-Sigma Modulation", IEEE Transactions on Circuit and System II Express Briefs, Volume 55, Issue 8, pp.728, 2008
- [2] Motoki Harada, Satoshi Hirano, Tomio Goto, Masaru Sakurai: "An Optimal Design Method of Delta-Sigma Modulator for Class-D Audio Amplifier", The 13<sup>th</sup> IEEE International Symposium on Consumer Electronics (ISCE 2009)
- [3] N.Takahashi, K.Ebihara, K.Yoshida, T.Nakata, K.Ohashi and K.Miyata: "Investigation of Simulated Annealing Method and its Application to Optimal Design of Die Mold for Orientation of magnetic Powder, IEEE Transactions on Magnetics, 32-3, 1210/1213, 1996
- [4] S.Kirkpatrick, C.D.Gelatt, Jr., M.P.Vecchi: "Optimization by Simulated Annealing", Science, 220-4598, 671/680, 1983