Maximum Likelihood Detection with Arbitrary Modulations in Cooperative Relay Channels

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Abstract**—** This paper proposes a simple combining technique with arbitrary modulations for a cooperative relay scheme based on a Detect-and-Forward (DEF) relay protocol. Here we present Maximum Likelihood (ML) criterion detection at the destination which considers individual symbol error rate (SER) to facilitate the detection in noisy relay networks. In particular, our proposed algorithm is flexible to signals with different modulation formats as detection is done on a symbol-by-symbol basis. If different modulations are used at the source and the relays, we propose that lower modulation constellation is used at the source. By computer simulations, the results show that significant Packet Error Rate (PER) performance can be achieved by the proposed scheme and we compare it against Cooperative-Maximum Ratio Combining (C-MRC) and Selection Combining (SC).

Keywords- Cooperative Relay; Detect-and-Forward; Maximum-Likelihood criterion detection.

I. INTRODUCTION

In recent years, there has been a growing interest in cooperative wireless communication. The basic idea is to assist the destination node through the multiple replicas of the same information transmitted from the source. Various relaying schemes have been proposed to explore the benefits of cooperative communication, mainly divided into three categories, including Decode-and-Forward (DF), Amplify-and-Forward (AF) [1]-[3] and Detect-and-Forward (DEF). Among these protocols, DEF is attractively simple in complexity where the relay detects the signals (hard-decision detection) and modulates before forwarding to the destination.

Many research papers in relay networks assume the modulations used by the source and the relays to be the same. In some favorable conditions, the source can use higher power and larger symbol constellations to optimize the channel resources. For such relay networks, signals from the source-destination (S-D) and relay-destination (R-D) links may not be necessarily belong to the same modulation formats. Thus, it is imperative to design a combining scheme which can address this problem effectively. One finds that a conventional maximal ratio combining (MRC) cannot be used for combining signals received in different modulation formats. Selection combining (SC) has been proposed to be utilized for this purpose [4] and yet, this strategy is far from being optimal. In [5], Soft-bit MRC is proposed which aims at combining signals with different modulations assuming perfect relays.

However, since [5] considers perfect relay channels, the solution may not be realistic in practical relay networks where there is potential erroneous detection at relays. For example, our previous work on Maximum Likelihood (ML) criterion detection with DEF protocol [6] simplifies the conventional ML-based detection which assumes sufficiently high signal-to-noise-ratio (SNR) approximation for S-R link. The authors in [7] have developed a piece-wise linear receiver approximating the ML criterion detection that requires knowledge of the average SNR of the first hop. However, this scheme cannot achieve full diversity for more than one relay. In [8], another combining technique namely Cooperative-MRC (C-MRC) is introduced that approximates the ML detector. However, C-MRC results in serious propagation error under *asymmetrical* networks when SNR of relay-destination (R-D) link is larger than that of source-destination (S-D) link or source-relay (S-R) link. In addition, unlike ML-based strategy, C-MRC cannot be used in relay networks with arbitrary modulation [9]. In [9], the authors proposed the performance of the hard-decision ML criterion detection-based combining technique under coded cooperative scheme. In [10] and [11], the authors have proposed a non-coherent combiner in uncoded cooperative relaying scheme using DEF protocol when channel state information (CSI) of S-R link is not available at the destination. In [4]-[11], the authors have derived sub-optimal receivers but leaving two key issues which need to be addressed in noisy relay networks: 1) exploiting effectively perfect knowledge of all links for optimal combining at the destination i.e., the error probability at the relay; 2) solution for combining noisy relayed signals with different modulation levels. In our work [12], we have proposed an ML-based combining strategy which exploits every symbol error probability for the detection at the destination in quadrature phase-shift keying (QPSK). To guarantee an optimal ML criterion detection, the destination needs to know the error characteristics of the S-R link (perfect CSI) in the form of relay error probabilities. However, [12] is not defined properly for higher modulation constellations but focusing on only binary signals with the same modulation constellation at the source and the relays.

In this paper, we extend the proposed algorithm in [12] to *M*-Quadrature Amplitude Modulations e.g., 16QAM. Unlike in C-MRC, the instantaneous CSI in the proposed scheme involves Q-function expression for each symbol in the modulation which provides accurate knowledge of S-R link. For simplicity, we analyze this ML performance with a simple DEF in uncoded cooperative relay networks and compare against the baseline C-MRC under *symmetrical* channels where all link SNRs are the same and *asymmetrical channels* that is where the R-D link is different from others. We also show through channel capacity analysis that the proposed scheme is superior to C-MRC in both network setups. In addition, our work here also investigates the proposed scheme when different modulations are used at the source and the relay under noisy relay channels as opposed to the solution in [5]. Through computer simulations we observe that the proposed scheme is not only practical to the different modulated signals but also shows a remarkable potential in achieving significant diversity gains with better packet error rate (PER) performance than that of C-MRC.

The organization of this paper is as follows: Section II is System Description and the proposed scheme, Simulation Results and Discussions are given in Section III, and finally in IV the paper is summarized. The derivation of the individual SER for 16QAM in Gray mapping is presented in the Appendix.

II. SYSTEM DESCRIPTION

A. *System Model*

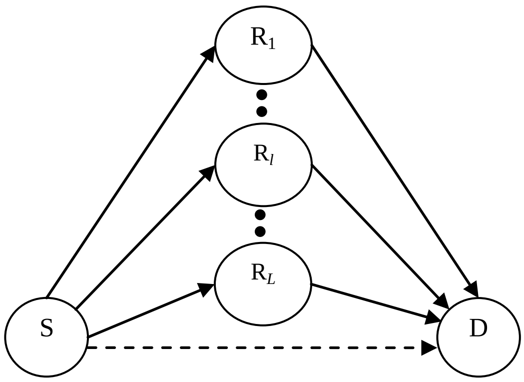

Figure 1: Block diagram of the cooperative relay system with multiple relay channels.

We consider a general case shown in Fig. 1, a source node (S) and a destination (D) with L relays $R_l,\ l \in \{1,2,...,L\}$ over flat Rayleigh fading channels. Assuming time division multiplexing for simplicity, the source transmits its signal x_s in the first timeslot to the destination and the relays with the average power E_s. Due to the broadcast nature of wireless channel, both the destination and all L relays receive noisy symbols of x_s. The received signals at the destination and at the l-th relay denoted by y_{sd} and $y_{sr,l}$ respectively can be written as

$$\begin{aligned} y_{sd} &= h_{sd} x_s + n_{sd} \\ y_{sr,l} &= h_{sr,l} x_s + n_{sr,l} \end{aligned} \quad (1)$$

where the subscripts indicate the node relation such that h_{sd} and $h_{sr,l}$ are independent complex-valued channel gains for the S-D link and S-R link of the l-th relay respectively. For simplicity, all channels are Quasi-static Rayleigh fading channels i.e., $h_{sd} \sim \mathcal{CN}(0,1)$ and $h_{sr,l} \sim \mathcal{CN}(0,1)$, where $\mathcal{CN}(\mu,\sigma^2)$ denotes a complex Gaussian random variable with mean μ and variance σ^2. n_{sd} and $n_{sr,l}$ are independent additive white Gaussian noise at the destination and the relay respectively which are modeled as $n_{sd} \sim \mathcal{CN}(0,\sigma_{sd}^2), n_{sr,l} \sim \mathcal{CN}(0,\sigma_{sr,l}^2)$ with variance equal to $N_0/2$ per dimension. We assume that the average SNR for all links are the same denoted as $\bar{\gamma} = E_s/N_0$, while the instantaneous SNR is represented as $\gamma_{sd} = |h_{sd}|^2 \bar{\gamma}$ and $\gamma_{sr,l} = |h_{sr,l}|^2 \bar{\gamma}$ respectively. The relay performs a hard-decision detection (DEF) and re-modulates the detected symbol as $x_{r,l}$ with the same average power E_s for re-transmissions in timeslot 2. The symbol received at the destination is given as

$$y_{rd,l} = h_{rd,l} x_{r,l} + n_{rd,l} \quad (2)$$

where $h_{rd,l} \sim \mathcal{CN}(0,1)$ and $n_{rd,l} \sim \mathcal{CN}(0,\sigma_{rd,l}^2)$ with variance equal to $N_0/2$ per dimension. The instantaneous SNR is $\gamma_{rd,l} = |h_{rd,l}|^2 \bar{\gamma}$.

At the destination, the received signals from the source and the relay node are combined in order to recover the original source data.

B. *Proposed ML-based Combining Strategy*

In this section, we generalize our work in [12] to *M*-QAM with different modulations at the source and the relay nodes. To better motivate the proposed ML combining strategy, let us take a closer look at C-MRC. In [8], the authors have proposed an improved version of MRC termed as C-MRC. The C-MRC output at the destination node is given by assuming independent relay channels

$$y_{cmrc} = h_{sd}^* y_{sd} + \sum_{l=1}^{L} \frac{\gamma_{\min,l}}{\gamma_{rd,l}} h_{rd,l}^* y_{rd,l} \quad (3)$$

where $\gamma_{\min,l} = \min(\gamma_{sr,l}, \gamma_{rd,l})$, $\gamma_{sr,l}$ and $\gamma_{rd,l}$ are instantaneous SNR of the S-R and R-D channels for the l-th relay node respectively. $\gamma_{\min,l}$ is tight approximation of the equivalent SNR of the S-R-D link at high SNR [8]. The usual intuitive meaning associated with (3) is that when $\gamma_{sr,l}$ is high, the detector places full confidence to the arriving signals from the relay. In case of low $\gamma_{sr,l}$, the confidence is weighted according to the ratio of both hops that is S-R-D link. In fact, from our knowledge, like its predecessor MRC, (3) cannot be easily used for signals with different modulations. Thus, we compare the proposed scheme also against the conventional SC which has been widely used to combine signals from different modulation constellations [4].

The proposed algorithm in [12] optimally combines the noisy signals received at the destination node, y_{sd} and

$y_{rd,l}$ by considering the effect of detection errors at the output of the *l*th relay. However, the focus is only on the combining method with the same modulation, QPSK at both the source and the relays. From [12], the corresponding joint ML decision criterion finds $\hat{x}_s$, an estimate of x_s and is defined as

$$\hat{x}_s = \underset{x_s, x_r \in \{\chi_s = \chi_{r,l}\}}{\arg\max} \; p_{sd}(y_{sd} \mid x_s) \times \prod_{l=1}^{L} \left\{ \begin{array}{l} P(x_{r,l} = x_s) p_{rd}(y_{rd,l} \mid x_{r,l} = x_s) \\ + \sum_{x_{r,l} \in \chi_{r,l}} P(x_{r,l} \neq x_s) p_{rd}(y_{rd,l} \mid x_{r,l}) \end{array} \right\} \quad (4)$$

where χ_s and $\chi_{r,l}$ denotes the finite set of the constellation at the source and the *l*-th relay respectively; we use capital P as the probability; $p_{sd}(y_{sd} \mid x_s)$ is the PDF of the source signal y_{sd} conditioned upon the transmitted signal x_s and $p_{rd}(y_{rd,l} \mid x_{r,l} = x_s)$ is the PDF of the relayed signal $y_{rd,l}$ conditioned on the equality of both transmitted symbols $(x_{r,l} = x_s)$. The bracketed term in (4) has to consider the error probability of the received signal $y_{sr,l}$ at the *l*-th relay accounting the individual SER of each signal point.

In QPSK modulation, the transmit symbol x_s which is labeled by two bits, (b_1, b_2) takes from the constellation set $\chi_s \in \{s_1, s_2, s_3, s_4\}$. Assuming the source and the relays use the same QPSK modulation i.e., $\chi_s = \chi_{r,l}$, the detection at the destination is performed jointly by the ML criterion and we can expand (4) as

$$\hat{x}_s = \underset{x_s, x_r \in \{\chi_s = \chi_{r,l}\}}{\arg\max} \; p_{sd}(y_{sd} \mid x_s) \times \prod_{l=1}^{L} \left\{ \begin{array}{l} [1 - (\varepsilon_1 + 2\varepsilon_2)] p_{rd}(y_{rd,l} \mid x_{r,l} = x_s) \\ + \varepsilon_1 p_{rd}(y_{rd,l} \mid x_{r,l} = x_s \varepsilon^{j\pi}) \\ + \varepsilon_2 p_{rd}(y_{rd,l} \mid x_{r,l} = x_s \varepsilon^{j\frac{\pi}{2}}) \\ + \varepsilon_3 p_{rd}(y_{rd,l} \mid x_{r,l} = x_s \varepsilon^{-j\frac{\pi}{2}}) \end{array} \right\} \quad (5)$$

where ε_1, ε_2 and ε_3 denote the symbol error probabilities from $s_1 \to s_3$, $s_1 \to s_2$ and $s_1 \to s_4$ respectively; ε_1 and $\varepsilon_2 = \varepsilon_3$ are analytically expressed as the Gaussian Q function where $Q(x) = \left(1/\sqrt{2\pi}\right) \int_x^\infty \exp\left(-t^2/2\right) dt$. In the bracketed term of (5), we include the multiplicative error term in exponential function, $\varepsilon^{j\phi}$ with the following equality $x_{r,l} = x_s \varepsilon^{j\phi}$ where $\phi \in \{0, \pi, \frac{\pi}{2}, -\frac{\pi}{2}\}$ denotes the phase changes that depends on the symbols transmitted from the relay. This means that (5) takes into account the fact that the relay does not operate error-free. In this paper, we employ closed–form expressions for the probability of error for each constellation symbol for QPSK as

$$\varepsilon_1 = P(x_{r,l} = x_s \varepsilon^{j\pi}) = \frac{1}{4} erfc^2 [\sqrt{\frac{E_s}{2N_0}}] = \left\{ Q\left(\sqrt{\frac{E_s}{N_0}} \right) \right\}^2 \quad (6)$$

$$\varepsilon_2 = P(x_{r,l} = x_s \varepsilon^{j\frac{\pi}{2}}) = \varepsilon_3 = P(x_{r,l} = x_s \varepsilon^{-j\frac{\pi}{2}}) = Q\left(\sqrt{\frac{E_s}{N_0}} \right) - \left\{ Q\left(\sqrt{\frac{E_s}{N_0}} \right) \right\}^2 \quad (7)$$

where erfc is the complementary error function. For higher modulation like 16QAM, we provide some expressions of the individual SER in the Appendix. When y_{sd} and $y_{rd,l}$ are received at the destination, by inserting s_1, s_2, s_3 or s_4 to x_s and examining how large the argument value in (5), we can determine the transmit signal point x_s from the finite set χ_s in QPSK constellation. The PDF expression in (5) can be represented by

$$p_{rd}(y_{rd,l} \mid x_{r,l} = x_s \varepsilon^{j\phi}) = \frac{1}{\sqrt{2\pi\sigma_{rd,l}^2}} \exp\left\{ -\frac{|y_{rd,l} - h_{rd,l} x_s \varepsilon^{j\phi}|^2}{2\sigma_{rd,l}^2} \right\} \quad (8)$$

where $\phi \in \{0, \pi, \frac{\pi}{2}, -\frac{\pi}{2}\}$. The analytical results presented thus far in previous works have been derived from studies which examined the SER problem assuming that the symbol error probability of each QPSK symbol is equally likely (average SER). Thus, these results cannot be treated as offering a complete ML solution. Note that another advantage in the proposed ML over C-MRC is its flexibility of combining different modulated signals from different nodes since each link can be treated independently (symbol-wise detection).

Next, we generalize (5) to *M*-QAM. From (5), we observe that there are $\chi_{r,l} - 1$ ways of making an incorrect decision and their impacts on detection at the destination are not necessarily the same. Thus, we can easily show the decision criterion for general *M*-QAM as

$$\hat{x}_s = \underset{x_s, x_r \in \{\chi_s \neq \chi_{r,l}\}}{\arg\max} \; p_{sd}(y_{sd} \mid x_s) \times \prod_{l=1}^{L} \left\{ \begin{array}{l} (1 - \sum_{\kappa=1:\chi_{r,l}-1} \varepsilon_\kappa) p_{rd}(y_{rd,l} \mid x_{r,l} = x_s) \\ + \sum_{\kappa=1:\chi_{r,l}-1} \varepsilon_\kappa p_{rd}(y_{rd,l} \mid x_{r,l}^{\kappa}) \end{array} \right\} \quad (9)$$

where $\varepsilon_\kappa, \kappa = \{1, 2, ..., \chi_{r,l} - 1\}$ is the symbol error probability for each symbol in *M*-QAM according to the modulation size in each relayed path and are expressed in Q-function as well. For example in the Appendix, we illustrate the derivations of ε_κ for some 16QAM symbols.

C. *Analysis of Combining Schemes*

In this sub-section, we analyze C-MRC and the proposed ML schemes in terms of their channel capacity. Here, we assume one relay node for simplicity. Let us denote the channel capacities of S-R, R-D and S-D links by $C_{sr}(\gamma_{sr}) = \log_2(1 + \gamma_{sr})$, $C_{rd}(\gamma_{rd}) = \log_2(1 + \gamma_{rd})$ and $C_{sd}(\gamma_{sd}) = \log_2(1 + \gamma_{sd})$ respectively, and the joint capacity of the combined signals at the destination during the cooperative phase by C_{tol}. The channel capacity unit is bit per channel use. The total capacities C_{tol} for C-MRC and the proposed scheme are as follows [13]

$$C_{tol}^{C-MRC} = C(\gamma_{sd} + \gamma_{rd})$$
$$C_{tol}^{ML} = C(\gamma_{sd}) + C(\gamma_{rd}) \tag{10}$$

Assuming the instantaneous SNR for each link $\gamma_{sr}, \gamma_{rd}, \gamma_{sd} \geq 0$, then we have

$$C(\gamma_{sr} + \gamma_{rd}) + C(\gamma_{sd}) \geq C(\gamma_{sd} + \gamma_{rd}) + C(\gamma_{sr}) \tag{11}$$

if and only if $(\gamma_{sd} \geq \gamma_{sr} \vee \gamma_{rd} = 0)$. (10) and (11) show that the variations in the relayed link reduces the total channel capacity. Particularly, the degradation in performance of C-MRC can be worse than that of the ML i.e., $C_{tol}^{ML} \geq C_{tol}^{C-MRC}$. We also prove this claim by computer simulations in what follows.

D. *Complexity Comparison*

The computational complexity of the receiver at the destination depends on the detection algorithms, the hardware architectures, and other factors. In this paper, we evaluate the computational complexity for our proposed scheme, C-MRC and SC based on the number of complex multiplications and additions. For convenience, we consider the required computations for the functions of equalization, detection and signal combining at the destination in a relay node scheme ($L = 1$) only. Here we assume QPSK modulation is used at the source and relay node. We define that each multiplication from two complex numbers takes four complex multiplications and two additions. If Euclidean distance metric calculation is employed, we need 46 complex multiplications and 16 additions to detect a symbol at the receiver. Thus, this becomes the baseline computational complexity for SC strategy. Due to space limitation, other derivations are omitted for brevity. Table 1 compares the number of required complex multiplications and additions for each scheme per symbol.

Table 1: The number of complex multiplications and additions at each scheme.

Complexity	SC	C-MRC	Proposed
Multiplication	46	50	230
Addition	16	28	80

Table 1 shows that the computational complexity increases with the order from SC<C-MRC<ML-based Combining (proposed). SC turns out to be the lowest but with a significant reduction in the error rate performance as shown in the following section. SC only uses one signal for detection at the receiver and hence, the computation is less. This outcome for our proposed scheme is expected since the additional complexity in the scheme is coupled with a significant error rate improvement compared against the conventional SC and C-MRC in various simulation setups as shown in the manuscript. The complexity of the proposed scheme is highest because the destination has to consider individual SER of making wrong decisions at relay nodes in the detection. Thus, the complexity of the proposed scheme increases as the modulation increases. However, to assist the detection at the destination, our proposed scheme only requires the average receive SNR of S-R link to compute individual SER of the modulation as shown in (6) and (7). Therefore, our proposed scheme still inherits an interesting trade-off between the error rate performance and the system complexity. Although C-MRC is simpler in the computational complexity, its biggest challenge is to have accurate instantaneous channel knowledge at the receiver. In practice, one needs accurate channel estimation and a high signaling overhead in C-MRC scheme to feedback the channel knowledge to the destination. In fact, there is no practical C-MRC approach ever proposed yet for combining different modulated signals.

III. SIMULATION RESULTS AND DISCUSSIONS

Table 2: Simulation parameters.

Information Bits	1008 per packet
Modulations	QPSK (4QAM) and 16QAM
Channel Model	Quasi-static Rayleigh Fading Channel
Relay Protocol	DEF
Relay Equalizer	Zero Forcing
Combining Protocol	Proposed ML / C-MRC/SC

Throughout the simulation works, we use the stipulated parameters in Table 2 unless otherwise stated. We analyze the Packet Error Rate (PER) against average SNR in decibel (dB). For convenience, we restrict our simulation work to QPSK and 16QAM modulations only. To reduce the computational complexity in the proposed ML for 16QAM, we adopt the max-log approximation. We assume the source and all relay nodes transmit with the same average power E_s resulting in the average SNR, $\bar{\gamma} = E_s / N_0$ (symmetrical network). For C-MRC, we also consider the destination has a perfect knowledge of S-R link (i.e., instantaneous SNR) and perfect channel estimation is assumed. In this simulation, we only consider blind cooperative relaying schemes where relay nodes always re-transmit to the destination whether the signal is correctly detected or contains errors. No automatic repeat request (ARQ) protocol is used to avoid the error propagation from the relay nodes to the destination.

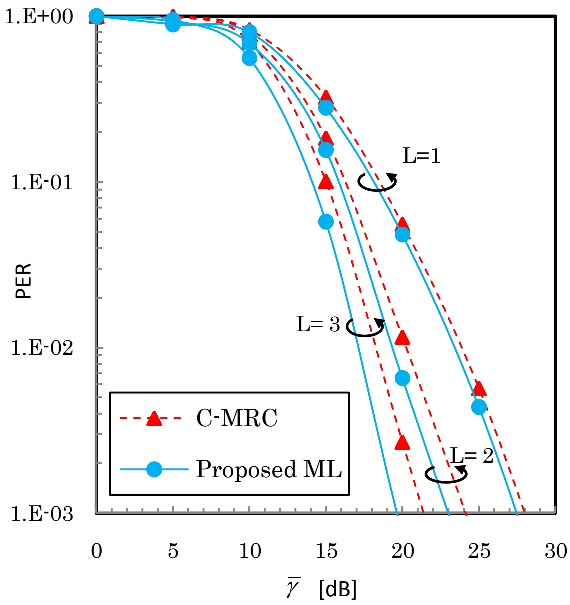

Figure 2: PER comparison with 16QAM at the source and the relay nodes between the proposed ML scheme and C-MRC (dashed lines) using DEF protocols for $L = 1, 2$ and 3 relays.

Fig. 2 shows the PER performance of the proposed scheme for 16QAM modulation at both the source and the relays, $\chi_s = \chi_{r,l}$ against the baseline for multiple relay nodes i.e., $L = 1, 2$ and 3. As expected, the proposed schemes outperform C-MRC (3) in all cases with 0.5dB, 1dB and 1.5dB gap at $\text{PER} = 10^{-3}$ for 1, 2 and 3 relay cases respectively. We can also observe that all the cooperative schemes achieve full order diversity as observed from the slopes of the curves i.e., $10^{-(L+1)}/10(\text{dB})$ (diversity order $\bar{\gamma}$ of $L+1$). This result demonstrates that the proposed algorithm has better accuracy of symbol detection due to the sufficient statistics of the received signals y_{sd} and $y_{rd,l}$. For this reason, the conditional probability $p_{rd}(y_{rd,l} \mid x_{r,l})$ can be computed using the observations $p_{rd}(y_{rd,l} \mid x_{r,l} \neq x_s)$. In practice therefore, with necessary CSI, the destination can optimally combine signals received from the source and noisy relays assisted by DEF protocol only.

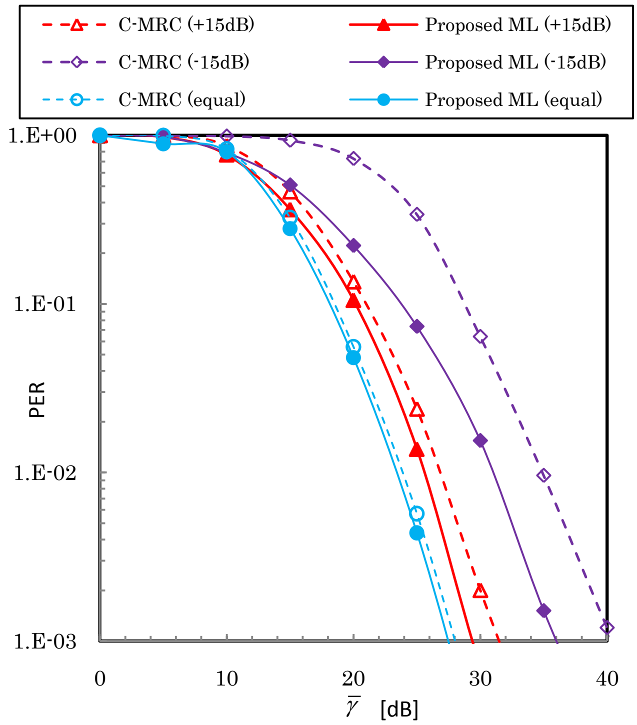

Figure 3: PER comparison with 16QAM at the source and the relay node between the proposed ML scheme (solid lines) and C-MRC (dashed lines) when the average SNR of R-D link, $\bar{\gamma}_{rd}$ varies at $L = 1$ relay case.

Next, in Fig. 3 we simulate the proposed scheme and C-MRC with the same 16QAM in both nodes under different R-D link quality. From here onwards we only simulate for one relay node $(L = 1)$. Thus, for convenience we remove the subscript l in the notation. We vary the average SNR for R-D link $\bar{\gamma}_{rd}$, and we keep the average SNR for S-D link and S-R link the same, $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}$. This scenario is feasible due to the nature of broadcast transmission of the source node with relays which are typically power-constraint nodes. We simulate the schemes at three different scenarios of R-D link quality: $\bar{\gamma}$ +15dB (+15dB), $\bar{\gamma}$ -15dB (-15dB) and $\bar{\gamma}_{rd} = \bar{\gamma}$ (equal). From Fig. 3, we find that the proposed scheme can outperform C-MRC when R-D link has sufficiently high SNR quality (+15dB) with marginal 1dB gap at PER= 10^{-3} and 2.5dB gap at PER= 10^{-2} for low SNR quality (-15dB). One way to explain this is that when R-D link has higher SNR compared to S-D link, the combined signal at the destination is dominated by the erroneous signal from the relayed link whose error is due to the detection error at the relay. Given that the relay has made a decision error and hence the source and the relay send contradicting information to the destination. As a result, when the R-D has very low SNR, the PER performance is degraded further compared to the case of equal SNR. In C-MRC, one can also refer to (3) that the sub-optimality of C-MRC is due to the weighted signal from the relayed link which becomes larger than that of the direct link. C-MRC effectiveness is largely conditioned on the link quality of R-D link over S-D link (direct path). In particular, although the received signals at the relays are noisy and only DEF is used at the relays, the proposed scheme improves achievable PER performance which becomes an added advantage compared to C-MRC. This result also confirms the channel capacity analysis in II-C.

Another feature of our proposed scheme in (9) is the feasibility aspect in combining arbitrary modulations. In Fig. 4, we simulate the proposed scheme with different modulations, QPSK and 16QAM at the relay node $(L = 1)$. For the comparison, we use selection combining (SC) with the same channel setup. We also simulate a scheme when no relay is used with BPSK modulation. A simulated lower bound with one perfect relay (i.e., error-less relay detection) is also included in this simulation. The results in Fig. 4 clearly show that the proposed scheme outperforms SC scheme with great margins. In both combining techniques, as expected, we can clearly see that there is a slight improvement in PER if lower modulation i.e., QPSK is used at the relay which is about 1dB gain at PER=10^{-2}. This result is expected due to the fact that lower modulation is less vulnerable to errors.

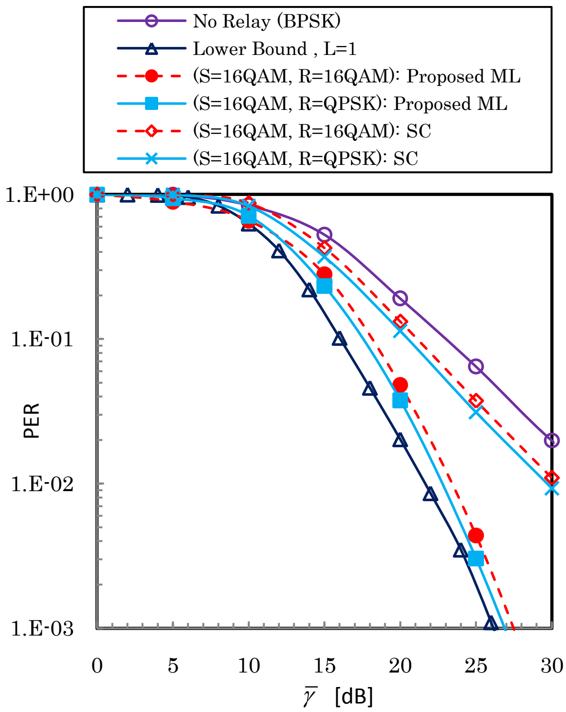

Figure 4: PER comparison proposed ML scheme and SC with 16QAM at the source and different modulation at the relay using DEF protocols for $L = 1$ relay.

Notwithstanding, Fig. 4 does not consider the same total transmission rate at the destination. In Fig. 5, in a fixed transmission rate scheme i.e.,

$\eta = ((\log_2 \chi_s)^{-1} + (\log_2 \chi_r)^{-1})^{-1}$ which means that η is the same for the cases in comparison, we simulate when the source and the relay use different modulation assignments, $\chi_s \neq \chi_r$. For simplicity, the scheme uses 2 sets of modulation combinations from QPSK and 16QAM. In the proposed scheme, since different modulations carry different number of bits per symbol, we propose to do bit-by-bit detection if mapping conversion is required at the relay node. To extract the bits from the symbols, symbol log-likelihood ratio (LLR) can be used [6]. Thus, regardless of the modulation constellations used at the relay, we can easily convert the mapping from QPSK to higher constellations or vice versa. For case 1 when S uses QPSK, the relay employs 16QAM (S=QPSK, R=16QAM). In case 2, the source uses 16QAM and the relay uses QPSK (S=16QAM, R=QPSK) which is identical to the curves in Fig. 4. From Fig. 5, the result clearly shows that the proposed scheme performs better when lower modulation is used at the source which is about 3dB improvement in the proposed scheme at PER $=10^{-3}$. The proposed scheme also easily achieves the full diversity gain of 2 for both cases. The same trend occurs in SC scheme with around 3dB improvement at PER$=10^{-2}$ but with lower diversity gain due to the error propagation from the relay. Higher modulations at the relay tend to be more susceptible to noisy channels. The 1dB loss in the simulation result is the direct outcome of the error propagation of the noisy channels. It is expected that higher *M*-QAM modulation is more susceptible to noise. In addition, in our proposed scheme, the ML detector places more weight on the signals coming from the source directly, thus giving less weight on the relayed link. This reduces the effect of the error propagation from the relay node. From all cases, we can draw a conclusion that assigning lower modulation like QPSK at the source is a better strategy to bring more performance improvement in relay networks. To prevent the deleterious effect of error propagation at the relay, it is important that the source node is assigned with lower modulation.

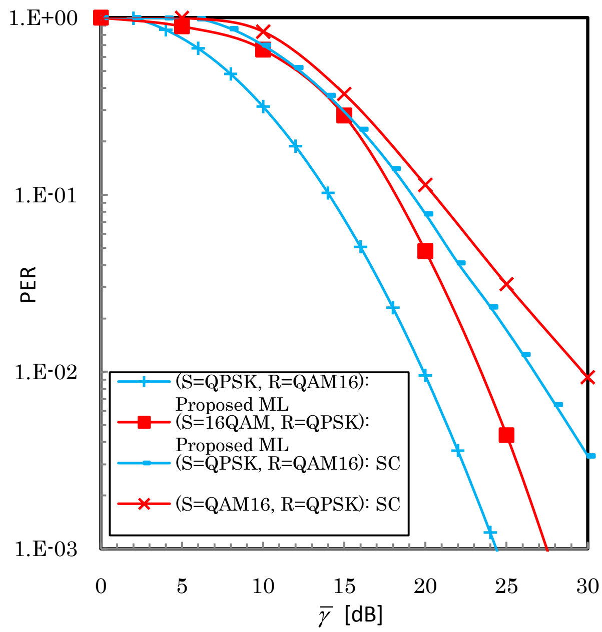

Figure 5: PER comparison proposed ML scheme and SC with different modulations at the source and the relay using DEF protocols for $L = 1$ relay.

IV. Conclusions

In this paper, an extension of ML-based combining strategy for cooperative relay scheme to arbitrary modulations is proposed. Since the proposed scheme accounts the potential errors at the relays for the detection at the destination, we can accurately model the transition probabilities for the erroneous transmission from noisy relays. Our work also investigates the PER performance of the proposed scheme when the source and the relays have different modulations. We found that it is better to use lower modulation at the source, thus reducing possible error propagation from the relays. Through computer simulation, we show that the proposed ML scheme is superior to the conventional C-MRC in PER performance for all cases under the *symmetrical* or *asymmetrical* channels with greater flexibility in implementation compared to C-MRC regardless of the modulation schemes.

Appendix

Derivation of Individual Symbol Error Rate (SER) of 16QAM Signals in Gray Mapping

In this section, we present the derivations of individual SER of 16QAM symbols in AWGN channels which become the side information to our proposed scheme (9). Note that our framework in (9) also suits well for other modulations like QPSK having quadrature error or with I-Q gain mismatch [14], [15], since it treats the error probability in a symbol-by-symbol basis. Employing the two-dimensional (2-D) Gaussian Q-function representation, we present closed-form expressions for the individual SER of 16QAM signals. Fig. 6 depicts the signal points for 16QAM with its decision boundaries as the dashed lines when Gray mapping is used. The constellation points of 16QAM are normalized with the factor $a = 1/\sqrt{10}$ to ensure that the average energy over all symbols is unity. Let us denote I and Q as the in-phase and quadrature components respectively. Since each complex symbol of 16QAM corresponds to four binary bits, (b_1, b_2, b_3, b_4) as presented in Fig. 6 we label the respective symbols accordingly.

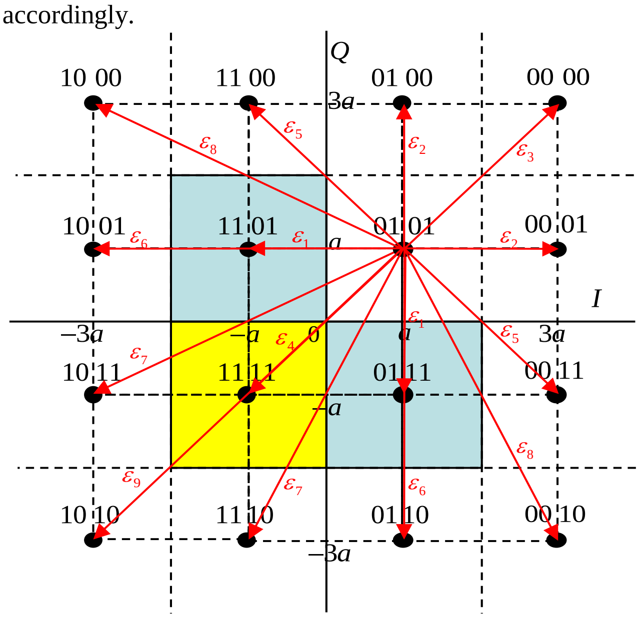

Figure 6: 16QAM symbols and symbol error probabilities.

Using similar derivations in QPSK [12], first we consider, for instance, the symbol (01 01) is transmitted from the source assuming the perfect CSI is available at the receiver side. If the receiver wrongly detects the symbol as (00 01), the symbol error probability for this particular symbol is calculated from the following integrations

$$\varepsilon_2 = \int_0^{\frac{2}{\sqrt{10}}} \int_{\frac{2}{\sqrt{10}}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(x - \frac{1}{\sqrt{10}})^2 + (y - \frac{1}{\sqrt{10}})^2}{2\sigma^2} \right) dxdy$$
$$= \frac{1}{2} erf[\frac{1}{2\sqrt{5}\sigma}] erfc[\frac{1}{2\sqrt{5}\sigma}]$$
$$= Q\left(\sqrt{\frac{E_s}{5N_0}}\right) - 2\left\{Q\left(\sqrt{\frac{E_s}{5N_0}}\right)\right\}^2 \quad (12)$$

where erf is the error function. For other symbols like (01 00) and (00 01), identical SER can be observed due to symmetry. Likewise, the calculation for ε_3 for symbol (00 00) which is located on the right top corner of the quadrant, can be found from the following integrations as

$$\varepsilon_3 = \int_{\frac{2}{\sqrt{10}}}^{\infty} \int_{\frac{2}{\sqrt{10}}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(x - \frac{1}{\sqrt{10}})^2 + (y - \frac{1}{\sqrt{10}})^2}{2\sigma^2} \right) dxdy$$
$$= \frac{1}{4} erfc^2[\frac{1}{2\sqrt{5}\sigma}] = \left\{Q\left(\sqrt{\frac{E_s}{5N_0}}\right)\right\}^2 \quad (13)$$

Similarly, for ε_4 which is the transition from the transmitted symbol (01 01) to symbol (11 11) as shown in yellow quadrant, it can be found from the following integration as

$$\varepsilon_4 = \int_{-\frac{2}{\sqrt{10}}}^{0} \int_{-\frac{2}{\sqrt{10}}}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(x - \frac{1}{\sqrt{10}})^2 + (y - \frac{1}{\sqrt{10}})^2}{2\sigma^2} \right) dxdy$$
$$= \frac{1}{4}\left(erf[\frac{1}{2\sqrt{5}\sigma}] - erf[\frac{10+\sqrt{5}}{10\sigma}] \right)^2$$
$$= \frac{1}{4}\left(2Q\left(\sqrt{\frac{21E_s}{5N_0}}\right) - 2Q\left(\sqrt{\frac{E_s}{5N_0}}\right) \right)^2 \quad (14)$$

Since some symbols like symbol (11 01) and (01 11) as shown in Fig. 6 are identical i.e., ε_1 , computation of these SERs can be reduced. Finally, other SERs can be straightforward in a similar fashion and they are not shown here for simplicity.

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