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Original Paper

Impact of Lead–time Decision in a Decentralized Supply Chain under Price and Lead-time Sensitive Demand

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Abstract: This paper studies the effect of lead-time decision on the performance of a decentralized supply chain that consists of one supplier, one retailer, and wherein the demand is sensitive to both retail price and lead-time. Three different scenarios based on lead-time decision making are studied and compared analytically and numerically. The lead-time is decided by the retailer and the supplier in the first and second scenarios, respectively, and it is centralized in the third scenario. The modeling considers holding and tardiness costs incurred by the difference between promised and realized delivery lead-times. The supply chain is analyzed using a power distribution function. This distribution is a parametric function that models the lead-time in general environment and has the same properties as exponential distribution for specific parameters. The optimal decision variables and expected profits are characterized and compared for the three scenarios. The relative decrease of total expected profits in the decentralized chains from that in the centralized model is observed and discussed. Furthermore, the effect of own price and lead-time sensitivity demand factors are studied numerically.

Key words: Supply chain management, Lead-time, Supplier, Retailer.

1 INTRODUCTION

In this paper, we study the impact of centralized and decentralized lead-time decision in a two-level supply chain management system, consisting of one supplier, one retailer, and wherein the demand is sensitive to both lead-time and retail price. In this study, the lead-time is defined as the interval from the moment a consumer places an order to the moment the order is received, including the time required for the intermediary process between the retailer and the supplier. When a consumer places an order to the retailer, a promised time to receive the order will be announced. Such time is defined as the promised delivery lead-time (PDL), which is also expressed in literature as *quoted lead time* or *planned lead time*. However, this PDL can be smaller or greater than the realized delivery lead-time (RDL), which is the exact interval of time to deliver the order of the consumer. The RDL corresponds to the *response time* or *cycle time* in literature. It is a stochastic variable and may deviate from the PDL due to many reasons such as high demand. As a consequence, the actor of the chain who decides the lead-time is faced with holding and tardiness costs incurred by the difference between the PDL and RDL.

We aim to evaluate and discuss the effect of decision leadership on the PDL in the decentralized chains. This problem is a Stackelberg game in which the supplier or the retailer can decide the PDL. However, the effect depending on who decides the PDL on the profitability of the players and the chain is not clear. Hence, by formulating the decentralized decision problem as a Stackelberg games with the supplier or the retailer as the leader, we can answer the following question: which leadership decision is more effective in achieving more profits for the players and for the chain? After answering this question, we will compare the results of the decentralized chains to a reference, the centralized chain. An indicator of deviation of decentralized performances, called the inefficiency of decentralized chain, is used. It gives the relative decrease of the entire expected profit of the decentralized models from that in a centralized chain. In addition, as the demand function depends on price and lead-time, we will numerically study the effect of own price and lead-time sensitivity demand factors on the different performances of the chains.

In actual global and competitive markets, the consumer benefits from a variety of choices. Therefore, it is not sufficient to consider the selling price as a unique competition factor in a supply chain. In this respect, the market actors have been investigating new competition criteria based on the consumers' attention. Sterling et al. [1] and Ballou et al. [2] reported that the rapidity and the regularity of delivery time have a particular importance in customer service. Such delivery time is related to the so called "lead-time" factor. Generally, lead-time depends on the efficiency and the capacity of the selling system. For example, So [3] reported that a retailer needs to provide sufficient capacity and guarantees the efficiency of his delivery system to achieve desired lead-time performances. This competition factor is widely discussed in supply

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chain management literature such as in Yano [4-6], Li [7], Hopp and Spearman [8], Li and Lee [9], Lederer and Li [10], Palaka et al. [11], So and Song [12], Song et al. [13], Cachon and Harker [14], Boyaci and Ray [15]. Recently, Liu et al. [16] have studied pricing and lead-time decisions in a two level decentralized supply chain consisting of one supplier and one retailer, in which the supplier decides the lead-time and faces related costs. Furthermore, Pekgun et al. [17] have compared centralized and decentralized supply chains under price and lead-time sensitive demand. In most of these studies, the supplier is a lead-time decision maker. However, consumers place their order to the retailer and obtain information from that retailer about the time of delivery. The retailer must inform the consumer owing to their direct relationship. On the other hand, this also raises a question regarding the announcement of delivery lead-time. The retailer and the supplier can share the lead-time information if they represent the same company and/or if they have the same economic orientation. However, the retail supply chain is not usually this case. From these reasons, it seems to be more suitable for the retailer to set the lead-time. Therefore, the scenario in which the retailer is a lead-time decision maker should be considered and studied. In industrial management related literature, this scenario has not been deeply studied. For that reason, we will focus on effect of lead-time decision on the performances of a retail supply chain. Furthermore, we will compare the optimal decision variables and expected profits in this scenario with those when the supplier is a lead-time decision maker.

The decentralized chain is based on a leader-follower model. However, in the centralized chain a single decision maker exists and the lead-time information is shared. Three different scenarios based on lead-time decision are studied and compared. In the first scenario, called Scenario 1, the retailer decides the PDL and the retail price to be quoted to the consumer, but the supplier determines the wholesale price. In a retail supply chain, this scenario is frequently used in practice when the supplier and the retailer are from different companies. For example, when a consumer places an order for furniture, the retailer quotes him a PDL and a price based on standard settings. The second scenario (Scenario 2) describes a supply chain in which the supplier is a leader and the retailer is a follower. The supplier determines the PDL and the wholesale price, but the retailer quotes the retail price. This scenario was studied in [16], where the authors consider the PDL as information provided by the supplier to the retailer. However, as mentioned above, it is more suitable for the retailer to announce the PDL to the consumer. The decentralization of lead-time decision in these two scenarios is considered because the supplier and the retailer may represent two different companies with no shared internal information. The performances of the decentralized lead-time decision will be compared to that of a centralized supply chain, which will be studied in the third scenario (called Scenario 3). The PDL may be considered as internal information between the supplier and the retailer, which was decided preliminary. Scenario 3 will be used as a benchmark for comparison. On the other hand, the problem cannot be solved under an exponential distribution. Instead, a new parametric distribution function of lead-time, called power distribution, is used. Under certain specific parameters, it has the same properties of an exponential distribution and allows us to derive different solutions analytically.

The paper is organized as follow: the second section formulates the models in the three scenarios. The third section describes the modeling of lead-time using exponential distribution. In last section, the power distribution of lead-time is modeled and the different scenarios are studied and compared. Furthermore, the impact of own price and lead-time sensitivity demand factors on the performances of the chain are discussed theoretically and numerically.

2 FORMULATION OF THE MODELS

Three different scenarios are studied to determine the optimal decision variables and expected profits in a two level supply chain, consisting of one supplier and one retailer. In Scenario 1, the retailer decides the lead-time, and this decision is taken by the supplier in Scenario 2 and it is centralized in Scenario 3. The supplier produces products at a constant production cost rate (c) including the transportation cost to the retailer or to the consumer. The supplier has ample capacity to satisfy any received demand. The retailer faces an administrative cost per unit (c_r). The actor of the chain who decides the lead-time faces lead-time costs incurred by the difference between the PDL and the RDL. If the RDL is less than the PDL, the product is kept in stock and a holding cost (h) per unit per unit time is introduced; however, he faces a tardiness cost (b) per unit per unit time, when the RDL exceeds the PDL. We assume a demand rate λ dependent cumulative distribution function (cdf) R_λ and a probability distribution function (pdf) r_λ for lead-time. The lead-time costs are defined as in [16] and [21], where they are expressed, for a given λ, by

$$C(l, R_\lambda) = h\int_0^l (l-t) r_\lambda(t)dt + b\int_l^\infty (t-l) r_\lambda(t)dt, \quad (1)$$

where the demand function λ is deterministic and linear in retail price and lead-time. It is expressed as

$$\lambda(p, l) = \lambda_0 - \alpha p - \beta l, \quad (2)$$

where λ_0, α and β are the base market potential, own price sensitivity demand factor, and own lead-time sensitivity demand factor, respectively. We define the standard waiting cost as $c_w = \frac{\beta}{\alpha}$ per unit of the PDL and the maximum retail price as $p^{max} = \frac{\lambda_0}{\alpha}$. The demand function is similar to that reported by Boyaci and Ray [15], Pekgun el al. [17], Tsay and Agrawal [18], and Balasubramanian and Bhardwaj [19].

In Scenario 1, to maximize their profits, the supplier decides his wholesale price w_{d1}; however, the retailer decides his lead-time l_{d1} and retail price p_{d1}. The optimization problem of the supplier is given by $max_{w_{d1}}\,\pi_{s1}(w_{d1}, p_{d1}(w_{d1}), l_{d1}(w_{d1})) = (w_{d1} - c)\lambda_{d1}(p_{d1}(w_{d1}), l_{d1}(w_{d1}))$,

where $p_{d1}(w_{d1})$ and $l_{d1}(w_{d1})$ are the optimal solutions for the following retailer's optimization problem. The index $d1$ refers to the decentralized chain in Scenario1. Then, for a given w_{d1}, the optimization problem of the retailer is expressed as

$$max_{p_{d1},l_{d1}}\,\pi_{r1}(p_{d1}, l_{d1}) = \left(p_{d1} - w_{d1} - c_r - C\left(l_{d1}, R_{\lambda_{d1}}\right)\right)\lambda_{d1}(p_{d1}(w_{d1}), l_{d1}(w_{d1})).$$

The supplier decides his wholesale price w_{d2} and lead-time l_{d2}. The problem is expressed as

$$max_{w_{d2}}\,\pi_{s2}(w_{d2}, l_{d2}) = \left(w_{d2} - c - C\left(l_{d2}, R_{\lambda_{d2}}\right)\right)\lambda_{d2}(p_{d2}(w_{d2}), l_{d2}(w_{d2})),$$

where the index d_2 refers to the decentralized chain in Scenario 2. The optimization of the retailer for a given w_{d2} is expressed as

$$max_{p_{d2},l_{d2}}\,\pi_{r2}(p_{d2}, l_{d2}) = (p_{d2} - w_{d2} - c_r)\lambda_{d2}(p_{d2}, l_{d2}).$$

In the third scenario, one of the chain' actors is a decision maker and the other one is a follower. The wholesale price is excluded from the optimization problem, as an internal variable. The total profit function of the centralized chain is given as

$$\pi_c(\lambda_c(p, l)) = \left(p^{max} - \frac{\lambda_c}{\alpha} - c_w l_c - c - c_r - C\left(l_c, R_{\lambda_c}\right)\right)\lambda_c(p, l),$$

where the index c refers to the centralized chain.

In all these scenarios, we assume that the right hand-sides of the optimization problems are positive.

3 EXPONENTIAL DISTRIBUTION

In the M/M/1 system, the service times are independent and identically exponentially distributed. As reported by Boyaci and Ray [15], the exponential distribution gives an important approximation of waiting times. Its cdf and pdf of lead-time are given by

$R_{\lambda_{d1}}(t) = 1 - e^{-(\gamma-\lambda_{d1})t}$ and

$r_{\lambda_{d1}}(t) = (\gamma - \lambda_{d1})e^{-(\gamma-\lambda_{d1})t}$ for $0 \le t \le \infty$,

respectively, where γ is the mean service rate.

Here, only Scenario 1 will be studied using the exponential distribution of lead-time. The other scenarios were studied by Liu et al. [16]. For a given wholesale price w_{d1}, the retailer profit function depends on three dependent parameters; the price, the lead-time, and the demand. To solve this technical problem, the retail price is expressed as a function of lead-time and the demand function. Then the optimal lead-time solution can be obtained for a given demand. Using Eq. (2), we obtain

$$p_{d1} = p^{max} - \frac{\lambda_{d1}}{\alpha} - c_w l_{d1}. \tag{3}$$

The optimization problem of the retailer can be rewritten as

$$max_{\lambda_{d1},l_{d1}}\,\pi_{r1}(\lambda_{d1}, l_{d1}) = \left(p^{max} - \frac{\lambda_{d1}}{\alpha} - c_w l_{d1} - w_{d1} - c_r - C\left(l_{d1}, R_{\lambda_{d1}}\right)\right)\lambda_{d1}.$$

Lemma 1 For a given wholesale price w_{d1} and demand λ_{d1}, there is a unique optimal lead-time $l_{d1}^*(\lambda_{d1})$, which depends on λ_{d1} and is expressed by

$$l_{d1}^*(\lambda_{d1}) = R_{\lambda_{d1}}^{-1}\left(\frac{b-c_w}{b+h}\right),$$

where $R_{\lambda_{d1}}^{-1}$ is the inverse of the distribution function $R_{\lambda_{d1}}$.

Proof: By differentiating $\pi_{r1}(\lambda_{d1}, l_{d1})$ for the lead-time l_{d1}, we obtain

$$\frac{\partial\pi_{r1}(\lambda_{d1}, l_{d1})}{\partial l_{d1}} = \left(-c_w - \frac{\partial C\left(l_{d1}, R_{\lambda_{d1}}\right)}{\partial l_{d1}}\right)\lambda_{d1} \text{ and}$$

$$\frac{\partial^2\pi_{r1}(\lambda_{d1}, l_{d1})}{\partial {l_{d1}}^2} = -\lambda_{d1}\frac{\partial^2 C\left(l_{d1}, R_{\lambda_{d1}}\right)}{\partial {l_{d1}}^2},$$

with $\frac{\partial C\left(l_{d1}, R_{\lambda_{d1}}\right)}{\partial l_{d1}} = -b + (b + h)R_\lambda(l_{d1})$ and $\frac{\partial^2 C\left(l_{d1}, R_{\lambda_{d1}}\right)}{\partial {l_{d1}}^2} = (b + h)r_{\lambda_{d1}}(l_{d1}) > 0$ for all l_{d1}. Thus, the retailer profit function is strictly concave in l_{d1} and the unique optimal lead-time $l_{d1}^*(\lambda_{d1})$ is given by $l_{d1}^*(\lambda_{d1}) = R_{\lambda_{d1}}^{-1}\left(\frac{b-c_w}{b+h}\right)$.

The ratio $\left(\frac{b-c_w}{b+h}\right)$ reflects the cost parameter [21]. The optimal lead-time is dependent on retail price through λ_{d1}. Note that if $b - c_w \le 0$, the optimal lead-time is zero. Using the exponential distribution, the optimal lead-time can be expressed if $b > c_w$ as $l_{d1}^*(\lambda_{d1}) = \frac{d}{\gamma-\lambda_{d1}}$, where $d = -ln\left(\frac{h+c_w}{b+h}\right)$. Substituting $l_{d1}^*(\lambda_{d1})$ in (1), gives $\left(l_{d1}, R_{\lambda_{d1}}\right) = \frac{hd+c_w}{\gamma-\lambda_{d1}}$. Thus, the retailer profit function is given by

$$\pi_{r1}(\lambda_{d1}) = \left(p^{max} - \frac{\lambda_{d1}}{\alpha} - \frac{(c_w+h)d+c_w}{\gamma-\lambda_{d1}} - w_{d1} - c_r\right)\lambda_{d1}.$$

Lemma 2 For a given wholesale price w_{d1}, a unique optimal demand λ_{d1}^* and retail price p_{d1}^* exist. The unique optimal demand is given by $\lambda_{d1}^* = \gamma - \phi^*$, where $\phi^*$is the solution of the cube equation $\phi^{*3} + A\phi^{*2} + B\phi^* + C = 0$, with $A \equiv \frac{\alpha(p^{max}-w_{d1}-c_r)}{2} - \gamma$, $B \equiv 0$, and $C \equiv -\frac{\alpha((c_w+h)d+c_w)\gamma}{2}$.

Proof: By differentiating the retailer profit function on the demand function λ_{d1} we obtain $\frac{\partial \pi_{r1}(\lambda_{d1})}{\partial \lambda_{d1}} = p^{max} - \frac{2\lambda_{d1}}{\alpha} - ((c_w+h)d+c_w)\left(\frac{1}{\gamma-\lambda_{d1}} + \frac{\lambda_{d1}}{(\gamma-\lambda_{d1})^2}\right) - w_{d1} - c_r$ and $\frac{\partial^2 \pi_r(\lambda_{d1})}{\partial \lambda_{d1}^2} = -\frac{2}{\alpha} - ((c_w+h)d+c_w)\left(\frac{1}{(\gamma-\lambda_{d1})^2} + \frac{\lambda_{d1}+\gamma}{(\gamma-\lambda_{d1})^3}\right) < 0$ for all λ_{d1}. Thus, the retailer profit function is strictly concave in λ_{d1} and the unique optimal demand function is given by $\lambda_{d1}^* = \gamma - \phi^*$, where ϕ^* is the solution of the third order equation $\phi^{*3} + A\phi^{*2} + B\phi^* + C = 0$, with $A \equiv \frac{\alpha(p^{max}-w_{d1}-c_r)}{2} - \gamma$, $B \equiv 0$, and $C \equiv -\frac{\alpha((c_w+h)d+c_w)\gamma}{2}$.

The cube equation on λ_{d1}^* has three possible solutions. However, the retailer profit function is concave in λ_{d1}^* , which guarantees the existence of a unique positive solution. This solution cannot be obtained analytically or numerically because it depends on the wholesale price w_{d1}, which is an unknown decision variable. Liu et al. [16] discussed nearly the same cube equation, where the constants depend on chain and distribution parameters only and can be solved numerically using the formulas reported in Spiegel and Liu [20]. They introduced some approximations to model the lead-time as function of demand. In this paper, a new approach based on the so called power distribution function is used to solve the problem analytically. Such a distribution has, under some conditions, the properties of exponential distribution of lead-time in the M/M/1 system. Its advantages will be discussed in detail in the next section.

4 POWER DISTRIBUTION

As mentioned in the previous section, the problem cannot be solved using an exponential distribution. Thus, a new approach based on the power distribution function is used. This distribution function of lead-time was introduced by Zhengping et al. in [21]. It is a parametric function which models a wide variety of distributions such as uniform and triangular distributions. In addition, we will show that the power distribution is more suitable in the context of modelling lead-time in a general industrial management environment. It has the same properties as the exponential distribution for specific parameters. The advantages of this distribution will be discussed after its definition. The cdf and the pdf of lead-time are expressed by $R_{\lambda_{d1}}(t) = \left(\frac{t}{\rho\lambda_{d1}}\right)^{\varpi}$ and $r_{\lambda_{d1}}(t) = \frac{\varpi t^{\varpi-1}}{(\rho\lambda_{d1})^{\varpi}}$ for $0 \le t \le \rho\lambda_{d1}$, respectively, where $\varpi > 0$ and $\rho > 0$ are the shape and the scale parameters, respectively. The interval $\rho\lambda_{d1} = T$ represents the longest possible lead-time for a job in the system, when the demand rate is λ_{d1}. The properties of the power distribution function of lead-times are summarized in the following points:

- The service mean rate and the demand are $1/\rho$ and λ_{d1}, respectively. They are analogous to γ and λ_{d1} , respectively, in the exponential distribution.
- Infinite lead-time is not allowed as in practice.
- It can be used in a different situation by varying the shape parameter ϖ. As shown in Fig. 1, the pdf of lead-time under various ϖ has different behaviors. For $\varpi = 0$ or $\varpi = \infty$, the optimal lead-time is deterministic and equal to 0. For $0 < \varpi < 1$, the pdf drops in lead-time as in the M/M/1 system. In this case, short lead-times have high probability, indicating a rapid delivery of the order. The cases where $\varpi = 1$ and 2 correspond to the uniform and the triangular distributions, respectively. For$\varpi > 1$, the pdf increases with lead-time, which indicates that long lead-time has high probability, in contrast to the exponential distribution. The order that the supplier receives from the retailer tends to remain in the system, which is consistent with common practice where deliveries normally take place near end of promised lead-time (PDL) or even beyond in some cases.

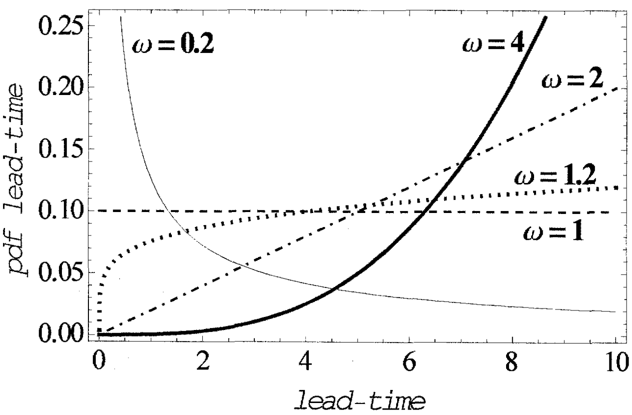

Fig. 1 Probability distribution function of lead-time for various shape parameters ϖ.

4.1 Retailer Decides the Lead-time

Using results of Lemma 1 and the expression of the power distribution function, the optimal lead-time can be expressed as $l_{d1}^*(\lambda_{d1}) = \rho\lambda_{d1}\tau$, where $\tau = \sqrt[\varpi]{\frac{b-c_w}{b+h}}$. The lead-time costs can be expressed by $(\lambda_{d1}) = \eta\rho\lambda_{d1}$, where $\eta = \frac{\tau^{\varpi+1}h+b(\tau^{\varpi+1}+\varpi-\tau-\tau\varpi)}{\varpi+1}$. Thus, the retailer profit function can be rewritten as $\pi_{r1}(\lambda_{d1}) = \left(p^{max} - \frac{\lambda_{d1}}{\alpha} - c_w\rho\lambda_{d1}\tau - w_{d1} - c_r - \eta\rho\lambda_{d1}\right)\lambda_{d1}$.

Lemma 3 For given wholesale price w_{d1}, a unique optimal demand function λ^* and retail price p^* exist. They are expressed as $\lambda_{d1}^* = \frac{\alpha(p^{max}-w_{d1}-c_r)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}$ and $p_{d1}^* = p^{max} - \frac{\lambda_{d1}^*}{\alpha}(1+\alpha c_w\tau\rho)$.

Proof: By differentiating the retailer profit function on the demand λ_{d1}, we obtain $\frac{\partial\pi_{r1}(\lambda_{d1})}{\partial\lambda} = p^{max} - w_{d1} - c_r - \frac{2\lambda_{d1}}{\alpha} - 2c_w\rho\lambda_{d1}\tau - 2\eta\rho\lambda_{d1}$ and $\frac{\delta^2\pi_{r1}(\lambda_{d1})}{\delta\lambda_{d1}{}^2} = -\frac{2}{\alpha} - 2c_w\rho\tau - 2\eta\rho < 0$. Thus, the retailer profit function is strictly concave in λ_{d1} and the unique optimal demand function is given by $\lambda_{d1}^* = \frac{\alpha(p^{max}-w_{d1}-c_r)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}$. Substituting l_{d1}^* and λ_{d1}^* in Eq. (3), the unique optimal retail price is given by $p_{d1}^* = p^{max} - \frac{\lambda_{d1}^*}{\alpha}(1+\beta\tau\rho)$.

Inserting the optimal demand in the supplier profit function, we obtain $\pi_{s1}(w_{d1}) = (w_{d1}-c)\frac{\alpha(p^{max}-w_{d1}-c_r)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}$.

Lemma 4 There is a unique optimal wholesale price w_{d1}^*, given by

$$w_{d1}^* = \frac{p^{max}-c_r+c}{2}. \tag{4}$$

Proof: By differentiating the supplier profit function on the wholesale price w_{d1}, we obtain $\frac{\partial\pi_{s1}(w_{d1})}{\partial w_{d1}} = \frac{-\alpha(2w_{d1}-p^{max}+c_r-c)}{2(1+\alpha c_w\rho\tau+\alpha\eta\rho)}$ and $\frac{\partial^2\pi_{s1}(w_{d1})}{\partial w_{d1}{}^2} = \frac{-\alpha}{1+\alpha c_w\rho\tau+\alpha\eta\rho} < 0$. Thus, the supplier profit function is strictly concave in w_{d1} and the unique optimal wholesale price is given by $w_{d1}^* = \frac{p^{max}-c_r+c}{2}$.

The optimal decision variables and expected profits are expressed by

$$l_{d1}^* = \frac{\rho\tau\alpha(p^{max}-c-c_r)}{4(1+\beta\rho\tau+\alpha\eta\rho)}, \tag{5}$$

$$\lambda_{d1}^* = \frac{\alpha(p^{max}-c-c_r)}{4(1+\beta\rho\tau+\alpha\eta\rho)}, \tag{6}$$

$$p_{d1}^* = p^{max} - \frac{\lambda_{d1}^*}{\alpha}(1+\beta\tau\rho), \tag{7}$$

$$\pi_{s1}^* = \frac{\alpha(p^{max}-c-c_r)^2}{8(1+\beta\rho\tau+\alpha\eta\rho)} = \frac{\lambda_{d1}^*}{2}(p^{max}-c-c_r), \tag{8}$$

$$\pi_{r1}^* = \frac{\alpha(p^{max}-c-c_r)^2}{16(1+\beta\rho\tau+\alpha\eta\rho)} = \frac{\lambda_{d1}^*}{4}(p^{max}-c-c_r). \tag{9}$$

The sum of the supplier and retailer profits is given by

$$\pi_{d1}^* = \pi_{r1}^* + \pi_{s1}^* = \frac{3\lambda_{d1}^*}{4}(p^{max}-c-c_r). \tag{10}$$

4.2 Supplier Decides the Lead-time

Using Lemma 1 in Liu et al. [16]; for a given w_{d2} and lead-time l_{d2}, the optimal retail price is given by $p_{d2}^* = \frac{p^{max}+c_r-c_wl_{d2}+w_{d2}}{2}$. Substituting p_{d2}^* in Eq. (2) and expressing the wholesale price as a function of demand, we obtain

$w_{d2}(\lambda_{d2}) = p^{max} - c_wl_{d2} - 2\lambda_{d2}/\alpha - c_r$.

Using the same methodology as in §4.1; for a given demand function λ_{d2}, there is a unique optimal lead-time $l_{d2}^*(\lambda_{d2})$, which depends on λ_{d2} and is given by $l_{d2}^*(\lambda_{d2}) = \rho\lambda_{d2}\tau$. Then, the supplier profit function is rewritten as $\pi_{s2}(\lambda_{d2}) = (p^{max} - c_r - c - \lambda_{d2}(c_w\rho\tau + 2/\alpha + \eta\rho))\lambda_{d2}$.

Lemma 5 Under power distribution, there is a unique optimal demand function λ_{d2}^* expressed by

$$\lambda_{d2}^* = \frac{\alpha(p^{max}-c-c_r)}{2(2+\beta\rho\tau+\alpha\eta\rho)}, \tag{11}$$

Proof: By differentiating the supplier profit function for the demand λ_{d2}, we obtain $\frac{\partial\pi_{s2}(\lambda_{d2})}{\partial\lambda_{d2}} = p^{max} - c - c_r - 2\lambda_{d2}(c_w\rho\tau + 2/\alpha + \eta\rho)$ and $\frac{\partial^2\pi_{2s}(\lambda_{d2})}{\partial\lambda_{d2}{}^2} = -2(c_w\rho\tau + 2/\alpha + \eta\rho) < 0$. Thus, the supplier profit function is strictly concave in λ_{d2} and the unique optimal demand function is given by $\lambda_{d2}^* = \frac{\alpha(p^{max}-c-c_r)}{2(2+\beta\rho\tau+\alpha\eta\rho)}$.

The unique optimal lead-time is expressed as

$$l_{d2}^* = \frac{\rho\tau\alpha(p^{max}-c-c_r)}{2(2+\beta\rho\tau+\alpha\eta\rho)}. \tag{12}$$

Substituting l_{d2}^* and λ_{d2}^* in w_{d2}^*, the unique optimal wholesale price can be given by

$$w_{d2}^* = p^{max} - c_r - \frac{(p^{max}-c-c_r)(2+\beta\rho\tau)}{2(2+\beta\rho\tau+\alpha\eta\rho)}. \tag{13}$$

From this, we obtain the unique optimal retail price as

$$p_{d2}^* = p^{max} - \frac{\lambda_{d2}^*}{\alpha}(\beta\rho\tau+1). \tag{14}$$

Finally, the retailer and supplier profit functions are given, respectively by

$$\pi_{r2}^* = \frac{\lambda_{d2}^{*2}}{\alpha} = \frac{\alpha(p^{max}-c-c_r)^2}{4(2+\beta\rho\tau+\alpha\eta\rho)^2}, \tag{15}$$

$$\pi_{s2}^* = (2+\beta\rho\tau+\alpha\eta\rho)\frac{\lambda_{d2}^{*2}}{\alpha} = (2+\beta\rho\tau+\alpha\eta\rho)\pi_{r2}. \tag{16}$$

The sum of the supplier and retailer profits is given by

$$\pi_{d2}^* = \pi_{r2}^* + \pi_{s2}^* = \frac{\alpha(3+\beta\rho\tau+\alpha\eta\rho)(p^{max}-c-c_r)^2}{4(2+\beta\rho\tau+\alpha\eta\rho)^2}. \tag{17}$$

4.3 Centralized Scenario

In this case, one of the chain' actors is a decision maker and the other one is a follower. The wholesale price is excluded from the optimization problem because it is an endogenous variable. This scenario is considered as a benchmark to be compared with the decentralized scenarios. As calculated in § 4.1, the optimal lead-time and lead-time cost are given by $l^*(\lambda) = \rho\lambda\tau$ and $C(\lambda) = \eta\rho\lambda$, respectively. The profit function of the centralized chain is given by

$$\pi(\lambda) = \left(p^{max} - \frac{\lambda}{\alpha} - c_w\rho\lambda\tau - c - c_r - \eta\rho\lambda\right)\lambda.$$

Using the same methodology as in Lemma 2, we obtain the optimal demand function, optimal price, optimal lead-time, and expected profit as

$$\lambda^* = \frac{\alpha(p^{max}-c-c_r)}{2(1+\beta\rho\tau+\alpha\eta\rho)} = 2\lambda_{d1}^*, \tag{18}$$

$$p^* = p^{max} - \frac{\lambda^*}{\alpha}(1+\beta\tau\rho), \tag{19}$$

$$l^*(\lambda) = \rho\tau\frac{\alpha(p^{max}-c-c_r)}{2(1+\beta\rho\tau+\alpha\eta\rho)}, \tag{20}$$

$$\pi^* = \frac{\lambda^{*2}}{\alpha}(1+\beta\rho\tau+\alpha\eta\rho) = \frac{\alpha(p^{max}-c-c_r)^2}{4(1+\beta\rho\tau+\alpha\eta\rho)}. \tag{21}$$

4.4 Comparison between the Scenarios

In this sub-section, the optimal decision variables and expected profits are compared for the three scenarios. First, the decentralized models will be compared to the reference centralized model. This comparison was reported in [16] and [22], in which the inefficiency of decentralized supply chain from the centralized one was used. This inefficiency is expressed by

$$q_{\pi_{di}} = 1 - \frac{\pi_{di}^*}{\pi^*} \text{ for } i = 1,2. \tag{22}$$

Note that the inefficiency of centralized model is zero. Equation 22 quantifies the regression of the performances in decentralized chain from that in the centralized one. Second, the performances of scenario 1 and 2 will be compared to quantify and discuss the impact of lead-time decision on the performances of the model.

4.4.1 Inefficiency of Decentralized Chain in Scenario 1

From the results of §4.1 and §4.3, the optimal demand function and the optimal lead-time in the centralized chain are double those in Scenario 1, where the retailer decides the lead-time, that is $\lambda^* = 2\lambda_{d1}^*$ and $l^* = 2l_{d1}^*$. However, the optimal retail price in Scenario 1 is higher than that of the centralized chain, that is $p_{d1}^* = \frac{p^{max}+p^*}{2} > p^*$. Concerning the comparison of total profits, we have $\pi_{d1}^* = \frac{3\pi^*}{4}$ and $q_{\pi_{d1}} = 0.25$. Therefore, when the retailer sets the lead-time, the total profits of both the retailer and supplier regress constantly 1/4 from that of an integrated system. The inefficiency of the decentralized model in Scenario 1 is independent of chain and distribution parameters.

4.4.2 Inefficiency of Decentralized Chain in Scenario 2

From the results of §4.2 and §4.3, the optimal demand function, the optimal lead-time, and the optima retail price in the centralized chain are higher than that in the case of Scenario 2, where the supplier decides the lead-time, and the following equation is satisfied.

$$\frac{\lambda_{d2}^*}{\lambda^*} = \frac{1+\beta\rho\tau+\alpha\eta\rho}{2+\beta\rho\tau+\alpha\eta\rho} = \frac{l_{d2}^*}{l^*} = \frac{p^{max}-p_{d2}^*}{p^{max}-p^*} < 1.$$

Concerning the comparison of total profits, we have $0.75 < \frac{\pi_{d2}^*}{\pi^*} = \frac{4}{3}\left[1 - \frac{1}{(\beta\rho\tau+\alpha\eta\rho+2)^2}\right] < 1$, which results in an inefficiency of decentralized supply chain in scenario 1 of $q_{\pi_{d2}} = \frac{1}{4+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2} < 0.25$. When the supplier sets the lead-time, the entire total profits of both retailer and supplier drops for a maximum of 1/4 of that of the integrated chain. This result depends on chain and distribution parameters; however, it is often less than that in the case when the retailer decides the lead-time.

4.4.3 Comparison between Scenarios 1 and 2

From results of §4.2 and §4.3, we have

$$\frac{\lambda_{d2}^*}{\lambda_{d1}^*} = \frac{l_{d2}^*}{l_{d1}^*} = \frac{2(1+\beta\rho\tau+\alpha\eta\rho)}{2+\beta\rho\tau+\alpha\eta\rho} > 1.$$

Thus, the retailer orders a higher quantity when the supplier decides the lead-time. It can be explained by the retailer not having to be responsible to compensate for the waiting cost for the consumer. As a consequence of high demand, long lead-time is required to complete the job. Concerning the retail prices, we have

$$\frac{p_{d2}^*-p^{max}}{p_{d1}^*-p^{max}} = \frac{\lambda_{d2}^*}{\lambda_{d1}^*} > 1,$$

which gives $p_{d2}^* < p_{d1}^*$. From Eqs. (9) and (15), we have

$$\frac{\pi_{r2}^*}{\pi_{r1}^*} = \frac{4(1+\beta\rho\tau+\alpha\eta\rho)}{(2+\beta\rho\tau+\alpha\eta\rho)^2} < 1.$$

This means that the retailer achieves more profits when he decides lead-time. The same result is found for the supplier, where

$$\frac{\pi_{s2}^*}{\pi_{s1}^*} = \frac{2(1+\beta\rho\tau+\alpha\eta\rho)(2+\beta\rho\tau+\alpha\eta\rho)}{(2+\beta\rho\tau+\alpha\eta\rho)^2} > 1.$$

Therefore, the chain actor who decides the lead-time achieves more profits. Concerning the total profits in the two scenarios, we have

$$\frac{\pi_{d2}^*}{\pi_{d1}^*} = \frac{4(3+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2)}{3(4+4(\beta\rho\tau+\alpha\eta\rho)+(\beta\rho\tau+\alpha\eta\rho)^2)} > 1.$$

4.5 Numerical Results

In this section, we numerically compute the optimal decision variables and expected profits under the power distribution function. Two different values of shape parameter will be used. The first one is set to $\varpi = 0.2$, where the pdf of lead-time drops as lead-time increases. This case imitates the properties of the exponential distribution. However, the second value is set to $\varpi = 1.2$, where the pdf of lead-time increases as lead-time increases. For all our simulations, the numerical values of chain parameters are set as $\lambda_0 = 100$, $\alpha = 1$ if β is varied, $\beta = 1$ if α is varied, $b = 2$, $h = 0.3$, $c_r = 5$, $c = 20$, $\varpi \epsilon \{0.2, 1.2\}$, and $\rho = 10$. The decision variables and expected profits that can be discussed theoretically will not be presented in the numerical results.

4.5.1 Effect of Own Price Sensitivity α

First, we have $\frac{\partial \tau}{\partial \alpha} = \frac{\beta}{\alpha^2 \varpi (b+h)} \left(\frac{b-\beta/\alpha}{b+h}\right)^{\frac{1-\varpi}{\varpi}} > 0$, $\frac{\partial(\alpha(p^{max}-c-c_r))}{\partial \alpha} = -c - c_r < 0$, $\frac{\partial \eta}{\partial \tau} = \frac{1}{\varpi+1}[(\varpi+1)(b+h)\tau^{\varpi} - b(\varpi+1)] = -\frac{\beta}{\alpha} < 0$, and $\frac{\partial(1+\beta\rho\tau+\alpha\eta\rho)}{\partial \alpha} = \beta\rho\frac{\partial \tau}{\partial \alpha} + \eta\rho + \alpha\rho\frac{\partial \eta}{\partial \tau}\frac{\partial \tau}{\partial \alpha} = \eta\rho > 0$. From this, it is easy to see in Scenarios 1 and 2 that $\frac{\partial \lambda_{di}^*}{\partial \alpha} < 0$, $\frac{\partial \pi_{ri}^*}{\partial \alpha} < 0$, and $\frac{\partial \pi_{si}^*}{\partial \alpha} < 0$ for $i = 1, 2$. Therefore, increasing the own price sensitivity demand factor αdecreases the demand function and the expected profits. The sensitivity of the other decisions variables to α will be discussed numerically. In all figures, the solid, the dashed and the dot-dashed lines correspond to Scenarios 1, 2 and 3, respectively.

4.5.1.1 Case of Shape Parameter $\varpi = 0.2$

For $\varpi = 0.2$, the pdf decreases as the lead-time increases as in the M/M/1 system. As shown in Fig. 2 (a), for low values of α, the wholesale price in Scenario 2 is higher than in Scenario 1. However, it converges in the two cases to the product cost rate c for $\alpha_c = \frac{\lambda_0}{c+c_r}$. In addition, as plotted in Fig. 2 (b), the retail price decreases as α increases. Its sensitivity to lead-time decision is weak. For such α_c, the optimal price is equal to $c + c_r$ and the others decision variables are zero. Furthermore, an important finding is that the lead-time is a non-monotone function for α. As shown in Fig. 2(c), it reaches its maximum for $\alpha = \alpha_{lmax}$, which depends on the setting of chain and distribution parameters. It is worth noting that the non-monotony of lead-time disappears for high tardiness cost b. Further, it is easy to see numerically that in contrast to the other parameters, α_{lmax} shifts right as own lead-time sensitive demand factor β increases. Finally, an infinity lead-time is not allowed in the chain, which is in accordance with the conditions in practice.

4.5.1.2 Case of Shape Parameter $\varpi = 1.2$

For $\varpi = 1.2$, the pdf increases as the lead-time increases. This means that long lead-times have high probabilities. This case is in contrast to the behavior of the pdf in the M/M/1 system. The dependence of the wholesale, the retail price, and the lead-time to the own price sensitivity demand factor α is plotted in Figs. 3 (a), (b), and (c), respectively. At low values of , the gap between the retail price in the centralized and decentralized decisions are bigger than in the case where $\varpi = 0.2$. In addition, α_{lmax} shifts to low values of α as the shape parameter increases.

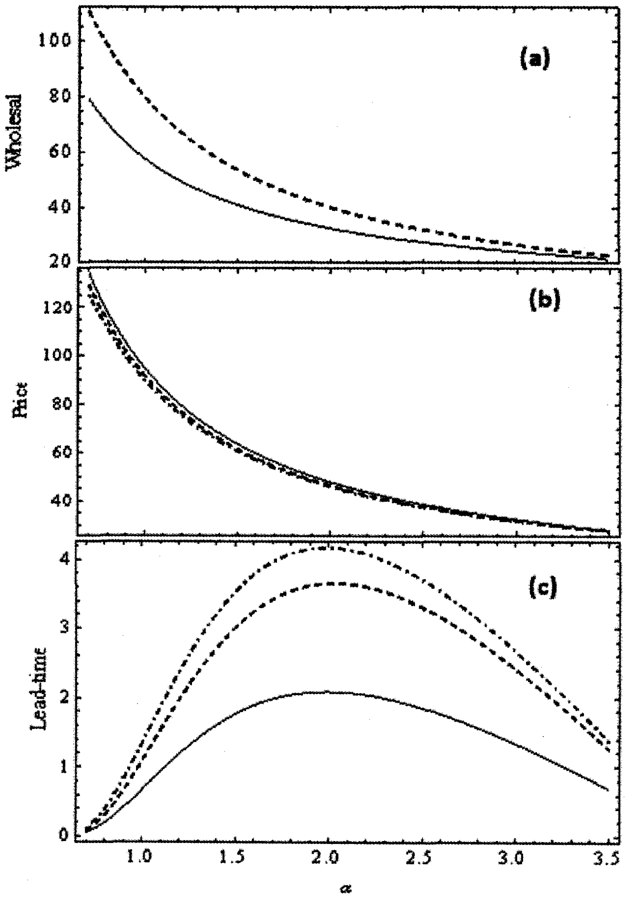

Fig. 2 Own price sensitive demand dependence of wholesale, retail price, and lead-time for a shape parameter $\varpi = 0.2$.

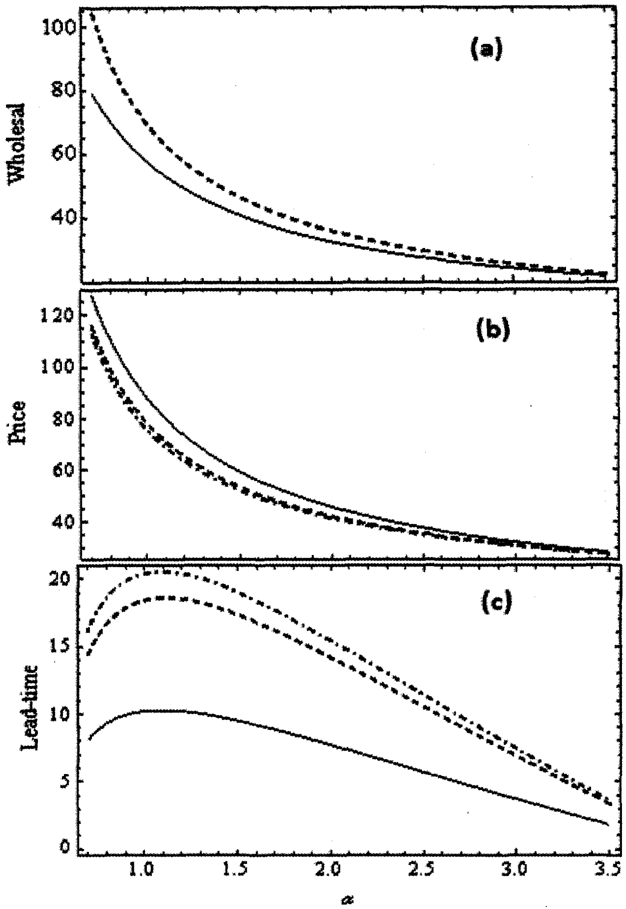

Fig. 3 Own price sensitive demand dependence of wholesale, retail price, and lead-time for a shape parameter $\varpi = 1.2$.

4.5.2 Effect of Own Lead-time Sensitivity β

First, we have $\frac{\partial \tau}{\partial \beta} = -\frac{1}{\alpha\varpi(b+h)}\left(\frac{b-\beta/\alpha}{b+h}\right)^{\frac{1-\varpi}{\varpi}} < 0$, $\frac{\partial \eta}{\partial \tau} = \frac{1}{\varpi+1}[(\varpi+1)(b+h)\tau^{\varpi} - b(\varpi+1)] = -\frac{\beta}{\alpha} < 0$, and $\frac{\partial(1+\beta\rho\tau+\alpha\eta\rho)}{\partial \beta} = \rho\tau + \beta\rho\frac{\partial \tau}{\partial \beta} + \alpha\rho\frac{\partial \eta}{\partial \tau}\frac{\partial \tau}{\partial \beta} = \rho\tau > 0$. Thus, it is easy to see in Scenarios 1 and 2 that $\frac{\partial l_{di}^*}{\partial \beta} < 0$ $\frac{\partial \lambda_{di}^*}{\partial \beta} < 0$, $\frac{\partial \pi_{ri}^*}{\partial \beta} < 0$, and $\frac{\partial \pi_{si}^*}{\partial \beta} < 0$ for $i = 1,2$. Therefore, increasing the own lead-time sensitivity demand factor β reduces the lead-time, the demand function, and the expected profits. We will discuss the behavior of the wholesale and retail prices with β numerically. In all figures, the solid, the dashed and the dot-dash lines correspond to Scenarios 1, 2 and 3, respectively.

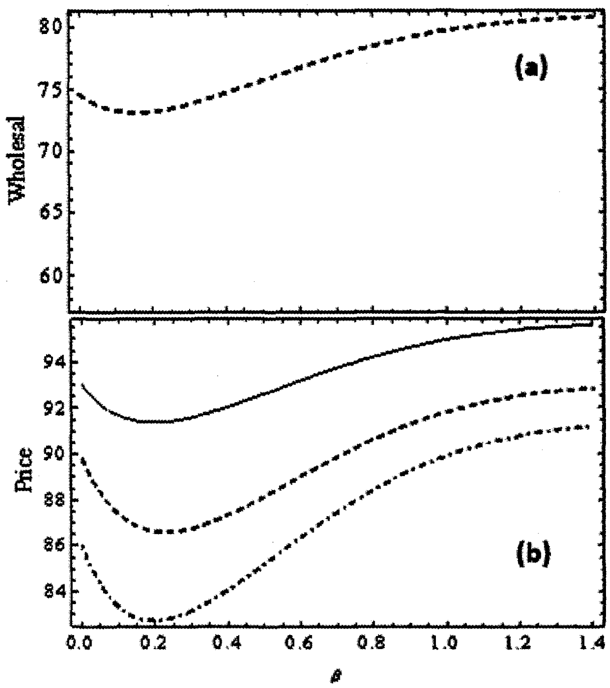

Fig. 4 Own lead-time sensitive demand dependence of wholesale and retail price for a shape parameter $\varpi = 0.2$.

4.5.2.1 Case of Shape Parameter $\varpi = 0.2$

The wholesale price in Scenario 1 is independent of the own lead-time sensitivity demand factor β. However, as shown in Fig. 4 (a), it is not monotone in Scenario 2 and is limited by a minimum wholesale price value w_{min}. Further, as shown in Fig. 4 (b), the retail price in the three scenarios is also not monotone and is limited by a minimum price p_{min} . The value of own lead-time sensitivity demand factor $\beta = \beta_{min}$ that corresponds to w_{min} and p_{min} is not the same and it depends on the chain and distribution parameters. This value depends strongly on the tardiness cost b and the own price sensitivity demand factor α. It is worth noting that the non-monotony of wholesale and retail price disappears for high tardiness cost b. In addition, in contrast to the other parameters, β_{min} shifts to high values as α increases. An important finding is that the wholesale and retail prices are limited with minimum values, under which the supplier and retailer cannot sell their products.

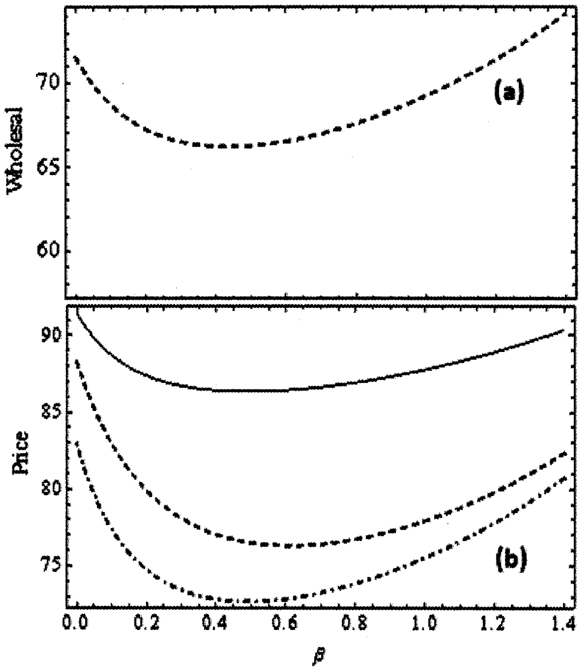

Fig. 5 Own lead-time sensitive demand dependence of lead-time for a shape parameter $\varpi = 1.2$.

4.5.2.2 Case of Shape Parameter $\varpi = 1.2$

In contrast to the wholesale and retail prices, increasing the shape parameter increases the lead-time for all values of β. The optimal demand and profits start with high values for high shape parameter, however they drop rapidly with increasing β more than in the case of low ϖ. Furthermore, as shown in Figs. 5 (a) and (b), β_{min} shifts to high values with increasing ϖ.

5 CONCLUSIONS

In a supply chain consisting of one supplier and one retailer and wherein the demand is sensitive to retail price and lead-time, three different scenarios based on lead-time decision are studied and compared theoretically and numerically. Using the power distribution function, when the retailer decides the lead-time, the entire expected profit regresses constantly 1/4 from that of the centralized model and the inefficiency of the decentralized chain is independent of chain and distribution parameters. However, when the supplier decides the lead-time, the inefficiency of the decentralized chain is less than 1/4, it depends on chain and distribution parameters, and it is often less than that when the retailer decides the lead-time. We also found that the chain actor who decides the lead-time achieves more profits than the other one, independently of chain and distribution parameters. Numerically, the consumers are found to be sensitive to the own price sensitive demand factor, where infinity lead-time is not allowed. The retailer is found to be sensitive to the own lead-time sensitive demand factor, where he cannot decrease his retail price under a minimum value. The two limits of lead-time and retail price are sensitive to the tardiness cost.

REFERENCES

[1] Sterling, J. U. and Lambert, D. M.: "Customer Service Research: Past Deficiencies, Present Solutions and Future Opportunities," Int. J. Phys. Distrib. Mater. Manage., Vol. 19, pp. 1-23 (1989)

[2] Ballou, R. H.: Business Logistics Management, Prentice Hall, Upper Saddle River, NJ (1998)

[3] So, K. C.: "Price and Time Competition for Service Delivery," Manuf. Serv. Oper. Manage. Vol. 2, pp. 392-409 (2000)

[4] Yano, C.: "Setting Planned Lead-times in Serial Production Systems with Tardiness Costs," Manage. Sci., Vol. 33, pp. 95-106 (1987)

[5] Yano, C.: "Stochastic Lead Times in Two-level Distribution Systems," Naval Res. Logist., Vol.34 pp. 831-843 (1987)

[6] Yano, C.: "Stochastic Lead-times in Two-level Assembly Systems," IIE Trans., Vol. 19, pp. 371-378 (1987)

[7] Li, L.: "The Role of Inventory in Delivery-time Competition," Manage. Sci., Vol. 38, pp. 182-197 (1992)

[8] Hopp, W. and Spearman, M.: "Setting Safety Lead Times for Purchased Components in an Assembly System," IIE Trans., Vol. 25, pp. 2-11 (1993)

[9] Li, L. and Lee Y. S.: "Pricing and Delivery-time Performance in a Competitive Environment," Manage. Sci., Vol. 40, pp. 633-646 (1994)

[10] Lederer, P. J. and Li, L.: "Pricing, Production, Scheduling, and Delivery-time Competition," Oper. Res., Vol. 45, pp. 407-420 (1997)

[11] Palaka, K., Erlebacher, S. and Kropp, D. H.: "Lead Time Setting, Capacity Utilization, and Pricing Decisions under Lead Time Dependent Demand," IIE Trans., Vol. 30, pp. 151-163 (1998)

[12] So, K. C. and Song, J.: "Price, Delivery Time Guarantees and Capacity Selection," Eur. J. Oper. Res., Vol. 111, pp. 28-49 (1998)

[13] Song, J., Yano, C. and Lerssisuriya, P.: "Contract Assembly: Dealing with Combined Supply Lead Time and Demand Quantity Uncertainty," Manuf. Serv. Oper. Manage., Vol. 2, pp. 287-296 (2000)

[14] Cachon, G. P. and Harker, P. T.: "Competition and Outsourcing with Scale Economies," Manage. Sci., Vol. 48, pp. 1314-1333 (2002)

[15] Boyaci, T. and Ray, S.: "Product Differentiation and Capacity Cost Interaction in Time and Price Sensitive Markets". Manuf. Serv. Oper. Manage., Vol. 5, pp. 18-36 (2003)

[16] Liu, L., Parlar, M. and Zhu, S. X.: "Pricing and Lead Time Decisions in Decentralized Supply Chains," Manage. Sci., Vol. 53, pp. 713-725 (2007)

[17] Pekgun, P., Griffin, P. and Keskinocak, P.: "Centralized vs. Decentralized Competition for Price and Lead-time Sensitive Demand," INFORMS, Pittsburgh, PA, Nov. 5-8 (2006)

[18] Tsay, A. A. and Agrawal, N.: "Channel Dynamics under Price and Service Competition," Manuf. Serv. Oper. Manage., Vol. 2, pp. 372-391 (2000)

[19] Balasubramanian, S. and Bhardwaj, P.: "When Not All Conflict Is Bad: Manufacturing-Marketing Conflict and Strategic Incentive Design," Manage. Sci., Vol. 50, pp. 489-502 (2004)

[20] Spiegel, R. M. and Liu, J. M.: Mathematical Handbook of Formulas and Tables. Schaum's Outline Series, 2nd ed., McGraw-Hill, New York (1999)

[21] Zhengping W., Burak, K., Scott, W. and Kum-Khiong, Y.: "Ordering, Pricing, and Lead-time under Lead-Time and Demand Uncertainty", (August 2009), http://myweb.whitman.syr.edu/

[22] Wang, Y., Jiang, L. and Shen, Z.: "Channel Performance under Consignment Contract with Revenue Sharing". Manage. Sci., Vol. 50, pp. 34-47 (2004)