LETTER

Automated Creation of Beamformer-Based Optimum DOA Estimation Algorithm Using Genetic Algorithm

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SUMMARY   A novel method for automatically creating an optimum direction-of-arrival (DOA) estimation algorithm for a given radio environment using a genetic algorithm (GA) is proposed. DOA estimation algorithms are generally described by parameters and operators. The performance of a DOA estimation algorithm is evaluated using root mean square error (RMSE) through computer simulations. A GA searches for the combination of parameters and operators that gives the lowest RMSE. Because a GA can treat only bit strings, Polish notation is used to convert bit strings into a DOA estimation algorithm. A computer simulation showed that the proposed method can create a new angle spectrum function. The created angle spectrum function has higher resolution than the Capon method.
key words: *array antenna, DOA estimation, genetic algorithm, genetic programming, Polish notation, combinatorial optimization*

1. Introduction

Direction-of-arrival (DOA) estimation technology using an array antenna has many applications such as location of wireless terminals, vehicle radar, and so on. Various DOA estimation algorithms have been proposed [1]. Each algorithm has advantages and disadvantages while individual applications have specific requirements such as low signal-to-noise ratio (SNR), small number of snapshots, high correlation among incoming waves, and low calculation cost. Therefore, it is necessary to properly use algorithms according to each requirement. To meet individual requirements, trial-and-error investigations by using computer simulation have been conducted.

On the other hand, significant developments in computer capability have enabled the development of repetition type optimization algorithms. A genetic algorithm (GA) is a powerful meta-heuristic repetition-type optimization algorithm, especially for combinatorial optimization problems [2]. When fitness is calculated from bit strings (which is called a chromosome in a GA), the GA finds the chromosome that minimize fitness. By adopting chromosomes to individual applications, a GA is applicable for any kind of combinatorial optimization problem. For example, an algorithm is optimized using a GA through tree-structure expression with Polish notation, which is known as genetic programming (GP) [3].

In antenna engineering, the design of an antenna struc-

ture using a GA has been proposed [4]–[6]. A GA was used to create an equivalent circuit [7]. A DOA estimation algorithm using a GA was also proposed [8], [9]. However, using a GA to create an optimum DOA estimation algorithm has not been proposed. We propose a method for creating optimum DOA estimation algorithms by using a GA. In Sect. 2, proposed procedure is described. In Sect. 3, proposed procedure is numerically evaluated. We conclude the paper in Sect. 4.

2. Proposed Method

2.1 Overview of Proposed Method

The proposed method consists of three steps: 1) A GA optimizes bit strings with respect to their fitness, 2) bit strings are transformed into angle spectrum functions by using Polish notation, and 3) DOA estimation simulation is conducted by using the angle spectrum functions to evaluate their performances as fitness. This procedure is shown in Fig. 1. By repeating this procedure, angle spectrum functions with good root mean square error (RMSE) can survive and a better angle spectrum function is created.

2.2 Transformation from Chromosomes into Angle Spectrum Function

A GA optimizes chromosomes, which consist of several genes. In the proposed method, genes correspond to parameters and operators, and a chromosome corresponds to an angle spectrum function.

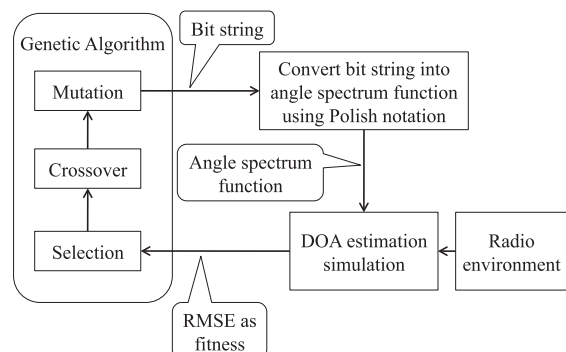Table 1 lists the assignment of genes to parameters and

Fig. 1   Summary of proposed method.

Table 1 Assignment of genes to parameters and operators.

Gene	Parameter or Operator
0	$f(\mathbf{A}) = \mathbf{A}^{-1}$:Inverse
1	$f(\mathbf{A}, \mathbf{B}) = \mathbf{A}^H \mathbf{B} \mathbf{A}$:Quadratic
2	$\mathbf{R}_{xx}$:Correlation Matrix
3	$\mathbf{a}(\theta)$:Mode Vector
4	$f(\mathbf{A}) = \mathbf{A}^H \mathbf{A}$
5	$f(\mathbf{A}, B) = \mathbf{A}/B$:Quotient (B is scalar)
6	1 :Unity
7	Blank

Fig. 2 Translation of chromosome to angle spectrum function through tree expression for Capon method.
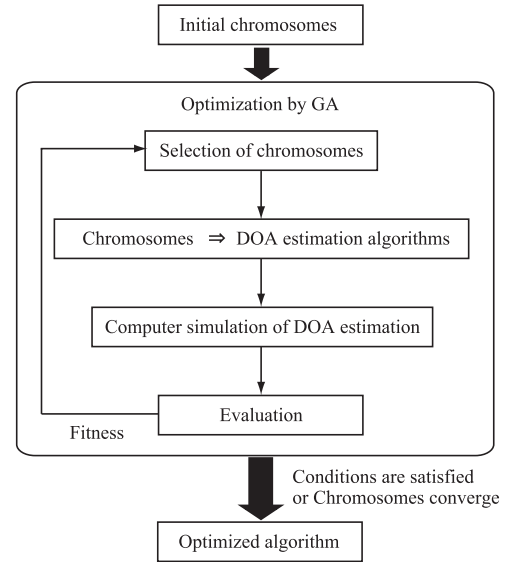
operators. A gene consist of 3 bits; therefore one gene represents $2^3 = 8$ kinds of parameters and operators. Genes are described in decimal expression in this table. Genes 2, 3, and 6 are parameters and 0, 1, 4 and 5 are operators. Gene 7 is blank.

Polish notation is an expression of a numerical formula for putting operators before operands (parameters). By using Polish notation, any numerical formula is expressed without using parentheses. In the proposed method, the Polish notation is used to form an angle spectrum function from a bit string. For example, $\mathbf{R}_{xx}^{-1}$ is transformed into '02'. Chromosomes '702', '072' and '027' also equal '02'.

By using the assignment of genes listed in Table 1 and Polish notation, the angle spectrum function of the Capon method P_{CP}:

$$P_{CP}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}_{xx}^{-1}\mathbf{a}(\theta)} \tag{1}$$

is converted into '01302' through the tree expression shown in Fig. 2. Chromosome '561302' also becomes the angle spectrum function of P_{CP}. In the same manner, the angle spectrum function of the beamformer method P_{BF}:

Fig. 3 Flowchart of proposed method.

$$P_{BF}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{R}_{xx}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)} \tag{2}$$

is transformed into '513243'.

2.3 Generating Optimum DOA Estimation Algorithm by Using GA

Figure 3 shows the detailed procedure of the proposed method. First, the GA generates first generation chromosomes, which are randomly selected. The number of chromosomes in one generation is called a "population". Next, chromosomes of the first generation are translated into angle spectrum functions, as described in the previous subsection. Then, DOA estimation simulation is conducted by using each angle spectrum function. The RMSE of each angle spectrum function is used as the fitness value of the chromosome in the GA. The next generation chromosomes are selected from the chromosome of the parent generation whose fitness is better. Through crossover and mutation, next generation chromosomes are then generated. Optimization using the GA is repeated until fitness conditions are satisfied or chromosomes converge. Finally, an optimum chromosome is obtained.

The mechanism of how the proposed method can create a new angle spectrum function is shown in Fig. 4. In this example, chromosomes of the parent generation are '513243' and '01302', which correspond to the angle spectrum functions of P_{BF} and P_{CP}. At a crossover stage, the GA randomly selects the crossover point. In this example, this is selected before the second gene. By the crossover, a part of the chromosome is exchanged. This operation means the exchange of the sub-tree in the tree expression. Since this operation is executed symbolically without considering mathematics, the resultant chromosome by crossover may not be calculable due to inconsistency in the number of operands or the size of the matrix. For example, chromosome '513202'
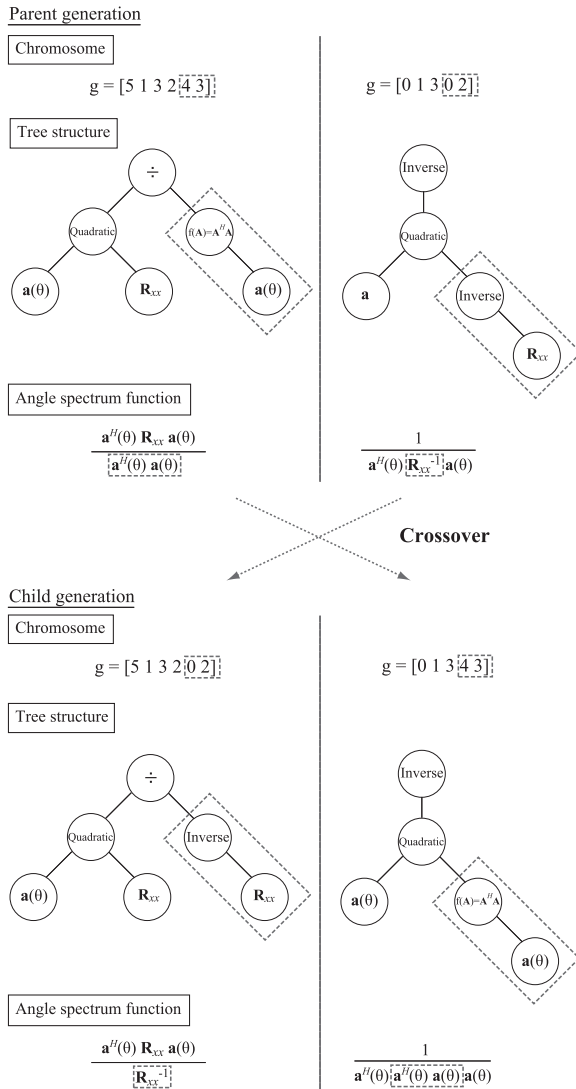
Parent generation

Chromosome

g = [5 1 3 2 4 3]

g = [0 1 3 0 2]

Tree structure

Angle spectrum function

$$\frac{\mathbf{a}^H(\theta)\, \mathbf{R}_{xx}\, \mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\, \mathbf{a}(\theta)}$$

$$\frac{1}{\mathbf{a}^H(\theta)\, \mathbf{R}_{xx}^{-1}\, \mathbf{a}(\theta)}$$

Crossover

Child generation

Chromosome

g = [5 1 3 2 0 2]

g = [0 1 3 4 3]

Tree structure

Angle spectrum function

$$\frac{\mathbf{a}^H(\theta)\, \mathbf{R}_{xx}\, \mathbf{a}(\theta)}{\mathbf{R}_{xx}^{-1}}$$

$$\frac{1}{\mathbf{a}^H(\theta)\, \mathbf{a}^H(\theta)\, \mathbf{a}(\theta)\, \mathbf{a}(\theta)}$$

Fig. 4 Creation of next generation angle spectrum functions by crossover.

Table 2 Parameters of GA.

Length of gene	3 bits
Length of chromosome	6 genes (18 bits)
Population size	50
Crossover rate	0.6
Mutation rate	0.01
Total number of trials	10000

Table 3 Simulation conditions.

Array configuration	Uniform linear array
Antenna element	isotropic
Element spacing	Half wavelength
Number of elements	8
Number of waves	2(uncorrelated, equal power)
Number of snapshots	100
SNR	20 dB

Table 4 Radio environment 1.

DOA of 1st wave	$\theta_1 = -80$ deg. to 65 deg.
DOA of 2nd wave	$\theta_2 = \theta_1 + 15$ deg.

Fig. 5 Convergence of genes.

in Fig. 4 is not calculable because a matrix comes in denominator. In such a case, fitness of 999 degrees is returned to the GA without conducting DOA estimation simulation.

DOA estimation algorithms are generally classified as spectral-based methods or parametric methods [1]. The range of the proposed method is limited to the spectral-based method. Spectral-based methods are classified as beamforming type or subspace-based types. In the assignment of genes listed in Table 1, the proposed method is limited to the beamforming type. However, by introducing genes that represents a noise subspace matrix, the proposed method can be extended to the subspace-based type.
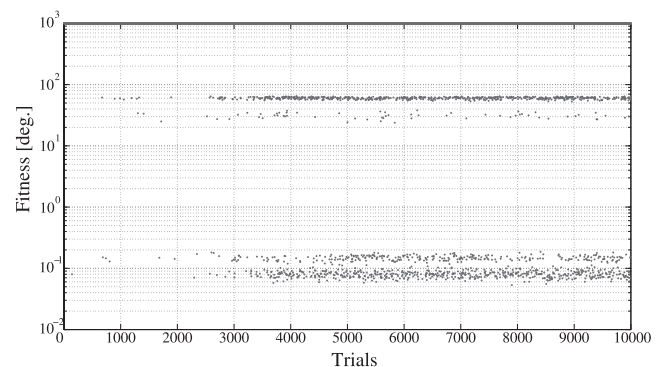
3. Demonstration

3.1 Creation of New DOA Estimation Algorithm by Using Proposed Method

To validate the effectiveness of the proposed method, we conducted a computer demonstration. Parameters of the GA are listed in Table 2. The array antenna and radio environment conditions are listed in Tables 3 and 4, respectively. In this calculation, the number of incoming waves was assumed to be 2. The average RMSE of each incoming waves was used as the fitness value.
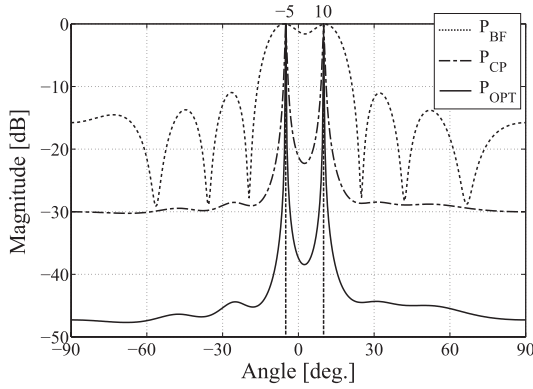
Convergence of the fitness value is shown in Fig. 5. The fitness values show the average RMSE of the corresponding angle spectrum function. A fitness of 999 degrees means that the corresponding chromosomes cannot be converted to angle spectrum functions due to the inconsistency of operators and operands at the Polish notation stage. A fitness value around 90 degrees means that the corresponding chromosomes are successfully converted to the angle spectrum function, but they do not act as a DOA estimation algorithm.

Table 5 Chromosomes searched using GA (Top 5 fitness values).

Chromosomes	Angle spectrum function	fitness [deg.]
013042	P_{OPT}	8.0281×10^{-2}
013702		1.4215×10^{-1}
013072	P_{CP}	1.4528×10^{-1}
013027		1.5082×10^{-1}
113732	constant times of P_{BF}	2.4290×10

Table 6 Radio environment 2.

DOA of 1st wave	$\theta_1 = -5$ deg.
DOA of 2nd wave	$\theta_2 = 10$ deg.

Fig. 6 Angle spectrum functions for -5- and 10-deg. incident waves.

Fig. 7 RMSE versus 1st DOA.

Fig. 8 Resolution of DOA estimation.

A fitness value under 1 degree means that the corresponding chromosomes are converted to angle spectrum functions and successfully act as a DOA estimation algorithm. After 5000 trials, chromosomes were converged.

The chromosomes that gave the top 5 fitness values are listed in Table 5. The angle spectrum function P_{OPT} that corresponds to the chromosome with the best fitness value is
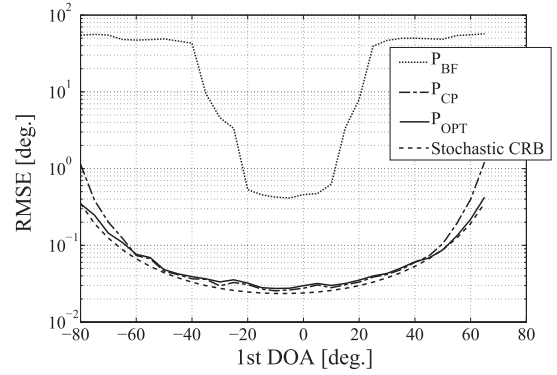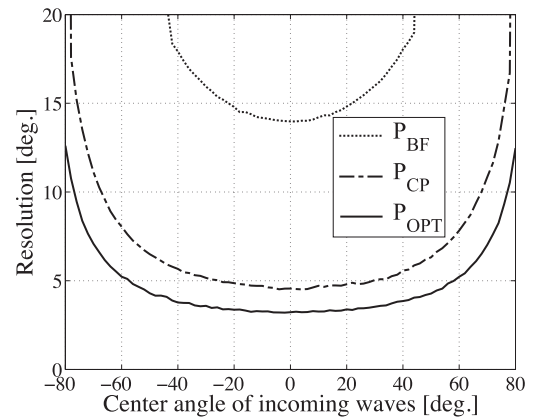
$$P_{OPT}(\theta) = \frac{1}{\mathbf{a}^H(\theta)(\mathbf{R}_{xx}^H \mathbf{R}_{xx})^{-1} \mathbf{a}(\theta)}. \tag{3}$$

The second-best algorithm was the Capon method. From this result, the proposed method can create a new angle spectrum function.

3.2 Validation of Created DOA Estimation Algorithm

At first, we evaluated the angle spectrum function under the same conditions listed in Table 3. In this calculation, the radio environment conditions are listed in Table 6. The angle spectrum functions are shown in Fig. 6. We found that the P_{OPT} has the sharpest angle spectrum.

Next, we examined the performance of the P_{OPT} under the simulation and radio environment conditions listed in Tables 3 and 4. The performance was evaluated using RMSE calculated through 100 independent trials. The results are shown in Fig. 7, which also shows the Cramer-Rao bound (CRB) [10]. We found that the P_{OPT} has the best RMSE for the endfire direction. This is because the angle

spectrum becomes broader for the endfire direction, thus the sharp spectrum of the P_{OPT} is effective. On the other hands, RMSE of the P_{OPT} for the broadside direction is as same as the Capon method. This is because the peak direction of the angle spectrum is unchanged although the angle spectrum of the P_{OPT} becomes sharper than the Capon method.

For several incoming waves, the resolution of DOA estimation is also an important index. Figure 8 shows the resolution, which is defined by the angle separation at which the peak of the spectrum function is split into two peaks when the separation of two incoming waves increased from 0 degrees. We found that P_{OPT} has the best resolution for all directions. This is because the P_{OPT} has the sharpest angle spectrum.

4. Conclusion

We proposed a method for automating the trial-and-error creation of DOA estimation algorithms by using a GA. Through computer simulation, we found that the proposed method can create a new angle spectrum function. The created angle spectrum function has better resolution than the Capon method.

In antenna design, simulator- and optimizer-based automated design is now commonly used. It is expected that the proposed method will become a beneficial tool in DOA

estimation algorithm design.

For future work, we will introduce a gene that represents a noise subspace matrix to extend the proposed method to a subspaced-based DOA estimation algorithm.

Acknowledgment

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