A unified description of Toyoura sand

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Summary. In this paper, the mechanical behavior of sand, was systematically described and modeled. Without losing the generality of the sand, a specific sand called as Toyoura sand, a typical clean sand found in Japan, has been discussed in detail. In the model, the results of conventional triaxial tests of the sand under different loading and drainage conditions were simulated with a fixed set of material parameters. The model only employs eight parameters among which five parameters are the same as those used in Cam-clay model. Once the parameters are determined with the conventional drained triaxial compression tests and undrained triaxial cyclic loading tests, and then they are fixed to uniquely describe the overall mechanical behaviors of the Toyoura sand, without changing the values of the eight parameters irrespective of what kind of the loadings or the drainage conditions may be. The capability of the model is discussed in a theoretical way.

1 Introduction

In recent years, research on constitutive model for soils has been developing very quickly. Some works in particular are worthy of mention in advance. Hashiguchi and Ueno (1977), Hashiguchi (1989) proposed the concept of “subloading”, making it possible to describe the overconsolidation of soils easily and efficiently. Asaoka et al. (1998), Asaoka et al. (2000a), and Asaoka et al. (2000b) proposed the concept of “superloading”, together with the concept of subloading, making it possible not only to describe overconsolidation, but also to explain the effect of the soil structure commonly observed in naturally deposited soils, which is one of the main reasons why soils may differ greatly from place to place. Zhang et al. (2007) proposed a new constitutive model for sand, in which in addition to the concept of superloading related to the soil skeleton structure (Asaoka et al., 1998) and the concept of subloading related to the density (Hashiguchi and Ueno, 1977), a new approach to describe the stress-induced anisotropy was introduced. Based on the model, mechanical behavior of fictional sand subjected to different loadings under different drainage conditions were simulated to verify if the model was suitable to describe the general behavior of the sand with one set of definite parameters (Zhang et al., 2010). Particular attention was paid to the description of the sand subjected to cyclic loading under undrained conditions, that is, for loose sand, liquefaction happens without transition from contractive state to dilative state; for medium dense sand, cyclic mobility occurs while for dense sand, liquefaction will not occur.

The influence of intermediate stress was carefully investigated which leads to the establishment of t_{ij} models (Nakai, 1989). Apart from the t_{ij} concept (Nakai and Mihara, 1984), Yao et al (2008) and Wan et al (2010) discussed the influence of intermediate stress and proposed a concept of transform stress (TS) by which it is also possible to describe this dependency. The advantage of TS concept is that the constitutive model established in p-q stress space can be easily extended to true triaxial stress space without any change or adding of material parameters. In this paper, the authors try to use the model (Zhang et al., 2007), to describe the overall mechanical behaviors of Toyoura Sand, a typical clean sand, in a unified way. In other words, all the behavior of Toyoura Sand, no matter what loading and drainage conditions may be, its mechanical behavior is described with a fixed set of material parameters. The model proposed by Zhang et al (2007) is firstly given in an infinitesimal strain level. Then the influence of intermediate principal stress is taken into consideration by adopting the TS concept. It is important to be emphasized here that the eight material parameters involved in the model, will be constant no matter what kind of loading or drainage conditions may be.

2 Constitutive Model of Sand

Here just a brief description of the yield surfaces is given as shown in Figure 1 and Figure 2.

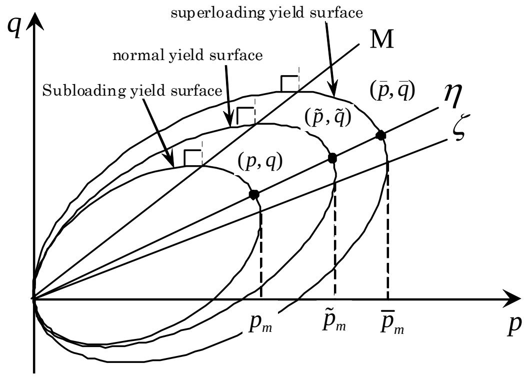
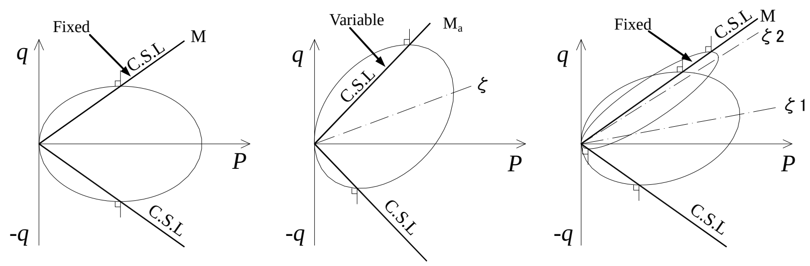


Fig. 1. Subloading, normal and superloading yield surfaces in *p*-*q* plane adopted in the present model



(a) Modified cam-clay model (b) SYS Cam-clay model (c) Proposed model

Fig. 2. Changes in the subloading yielding surfaces at different anisotropy ζ

The similarity ratio of the superloading yield surface to normal yield surface R^* and the similarity ratio of the superloading yield surface to subloading yield surface R are the same as those in the work by Asaoka et al. (2002), namely,

$$R^* = \frac{\tilde{p}}{\bar{p}} = \frac{\tilde{q}}{\bar{q}}, \quad 0 < R^* \le 1 \tag{1}$$

$$R = \frac{p}{\bar{p}} = \frac{q}{\bar{q}}, \quad 0 < R \le 1 \tag{2}$$

$$\frac{\bar{q}}{\bar{p}} = \frac{\tilde{q}}{\tilde{p}} = \frac{q}{p} \quad \text{(Similarity of yield surfaces)} \tag{3}$$

where (p, q), $(\tilde{p}, \tilde{q})$ and $(\bar{p}, \bar{q})$ represent the present stress state, the corresponding normally consolidated stress state and the structured stress state at p-q effective stress space respectively, which is only related to a conventional triaxial stress ($\sigma_{22} = \sigma_{33}$, $q = \sigma_{11} - \sigma_{33}$).

The normally yield surface is given in the following form as:

$$f = \ln\frac{\sigma_m}{\sigma^*_{m0}} + \ln\frac{M^2 - \varsigma^2 + \eta^{*2}}{M^2 - \varsigma^2} + \ln R^* - \ln R - \frac{\varepsilon_v^p}{C_p} = 0 \tag{4}$$

$$\eta^* = \sqrt{\frac{3}{2}\hat{\eta}_{ij}\hat{\eta}_{ij}}\,,\ \hat{\eta}_{ij} = \eta_{ij} - \beta_{ij}\,,\ \eta_{ij} = \frac{S_{ij}}{\sigma_m}\,,\ \sigma_m = p = \sigma_{ii}/3 \tag{5}$$

$$\eta = \sqrt{\frac{3}{2}\eta_{ij}\eta_{ij}}\,,\ \varsigma = \sqrt{\frac{3}{2}\beta_{ij}\beta_{ij}} \quad C_p = \frac{\lambda - \kappa}{1 + e_0} \tag{6}$$

The consistency equation for the subloading yield surface can then be given as:

$$df = 0 \Rightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \beta_{ij}} d\beta_{ij} + \frac{1}{R^*} dR^* - \frac{1}{R} dR - \frac{1}{C_p} d\varepsilon_v^p = 0 \tag{7}$$

It is very clear from Figure 2 that the flat ratio of the elliptical yield surface changes with the value of anisotropy.

1) Evolution rule for stress-induced anisotropic stress tensor β_{ij}

The following evolution rule for the anisotropic stress tensor is defined as:

$$d\beta_{ij} = \Lambda \frac{\sqrt{6}Mb_r(M - \varsigma)\hat{\eta}_{ij}}{C_p(M^2 - \varsigma^2 + \eta^{*2})\sigma_m} \tag{8}$$

From Equation(9), it is known that anisotropy will stop its development at the time when ς approaches to M .

2) Evolution rule for degree of structure R^*

$$dR^* = \Lambda \frac{2aMR^*(1 - R^*)\eta^*}{C_p(M^2 - \varsigma^2 + \eta^{*2})\sigma_m} \tag{10}$$

Here, a is a parameter that controls the rate of the collapse of the structure during shearing. From the definition, it is clear that the structure of a soil will never be regained once it has been lost.

3) Evolution rule for degree of overconsolidation R

The changing rate of overconsolidation is assumed to be controlled by two factors, the plastic component of stretching and the increment in anisotropy.

$$dR = \Lambda \frac{-mM\ln R\sqrt{6\eta^{*2} + \frac{1}{3}(M^2 - \eta^2)^2}}{C_p(M^2 - \varsigma^2 + \eta^{*2})\sigma_m}\left[\frac{(\sigma_m/\sigma_{m0})^2}{(\sigma_m/\sigma_{m0})^2 + 1}\right] + R\frac{\eta}{M}\frac{\partial f}{\partial \beta_{ij}} d\beta_{ij} \tag{11}$$

$$\Lambda = \frac{\dfrac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\varepsilon_{kl}}{h_p + \dfrac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \dfrac{\partial f}{\partial \tilde{\sigma}_{kl}}} \tag{12}$$

$$h_p = \frac{1}{C_p(M^2 - \varsigma^2 + \eta^{*2})\sigma_m}[M_s^2 - \eta^2] \tag{13}$$

where, m is a parameter that controls the losing rate of overconsolidation R during shearing.
The loading criteria are given as:

$$\begin{cases} \Lambda > 0 & loading \\ \Lambda = 0 & neutral \\ \Lambda < 0 & unloading \end{cases} \tag{14}$$

The above Equations were discussed in conventional triaxial stress space. Then the TS concept proposed by Yao et al (2008) was adopted to taken into consideration of the intermediate stress dependency. The detailed description about this model in TS space is omitted because of the same form of the equations in the conventional triaxial stress space. Details can be referred to the works by Yao et al (2008) and Wan et al (2010). In the following section, all the calculations are carried out in TS space.

3 Determination of material parameters

Among the eight parameters involved in the model, five parameters, M , N , λ , κ and v are the same as in the Cam-clay model. The other three parameters and their functions are listed as follows.
a : Parameter that controls the collapse rate of structure
m : Parameter that controls the losing rate of overconsolidation
b_r : Parameter that controls the developing rate of stress-induced anisotropy

Table 1 Material parameters of Toyoura Sand

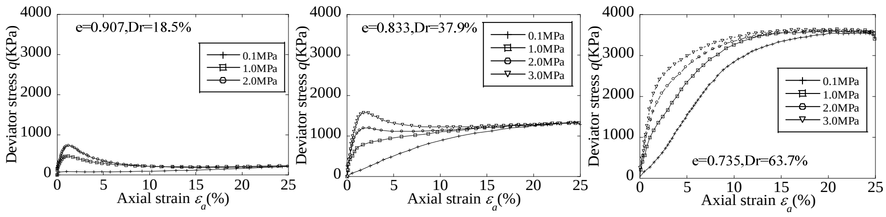
Compression index $\boldsymbol{\lambda}$	0.050
Swelling index $\boldsymbol{\kappa}$	0.0064
Critical state parameter **M**	1.30
Void ratio **N** (*p*=98 kPa on *N.C.L.*)	0.87
Poisson's ratio ***v***	0.30
Degradation parameter of overconsolidation state ***m***	0.01
Degradation parameter of structure ***a***	0.50
Evolution parameter of anisotropy $\boldsymbol{b_r}$	1.50

These three parameters have clear physical meanings and can be determined by undrained triaxial cyclic loading tests and drained triaxial compression tests. Parameter m can be determined based on the losing rate of overconsolidation of soil sample in conventional triaxial compression tests. Parameter a can be determined based on the collapse rate of the structure of the soil sample formed in its natural depositary process, as shown in the work by Asaoka et al (2002). Parameter b_r can be determined based on the developing rate of the stress-induced anisotropy of the soil sample in undrained triaxial cyclic loading tests. The detailed can be referenced to the work by Zhang et al (2010, 2011). The material parameters of Toyoura Sand used in this paper is shown in table 1.

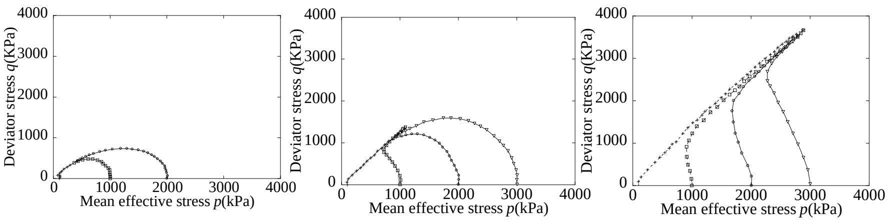
4 Verification of the model by tests

4.1 Confining-stress dependency of sand in undrained monotonic loading test

It is reported in the research by Verdugo and Ishihara (1996) that under the same void ratio, the sand behaves like a loose sand if the confining stress is large and the sand behaves like dense sand if the confining stress is small. Such a phenomenon originally defined by Ishihara (1993) is called as "confining-stress dependency of sand". The mechanical behavior of sands with the same density but different confining stresses can also be reproduced uniquely with one set of the same material parameters in all different conditions by simulation as shown in Figure 5. The simulated results on the whole coincide well with the test results by Verdugo and Ishihara (1996) in Figure 4 quantitatively and qualitatively.

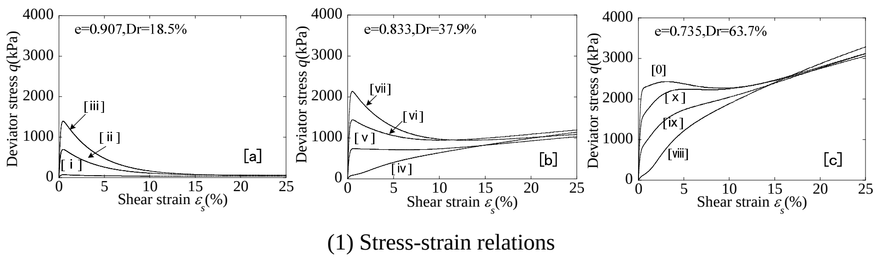


(1) Stress-strain relations

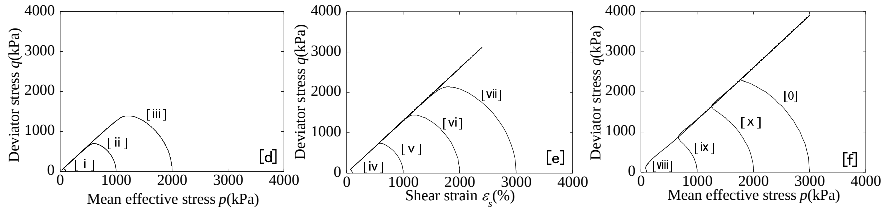


(2) Effective stress paths

Fig. 4. Test results of stress paths and stress-strain relations of Toyoura Sand with the same void ratio but different confining stress in undrained triaxial compression test (Verdugo and Ishihara, 1996)



(1) Stress-strain relations

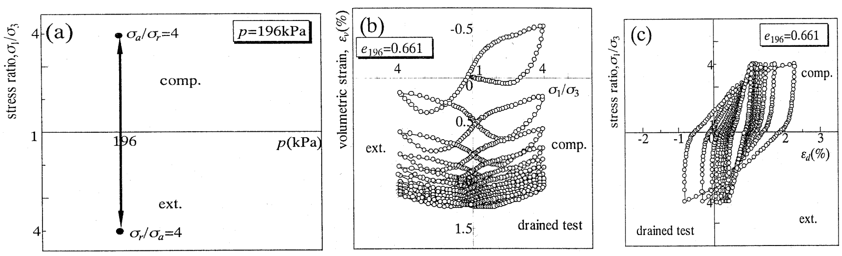


(2) Effective stress paths

Fig. 5. Simulation of the test results in Figure 4

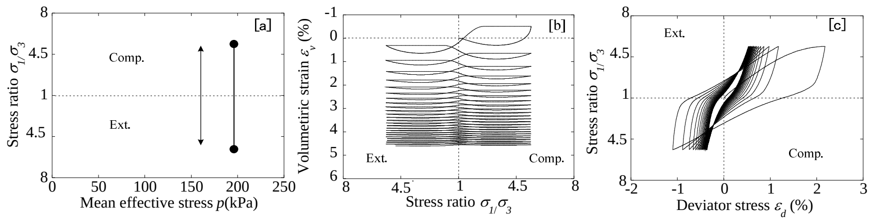
4.2 Sand subjected to drained cyclic loading

The behaviors of dense sand subjected to drained cyclic loading under constant mean effective stress were simulated. As can be seen in Figure 7, the overall characteristics of the sand predicted by the present model, for instance, the changes in dilatancy and stress-strain relations, agree qualitatively well with the test results in Figure 6 by Hinokio (2000), but showing a slight over-estimation of volume strain.

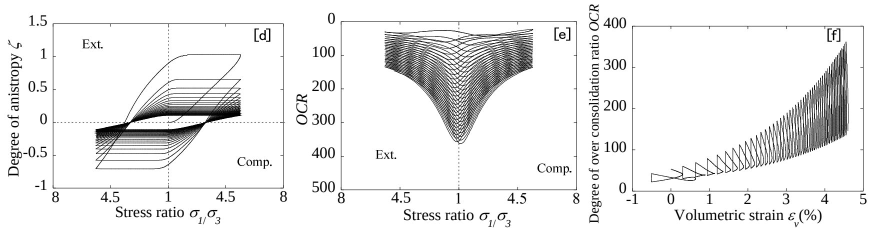


(a) Loading (b) Volumetric strain-stress ratio relation (c) Stress-strain relation

Fig. 6. Test results of dense sand in drained cyclic loading tests (Hinokio, 2000)



(a) Loading (b) Volumetric strain- stress ratio relation (c) Stress-strain relation



(e) Change of anisotropy (f) Change of *OCR* with stress ratio (g) Change of *OCR* with

Fig. 7. Simulation of the test results in Figure 6

5 Performance of the model

5.1 Influence of density

Eight groups of sands with different densitiesare were simulated in cyclic loading tests with a confining stress of 196 kPa. The amplitude of the cyclic loading in shear stress ratio $(q / 2p_0)$ is 0.12. Figure 8 shows the stress paths and stress-strain relations of the sands with different densities in undrained tests. It is clear from the figures that very loose sands ([1] & [2]) generate a large failure strain along the path directly towards the zero effective stress state without transition from contractive state to dilative state. For relatively loose sands ([3] & [4]), they also generate large failure strain at last but transition from contractive state to dilative state. However, for medium dense sands ([5] & [7]), cyclic mobility occurs and the strain increases gradually to a relatively larger scale. On the other hand, the dense sand ([8]) only generates a small amount of strain and never shows cyclic mobility. Therefore, the mechanical behavior of sand subjected to undrained cyclic loading can be uniquely and properly described by the constitutive model under the condition that all the material parameters are kept constant.

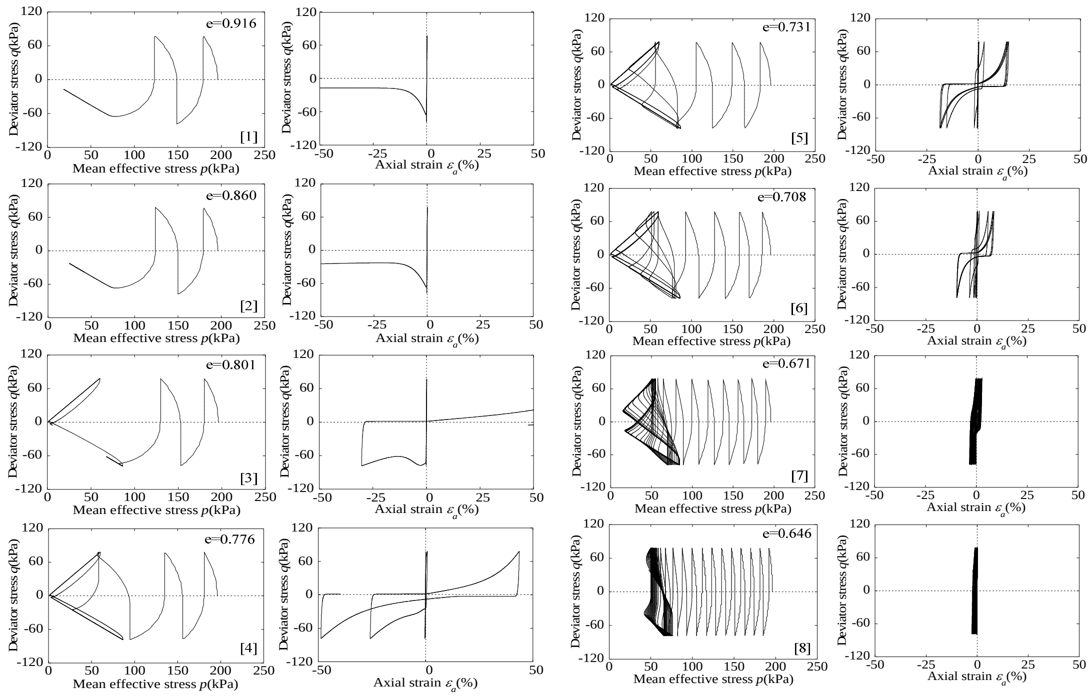


Fig. 8. Stress paths, stress-strain relations of the sand specimens with different densities subjected to cyclic triaxial test under undrained condition

5.2 Influence of stress-induced anisotropy

The stress-induced anisotropy has great influence on the mechanical behavior of sand subjected to consequential loadings. This influence can be investigate through two sand samples [r] and [s], which have almost the same properties in the state variables but different stress-induced anisotropy ς_0 at the beginning of undrained triaxial cyclic loading tests. A different stress-induced anisotropy is obtained by different loading direction of the samples [r] and [s] as shown in Figure 9. Then different behaviors of the samples with different stress-induced anisotropy under undrained condition by simulation are shown in Figure 10. Further discussion about the influence of the stress-induced anisotropy can be referred to the work by Ye et al (2011).

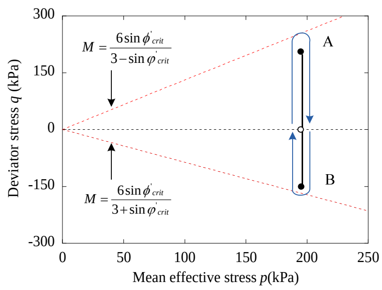
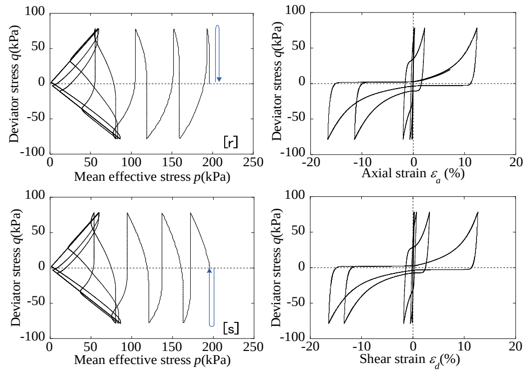


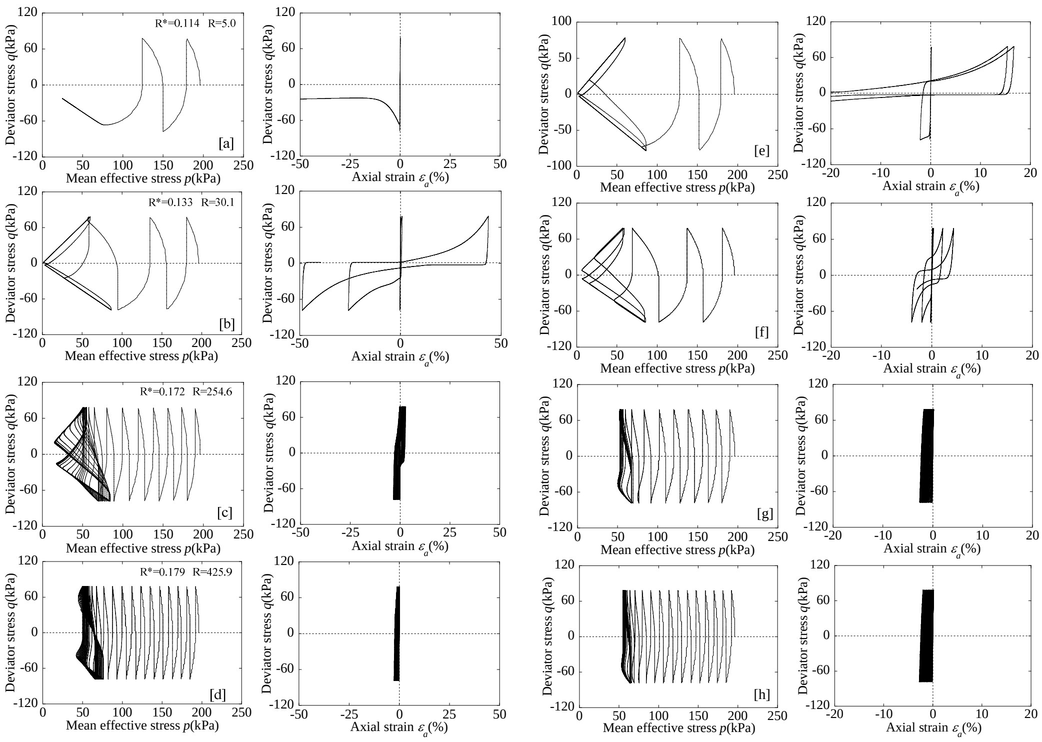
Fig. 9. Prescribed loading path under drained condition



(a) Effective stress paths (b) Stress-strain relations

Fig. 10. Influence of initial anisotropic

5.3 *Influence of structure*



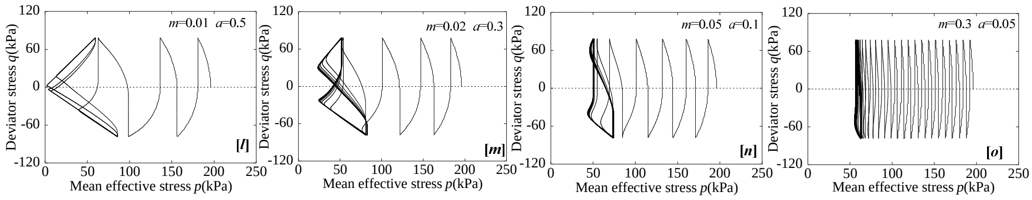
(1) Considering the structure effect (2) Without the structure effect

Fig. 11. Influence of the structure

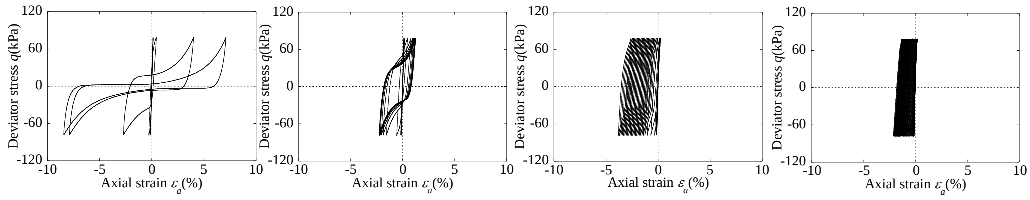
Four sands [2], [4], [7] and [8] are considered to identify the influence of the structure. Sands [2], [4], [7] and [8] with the structure, are marked by [a], [b], [c] and [d]. While the sands without the structure marked by [e], [f], [g] and [h] are almost the same as the sands marked by [2], [4], [7] and [8], that is R^* is always equal to 1.0. As to the mechanical behavior of the sands with the structure shown in Figure 11(1), very loose sand will fail along the way directly towards the original, the zero effective stress state in the effective stress space, before the cyclic mobility has a chance to occur; Medium dense sand shows cyclic mobility; Dense sand will never show cyclic mobility. Comparing the behavior of the set of sands without the structure as shown in Figure 11(2) with those in Figure 11(1), it is implicated that the liquefaction behavior of loose sand is impossible to be described properly when the structure is not considered. For dense sand, though the structure does not affect the behavior of the sand in the cyclic mobility region so much as the loose sand, the shape of the effective stress path is affected more or less.

5.4 Difference between clayey soils and sandy soils

According to the works by Asaoka et al. (2000a), the difference between clayey soils and sandy soils depends on the rate of loss in overconsolidation and the rate of the collapse of the structure during static shearing. For sandy soils, the rate of loss in overconsolidation is very slow, while the rate of the collapse of the structure is very fast. On the contrary, for clayey soils, the rate of loss in overconsolidation is very fast, while the rate of the collapse of the structure is very slow.
A set of soils with the same initial conditions but different values for a and m, which control the changing rates of overconsolidation and the collapse of the structure, are investigated for their behavior when subjected to cyclic loading under undrained conditions. Figure 12 shows the different behavior of soils from sandy soil to clayey soil subjected to cyclic loading under undrained condition. From the figure, it is very clear that by changing parameters a and m, the difference between sandy soils and clayey soils can be easily and uniquely identified.



(1) Effective stress paths



(2) Stress-strain relations

Fig. 12. Difference between clay and sand

6 Conclusions

In this paper, the capability of the model to give a unified description of the overall behaviors with fixed values of eight parameters is verified. By introducing the TS concept, the influence of intermediate principal stress on mechanical behavior of sand can be properly taken into consideration, without changing any value of the material parameters if compared with those adopted in conventional triaxial stress condition.

It cannot say that the model can perfectly describe the various behaviors of Toyoura Sand, but that the model can give a unified description of Toyoura Sand with quite satisfactory accuracy only by using eight material parameters with fixed value. On the other hand, in some very specific aspects, such as the non-coaxial property, which have been pursued for years by many researchers, should be done in future to verify the applicability of the model.

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