# A block-coded modulation method for one-way multi-mode data transmission 

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#### Abstract

This paper presents a multi-mode block-coded modulation (BCM) method that enables both multi-mode transmission and coding gain. In multi-mode transmission, the property of data is changed for each mode, and the data are adaptively transmitted. In our method, all modes are constructed by separate multi-level block codes, wherein the number of levels, set partition, rate, and distance properties of each mode may differ from those of the other modes. Different modes may also have different signal constellations, that is, different modulation methods. The code length is the same for all modes, and the mode can be switched at each block boundary if desired. This method uses one-way transmission where the transmitter has no feedback information from the receiver. In decoding, the maximum likelihood decoding (MLD) is carried out using Viterbi algorith$m$ (VA). The decoding of all modes, including decoding of the mode-index, can be accomplished using one trellis diagram. This paper describes the multi-mode encoding and decoding method, and analyzes the distance properties and upper bounds for inter- and intra-modes. It is clarified at which distance the code should be increased to reduce the decoding mode error. Three design examples are presented for unequal error protection (UEP)/equal error protection (EEP), variable-rate, and hybrid AWGN/flat-fading codes.


Keywords- Multi-mode transmission, multi-level block coded modulation, Viterbi decoding, mode-index code.

## I. Introduction

ALONG with the growth of various communication and broadcasting applications, high-data-rate and highcapacity transmission is being increasingly demanded. This demand appears not only in office and backbone transmission but also in private-user communication because of the rapid expansion of internet access. It is therefore important to develop effective means of communication and apply it to home use that will lower the cost of users' equipment. Such means of communication will require a high-capacity transmission method that uses low-cost equipment. Coded modulation $[1-3]$ produces a large coding gain with a relatively high coding rate and is one of the candidates to enable this broadband and efficient transmission.

Adaptive transmission [4-6] has been extensively investigated as one of the methods that can accomplish more effective communication. It is a transmission system that changes its parts according to the state transition of other parts of the system. The adaptive transmission system thus protects its transmission links from the degradation caused by changes in the states of its parts.
There are mainly two types of adaptive transmission:

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two-way and one-way. In a two-way flexible transmission system, various state transitions can be accommodated, such as changes in propagation distortion and quality and the quantity of data queued at each transmitter and receiver. A one-way flexible transmission system in contrast cannot use as much information about state transitions as can a two-way transmission system because it has no interactive connection for the transfer of the state transitions. It does not accommodate changes in the state of the propagation path or the receiver, but it does accommodate changes in the state of the transmitter, such as the data transmission rate, the quantity of data queued, and the quality of service (QoS) of the sending data transmission. Its structure is much simpler than that of a two-way transmission system. In a one-way system, only the transmitter adapts to changes in the traffic. This system is good when we want to give home users a simple system that provides effective transmission.

We previously used block-coded modulation (BCM) as a way of adaptive transmission, namely, one-way multirate transmission [7]. However, on our previous study, we only described adaptive multi-rate transmission, and did not investigate the distance characteristics of the code.

In light of the above situation, we have developed a one-way multi-mode transmission method using multi-level BCM [2, 8-13] with a constant frame length. Using a transmission mode in BCM enables multi-mode transmission not only at a variable rate but also with variable quality and variable distance of codes. These various modes of transmission are made possible by changing at the transmitter parts of the BCM encoder, such as the level of the serial/parallel (S/P) converter and encoder, multilevel encoding, modulation, and the set-partitioned mapper. Maximum-likelihood decoding (MLD) is carried out at the receiver using a Viterbi decoder with an extended trellis diagram having states for all modes. We thus can obtain a one-way multi-mode transmission that has a large coding gain. We also describe the distance characteristics of the proposed BCM in regard to reducing the mode error probability at the receiver. Several distances, such as the minimum squared Euclidean distance (MSED), the minimum symbol distance (MSD), and the minimum product distance (MPD) for additive white Gaussian noise (AWGN) and flat fading noise, are analyzed. We present the results of computer simulations using 16QAM and QPSK to demonstrate the effectiveness the proposed method.

This paper is organized as follows: in Section II, the basic idea behind the one-way multi-mode transmission method is described. The distance parameters of BCM are analyzed in Section III, and the upper bound of the code is
derived in Section IV. In Section V, the computer simulation results are presented to demonstrate the effectiveness of the method. The application of this method to two-way transmission to obtain a more flexible system is briefly described in Section VI. Section VII concludes the paper.

## II. Multi-mode code construction

In multimedia communications, the transmission data requires various types of specs such as a fast or a slow rate, QPSK or QAM modulation, and several volumes of link-margin. It is useful to transmit these multimedia data from a single transmitter to single receiver because of the simplicity of the system. Therefore, the use of multi-mode transmission accommodating some types of data specs is a convenient method. However, when the multi-mode signals are constructed in single transmitter, the inter-mode distance of Hamming or Euclid usually becomes short compared to the single-mode distance, and the transmission quality is degraded. For example, in BPSK and QPSK hybrid transmission, both constellations are stacked to construct a multi-mode signal. Then, the bit error rate (BER) increases because it is dominated by the cross-Euclidean distance between BPSK and QPSK, which is much shorter than self-distances. Thus, encoding the multi-mode transmission is a valid operation. In multi-mode transmission, a mode-index is often used and transmitted with data. When encoding, the mode-index and the data are usually encoded separately in the transmitter, and decoded in order of mode-index and data in the receiver. However, in this method, since the mode-index and data have no interconnection, the maximum-likelihood decoding (MLD) including the mode-index can hardly be carried out, and the data transmission rate cannot be increased to keep the correct decoding of mode. Therefore, to achieve hybrid transmission without any performance degradation, we propose one-way multi-mode transmission using BCM. This enables both multi-mode transmission and coding gain. The MLD with Viterbi algorithm (VA), in which the decoding of all modes can be done by using one trellis diagram, is carried out.
Fig. 1(a) shows the encoder for multi-mode transmission. The coding gain is obtained by multi-level encoding. First, we determine the sending mode of a frame from the data characteristics, such as the contents, quantity of queue sequences, and required quality. Next, based on the sending mode, the number of bit levels in the serial/parallel converter and multi-level encoder is changed. Then, the data at each level is encoded, and the coded bits are mapped into the set-partitioned [1] signal constellation corresponding to the mode. Finally, sending signals are composed. As shown in Fig. 1(a), we can change some or all of the modevariable components, including the serial/parallel converter, encoder, and signal constellation, for each mode. The index of the sending mode can also be encoded and included as a part of or a complete one-level code, in which the inserted level is arbitrary. This extends the Hamming distance between different modes and decreases the error rate.


Fig. 1. (a) Multi-mode encoder and (b) hybrid Viterbi decoder.

Fig. 1(b) shows a hybrid Viterbi decoder [14, 15] that uses a trellis diagram. The VA can be used not only for the trellis-coded modulation (TCM) but also for a certain BCM structure as described in $[3,16,17]$. If each mode and all data bits have the same generation probability, this Viterbi decoding in the multi-level BCM is MLD. The number of trellis states comes from the code structure, such as the depth of the multi-level and the nature of each code. When the multi-level code includes an encoded mode index, the trellis diagram can be divided into sub-trellis areas by index-code words, as shown in Fig. 1(b). Naturally, there may be conjunction, separation, and crossover between the sub-trellis areas. If the encoded mode index does not exist, a normal trellis diagram for multi-level BCM is composed.

Since the VA is used, the complexity of MLD decoding is reduced similar to a single-mode VA. Let $m$ be a number of modes in a block code, and $k_{m}$ be a number of information bits in mode $m$. Then, the decoding complexity on MLD with a full search algorithm becomes $O\left(\sum^{m} 2^{k_{m}}\right)$, while that of the VA for multi-mode becomes $O(E)$ [18], where $E$ is the number of edges in the trellis diagram in Fig. (b) that counts the parallel paths as plural edges. This $O(E)$ is almost always less than $O\left(\sum^{m} 2^{k_{m}}\right)$ as well as the single-mode VA. Thus, after the trellis diagram is drawn, our method obtains the reduction of decoding complexity for MLD.

The above system comprised of an encoder and a decoder enables one-way multi-mode transmission. Since this system can be implemented with a similar configuration of conventional multi-level BCM and Viterbi decoding, it is
easy to compose such an encoder and a decoder.

## III. Distance characteristics of codes

As described in $[1,19,20]$, the major parameters for evaluating a code are the minimum squared Euclidean distance (MSED), the minimum symbol distance (MSD), and the minimum product distance (MPD). These distances affect the BER of the codes, and the longer these distances are, the better the BER performance becomes. The MSED is mainly regarded as a parameter for evaluating the additive white Gaussian noise (AWGN) channel, and the MSD and MPD are parameters for evaluating the wireless flatfading channel. In this paper, we analyze these distances to evaluate the code.

To do so, we first define the code notation $[8,21]$. Let $C_{0}$ be a multi-level block code with code length $L$. When $C_{0}$ is attached to a $2^{l}$-point signal constellation by using set-partitioning, one symbol of $C_{0}$ consists of $l$ bits. If the number of levels of the multi-level code is assumed to be $n, n$ becomes $n \leq l$, and $C_{0}$ is given by

$$
\begin{equation*}
C_{0}=C_{01} C_{02} \cdots C_{0 n} \tag{1}
\end{equation*}
$$

where $C_{0 i}$ is the $i$-th component code of $C_{0}$. Let $\boldsymbol{v}$ be the code word in $C_{0}$, and $v^{j}$ be the $j$-th component $(1 \leq j \leq L)$ of the code word, then $\boldsymbol{v}$ can be expressed as

$$
\begin{equation*}
\boldsymbol{v}=\left(v^{1}, v^{2}, \cdots, v^{L}\right) . \tag{2}
\end{equation*}
$$

Let $v^{j(i)}$ be the $j$-th $(1 \leq j \leq L)$ component of the $i$-th component code, $C_{0 i}(1 \leq i \leq n)$, then $v^{j}$ and code word $\boldsymbol{v}^{(i)}$ of component code $C_{0 i}$ become

$$
\begin{align*}
v^{j} & =v^{j(1)} v^{j(2)} \cdots v^{j(n)}  \tag{3}\\
\boldsymbol{v}^{(i)} & =\left(v^{1(i)}, v^{2(i)}, \cdots, v^{L(i)}\right) \tag{4}
\end{align*}
$$

Then, $C_{0}$ is given by

$$
\begin{equation*}
C_{0}=\left\{\boldsymbol{v}^{(1)} \boldsymbol{v}^{(2)} \cdots \boldsymbol{v}^{(n)} \quad: \quad 1 \leq i \leq n\right\} \tag{5}
\end{equation*}
$$

We will now apply the code notation to multi-mode BCM. Taking $m$ as the number of modes in the code, we can denote the code parameters for each mode as listed in Tab. I. The MSED of the code is derived from $m$ as follows. When a decoding error occurs at the receiver, there are two possible types of errors: bit errors in the correct mode and bit errors in the incorrect mode. Let $d_{I}^{2}$ be the minimum squared Euclidean distance between code words in the same modes (intra-mode distance), and $d_{D}^{2}$ be that between code words in different modes (inter-mode distance). The bit error rate in the intra-mode can thus be reduced by increasing $d_{I}^{2}$. Arranging the structure of each level code of $C_{k}$ and set-partition $S P_{k}$ enables increasing $d_{I}^{2}$. This is the same as extending the MSED in conventional multi-level BCM. From Tab. I, $d_{I}^{2}$ is given by

$$
\begin{equation*}
d_{I}^{2}=\min _{\left(\boldsymbol{u}_{k}, \boldsymbol{v}_{k}\right) \in C_{k}, 1 \leq k \leq m}\left[\sum_{j=1}^{L}\left|M_{k}\left(u_{k}^{j}\right)-M_{k}\left(v_{k}^{j}\right)\right|^{2}\right], \tag{6}
\end{equation*}
$$

where $M_{k}\left(u_{k}^{j}\right)$ is the signal vector of $u_{k}^{j}$, which is the $j$ th component of code word $\boldsymbol{u}_{k}$. In the same way, the bit error rate of an inter-mode can be reduced by increasing $d_{D}^{2}$, which is given by

$$
\begin{equation*}
d_{D}^{2}=\min _{\boldsymbol{u}_{h} \in C_{h}, \boldsymbol{v}_{k} \in C_{k}}\left[\sum_{j=1}^{L}\left|M_{h}\left(u_{h}^{j}\right)-M_{k}\left(v_{k}^{j}\right)\right|^{2}\right] \tag{7}
\end{equation*}
$$

where $1 \leq(h, k) \leq m$ and $h \neq k$. Here, $d_{D}^{2}$ must be large to eliminate the mode error, because the number of error bits tends to be high when a code is decoded in a different mode. This phenomenon will be demonstrated in the examples in Section V. When the mode-index code is used at level $i$ of codes, all of the component codes from $C_{1 i}$ to $C_{m i}$ include mode-index code words. Since each code word in the mode-index component code is assigned a different mode, $d_{D}^{2}$ becomes larger than the minimum squared Euclidean distance of mode-index code at level $i$. Therefore, to increase $d_{D}^{2}$, we should adjust the mode-index code and set-partitioning at level $i$. If the code does not have a mode-index component code, $d_{D}^{2}$ is calculated from Eq. (7). However, when $L$ is large, this calculation is very complex, so it is convenient to use a mode-index code to set the code distance.

From Eqs. (6) and (7), the MSED of the code, $d_{e}^{2}$, is given by

$$
\begin{equation*}
d_{e}^{2}=\min \left[d_{I}^{2}, d_{D}^{2}\right] . \tag{8}
\end{equation*}
$$

Increasing $d_{e}^{2}$ improves the BER performance mainly in AWGN environments.

We will now consider MSD $\delta$ and MPD $\Delta_{p}^{2}$ for fading environments. These parameters determine the BER performance in fading environments. Large $\delta$ and $\Delta_{p}^{2}$ improve the BER performance in a high- and a low-SNR area, respectively. In the same way as the MSED, these parameters can be divided into those for code words in intra-modes and those for code words in inter-modes. The MSD for the intra-modes is denoted as $\delta_{I}$ and that for inter-modes is denoted as $\delta_{D}$. Note that when calculating $\delta_{D}$, the same code symbol, $u^{j}(1 \leq j \leq L)$, for different modes is sometimes counted as a different symbol, because modulation and set-partitioning $M_{k}\left(u^{j}\right)$ can be different in different modes. Here, $\delta_{I}$ is given by

$$
\begin{align*}
\delta_{I} & =\min _{\left(\boldsymbol{u}_{k}, \boldsymbol{v}_{k}\right) \in C_{k}, 1 \leq k \leq m}\left[\delta\left(\boldsymbol{u}_{k}, \boldsymbol{v}_{k}\right): \boldsymbol{u}_{k} \neq \boldsymbol{v}_{k}\right]  \tag{9}\\
& =\min _{\boldsymbol{u}_{k} \in C_{k}, 1 \leq k \leq m}\left[\delta_{H}\left(\boldsymbol{u}_{k}^{(i)}\right): 1 \leq i \leq n_{k}\right] \tag{10}
\end{align*}
$$

where $\delta\left(\boldsymbol{u}_{k}, \boldsymbol{v}_{k}\right)$ is the number of different symbols between $\boldsymbol{u}_{k}$ and $\boldsymbol{v}_{k}, \delta_{H}\left(\boldsymbol{u}_{k}^{(i)}\right)$ is the minimum Hamming distance of the $i$-th component code in mode $k$, and $n_{\text {index }}$ is the level number of the mode-index code. Similarly, $\delta_{D}$ is given by

$$
\begin{equation*}
\delta_{D}=\min _{\boldsymbol{u}_{h} \in C_{h}, \boldsymbol{v}_{k} \in C_{k}}\left[\delta\left(\boldsymbol{u}_{h}, \boldsymbol{v}_{k}\right)\right], \tag{11}
\end{equation*}
$$

where $1 \leq(h, k) \leq m$ and $h \neq k$. If the mode-index component code is included in the code, then

$$
\begin{equation*}
\delta_{D} \geq \min _{\boldsymbol{u}_{h} \in C_{h}, \boldsymbol{v}_{k} \in C_{k}}\left[\delta_{H}\left(\boldsymbol{u}_{h}^{\left(n_{\text {index }}\right)}, \boldsymbol{v}_{k}^{\left(n_{\text {index }}\right)}\right)\right] \tag{12}
\end{equation*}
$$

TABLE I
Parameters of multi-mode code

|  | mode 1 | $\cdots$ | mode $k$ |
| :--- | :---: | :---: | :---: |
| $\cdots$ | $\cdots$ | mode $m$ |  |
| code level | $n_{1}$ | $n_{k}$ | $n_{m}$ |
| multi-level code | $C_{1}=C_{11} C_{12} \cdots C_{1 n_{1}}$ | $C_{k}=C_{k 1} C_{k 2} \cdots C_{k n_{k}}$ | $C_{m}=C_{m 1} C_{m 2} \cdots C_{m n_{1}}$ |
| modulation method | $M_{1}(\boldsymbol{u})$ | $M_{k}(\boldsymbol{u})$ | $M_{m}(\boldsymbol{u})$ |
| set-partitioning | $S P_{1}$ | $S P_{k}$ | $S P_{m}$ |
| codeword | $\boldsymbol{u}_{1}$ | $\boldsymbol{u}_{k}$ | $\boldsymbol{u}_{m}$ |
| codeword of level $i$ | $\boldsymbol{u}_{1}^{(i)}$ | $\boldsymbol{u}_{k}^{(i)}$ | $\boldsymbol{u}_{m}^{(i)}$ |
| codeword of $j$-th symbol | $u_{1}^{j}$ | $u_{k}^{j}$ | $u_{m}^{j}$ |

Thus, $\delta_{D}$ can be increased by extending the minimum Hamming distance of the mode-index code. Increasing $\delta_{D}$ with the use of a mode-index code is easier than increasing $\delta_{I}$ because $\delta_{D}$ can be enlarged only by arranging the mode-index code as shown in Eq. (12). The MSD of the code is represented by

$$
\begin{equation*}
\delta=\min \left[\delta_{I}, \delta_{D}\right] \tag{13}
\end{equation*}
$$

Denoting $\Delta_{P I}^{2}$ and $\Delta_{P D}^{2}$ as the minimum product distance of the codes in the intra-modes and inter-modes, respectively, gives
$\Delta_{P I}^{2}=\min _{\left(\boldsymbol{u}_{k}, \boldsymbol{v}_{k}\right) \in C_{k}, 1 \leq k \leq m}\left[\prod_{j=1, u_{k}^{j} \neq v_{k}^{j}}^{L}\left|M_{k}\left(u_{k}^{j}\right)-M_{k}\left(v_{k}^{j}\right)\right|^{2}\right]$

$$
\begin{align*}
\Delta_{P D}^{2}=\min _{\boldsymbol{u}_{h} \in C_{h}, \boldsymbol{v}_{k} \in C_{k}}[ & \left.\prod_{j=1, u_{h}^{j} \neq v_{k}^{j}}^{L}\left|M_{h}\left(u_{h}^{j}\right)-M_{k}\left(v_{k}^{j}\right)\right|^{2}\right],  \tag{14}\\
& (1 \leq(h, k) \leq m, h \neq k) . \tag{15}
\end{align*}
$$

Therefore, MPD $\Delta_{P}^{2}$ of the code becomes

$$
\begin{equation*}
\Delta_{P}^{2}=\min \left[\Delta_{P I}^{2}, \Delta_{P D}^{2}\right] . \tag{16}
\end{equation*}
$$

Eqs. (8), (13), and (16) show that the BER performance is not greatly improved if either the inter- or intra-mode distance is quite large. Thus, the mode-index code should be constructed with a similar gain to intra-mode. If $L$ is large and $m$ is small, encoding the mode-index code using the overall $n_{\text {index }}$ level gives a large inter-mode distance. However, the data rate becomes low and the overall BER can not be improved because only the inter-distances are large. In this case, therefore, it is desired that the modeindex code uses a part of the $n_{\text {index }}$ level (e.g., the first half), and the rest is used to transmit data bits. In contrast, when $L$ is small and $m$ is large, the mode-index code should use the overall $n_{\text {index }}$ level, which is assigned at multiple levels of $l$.

## IV. Upper bound of BER performance

This section describes the upper bound of the code error performance [17]. The upper bound probability, $P_{b}$, of the
average BER performance in a high-SNR region is given by [19] as

$$
\begin{equation*}
P_{b} \leq \sum_{\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right) \in C} \sum_{\boldsymbol{x}_{1}} b\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right) p\left(\boldsymbol{x}_{1}\right) P\left(\boldsymbol{x}_{1} \rightarrow \boldsymbol{x}_{2}\right), \tag{17}
\end{equation*}
$$

where $p\left(\boldsymbol{x}_{1}\right)$ is the generation probability of $\boldsymbol{x}_{1}$, and $P\left(\boldsymbol{x}_{1} \rightarrow \boldsymbol{x}_{2}\right)$ is the pairwise error probability. In Eq. (17), $b\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right)$ is the Hamming distance between $\boldsymbol{x}_{1}$ and $\boldsymbol{x}_{2}$. Since the code has multi-modes, which can have a different number of bits in different modes, we must take this fact into account when calculating $b\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right)$. If $\boldsymbol{x}_{1}$ and $\boldsymbol{x}_{2}$ are the code words in the same mode or have the same bits in different modes, calculating the Hamming distance is straightforward. When they are in different modes and have a different number of bits, we count the error bits as follows: all of the over-and-short bits between $\boldsymbol{x}_{1}$ and $\boldsymbol{x}_{2}$ are counted as error bits in $b\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right)$. Therefore, the BER is redefined as these error bits divided by the sent bits, so that the BER can be more than one. This means that the BER is improved when different modes have a similar number of bits. However, we do not exclude the codes that have different bits because one-way transmission with a variable number of bits for each mode is an effective transmission system. To eliminate the mode error, mode-index codes should be inserted at a large Hamming distance, as shown in Eq. (12).

Consequently, decomposing Eq. (17) by the mode, $P_{b}$ can be represented by

$$
\begin{equation*}
P_{b} \leq \sum_{i=0}^{m} \sum_{j=0}^{m}\left[\sum_{\boldsymbol{x}_{1} \in C_{i}} \sum_{\boldsymbol{x}_{2} \in C_{j}} b\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right) p\left(\boldsymbol{x}_{1}\right) P\left(\boldsymbol{x}_{1} \rightarrow \boldsymbol{x}_{2}\right)\right] . \tag{18}
\end{equation*}
$$

Decomposing Eq. (18), the upper bounds for identical modes and for different modes are obtained as

$$
\begin{align*}
P_{b} & \leq P_{b I}+P_{b D},  \tag{19}\\
P_{b I} & =\sum_{i=0}^{m}\left[\sum_{\boldsymbol{x}_{1} \in C_{i}} \sum_{\boldsymbol{x}_{2} \in C_{i}} b\left(\boldsymbol{x}_{1}, \boldsymbol{x}_{2}\right) p\left(\boldsymbol{x}_{1}\right) P\left(\boldsymbol{x}_{1} \rightarrow \boldsymbol{x}_{2}\right)\right], \tag{20}
\end{align*}
$$

|  |  | s2 | S (L-1) | sL |
| :---: | :---: | :---: | :---: | :---: |
| 11 | $\bar{c}_{1}$ | $\mathrm{c}_{1}$ | $\bar{c}_{1}$ | $\widehat{c}_{1}$ |
| 12 | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{\text {L-1 }}$ | $\mathrm{c}_{2}$ |
| 13 | $\mathrm{a}_{\mathrm{L}}$ | $\mathrm{a}_{\text {L+1 }}$ | $\mathrm{a}_{\text {2L-2 }}$ | $\mathrm{a}_{2 \mathrm{~L}-1}$ |
|  |  | $\underline{a}_{2}^{2 L+1}$ |  | $\mathrm{a}_{3} \text { 3L-1 }$ |
| $\mathrm{c}_{1}=0($ mode 1$), 1($ mode 2$)$ |  |  |  |  |
| $\mathrm{c}_{2}=\mathrm{a}_{1} \oplus \mathrm{a}_{2} \oplus \cdots-\cdots-\cdots \oplus \mathrm{a}_{\mathrm{L}-1}$ |  |  |  |  |

(a)

(b)

(c)

Fig. 2. Structure of two-mode hybrid block code for UEP and EEP (code 1). (a) code, (b) hybrid set-partitioning, and (c) trellis diagram.

Here, $P_{b I}$ is the upper bound of the decoding-error probability in the intra-modes, and $P_{b D}$ is that in inter-modes.

## V. Examples

In this section, we will describe different codes based on the proposed method.

## A. Hybrid code of UEP and EEP

First, we will consider simple hybrid BCM with two modes, namely, unequal error protection (UEP) and equal error protection (EEP), called hereinafter "code 1".

Fig. 2(a) shows a multi-level code defined by length $L$. It is a four-level code with two modes, both of which have the same structure. Code $l 1$ is a mode-index component code, which is a repetition code of all zeros or all ones with Hamming distance $L$. Code $l 2$ is a $(L, L-1,2)$ parity check code, and codes $l 3$ and $l 4$ are uncoded ( $L, L, 1$ ) codes. Here, $(N, K, d)$ describes the number of bits $(N)$, the number of information bits $(K)$, and the Hamming distance $(d)$. To implement UEP and EEP, set-partitioning is changed for every mode as shown in Fig. 2(b). A 16QAM constellation is used for code 1 . The first bifurcation cor-

TABLE II
Code characteristics of each mode in code 1.

|  | mode $1 \quad$ mode 2 |  |
| :--- | :---: | :---: |
| code level | 4 |  |
| $l 1$ code | mode-index code |  |
| $l 2$ code | $(L, L-1,2)$ |  |
| $l 3$ and $l 4$ code | $(L, L, 1)$ |  |
| modulation | 16 QAM |  |
| signal partition | $S P_{1} \quad S P_{2}$ |  |
| trans. rate (bit/sym) | $(3 L-1) / L$ |  |
| $d_{D}^{2}$ | $L \Delta^{2}$ |  |
| $d_{I}^{2}$ of $l 2$ | $4 \Delta^{2}$ |  |
| $\frac{4 \Delta^{2}}{d_{I}^{2}}$ of $l 2$ | $4 \Delta^{2}$ |  |
| $d_{I}^{2}$ of $l 3$ and $l 4$ | $4 \Delta^{2}$ |  |

responds to both modes because $l 1$ is a mode-index code. After level $l 1$, the way of set-partitioning is divided into $S P_{1}$ for mode 1 and $S P_{2}$ for mode 2, which are based on different concepts. $S P_{1}$ is a conventional set-partitioning in which the Euclidean distance doubles in proportion to the code level, while $S P_{2}$ is set to extend the average Euclidean distance of $l 2$. Therefore, we can carry out UEP by sending the mode-2 code. The tradeoff when transmitting with $S P_{2}$ is that the Euclidean distance of $l 3$ and $l 4$ shortens $1 / \sqrt{2}$ times compared to that when transmitting with $S P_{1}$.

If a mode-index code is used, it should be used at a higher level, such as $l 1$. When the mode-index code is inserted at $l 1$, as in the above example with code 1 , we can divide the way of set-partitioning for each mode after $l 2$. This division increases the degree of design freedom of set-partitioning for a multi-mode. There is also another advantage of inserting a mode-index code at a higher level. Typically, a component code at a higher level in a conventional multi-level code is composed with a lower-rate code to set the Hamming distance, because the Euclidean distance corresponding to upper levels in set-partitioning is short compared to that corresponding to lower levels. Therefore, the loss in transmission rate caused by inserting the mode-index code is relatively low when the mode-index code is allocated at a higher level.

Tab. II lists the code characteristics, where $\Delta$ is the distance between the 16QAM signal points shown in Fig. 2(b) and $\overline{d_{I}^{2}}$ is the average of the squared Euclidean distances in the intra-mode when the generation of code words is equally likely. Since $\overline{d_{I}^{2}}$ of $l 2$ in $S P_{2}$ is 1.5 times as large as that in $S P_{1}$, the BER performance in $l 2$ of mode 2 will be better than that of mode 1 . Since $d_{D}^{2}$ depends on $L$, as shown in Tab. II, the error rate of the decoded mode should decrease in proportion to $L$.

The VA using the trellis diagram shown in Fig. 2(c) a simple diagram with four states - is used for decoding. The output labels of $A_{1}-A_{4}$ are the signal sets having the same branch rules as follows:
$A_{1}=(0000,0001,0010,0011), A_{2}=(0100,0101,0110,0111)$


Fig. 3. Equivalent baseband system block diagram for (a) AWGN and (b) fading environments.
$A_{3}=(1000,1001,1010,1011), A_{4}=(1100,1101,1110,1111)$,
where each signal set has four points. There are, therefore, four parallel paths in the branches. From the received signals, we calculate the minimum metric for $A_{1}-A_{4}$. This calculation also determines the signal points for sets $A_{1}-$ $A_{4}$. Then the sum of the metrics for $s 1$ to $s L$ in Fig. 2(a) is calculated based on the trellis diagram in Fig. 2(c), by using the VA. Since this trellis diagram has all states of both modes 1 and 2, multi-mode decoding can be carried out simply by processing the Viterbi decoding. This is the advantage of this scheme that simplifies multi-mode decoding. Furthermore, the mode can be switched at every code frame.

We evaluated the performance of code 1 by computer simulation using the system shown in Fig. 3(a). In the following, we assume that there is perfect clock and frame synchronization.

Fig. 4 shows the simulation results; Fig. 4(a) shows the BER performance in an AWGN environment where $L$ is 6 , and the generation probability, $p$, of mode 2 is 0.5 . The upper bounds were calculated from Eqs. (19)-(21). The BER curves obtained in the simulation asymptotically accord with the calculated upper bounds, although they are not yet close. The transmission rate is $17 / 6 \mathrm{bit} / \mathrm{symbol}$ at $L$ of 6 , and the BER performance is almost the same as that in uncoded 8QAM, which has a similar rate of 3 bit/symbol. This means that code 1 has a multi-mode characteristics without BER degradation.

In the following, the BER of $l 2$ for both the transmitted modes was calculated. The results when $L$ is 16 and $p$ is 0.5 are shown in Fig. 4(b). Since mode 2 is an UEP code protecting $l 2$, the $l 2 \mathrm{BER}$ when mode 2 is transmitted decreases. This code can thus generate hybrid transmission with UEP and EEP.

Fig. 4(c) shows the BER versus code length $L$. Naturally, the BER of inter-modes improves in proportion to $L$ because $d_{D}^{2}$ depends on $L$. Therefore, extending $d_{D}^{2}$, e-


Fig. 4. Simulation results for hybrid code 1. (a) $L=6$ with upper bounds, (b) BER of $l 2$ with $L=16$, and (c) BER versus $L$.

TABLE III
Code characteristics of each mode in code 2.

|  | mode 1 | mode 2 | mode 3 | mode 4 |
| :--- | :---: | :---: | :---: | :---: |
| code level | 4 | 4 | 4 | 2 |
| $l 1$ code |  | mode-index code |  |  |
| $l 2$ code |  | $(L, L-1,2)$ |  |  |
| $l 3$ and $l 4$ code | $(L, L, 1)$ | $\left(\frac{2 L}{3}, \frac{2 L}{3}, 1\right)\left(\frac{L}{3}, \frac{L}{3}, 1\right)$ | - |  |
| modulation |  |  |  |  |
| signal partition |  |  |  |  |
| trans. rate | 16 QAM | QPSK and 16QAM | QPSK |  |
| (bit/sym) | $\frac{3 L-1}{L}$ | $\frac{7 L / 3-1}{L}$ | $\frac{5 L / 3-1}{L}$ | $\frac{L-1}{L}$ |
| $d_{D}^{2}$ |  | $2 L P_{1}^{2} / 3$ |  |  |
| $d_{I}^{2}$ | $4 \Delta^{2}$ | $4 \Delta^{2}$ | $4 \Delta^{2}$ | $18 \Delta^{2}$ |

specially the Hamming distance of the mode-index code, reduces the mode error. However, because $d_{I}^{2}$ is constant (as listed in Tab. II), the BER of identical modes is almost constant. Hence, the results of the computer simulation agree with the distance characteristics and upper bounds described in Sections II and III.

On the BER performance shown in Fig. 4(a), the overall BER is dominated by that of the intra-mode as $E_{b} / N_{0}$ becomes large, though the BER of the inter-mode is lower. This means the mode-index code is too strong. As described in Section III, it may be better that the modeindex code at $l 1$ is shortened and the encoded data bits are inserted there. The corresponding trellis diagram has an increased number of states and edges. However, the transmission rate also increases.

## B. Variable-rate code

In the following, we demonstrate a one-way hybrid code with a variable-transmission rate. Fig. 5 shows four-mode BCM, called "code 2", that has four transmission rates with code length $L$, where $L^{\prime}=L / 3$. The code characteristics are listed in Tab. III. The main feature of code 2 is that a transmission can be carried out at four different rates by changing the depth of the code level and the modulation method. The component code in $l 1$ is the mode-index code, which is a repetitive parity code with a $2 L / 3$ Hamming distance, while in $l 2$ is an $(L, L-1,2)$ parity code. The number of bits in uncoded $l 3$ and $l 4$ codes is changed for each mode. For mode $k(1 \leq k \leq 4), 2 L(4-k) / 3$ bits are allocated in $l 3$ and $l 4$, and this difference enables variable-rate transmission using four different rates.

A multiple-assigned 16QAM signal constellation is used for code 2, as shown in Fig. 5(b). The 16QAM and QPSK constellations are stacked, so the system only needs a 16QAM modulator. The use of a single-constellation modulator for multiple constellations simplifies the system, which is its main advantage. Using set-partitioning and a mode-index code, we can obtain an equivalent Euclidean distance of the code even with a stacked signal constellation. Fig. 5(c) shows the set-partitioning of code 2. In $l 2$, there are dual-signal maps in the same partition, namely, QPSK and 16QAM constellations. However, the dualsignal points are distinguishable because the modulation method is fixed uniquely for each symbol after the trans-
mission mode and the code word of the mode-index code are selected. Therefore, decoding the mode-index code correctly is important for the correct decoding of the whole code at the receiver.

In Fig. 5(d), the trellis diagram does not have a definite sub-trellis area for each mode as it does in example A in Fig. 2(c) because it has a conjunction and a disjunction of branches. However, since the code is constructed by relatively simple component codes, the trellis is not complex; $00-11$ is a two-bit output corresponding to the QPSK symbol, and $A_{1}-A_{4}$ (which has four parallel paths) is a four-bit output corresponding to the 16QAM symbol as in Eq. (22).

We carried out a computer simulation using the system in an AWGN environment, as shown in Fig. 3(a). Note that if a mode error occurs in decoding, i.e., an over-andshort bit error, there appears a gap in the number of bits in $l 3$ and $l 4$ between the transmitted and decoded data. However, to simplify the simulation, we assume that no bit lag in the next frame is generated even if the mode error occurs.

Fig. 6 (a) shows the BER versus average $E_{b} / N_{0}$ where $L$ is 6 and the generation probabilities of modes 1 to 4 are, respectively, $0.8,0.1,0.05$, and 0.05 . Compared with the upper bound, all of the three simulation results get asymptotically close to the bounds. To compare the performance of each mode, we calculated the BER of every mode as shown in Fig. 6(b), where $L$ is 15 . In the figure, modes 1 to 4 indicate the generated modes at the transmitter, and the BER is calculated by including the decoded error bit$s$ both in the intra-mode and in the inter-mode from the transmitted mode as mentioned above. At the point where the BER is $10^{-5}$, the performance of mode 1 has a 0.56 dB gain relative to 8QAM, which has a similar transmission rate of $3 \mathrm{bit} / \mathrm{sym}$. Identically, modes 2 and 3 have 0.22 and 0.67 dB gains, respectively, for mode 1 , and mode 4 has a 1.78 dB gain relative to BPSK with a similar rate of 1 bit/sym. This means that the BER can be improved with the same signal power by selecting mode 4 at the transmitter. When the transmitter has a margin for the data transmission rate, efficient transmission can be generated by adaptively selecting mode 4 . The implementation of this adaptive transmission is easy because it only requires selecting the mode for every frame unit at the transmitter.

In this simulation, we calculated the BERs where the generation probability of mode 1, i.e., 16QAM symbols , was high. However, if the probability of modes 4 or 3 becomes high, the average rate becomes low, then the average $E_{b} / N_{0}$ for the same SNR becomes high, and the performance degrades. This degradation problem is solved by changing the amplitude ratio of 16QAM and QPSK as shown in Fig. 7. This operation decreases the average $E_{b} / N_{0}$ in these situations. If a more complicated modem is used, we can arrange the signal design by using 20 -point QAM.

As demonstrated in example B, using the proposed BCM method enables adaptive variable-rate transmission, which has various combinations of rates.


Fig. 5. Four-mode variable-rate hybrid code (code 2). (a) code structure, (b) constellation of multiple-assigned 16QAM, (c) set-partitioning, and (d) trellis diagram.

## C. Hybrid code for $A W G N$ and flat fading environments

We have developed a hybrid code with a variable rate for both AWGN and flat fading environments. Fig. 8 shows a block code called "code 3 ", where $L^{\prime}=L / 3$. It consists of four modes with length $L$ that have different numbers of bits and code distances. Tab. IV lists the characteristics of code 3. This code has a multi-transmission rate function by changing the depth of the code level in the same way as code 2 , and it exhibits good performance in both AWGN and flat fading environments by adjusting $d_{I}^{2}$ and $\delta_{I}$ of each mode. The mode-index code of $l 1$ is the same as code 2 . The component code of $l 2$ is the ( $L, L-1,2$ ) parity code for modes 1,2 , and 4 , and ( $L, L, 1$ ) uncoded for mode 3 , while the component codes of $l 3$ and $l 4$ exist only in modes 1 and 2 . In mode 1 , they are both ( $L, L, 1$ ) uncoded; and in mode 2 , they are ( $L, L-1,2$ ) parity codes. The transmission rate differs significantly between modes 1 and 2 , and modes 3 and 4 because of the difference in $l 3$
and 14 . Although modes 3 and 4 have lower transmission rates, they have more than twice the Euclidean length of $d_{I}^{2}$ than do modes 1 and 2, and, as a result, their performance in AWGN environments improves when $d_{D}^{2}$ is sufficiently large. As mentioned in Section III, the BER performance in fading environments is determined by MSD $\delta$. From Eq. (13), since $\delta_{D}$ is easily extended by extending $L$, the BER performance in fading environments depends on $\delta_{I}$. Therefore, the BER performance will be improved by transmitting modes 2 and 4 because $\delta_{I}$ is two, which is twice as large as that on modes 1 and 3 .

Fig. 8(b) shows a signal constellation, $S P_{2}$, for modes 2 and 4. In Tab. IV, $S P_{1}$ for modes 1 and 3 is the same constellation as that in example B in Fig. 5(b). These two constellations are both set-partitioned and only have different dispositions of $l 3$ and $l 4$ corresponding to the code structure. The trellis diagram of code 3 is shown in Fig. 8(c), where the branch outputs, $A_{1}$ and $A_{2}$, are given in Eq. $(22), b_{1}=(00,01)$ and $b_{2}=(10,11)$. Since the component


Fig. 6. Simulation results for variable-rate code. (a) $L=6$ with upper bounds and (b) BER of each mode with $L=15$.
code of $l 1$ is the same as in code 2 , the basic trellis structure is also similar to that shown in Fig. 5(d).

We carried out a computer simulation using the system shown in Fig. 3 (a) to compare the results with the upper bounds in an AWGN environment. We assumed that the generation probabilities of modes 1 to 4 were, respectively, $0.45,0.45,0.05$, and 0.05 in the following. Fig. 9(a) shows the BER when L was 6 . Although the upper bounds are not yet close, all of the simulation results asymptotically approach them. Fig. 9(a) shows that the BER curves of the simulation result for the intra-mode and the inter-mode is not consistent with those of the upper bounds. However, it is shown that the BERs of the simulation are approaching, and the BER for the inter-mode will become lower at a high $E_{b} / N_{0}$ area than that for the inter-mode, and the simulation and upper bound become consistent. As shown in Figs. 4(a), 6(a), and 9(a), the upper bounds estimated using Eqs. (19)-(21) agree with the simulation results.

Fig. 9(b) shows the BER of each transmitted mode. The


Fig. 7. Set-partitioned 16QAM changing multiple-assigned points.

TABLE IV
Code characteristics of each mode in code 3.

|  | mode 1 | mode 2 | mode 3 | mode 4 |
| :---: | :---: | :---: | :---: | :---: |
| code level | 4 | 4 | 2 | 2 |
| $l 1$ code | mode-index code |  |  |  |
| 12 code |  | 1,2) | ( $L, L, 1$ ) | ( $L, L-1,2)$ |
| 13 and 14 code | ( $L, L, 1$ ) | , L-1,2) | - | - |
| modulation |  |  | QPSK |  |
| signal partition | $S P_{1}$ | $S P_{2}$ | $S P_{1}$ | $S P_{2}$ |
| trans. rate <br> (bit/sym) | $\frac{3 L-1}{L}$ | $\frac{3 L-3}{L}$ | 1 | $\frac{L-1}{L}$ |
| $d_{D}^{2}$ | $2 L \Delta^{2} / 3$ |  |  |  |
| $d_{I}^{2}$ | $4 \Delta^{2}$ | $4 \Delta^{2}$ | $9 \Delta^{2}$ | $18 \Delta^{2}$ |
| $\delta_{D}$ | $2 L / 3$ |  |  |  |
| $\delta_{I}$ | 1 | 2 |  | 2 |
| $\Delta_{P D}^{2}$ | $\left(2 L \Delta^{2} / 3\right)^{2 L / 3}$ |  |  |  |
| $\Delta_{P I}^{2}$ | $4 \Delta^{2}$ | $16 \Delta^{4}$ | $9 \Delta^{2}$ | $324 \Delta^{4}$ |

results, where $L$ is 15 , show that the BER performance can be roughly divided into that of modes 1 and 2 , and that of modes 3 and 4 . Mode 4 has a 0.78 dB gain relative to BPSK when BER is $10^{-4}$, and modes 1 and 2 have 0.33 and 0.78 dB gains, respectively, relative to 8QAM. This performance difference comes from the difference in the lengths of $d_{I}^{2}$ and $d_{D}^{2}$ shown in Tab. IV. When $L$ is 15 , the minimum squared Euclidean distance, $d^{2}$, of each mode is $4 \Delta^{2}$ for modes 1 and 2 , which is derived from $d_{I}^{2}$ in Eq. (8), $9 \Delta^{2}$ for mode 3 , which is derived from $d_{I}^{2}$, and $10 \Delta^{2}$ for mode 4 , which is derived from $d_{D}^{2}$. Since there is a large difference between modes 1 and 2, and modes 3 and 4, the performance is divided into two types according to these distances. In Fig. 9(b), the BER of mode 4 increased slightly compared to that of mode 3 in contrast to the Euclidean distance characteristics. This was caused by a decline of the average transmission rate from 1 to $14 / 15$; that is, by about a 0.3 dB degradation.

As shown in Fig. 9(b), in AWGN environments, modes 3 and 4 have better BERs than modes 1 and 2 with the same $E_{b} / N_{0}$, although the transmission rate of modes 3 and 4 is lower. Therefore, we can achieve adaptive transmission using code 3 in terms of the rate and BER by selecting an appropriate mode. Similarly to example B , if the generation probabilities of modes 3 or 4 are high, it is effective to


Fig. 9. Simulation results for variable-rate hybrid code for AWGN and fading environments. (a) $L=6$ with upper bounds and (b) BER with $L=15$ in AWGN. (c) $f_{D} T_{s}=1 / 200$ and (d) $f_{D} T_{s}=1 / 40$ in Rayleigh fading.
exchange the amplitude ratio as shown in Fig. 7.
We evaluated the BER performance in Rayleigh fading environments using the simulation system shown in Fig. 3(b). A pilot-symbol assisted modulation (PSAM) [23, 24] method using a fast Fourier transform (FFT) [25] was used for the fading estimation and compensation. The span and depth of the block interleaver was $L \times 15$. The BER versus the average $E_{b} / N_{0}$ in a relatively slow fading environment, where the normalized fading pitch, $f_{D} T_{s}$, is $1 / 200$, was calculated. As shown in Fig. 9(c), the performance of mode 4 greatly improved. When BER was $10^{-4}$, the gain of mode 4 was about 16.6 dB relative to BPSK with a similar rate. Mode 2 also had a 9.6 dB gain relative to 8QAM. As mentioned above, this is because minimum symbol distance $\delta$ of modes 2 and 4 is two. Since $\delta$ of modes 1 and 3 is one, their BERs decreased slowly versus $E_{b} / N_{0}$. Fig. 9(d) shows the BER versus average $E_{b} / N_{0}$ in a relatively fast fading environment where the normalized fading pitch is $1 / 40$. Mode 4 in the simulation showed good performance even in the fast fading environment when PSAM was used. As shown in Figs. 9(b), 9(c), and 9(d), the performance of
each mode of code 3 changes according to the noise environment because the distance characteristics of each mode are different. Therefore, using code 3 is effective in a transmission environment where AWGN and fading are mixed. For example, we can use code 3 in uplinks in mobile communications. The mobile station uses modes 1 and 3 to achieve an efficient transmission rate when it is not moving, that is, in an AWGN environment, and it uses modes 2 and 4 to maintain good performance levels in a fading environment when it is moving. Besides the adaptation for transmission environments, we can adaptively select the transmission rate, namely, fast modes 1 and 2, and slow modes 3 and 4.

## VI. Two-way multi-mode transmission

In this section, the method proposed in this paper is enhanced to be applicable to two-way multi-mode transmission. As described in Section I, the transmission mode can be adaptively changed according to certain states, such as receiver states and propagation conditions of a transmission channel, by using two-way transmission. More effec-

(a)

(b)

(c)

Fig. 8. Four-mode variable-rate hybrid code for AWGN and fading environments (code 3). (a) code structure, (b) constellation of multiple-assigned 16QAM for modes 2 and 4, and (c) trellis diagram.


Fig. 10. Two-way multi-mode system using feedback loop.
tive transmission is possible with the two-way system. A two-way adaptive system requires a reverse link to transmit the state. Although there are many methods for the reverse link, we will consider two simple methods. One is using a duplicate system in the reverse direction. A mode selector quotes some parameters from the decoder in the reverse direction and determines the mode. The other is, as shown in Fig. 10, returning the parameters from the decoder to the mode selector of the encoder by any means. The values, such as the remaining path metric of a Viterbi decoder, the metric balance of the remaining path for the second path, and the decoded mode number, can be used as parameters to select the sending mode. Since a small path metric means that the propagation conditions of a transmission channel are good, an encoder can select the mode that has many modulation points and a faster rate. The receiver state module in Fig. 10 outputs certain values as reference parameters for the mode selector in the encoder. As a simple example of an output of the receiver state module, the module adjusts the value of the path metric of the Viterbi decoder to control the sending mode, as shown in Fig. 10.

The two-way multi-mode system based on the proposed one-way method enables more flexible transmission with a relatively simple structure.

## VII. Conclusion

In this paper, we described a one-way multi-mode transmission method using multi-level BCM. The concept behind the method and the way to construct a multi-mode code were described. At the encoder, modes of various types are generated by changing the elements of the encoder, and all modes are combined as a block code. At the decoder, Viterbi decoding using an extended trellis diagram that has all the mode branches is carried out for simple and MLD decoding.

Inserting a mode-index component code into codes enables easy estimation of the code distance and reduces the mode error. We investigated the code performance from the point of view of the distance characteristics of the multi-mode code and the upper-bound probabilities

We have shown that the method can be used to construct a variety of codes by changing the coding elements, and to obtain a coding gain by using BCM. The method can be used for many other applications, such as producing a variable-length code for different modes. In addition to

Viterbi decoding, multi-stage decoding $[2,21]$ and turbodecoding [26] for multi-level BCM [27] will also be valid because the proposed method is a type of multi-level BCM.

The new method was extended to cover a two-way system that increases the flexibility of adaptive transmission. Thus, this method is a versatile tool for use in adaptive transmission, where it can be applied in a number of different ways.

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