【カテゴリーⅠ】

日本建築学会構造系論文集　第546号，31-38，2001年8月
J. Struct. Constr. Eng., AIJ, No. 546, 31-38, Aug., 2001

A THREE-PARAMETER DISTRIBUTION USED FOR STRUCTURAL RELIABILITY EVALUATION

構造信頼性評価における三つのパラメータを有する確率分布形

Yan-Gang ZHAO, Tetsuro ONO**, Hideki IDOTA* and Tomiyuki HIRANO****

趙　衍　剛，小野徹郎，井戸田 秀樹，平野富之

It is a fundamental task to determine the distribution of basic random variables in structural reliability evaluation. In the present paper, a three-parameter distribution, directly defined in terms of mean value, deviation and skewness, is suggested. The new distribution can be applied (1) as a candidate distribution in fitting the statistical data of basic variables and generally presenting two-parameter distributions with small skewness, (2) to realize normal transformation and generate random samplings for random variables with unknown cumulative distribution functions in order to include them into structural reliability analysis, and (3) to provide a moment reliability index for the cases where the first-three moments of the performance function can be easily obtained. Some numerical examples are presented, the simplicity, generality and flexibility of the distribution are investigated, the applicability and efficiency of the distribution are demonstrated, and the distributions of some basic random variables are discussed.

Key Words: *Structural reliability, probability moment, probabilistic uncertainty, probability distribution, FORM*

構造信頼性、確率モーメント、確率不確定性、確率分布形、１次信頼性解析法

1. INTRODUCTION

In structural reliability evaluation, the basic random variables, which represent uncertain quantities, such as loads, environmental factors, material properties, structural dimensions and variables introduced in order to account for modeling and prediction errors, are generally assumed to have a known cumulative distribution function (CDF) or probability density function (PDF). It is a fundamental task to determine the distribution of the basic random variables.

In order to determine the distribution of a basic random variable, the basic method is to fit the histogram of the statistical data of the variable by selecting a candidate distribution[1]. A Bayesian approach in which the distribution is assumed to be a weighted average of all candidate distributions in which the weights representing the subjective probabilities of each candidate being the true distribution, was suggested by Der Kiureghian & Liu[2]. The problem arises as to how to select the candidate distributions and the weights.

As candidate distributions selected to fit the statistical data of a basic random variable, two-parameter distributions such as the well known normal, lognormal, Weibull and Gamma distributions are often used, in which the parameters of the candidate distribution are generally determined from the mean value and deviation of the statistical data. After the two parameters are determined, the distribution form and the high-order dimensionless central moments, such as skewness, will be determined, and which may not be the same as those of the statistical data of the random variable.

Using the practical data collected by Ono et al.[3], two histograms representing the uncertainty included in the properties of H-shape structural steel are shown in Fig. 1, where Fig.1.a corresponds to the section area and Fig.1.b the residual stress at the flange. From Fig.1, one can see that the coefficient of variation corresponding to Fig.1.a is very small (0.0514) while the skewness is so large (0.7085), and the coefficient of variation corresponding to Fig.1.b is very large (0.7492) while the skewness is not quite large (0.823). The skewness of normal, lognormal, Weibull and Gamma distributions that have the same mean value and deviation of the data in Fig.1.a are readily obtained as 0, 0.1555, -0.9121 and 0.1024 respectively, and none of them can match the true skewness (0.7085) of the data. Similarly, the skewness of normal, lognormal, Weibull and Gamma distributions that have the same mean value and deviation of the data in Fig.1.b are obtained as 0, 1.7819, 0.6834 and 1.4984 respectively. One can also see that none of them can

* Assoc. Prof., Dept. of Architecture, Nagoya Institute of Technology, Dr. Eng.　名古屋工業大学社会開発工学科建築系　助教授・工博
** Prof., Dept. of Architecture, Nagoya Institute of Technology, Dr. Eng.　名古屋工業大学社会開発工学科建築系　教授・工博
*** Engineer, Dept. of Struc. Eng., Nikken Sekkei Co. Ltd., M. Eng.　日建設計㈱　工修

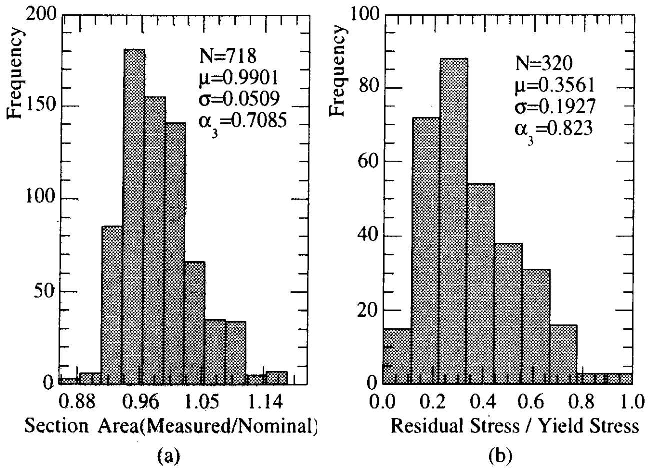

Fig. 1 Two Histogram Examples of Practical Data

match the true skewness (0.823) of the data. Therefore, the two-parameter distributions are not flexible enough to reflect the skewness of statistical data of a random variable, and distributions which can be determined by effectively using the information of skewness as well as the mean value and deviation of the statistical data are required.

In the present paper, a three-parameter distribution, directly defined in terms of mean value, standard deviation and skewness, is suggested. The new distribution, having characteristics of simplicity, generality and flexibility, can be applied as a candidate distribution in fitting the statistical data of basic random variables and generally presenting two-parameter distributions with small skewness, and to realize normal transformation and generate random samplings for random variables with unknown CDFs in order to include them into structural reliability analysis.

2. A THREE-PARAMETER DISTRIBUTION

As ground rules for definition of the three-parameter distribution, the following requirements are stipulated:

(1) Flexibility. -- The distribution should be flexible enough to effectively reflect the characteristics of skewness, i.e., beside the two parameters determined form the mean value and deviation, the third parameter should effectively include the influence of skewness.

(2) Simplicity. -- The procedure needed for determining the parameter and computing the PDF/CDF of the distribution should be convenient.

(3) Generality. -- The distribution should be generally effective for a large range of skewness, and able to be used to generally present two-parameter distributions using their first three moments.

In order to satisfy the three requirements above, a three-parameter distribution is defined using the following CDF and PDF:

$$F(x)=\Phi\left[\frac{1}{2\lambda}\left(\sqrt{1+2\lambda^2+4\lambda\left(\frac{x-\mu}{\sigma}\right)}-\sqrt{1-2\lambda^2}\right)\right] \tag{1}$$

$$f(x)=\frac{\phi\left[\frac{1}{2\lambda}\left(\sqrt{1+2\lambda^2+4\lambda\left(\frac{x-\mu}{\sigma}\right)}-\sqrt{1-2\lambda^2}\right)\right]}{\sigma\sqrt{1+2\lambda^2+4\lambda\left(\frac{x-\mu}{\sigma}\right)}} \tag{2}$$

in which Φ and ϕ are the CDF and PDF of a standard normal random variable, μ, σ and λ are the three parameters of the distribution.

In order to make Eqs. 1 and 2 operable, λ should be limited in the range of $|\lambda|\le 1/\sqrt{2}$. The random variable x is defined in the following ranges:

$$-\infty\le\frac{x-\mu}{\sigma}\le-\frac{1}{4\lambda}-\frac{1}{2}\lambda \qquad \text{for } \lambda<0 \tag{3a}$$

$$-\frac{1}{4\lambda}-\frac{1}{2}\lambda\le\frac{x-\mu}{\sigma}\le\infty \qquad \text{for } \lambda>0 \tag{3b}$$

From Eqs. 1 and 2, the relationship between the random variable x and the standard normal random variable u can be easily understood as

$$u=\frac{1}{2\lambda}\left(\sqrt{1+2\lambda^2+4\lambda\left(\frac{x-\mu}{\sigma}\right)}-\sqrt{1-2\lambda^2}\right) \tag{4}$$

$$x=\sigma\left(-\lambda+\sqrt{1-2\lambda^2}\,u+\lambda u^2\right)+\mu \tag{5}$$

Using Eq. 5, the first three moments of x are obtained as:

$$E[x]=\mu\ ,\quad E\left[(x-\mu)^2\right]=\sigma^2 \tag{6}$$

$$E\left[\left(\frac{x-\mu}{\sigma}\right)^3\right]=6\lambda-4\lambda^3 \tag{7}$$

From Eqs. 6 and 7, one can see that the mean value and the standard deviation of x are equal to the parameters μ and σ, respectively of the distribution. The third dimensionless central moment, i.e., the skewness, is only a function of parameter λ and is independent of μ and σ. Therefore, the distribution can be determined by three parameters, mean value μ, standard deviation σ and skewness α_3 of a random variable.

According to Eq.4, the normal transformation is simply realized using only the first three moments of the variable. Equation 4 is essentially the third-moment standardization function[4].

For $|\lambda|<<1$, the item of λ^3 in the right of Eq. 7 can be neglected and then, the parameter λ can be simply approximated as

$$\lambda=\frac{1}{6}\alpha_3 \tag{8}$$

In order to investigate this approximation, the variations of parameter λ with respect to skewness α_3 obtained using both Eqs. 7 & 8 are depicted in Fig. 2. From Fig. 2, one can see that when the absolute value of skewness is small (e.g. $|\alpha_3|\le 1$), λ is approximately proportional to α_3, and Eq. 8 approximates Eq.7 very well in the range of $|\alpha_3|\le 1$.

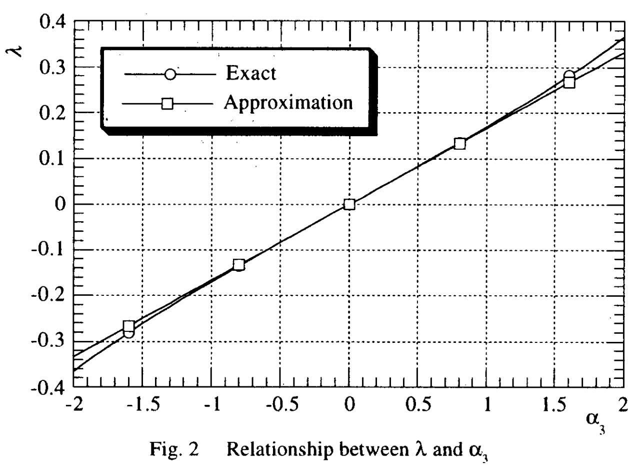

Fig. 2 Relationship between λ and α_3

Substitute Eq.8 into the Eqs. 1 and 2, a simple three-parameter distribution can be obtained.

$$F(x)=\Phi\left[\frac{1}{\alpha_3}\left(\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3\left(\frac{x-\mu}{\sigma}\right)}-\sqrt{9-\frac{1}{2}\alpha_3^2}\right)\right] \tag{9}$$

$$f(x)=\frac{3\phi\left[\frac{1}{\alpha_3}\left(\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3\left(\frac{x-\mu}{\sigma}\right)}-\sqrt{9-\frac{1}{2}\alpha_3^2}\right)\right]}{\sigma\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3\left(\frac{x-\mu}{\sigma}\right)}} \tag{10}$$

Definition range of x:

$$-\infty\le\frac{x-\mu}{\sigma}\le-\frac{3}{2\alpha_3}-\frac{1}{12}\alpha_3 \quad \text{for } \alpha_3<0 \tag{11a}$$

$$-\frac{3}{2\alpha_3}-\frac{1}{12}\alpha_3\le\frac{x-\mu}{\sigma}\le\infty \quad \text{for } \alpha_3>0 \tag{11b}$$

Particularly, for α_3=0, the definition range of x can be easily understood as $-\infty<(x-\mu)/\sigma<\infty$. For α_3=-1, the definition range of x can be obtained as $-\infty<(x-\mu)/\sigma<1.583$ and for α_3=1, $-1.583<(x-\mu)/\sigma<\infty$.

The relationships between x and u are expressed as:

$$u=\frac{1}{\alpha_3}\left(\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3\left(\frac{x-\mu}{\sigma}\right)}-\sqrt{9-\frac{1}{2}\alpha_3^2}\right) \tag{12}$$

$$x=\sigma\left(-\frac{1}{6}\alpha_3+\frac{1}{3}\sqrt{9-\frac{1}{2}\alpha_3^2}\,u+\frac{1}{6}\alpha_3u^2\right)+\mu \tag{13}$$

The first three moments are obtained as:

$$E[x]=\mu\ ,\quad E\left[(x-\mu)^2\right]=\sigma^2\ ,\quad E\left[\left(\frac{x-\mu}{\sigma}\right)^3\right]\approx\alpha_3 \tag{14}$$

From Eq. 14, one can understand that the mean value, standard deviation and skewness of x are equal to the three parameters μ, σ and α_3, respectively, of the distribution. That is to say, the distribution is directly defined in terms of mean value, standard deviation and skewness of a random variable. Since the distribution in Eqs. 9 and 10 is defined in terms of the CDF and PDF of a standard random variable which is considered most common in practical use, the CDF/PDF are convenient to compute. Therefore, the distribution satisfies the requirements of simplicity described previously.

For a standardized random variable $x_s=(x-\mu)/\sigma$, the standard form of Eqs. 9 and 10 can be easily given as

$$F(x_s)=\Phi\left[\frac{1}{\alpha_3}\left(\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3x_s}-\sqrt{9-\frac{1}{2}\alpha_3^2}\right)\right] \tag{15}$$

$$f(x_s)=\frac{3\phi\left[\frac{1}{\alpha_3}\left(\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3x_s}-\sqrt{9-\frac{1}{2}\alpha_3^2}\right)\right]}{\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3x_s}} \tag{16}$$

Since Eqs. 9 and 10 are very simple and are defined directly in terms of the mean value, standard deviation and skewness of a random variable, the distribution is proposed as the three-parameter distribution in the present paper.

Particularly, when α_3 approaches 0, the limit of Eq. 9 can be obtained as

$$\underset{\alpha_3\to0}{limit}\left[F(x)\right]=\Phi\left[\frac{x-\mu}{\sigma}\right] \tag{17}$$

That is to say, the distribution approaches normal distribution when the skewness approaches 0.

The PDFs of the standard three-parameter distribution for α_3>0 and α_3<0 are shown in Figs. 3 and 4. When α_3=0, the PDF degenerates as that of the standard normal distribution and is depicted as a thick solid line in Figs. 3 and 4. From Figs. 3 and 4, one can see that the distribution reflects the characteristics of skewness obviously. Therefore, the distribution satisfies the requirements of flexibility described previously. As investigated in the ensuing sections, the distribution also satisfies the requirements of generality described previously.

3. APPLICATIONS AND INVESTIGATIONS

3.1 Application as a Candidate Distribution

In order to investigate the efficiency of the proposed three-parameter distribution in fitting statistical data of a random variable, the first example uses the practical data of H-shape structural steel described in the introduction. The fitting result of the histogram of the ratio between measured values and nominal values for the section areas is shown in Fig. 5, in which the PDFs of the normal and lognormal distributions whose mean values and deviations are equal to those of the data, and the PDF of the proposed three-

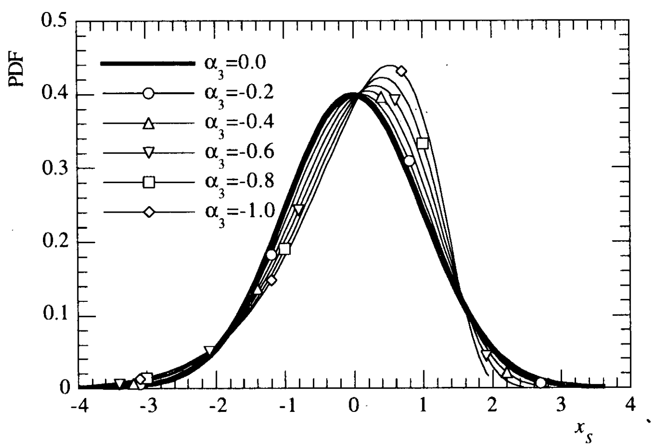

Fig. 3 PDF of the Standard Three-Parameter Distribution (α_3<0)

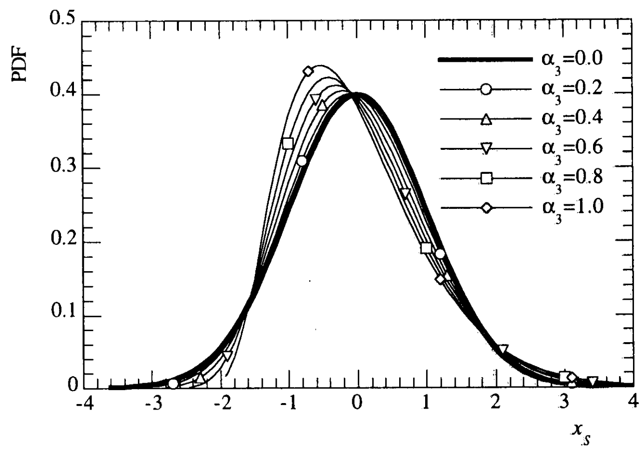

Fig. 4 PDF of the Standard Three-Parameter Distribution (α_3<0)

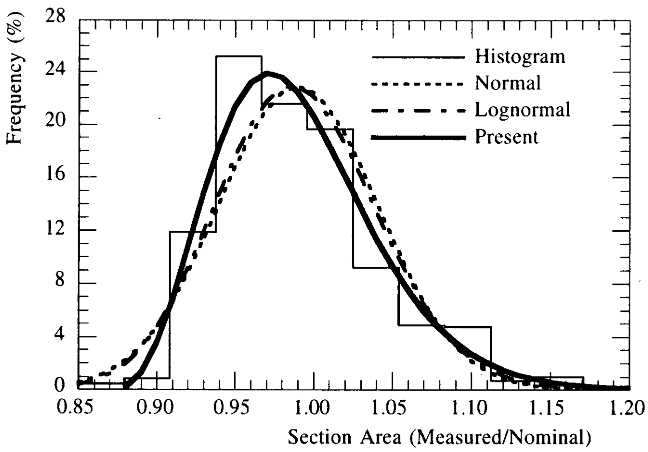

Fig. 5 Data Fitting for the Section Area of H-Shape Structural Steel

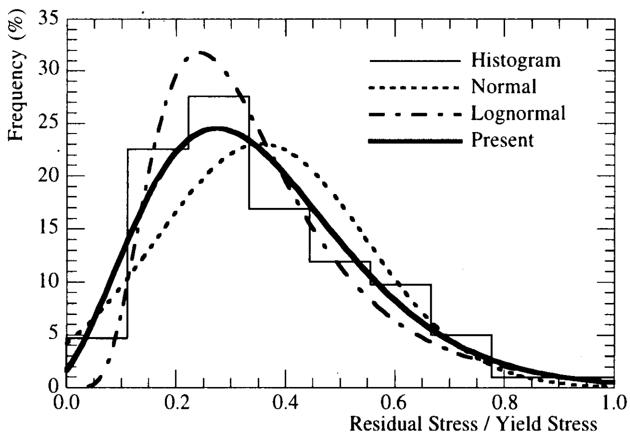

Fig. 6 Data Fitting for the Residual Stress of H-Shape Structural Steel

Table 1 Results of χ^2 Test for Section Area

Intervals	Freq.	Predicited Frequency			Goodness of fit		
		Nor.	Log.	Pres.	Nor.	Log.	Pres.
<0.908	9	38.7	35.3	17.1	22.8	19.6	3.83
0.908-0.938	85	69.3	72.2	85.9	3.54	2.27	0.01
0.938-0.967	181	123	129	154	26.8	20.8	4.67
0.967-0.996	155	160	162	166	0.13	0.28	0.79
0.996-1.025	141	150	146	131	0.54	0.17	0.72
1.025-1.054	66	102	97.5	83.1	12.9	10.2	3.54
1.054-1.083	35	50.5	49.1	44.5	4.77	4.06	2.03
1.083-1.113	34	18.2	19.3	21.1	13.6	11.2	7.90
>1.113	5	4.75	5.95	8.96	6.67	2.28	0.38
Sum	718	718	718	718	91.8	70.8	23.9

Note: Nor. = Normal, Log. = Lognormal, Pres. = Present

Table 2 Results of χ^2 Test for Residual Stress

Intervals	Freq.	Predicited Frequency			Goodness of fit		
		Nor.	Log.	Pres.	Nor.	Log.	Pres.
<0.111	15	32.6	6.54	23.5	9.48	10.9	3.08
0.111-0.222	72	45.4	73.2	61.9	15.6	0.02	1.66
0.222-0.333	88	67.0	95.9	77.1	6.57	0.66	1.54
0.333-0.444	54	71.6	66.0	65.8	4.33	2.17	2.11
0.444-0.555	38	55.4	37.1	44.2	5.44	0.02	0.88
0.555-0.667	31	31.0	19.5	25.2	0.00	6.75	1.34
0.667-0.778	16	12.5	10.1	12.6	0.96	3.40	0.89
0.778-0.889	3	3.67	5.29	5.76	0.12	0.99	1.32
>0.889	3	0.91	6.32	3.90	4.81	1.75	0.21
Sum	320	320	320	320	47.3	26.7	13.0

Note: Nor. = Normal, Log. = Lognormal, Pres. = Present

parameter distribution Eq. 10 whose mean value, deviation and skewness are equal to those of the data, are depicted. Figure 5 reveals the following:

(1) The PDF of the normal distribution has the greatest difference from the histogram of the statistical data among the three distributions. Since normal distribution is a symmetric distribution (0 skewness), it obviously can not be used to fit the histogram that has such a large skewness (0.7085).

(2) The PDF of the lognormal distribution is also very different from the histogram of the statistical data. Although the lognormal distribution can reflect skewness in some degree, the skewness is dependent on the coefficient of variation. Since the coefficient of variation for this data is very small (0.051), the skewness of the lognormal distributions corresponding to this coefficient of variation is too small (0.1555) to match that of the data (0.7085).

(3) Since the skewness of the three-parameter distribution is equal to that of the statistical data, it fits the histogram much better than the normal and lognormal distributions.

The results of the Chi-square test of this data are listed in Table 1, in which the goodness of the fit test is obtained using the following equation[11]:

$$T = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{18}$$

where O_i and E_i are the observed and expected frequencies, respectively. k is the number of categories used , and T is the goodness of the fit test.

From Table 1, one can see that the goodness of the fit test of the proposed distribution (23.86) is much smaller than those of normal (91.80) and lognormal (70.76) distributions. That is to say, the proposed distribution is more suitable to this statistical data.

Similarly, the fitting result of the histogram of the ratio between measured residual stress and yield stress is shown in Fig. 6, the results of the Chi-square test of this data are listed in Table 2. From Fig. 6, one can see that since the skewness of the lognormal distributions corresponding to this data (1.7819) is much larger than that of the data (0.7492), the proposed distribution fits the histogram much better than the normal and lognormal distributions. From Table 2, one can see that the goodness of the fit test of the proposed distribution (13.03) is much smaller than those of normal (47.32) and lognormal (26.77) distributions. That is to say, the proposed distribution is more suitable to this statistical data.

3.2 Distributions of Some Variables Used in Structural Reliability

As an application in presenting the distributions of some random variables used in structural reliability, the uncertainties included in some properties of structural steel are analyzed using the statistical data collected by Ono et al.[3]. The data were collected from 1030 papers and reports published in the journals and reports of the following institutions during the past 30 years[3]:

Architecture Institute of Japan (AIJ),
Japanese Society of Civil Engineers (JSCE),
Japanese Society of Steel Construction (JSSC),
American Society of Civil Engineers (ASCE),
Welding Research Council, etc.

Therefore, the data have quite good generality to reflect the uncertainties of the basic variables. The statistical results for some basic variables are listed in Table 3. In Table 3, the yield stress σ_y, ultimate stress σ_u, Young's modulus E and elongation correspond to SS41 material[5] without distinction of the section types; the section area and thickness correspond to rolled and welded H-shape steel without distinction of the kind of steel; the residual stress σ_R corresponds to the part of the flange of the welded H-shape steel, and Poisson's ratio is for all kinds of steel and all types of sections because the amount of data are very limited.

Since these variables were generally treated as normal or lognormal distributions[3], in order to investigate which distribution fits the statistical data better, the results of the Chi-square test for normal, lognormal and the three-parameter distributions are listed in Table 3. In Table 3, the goodnesses of the fit test are obtained from Eq. 18, and the smallest goodnessses of fit among the three distributions are underlined. From Table 3, one can see that

(1) The skewnesses of all the random variables are positive. According to the statistical experience of the writers of the present paper, the basic random variables presenting the uncertainties included in structural properties generally have positive skewnesses.

(2) For Young's modulus, the goodness of fit of the normal distribution (20.8) is smaller than those of lognormal (25.4) and the three-parameter (27.0) distributions, while for elongation, the goodness of fit of the lognormal distribution (7.24) is smaller than those of normal (61.3) and the three-parameter (10.5) distributions. That is to say, the normal and lognormal distributions are suitable to Young's modulus and elongation, respectively.

(3) For yield stress, Poisson's ratio, section area and residual stress, the goodness of fit test of the proposed three-parameter distribution (13.4, 12.9, 23.9, 13.9) are much smaller than those of the normal (72.1, 31.5, 91.8, 47.3) and lognormal (18.3, 16.1, 70.8, 26.7) distributions. It means that the three-parameter distribution is more suitable for these variables than normal and lognormal distributions.

(4) For ultimate stress and thickness, although the goodness of fit test of the three-parameter distributions is relatively small among the three distributions, the goodness of fit test is still quite large(55.1 and 40.9). This may be because the first three moments are not enough to express the probability characteristics of the random variables. The use of higher order moments may be required and further studies are needed.

Table 3 Distributions of Some Basic Random Variables

Basic Variables	N	k	Moments			Goodness of fit test		
			μ	σ	α_3	nor.	log.	pres.
σ_y (t/cm^2)	2195	8	3.055	0.364	0.512	72.1	18.3	<u>13.4</u>
σ_u (t/cm^2)	1932	7	4.549	0.317	0.153	74.3	<u>49.2</u>	55.1
E (10^{3}t/cm^2)	626	6	2.082	0.096	0.163	<u>20.8</u>	25.4	27.0
Elongation (%)	1572	6	28.22	5.216	0.491	61.3	<u>7.24</u>	10.5
Poisson's Ratio	165	6	0.283	0.029	0.639	31.5	16.1	<u>12.6</u>
Section Area*1	718	9	0.990	0.051	0.709	91.8	70.8	<u>23.9</u>
σ_R *2	320	9	0.356	0.193	0.823	47.3	26.7	<u>13.9</u>
Thickness*1	884	7	0.986	0.045	0.649	115	84.8	<u>40.9</u>

Note: *1 Measured value/Nominal value
*2 Measured residual stress/ Measured yield stress
Nor. = Normal, Log. = Lognormal, Pres. = Present

3.3 Comparison with some two-parameter distributions

Using the first three moments of a specific two-parameter distribution, a three-parameter distribution can be easily defined with the aid of Eqs. 9 and 10 or Eqs. 15 and 16, and the three-parameter distribution can be considered as an approximation or a presentation of the two-parameter distribution since they have the same first three moments. In order to investigate the generality and flexibility of the proposed three-parameter distribution, the second example considers the comparison of PDFs with the Gamma, Weibull and lognormal distributions, all of which are two-parameter distributions.

The results of the comparison are depicted in Figs. 7, 8 and 9, respectively. In Figs. 7, 8 and 9, the PDFs of the Gamma, Weibull and lognormal distributions are depicted as thin solid lines and those of the three-parameter distribution, which is defined using the same first three moments of the corresponding two-parameter distribution, are depicted as thick dash lines. For each two-parameter distribution, the comparisons are conducted in four cases of different values of coefficients of variation V=0.1, 0.2, 0.3 and 0.4 , which corresponds to the mean values of μ= 25, 30, 35 and 40 respectively.

Figures. 7, 8 and 9 reveal the following:

(1) Except in the case in which V=0.4 of the lognormal distribution, the thick dash lines almost coincide with the thin solid lines in all the investigated cases of the three two-parameter distributions. That is to say, the three-parameter distribution can be generally used to approximate or present a two-parameter distribution using the first three moments of the distribution.

(2) For the case in which V=0.4 of the lognormal distribution, the differences between the thick dash line and the thin solid line are significant when x is smaller than 30. In this case the skewness is equal to 1.264 which is beyond the simplification condition $|\alpha_3| \le 1$ of Eqs. 9 and 10. This is a caution when one uses the proposed three-parameter distribution.

3.4 Application as a moment reliability index

For a performance function $G(\mathbf{X})$, where $\mathbf{X}$ is the vector of basic random variables, if the first three moments of $G(\mathbf{X})$ can be obtained, the probability of failure, which is defined as the probability of $G \le 0$ can be readily obtained using the proposed three-parameter distribution.

For standardized random variable x_s=$(G$-$\mu_G)/\sigma_G$, a standard normal ran-

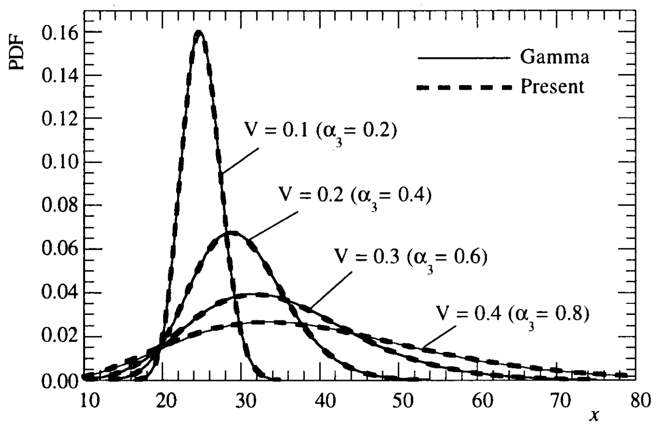

Fig. 7 Comparison with Gamma Distribution

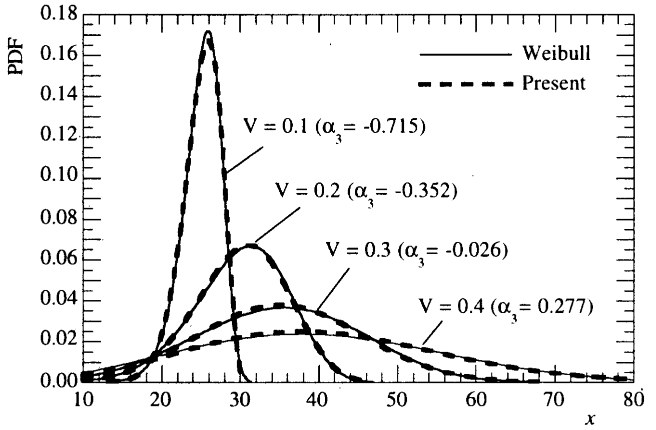

Fig. 8 Comparison with Weibull Distribution

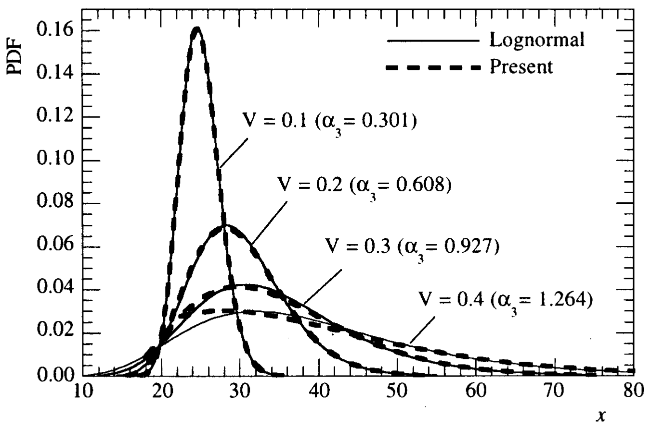

Fig. 9 Comparison with Lognormal Distribution

dom variable u can be transformed using the relationship Eq. 12:

$$u = \frac{1}{\alpha_3}\left(\sqrt{9+\frac{1}{2}\alpha_3^2+6\alpha_3 x_s} - \sqrt{9-\frac{1}{2}\alpha_3^2}\right) \tag{19}$$

Since

$$Prob\left[G \le 0\right] = Prob\left[x_s \le -\frac{\mu_G}{\sigma_G}\right] = Prob\left[x_s \le -\beta_{SM}\right] \tag{20}$$

a third-moment reliability index can be given as

$$\beta_{TM} = \frac{1}{\alpha_{3G}}\left(\sqrt{9-\frac{1}{2}\alpha_{3G}^2} - \sqrt{9+\frac{1}{2}\alpha_{3G}^2 - 6\alpha_{3G}\beta_{SM}}\right) \tag{21}$$

where β_{SM} is the second-moment reliability index, α_{3G} is the skewness of G.

If $G(\mathbf{X})$ is approximated as a second-order surface, the first three moments can be easily obtained and Eq. 21 can be directly used as a second-order third-moment reliability index. In many cases, the performance function is expressed as a linear sum of independent random variables in the original space:

$$G(\mathbf{X}) = \sum_{j=1}^{n} a_j x_j \tag{22}$$

the first three moments of $G(\mathbf{X})$ can also be easily obtained as

$$\mu_G = \sum_{j=1}^{n} a_j \mu_j \tag{23a}$$

$$\sigma_G^2 = \sum_{j=1}^{n} a_j^2 \sigma_j^2 \tag{23b}$$

$$\alpha_{3G}\sigma_G^3 = \sum_{j=1}^{n} a_j^3 \sigma_j^3 \alpha_{3j} \tag{23c}$$

then the third-moment reliability index can be easily obtained using Eq. 21.

The third example considers the following performance function, a plastic collapse mechanism of a one-bay frame[6].

$$G(\mathbf{X}) = x_1 + 2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 \tag{24}$$

The variables x_i are statistically independent and lognormally distributed and have means of $\mu_1=...=\mu_4=120$, $\mu_5=50$ and $\mu_6=40$, respectively, and standard deviations of $\sigma_1=...=\sigma_4=12$, $\sigma_5=15$ and $\sigma_6=12$, respectively.

Because all of the random variables in the performance function shown in Eq. 24 have a known PDF/CDF, the reliability index can be readily obtained using the First-Order Reliability Method(FORM)[7]. The FORM reliability index is $\beta_F=2.348$, which corresponds to a failure probability of $P_F=0.00943$. The true value of the failure probability is $P_F=0.0121$[6].

The skewnesses of variables x_i can be easily obtained as $\alpha_{31}=...=\alpha_{34}=0.301$, $\alpha_{35}=\alpha_{36}=0.927$, respectively. Using Eq.23, the mean value, standard deviation and skewness of $G(\mathbf{X})$ are obtained as $\mu_G=270$, $\sigma_G=103.27$ and $\alpha_{3G}=-0.5284$. Using the first three moments of $G(\mathbf{X})$, the second- and third-moment reliability index are readily obtained as $\beta_{SM}=2.6145$ and $\beta_{TM}=2.2674$ respectively. The probability of failure corresponding to the third-moment reliability index is equal to 0.0117, which is closer to the true value 0.0121 than that of FORM.

3.5 Application to Reliability Analysis Including Variables with unknown CDF

In first- and second-order reliability methods(FORM/SORM), the basic random variables are assumed to have a known CDF because the normal transformation (x-u transformation) and its inverse transformation (u-x transformation) are generally realized by using the CDF of the random variables. In practical applications, the cumulative distribution functions of some random variables are unknown, and the probabilistic characteristics of these variables may be expressed using only statistical moments. In order to include the random variables with unknown CDF into FORM/SORM, the x-u and u-x transformations can be easily realized using μ, σ, and α_3 instead of the cumulative distribution function with the aid of Eqs. 12 and 13, in which the relationship between x and u is expressed as an explicit function of μ, σ, and α_3. This implies that all the random variables with unknown CDFs are assumed to obey the proposed three-parameter distribution.

Furthermore, since random variable x is expressed as an explicit function of u as shown in Eq. 12, random samples can be easily generated based on those of normal random variable u, which are considered to be among the most common in practical use. By this method, the random variables with unknown CDFs can also be included into Monte-Carlo simulation of structural reliability analysis.

The fourth example considers the following simple performance function, a simple compressive state of a structural column.

$$G(\mathbf{X}) = Ax_1 x_2 - x_3 \tag{25}$$

Where A is the nominal section area, x_1 is a random variable presenting the uncertainty included in A, x_2 is yield stress and x_3 is a compressive load. Assume the column is made of H-shape structural steel with a section of H300×200[8] and material of SS41[5], then A=72.38cm^2. The CDFs of x_1 and x_2 are unknown, the only information about them are their first three moments as listed in Table 3, i.e., μ_1=0.990, σ_1=0.051, α_{31}=0.709, μ_2=3.055 t/cm^2, σ_2=0.364, α_{32}=0.512. x_3 is assumed as a lognormal variable with mean value μ_3=150t and standard deviation σ_3=45.

Although the CDFs of x_1 and x_2 are unknown, since their first three moments are known, the x-u and u-x transformations can be easily realized using Eqs. 12 and 13 instead of Rosenblatt transformation and FORM can be readily conducted with results of β_F=1.2635, P_f=0.1032. Furthermore, using Eq.13, the random sampling of x_1 and x_2 can be easily generated without using their CDFs, and the Monte-Carlo simulation can be thus easily conducted. The probability of failure is obtained as P_f=0.1012 when the number of samplings is taken to be 10,000.

4. CONCLUSIONS

A three-parameter distribution is suggested. The new distribution, having characteristics of simplicity, generality and flexibility, can be applied in many aspects of structural reliability. From the investigation of this paper, it is found that:

(1) The distribution can be used as a candidate distribution in fitting the statistical data of basic random variables. Since the distribution reflects the skewness of a random variable effectively, it generally fits the histograms of basic variables better than two-parameter distributions.

(2) Some two-parameter distributions, such as Gamma, Weibull and lognormal distributions that have small skewnesses, can be generally presented by the proposed three-parameter distribution.

(3) For some performance functions, such as second-order performance functions in standard space, or linear performance functions in original space, of which the first-three moments can be easily obtained, the proposed distribution can be easily applied to obtain a moment reliability index.

(4) Since the random variable x that obeys the proposed distribution is expressed as an simple explicit function of u in terms of the first-three moments, it is quite easy to apply it to include random variables with unknown CDFs into FORM/SORM analysis and Monte-Carlo simulation.

(5) The basic random variables presenting the uncertainties included in structural properties generally have positive skewnesses. Normal and lognormal distributions are suitable to Young's modulus and elongation, the proposed distribution is generally suitable to yield stress, Poisson's ratio, section area and residual stress.

(6) The range of the third dimensionless central moment, i.e., the skewness α_3, for which the proposed three-parameter distribution is operable, is $|\alpha_3| \leq 1$. Further study is required for random variables that have an extremely large absolute value of skewness.

ACKNOWLEDGEMENTS

The derivation of Eq. 8 directly from Eq. 7 is suggested by the reviewers of this paper. The writers wish to thank the reviewers for their critical comments and suggestions.

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和文要約

1. 序

構造信頼性解析では、外力や抵抗に含まれている不確定性は一般に確率変数として表され、確率変数の分布形を仮定・決定することは構造信頼性解析および構造信頼性設計の肝要なステップとなる。確率変数の分布形を決定するために、観測データにフィットする予想分布として幅広く用いられる正規、対数正規分布などの分布形はほとんど平均値と標準偏差の二つのパラメータで決められる。一旦平均値と標準偏差が決定すると、分布形の高次モーメントも決められ、分布形の重要な特徴としての歪度などは自由に選択できないことは大きな欠点である。本研究では統計データを精度よくフィットするために平均値、標準偏差、歪度の三つのパラメータで決められる分布形を提示することを試みる。

2. 三つのパラメータを有する分布形の提示

三つのパラメータを有する分布形を式(1)の累積確率分布関数(CDF)および式(2)の確率密度関数(PDF)で定義する。この分布形を有する確率変数xと標準正規確率変数uの関係は式(4)と式(5)のように得られ、xの3次までのモーメントは式(6)と式(7)のように求められる。式(6)と式(7)より、xの平均値と標準偏差はそれぞれパラメータμとσに等しいことが分かる。歪度には平均値μと標準偏差σを含めず，歪度はパラメータλだけの関数である。即ち，式(1)(2)の分布形は平均値，標準偏差および歪度の三つのパラメータで決められる。

パラメータλの絶対値が小さいとき，λと歪度α_3の間には式(8)の簡単な線形関係で表わされる。式(8)を前節の各式に代入することにより式(9)と式(10)の分布形が得られる。xの3次までのモーメントは式(14)のように求められ、xの平均値，標準偏差および歪度がそれぞれパラメータのμ, σ, α_3に等しいことが分かる。

$\alpha_3>0$および$\alpha_3<0$のときの標準型の確率密度関数はそれぞれFig.3とFig.4に示す。式(9)と式(10)の分布形はは直接平均値μと標準偏差σおよび歪度α_3で定義されており，パラメータを決める手間も必要としない。本研究では式(9)と式(10)を三つのパラメータを有する分布形として提示する。この分布形はα_3が0に近付くことにつれて正規分布の分布形に近付く。

3. 応用と考察

3.1 実測データによる分布形の考察

H型鋼の断面積に関する718個のデータ及び残留応力に関する320個のデータをフィットする正規、対数正規分布および本提案分布の確率密度関数をFig. 5とFig. 6に示す。提案分布は正規、対数正規分布より明らかにこれらのデータをよくフィットすることが分かる。正規、対数正規分布および本提案分布に対する検定の結果をTable 1及びTable 2に示す。正規、対数正規分布より、本提案分布の適応度かなり小さい、提案分布は正規，対数正規分布より明らかにこれらのデータを良く適応していることが分かる。

3.2 鋼構造信頼性解析における構造特性の分布形

鋼構造信頼性解析における構造特性として、降伏応力、極限応力、ヤング率、伸び率、ポアソン比、断面積、残留応力などの統計データについて、3次までのモーメントおよび検定結果をTable 3に示す。正規分布，対数正規分布はそれぞれヤング率と伸び率の統計データによく適応し、提案分布は降伏応力、ポアソン比、断面積、残留応力などの統計データによく適応していることが判る。

3.3 二つのパラメータを有する分布形との比較

既存の二つのパラメータを有する分布形との比較により提案分布形の一般性を検討する。変動係数が0.1, 0.2, 0.3 と0.4の４ケースのGamma，Weibullおよび対数正規分布の確率密度関数と提案した確率密度関数の比較をそれぞれFig.7, Fig. 8, Fig.9に示す。Fig.7, Fig. 8, Fig.9により，ほとんどの場合では細い実線と太い破線はほぼ重なっている。即ち，α_3が大きくないとき，提案分布は既存分布を含む一般的な分布として使うことができる。

3.4 モーメント信頼性指標としての応用

式(12) を限界状態関数 $Z=G(\mathbf{X})$ の標準正規化関数として取り扱い、３次モーメント信頼性指標 β_{TM}は式(21)のように得られる。式(24)の１層１スパン骨組の終局限界状態関数に対して､限界状態関数の3次までのモーメントは容易に計算でき、３次モーメント信頼性指標は 2.6145 として得られ、この結果が精算値にかなり近いことが判る。

3.5 分布形が分からない確率変数の取り入れ

分布形が分からない確率変数に対してその統計データから必ず平均値、標準偏差、歪度等のモーメントが計算できる。分布形の変わりに、式(12)と式(13)のx-uとu-x変換を用いれば、分布形が分からない確率変数をFORM/SORMに取り入れることができる。されに、式(13)のxはuの簡単な陽的関数なので、標準正規確率変数uの乱数を利用して、xの乱数も容易に発生でき、MCSにも適用できる。式(25)の限界状態関数には、 x_1, x_2 の分布形が未知であり，統計データからその3次までのモーメントが得られている。 x_1, x_2 の分布形が未知であるものの、式(12)と式(13) のx-uとu-x変換を用いて、 FORMの解析結果はP_f=0.1032のように得られる。式(13)によりx_1, x_2の乱数を発生し、サンプル数が 10,000 で得られたMCS の結果はP_f=0.1012 となる。

4. まとめ

1. 提案分布形は予想分布として応用することができ､二つのパラメータを有する分布形より統計データをよく対応することが判る。
2. 歪度が小さいときの既存のGamma、Weibull、対数正規分布などの二つのパラメータを有する分布形を代表することができる。
3. 三つのパラメータを有する分布形より、３次モーメント信頼性指標が容易に得られる。
4. 提案分布形に基づく標準正規化手法を用いて､分布形が分からない確率変数を FORM/SORM 及びMCS に取り入れることができる。
5. 構造特性の不確定性を表す確率変数は一般に正的歪度を有する｡提案分布は降伏応力、ポアソン比、断面積、残留応力などの統計データによく適応している。
6. 提案分布形の適用範囲は$|\alpha_3| \leq 1$ である。

(2000年11月９日原稿受理，2001年３月８日採用決定)