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Effective Design of Transmit Weights for Nonregenerative Multiuser MIMO Relay Downlink System

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SUMMARY In this paper, we focus on the cancellation of interference among Destination Users (DU's) and the improvement of achievable sum rate of the nonregenerative multiuser Multiple-Input Multiple-Output (MIMO) relay downlink system. A novel design method of transmit weight is proposed to successively eliminate the interference among DU's, each of which is equipped with multiple receive antennas. We firstly investigate the transmit weight design for the Amplify-and-Forward (AF) relay scheme where the Relay Station (RS) just retransmits the received signals from Base Station (BS), then extend it to the joint design scheme of transmit weights at the both BS and RS. In the proposed joint design scheme, through the comparison of lower bound of achievable rate, an effective DU selection algorithm is proposed to generate the transmit weight at the RS and obtain the multiuser diversity. Dirty Paper Coding (DPC) technique is employed to remove the interference among DU's and ensures the achievable rate of downlink. Theoretical derivation and simulation results demonstrate the effectiveness of the proposed scheme in obtaining the achievable rate performance and BER characteristics.
key words: *multiuser MIMO, dirty paper coding, relay communication, achievable rate*

1. Introduction

Wireless relay communication has attracted considerable interests due to its improvement of the coverage and the enhancement of the spectral efficiency [1]–[3]. In general, wireless relay techniques are categorized into regenerative and nonregenerative schemes. A regenerative relay requires the decoding of information bits from BS with re-encoding, i.e., Decode and Forward (DF), which results in high cost and latency. A nonregenerative relay like AF just performs the linear processing for the received signals from BS, which does not need any digital signal processing at RS, but only forwards the received signals to the DU's.

Relaying techniques over Single-Input Single-Output (SISO) channels have been well studied in [4], [5] and extended to the case of MIMO relay channels in [6]–[9]. Multiuser MIMO downlink relay system, where multiple DU's communicate with the BS through the RS using the same frequency, time and space, has the advantage of high data rate and large system coverage with supporting multiple DU's [10]–[16]. Either the SISO or MIMO single user relay scheme is not adequate for the multiuser MIMO relay downlink. In single user MIMO relay schemes, the transmit weight is designed just to remove the spatial multiplex interference [6], [7], however, in multiuser MIMO relay schemes, it is designed not only to cancel the spatial multiplex interference but also to exclude the interference among DU's to maintain the high capacity between BS and RS. On the other hand, for the link between RS and DU's, preventing the loss of system rate caused by the interferences among DU's is crucial [10]–[13]. In [10] and [11], the transmit weights are designed to decompose the multiuser MIMO relay system into SISO sub-channels, where the DPC technique based on QR Decomposition (QRD) is employed to remove the interference among DU's and at each sub-channel Water Filling (WF) scheme is used to distribute the transmit power on effective channels. However, these schemes are limited to the case of single receive antenna for each DU. A Generalized WF theorem is proposed in [12] to optimize the system rate by using the theory of convex optimization problem. The case in which each DU is equipped with multiple receive antennas is introduced in [12], but each receive antenna at DU independently detects the received signal, since the DPC technique is employed over the strictly lower triangular matrix obtained by the QRD. In addition in [12] the multiuser diversity is not considered. Actually, for the cellular system, the effective DU's should be optimally selected and the communication resources such as time, frequency, etc. are to be assigned ideally. In this situation, the schemes in [10], [11] and [12] consume the great computational load, since all possible DU combinations need to be calculated. Therefore, it is very difficult for those schemes in previous works to get the multiuser diversity.

In this paper, we propose a novel transmit weight design for the nonregenerative multiuser MIMO relay downlink system, where each DU is equipped with multiple antennas. We carefully design the transmit weights for the AF relay and non-AF relay cases, where the DPC technique is employed at the BS for ensuring the achievable rate under the fixed transmit power constraint. Moreover, the user selection scheduling with low complexity is proposed to obtain the multiuser diversity, where the lower bound of achievable rate of each DU is estimated in order to determine the effective DU's.

The contents of this paper are organized as follows. Section 2 presents the proposed system model of multiuser MIMO relay downlink system. In Sect. 3, we introduce the

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transmit weight design for the case of AF relay, and then extend it to the non-AF relay case, where some additional intelligences such as the signal processing at RS and the user selective algorithm are proposed. We give the simulation results in Sect. 4, where the proposed scheme shows the excellent performance compared with the conventional ones. The conclusions are given in Sect. 5. Some of the notations in this paper are illustrated as follows; vectors and matrixes are expressed by bold letters, we use $tr[\cdot]$, $[\cdot]^T$ and $[\cdot]^H$ as the trace, transpose and conjugate transpose of matrix, respectively.

2. System Model

Figure 1 illustrates the multiuser MIMO relay downlink model in this paper, where the BS is equipped with N_s transmit antennas and communicates with the RS with N_r antennas.

A MIMO channel between the BS and RS is denoted as $\mathbf{H}_1 \in \mathbb{C}^{N_r \times N_s}$. The BS transmits the signals with the transmit weight $\mathbf{M} \in \mathbb{C}^{N_s \times N_s}$ to the RS. The RS reprocesses the received signals with the transmit weight matrix $\mathbf{F} \in \mathbb{C}^{N_r \times N_r}$ and then broadcasts to N_d DU's, where the k-th DU is equipped with n_d^k $(k = 1, 2, \cdots N_d)$ receive antennas.

The broadcast channel between the RS and DU's is denoted as

$$\mathbf{H}_2 = [\mathbf{H}_{2,1}^T \cdots \mathbf{H}_{2,k}^T \cdots \mathbf{H}_{2,N_d}^T]^T \in \mathbb{C}^{N_{\Sigma_d} \times N_r} \tag{1}$$

where $N_{\Sigma_d} = \sum_{k=1}^{N_d} n_d^k = N_r = N_s$, and $\mathbf{H}_{2,k} \in \mathbb{C}^{n_d^k \times N_r}$ corresponds to the channel of the k-th DU. In this paper, we assume that the BS generates the transmit signals using the Channel State Information (CSI) of $\mathbf{H}_1$ and $\mathbf{H}_2$ obtained through the feedback from RS and $\mathbf{H}_2$ is also available at RS. In addition, we also assume the half-duplex time division transmissions using the 1st and 2nd time slots, those are assigned to BS to RS and RS to DU's respectively. Accordingly, the direct links between BS and DU's are not utilized [11]–[13].

In the first time slot, the transmit signal vectors are processed by the transmit weight $\mathbf{M}$ at the BS

$$\mathbf{M} = [\mathbf{M}_1 \cdots \mathbf{M}_k \cdots \mathbf{M}_{N_d}] \tag{2}$$

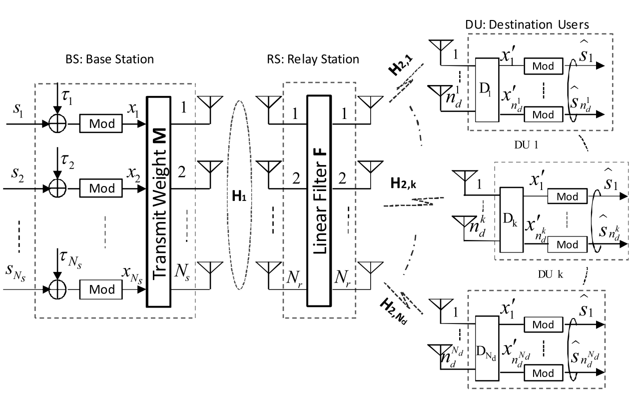

Fig. 1 Multiuser MIMO relay downlink model.

where $\mathbf{M}_k \in \mathbb{C}^{N_s \times n_s^k}$ is the transmit weight for the k-th DU and n_s^k denotes the number of transmit streams for the k-th DU.

The $N_r \times 1$ received signal vector at RS is expressed as

$$\mathbf{Y}_R = \mathbf{H}_1 \mathbf{M} \mathbf{X} + \mathbf{W}_1 \tag{3}$$

where $\mathbf{X} = [\mathbf{x}_1^T \cdots \mathbf{x}_k^T \cdots \mathbf{x}_{N_d}^T]^T \in \mathbb{C}^{N_s \times 1}$ with $\mathbf{x}_k \in \mathbb{C}^{n_s^k \times 1}$ is the transmit signal vector from BS and $\mathbf{W}_1 \in \mathbb{C}^{N_r \times 1}$ is the Additive White Gaussian Noise (AWGN) with zero mean and variance $\sigma_1^2 \mathbf{I}$. The transmit power constraint at BS can be denoted as

$$tr(\mathbf{M}\mathbf{M}^H) = P_1 \tag{4}$$

where P_1 is the total transmit power from BS.

In the second time slot, the RS reprocesses the received signal vector $\mathbf{Y}_R$ with the transmit weight $\mathbf{F}$ and then forwards the data streams to DU's through the multiuser MIMO broadcast channels. Thus $n_d^k \times 1$ receive signals at k-th DU can be written as

$$\mathbf{y}_k = \mathbf{H}_{2,k}\mathbf{F}\mathbf{H}_1\mathbf{M}\mathbf{X} + \mathbf{H}_{2,k}\mathbf{F}\mathbf{W}_1 + \mathbf{w}_{2,k} \tag{5}$$

The noise term $\mathbf{w}_{2,k} \in \mathbf{W}_2 \in \mathbb{C}^{N_{\Sigma_d} \times 1}$ also follows Gaussian distribution with zero mean and variance $\sigma_2^2 \mathbf{I}$, and the transmit power at RS is constrained by

$$tr(\mathbf{F}\mathbf{H}_1\mathbf{M}\mathbf{M}^H\mathbf{H}_1^H\mathbf{F}^H) + tr(\mathbf{F}\mathbf{F}^H)\sigma_1^2 = P_2 \tag{6}$$

where P_2 is the total transmit power from RS and we assume that

$$\begin{aligned} &E(\mathbf{X}\mathbf{X}^H) = \mathbf{I}; E(\mathbf{X}\mathbf{W}_1^H) = E(\mathbf{W}_1\mathbf{X}^H) = \mathbf{0} \\ &E(\mathbf{W}_1\mathbf{W}_1^H) = \sigma_1^2\mathbf{I}; E(\mathbf{W}_2\mathbf{W}_2^H) = \sigma_2^2\mathbf{I} \end{aligned} \tag{7}$$

3. Design of Transmit Weights for Multiuser MIMO Relay Downlink System

We propose the transmit weight designs for the AF relay case in Sect. 3.1 and non-AF relay in Sect. 3.2 respectively under the assumption of given DU's. The effective user selection algorithm is discussed in Sect. 3.3.

3.1 Transmit Weights Design for AF Relay System

Firstly, we consider the case of AF relay system, that is, the received signals at RS is simply amplified and retransmitted to DU's. We define the whole channel matrix of relay system in Fig. 1 as $\mathbf{H}_G \in \mathbb{C}^{N_{\Sigma_d} \times N_s}$ which is also expressed as

$$\mathbf{H}_G = \mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{M} = \widehat{\mathbf{H}}\mathbf{M} \tag{8}$$

where $\widehat{\mathbf{H}} \in \mathbb{C}^{N_{\Sigma_d} \times N_s}$ is further denoted as

$$\widehat{\mathbf{H}} = [\widehat{\mathbf{H}}_1^T \cdots \widehat{\mathbf{H}}_k^T \cdots \widehat{\mathbf{H}}_{N_d}^T]^T \tag{9}$$

and $\widehat{\mathbf{H}}_k$ corresponds to the k-th DU.

In the prior works [10]–[12], the DPC technique based on QRD is used to cancel the interference among DU's. However, those strategies are equal to the cases with single receive antenna in each DU, and consume the great calculation load when they come to the selection of effective DU's which will be discussed in Sect. 3.3.

To ensure each DU being able to accommodate multiple receive antennas, we give the proposed solutions as follows. For the cancellation of interference among DU's, we design the transmit weight $\mathbf{M}$ at BS and $\mathbf{F}$ at RS as

$$\begin{aligned} \mathbf{M} &= \widehat{\mathbf{M}}\mathbf{\Gamma}_1^{1/2}, \quad trace(\mathbf{\Gamma}_1) = P_1 \\ \mathbf{F} &= \mathbf{I}_{N_r \times N_r}\mathbf{\Gamma}_2^{1/2}, \quad trace(\mathbf{\Gamma}_2) = P_2 \end{aligned} \tag{10}$$

where $\mathbf{\Gamma}_1 = diag(\mathbf{p}_1^1, \cdots \mathbf{p}_1^k, \cdots, \mathbf{p}_1^{N_d})$, $p_{1,i}^k \in \mathbf{p}_1^k \in \mathbb{R}^{1\times n_d^k}(i = 1, 2, \cdots, n_d^k)$ and $\mathbf{\Gamma}_2 = diag(\mathbf{p}_2^1, \cdots \mathbf{p}_2^k, \cdots, \mathbf{p}_2^{N_d})$, $p_{2,i}^k \in \mathbf{p}_2^k \in \mathbb{R}^{1\times n_d^k}$ $(i = 1, 2, \cdots, n_d^k)$ are the power allocation matrix at BS and the amplification matrix at RS, respectively. $p_{1,i}^k$ and $p_{2,i}^k$ are the transmit powers for the i-th stream of k-th DU. The matrix $\widehat{\mathbf{M}}$ can be written as $\widehat{\mathbf{M}} = [\widehat{\mathbf{M}}_1 \cdots \widehat{\mathbf{M}}_k \cdots \widehat{\mathbf{M}}_{N_d}] \in \mathbb{C}^{N_s\times N_s}$.

Next, for the k-th DU, let us define the aggregate channel matrix $\widetilde{\mathbf{H}}_k$ of previous k-1 DUs' and its corresponding singular value decomposition (SVD) as

$$\widetilde{\mathbf{H}}_k = \begin{bmatrix} \widehat{\mathbf{H}}_1 \\ \vdots \\ \widehat{\mathbf{H}}_{k-1} \end{bmatrix} = \mathbf{U}_k\mathbf{\Lambda}_k[\mathbf{V}_k^1\mathbf{V}_k^0]^H \tag{11}$$

where $\mathbf{V}_k^0$ represents the $N_s - \sum_{j=1}^{k-1} n_d^j$ right singular vectors and forms an orthogonal basis for the null space of $\widetilde{\mathbf{H}}_k$, thus its columns are the candidate weights of $\widehat{\mathbf{M}}_k$ $(k = 2, \cdots, N_d)$ corresponding to the k-th DU. For the first DU, $\widehat{\mathbf{M}}_1$ is achieved by the eigenvectors which ensure the maximum transmit gain. In addition, in Eq. (11), we can also use the QRD of $\mathbf{I} - \widetilde{\mathbf{H}}_k^H\widetilde{\mathbf{H}}_k$ to obtain the null space of matrix $\widetilde{\mathbf{H}}_k$ to reduce the computational complexity [17]. Thus, the whole system channel matrix in (8) is expressed as

$$\mathbf{H}_G = \begin{bmatrix} \widehat{\mathbf{H}}_1\mathbf{M}_1 & \widehat{\mathbf{H}}_1\mathbf{M}_2 & \cdots & \widehat{\mathbf{H}}_1\mathbf{M}_{N_d} \\ \mathbf{0} & \widehat{\mathbf{H}}_2\mathbf{M}_2 & \cdots & \widehat{\mathbf{H}}_2\mathbf{M}_{N_d} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \widehat{\mathbf{H}}_{N_d}\mathbf{M}_{N_d} \end{bmatrix} \tag{12}$$

It is noticed that the DPC technique cannot be directly employed on the block triangular matrices of (12), because the elements (interferences) below the diagonal of $\widehat{\mathbf{H}}_k\mathbf{M}_k$ cannot be removed. So we carefully design the decoder matrix $\mathbf{D}$ at DU's side as

$$\mathbf{D} = diag(\mathbf{G}_1\mathbf{h'}_{11}^H, \cdots, \mathbf{G}_k\mathbf{h'}_{kk}^H, \cdots, \mathbf{G}_{N_d}\mathbf{h'}_{N_dN_d}^H) \tag{13}$$

where $\mathbf{h'}_{pq} = \widehat{\mathbf{H}}_p\mathbf{M}_q$ $(p,\ q = 1, 2 \cdots N_d)$ and $\mathbf{G}_k = diag^{-1}(\mathbf{B}_k)\mathbf{B}_k^{H^{-1}}$, and the triangular matrix $\mathbf{B}_k$ is obtained by the Cholesky decomposition $\mathbf{B}_k^H\mathbf{B}_k = chol(\mathbf{h'}_{kk}{}^H{}_{kk}\mathbf{h'}_{kk})$. Consequ-ently, the effective system channel matrix $\mathbf{H}_E \in \mathbb{C}^{N_{\Sigma d}\times N_s}$ can be expressed as follows

$$\begin{aligned} &\mathbf{H}_E = \mathbf{D}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{M} = \mathbf{D}\mathbf{H}_G = \mathbf{D'}\mathbf{B'} \\ &\mathbf{D'} = diag[\mathbf{G}_1\mathbf{B}_1^H \cdots \mathbf{G}_k\mathbf{B}_k^H \cdots \mathbf{G}_{N_d}\mathbf{B}_{N_d}^H] \\ &\mathbf{B'} = \begin{bmatrix} \mathbf{B}_1 & \left(\mathbf{B}_1^H\right)^{-1}\mathbf{h'}_{11}^H\mathbf{h'}_{12} & \cdots & \left(\mathbf{B}_1^H\right)^{-1}\mathbf{h'}_{11}^H\mathbf{h'}_{1N_d} \\ \mathbf{0} & \mathbf{B}_2 & \cdots & \left(\mathbf{B}_2^H\right)^{-1}\mathbf{h'}_{22}^H\mathbf{h'}_{2N_d} \\ \mathbf{0} & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{N_d} \end{bmatrix} \mathbf{\Gamma}_1^{1/2}\mathbf{\Gamma}_2^{1/2} \end{aligned} \tag{14}$$

where Cholesky decomposition of $\mathbf{h'}_{kk}^H\mathbf{h'}_{kk}$ generates the triangular matrix $\mathbf{B}_k$, which is the necessary condition for implementing the DPC in the multiuser MIMO scenario. Therefore, the DPC technique can be used to subtract the interferences in advance from DU's. The transmit signal at BS is generated based on $\mathbf{B'}$ and denoted as

$$x_i = \mathrm{mod}\,[s_i + \tau_i]\,, \tau_i = -\frac{1}{b'_{i,i}}\sum_{i<j} b'_{i,j}x_j; x_i \in \mathbf{X}, \tag{15}$$

where $b'_{i,j}$ is the entry of $\mathbf{B'}$, and we use the same modulo operation as in [18].

Consequently, The effective SINR for the i-th data stream at k-th DU is expressed as

$$SINR_{AF}^k = \frac{p_{1,i}^k p_{2,i}^k \omega_{1,i}^k}{\left\|\mathbf{H}_{2,k}\mathbf{F}\right\|^2 \sigma_1^2/n_d^k + \sigma_2^2} \tag{16}$$

where $\omega_{1,i}^k = \left|b_{i,i}^k\right|^2$, $i = 1, \cdots, n_d^k$ and $b_{i,i}^k$ is the diagonal entry of $\mathbf{B}_k$in (14).

The achievable rate of the system can be expressed as

$$\begin{aligned} R_{AF} &= \frac{1}{2}\max\left[\sum_{k=1}^{N_d}\sum_{i=1}^{n_d^k}\log_2(1 + SINR_{AF}^k)\right] \\ &= \frac{1}{2}\max_{\substack{p_{1,i}^k, p_{2,i}^k \\ k=1,\cdots,N_d}}\left[\sum_{k=1}^{N_d}\sum_{i=1}^{n_d^k}\log_2\left(1 + \frac{p_{1,i}^k p_{2,i}^k \omega_{1,i}^k}{\|\mathbf{H}_{2,k}\mathbf{F}\|^2\sigma_1^2/n_d^k + \sigma_2^2}\right)\right] \end{aligned}$$

subjected to

$$\begin{aligned} &tr(\mathbf{F}\mathbf{H}_1\mathbf{M}\mathbf{M}^H\mathbf{H}_1^H\mathbf{F}^H) + tr(\mathbf{F}\mathbf{F}^H)\sigma_1^2 \le P_2, \\ &\sum_{k=1}^{N_d}\sum_{i=1}^{n_d^k} p_{1,i}^k \le P_1, p_{1,i}^k \ge 0, p_{1,i}^k \in \mathbf{p}_1^k \end{aligned} \tag{17}$$

3.2 Joint Weight (JD) Design for Non-AF Relay System

The design method in the AF relay has the low complexity, but the RS cannot obtain the multiuser diversity. To obtain the multiuser diversity without leading to the great computation load, we carefully design the transmit weights at both BS and RS where the receive signal is not only amplified but also transformed in phase through the transmit weight $\mathbf{F}$ at RS.

Firstly, we apply the SVD to the matrix $\mathbf{H}_1$ corresponding to the first link.

$$\mathbf{H}_1 = \mathbf{U}_1\mathbf{\Lambda}_1^{1/2}\mathbf{V}_1^H \tag{18}$$

where $\mathbf{U}_1 \in \mathbb{C}^{N_r\times N_r}$ and $\mathbf{V}_1 \in \mathbb{C}^{N_s\times N_s}$ are unitary matrixes, and $\mathbf{\Lambda}_1 = diag(\lambda_1^1, \cdots, \lambda_1^k, \cdots, \lambda_1^{N_d})$ with $\lambda_{1,i}^k \in \lambda_1^k \in \mathbb{R}^{1\times n_d^k}$ is the diagonal matrix composed of the eigenvalues of $\mathbf{H}_1\mathbf{H}_1^H$. We design the transmit weights $\mathbf{M}$ at BS and $\mathbf{F}$ at RS as follows

$$\begin{aligned} \mathbf{M} &= \mathbf{V}_1\mathbf{\Gamma}_1^{1/2}, \quad trace(\mathbf{\Gamma}_1) = P_1 \\ \mathbf{F} &= \mathbf{F}_2\mathbf{F}_1, \qquad trace(\mathbf{\Gamma}_2) = P_2 \end{aligned} \tag{19}$$

where $\mathbf{F}_1 = \mathbf{U}_1^H\mathbf{\Gamma}_2^{1/2}$ operates on the first link $\mathbf{H}_1$ and $\mathbf{F}_2 = [\mathbf{F}_{2,1}, \cdots \mathbf{F}_{2,k}, \cdots, \mathbf{F}_{2,N_d}] \in \mathbb{C}^{N_s\times N_s}$ does on the second link $\mathbf{H}_2$. $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ have the same expressions as in (10). Similarly, we construct the previous k-1 DUs' aggregate matrix of k-th DU as

$$\widetilde{\mathbf{H}}_{2,k} = \begin{bmatrix} \mathbf{H}_{2,1} \\ \vdots \\ \mathbf{H}_{2,k-1} \end{bmatrix} = \mathbf{U}_{2,k}\mathbf{\Lambda}_{2,k}[\mathbf{V}_{2,k}^1 \mathbf{V}_{2,k}^0]^H \tag{20}$$

The columns of $\mathbf{V}_{2,k}^0$ constitute the transmit weight $\mathbf{F}_{2,k} \in \mathbb{C}^{N_s\times n_s^k}$ of k-th ($k = 2, \cdots, N_d$) DU and $\mathbf{F}_{2,1}$ is composed of the eigenvectors of $\mathbf{H}_{2,1}$. Thus, the system whole channel matrix is expressed as

$$\mathbf{H}_G = \begin{bmatrix} \mathbf{H}_{2,1}\mathbf{F}_{2,1} & \mathbf{H}_{2,1}\mathbf{F}_{2,2} & \cdots & \mathbf{H}_{2,1}\mathbf{F}_{2,N_d} \\ \mathbf{0} & \mathbf{H}_{2,2}\mathbf{F}_{2,2} & \cdots & \mathbf{H}_{2,2}\mathbf{F}_{2,N_d} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_{2,N_d}\mathbf{F}_{2,N_d} \end{bmatrix} \mathbf{\Gamma}_2^{1/2}\mathbf{\Lambda}_1^{1/2}\mathbf{\Gamma}_1^{1/2} \tag{21}$$

Similarly, to make the DPC technique applicable, the decoder matrix $\mathbf{D}$ at DU's is designed as

$$\mathbf{D} = diag[\mathbf{G}_1\mathbf{h'}_{11}^H \cdots \mathbf{G}_k\mathbf{h'}_{kk}^H \cdots \mathbf{G}_{N_d}\mathbf{h'}_{N_dN_d}^H] \tag{22}$$

where $\mathbf{h'}_{pq} = \mathbf{H}_{2,p}\mathbf{F}_{2,q}$, $\mathbf{G}_k = diag^{-1}(\mathbf{B}_k)\mathbf{\Lambda}_1^{-1/2}\mathbf{B}_k^{H^{-1}}$ and $\mathbf{B}_k$ is obtained by the Cholesky decomposition $\mathbf{B}_k^H\mathbf{B}_k = chol(\mathbf{h'}_{kk}^H\mathbf{h'}_{kk})$.

The effective system channel matrix $\mathbf{H}_E$ is denoted as follows

$$\begin{aligned} &\mathbf{H}_E = \mathbf{D}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{M} = \mathbf{D}\mathbf{H}_G = \mathbf{D}'\mathbf{B}' \\ &\mathbf{D}' = diag[\mathbf{G}_1\mathbf{B}_1^H \cdots \mathbf{G}_k\mathbf{B}_k^H \cdots \mathbf{G}_{N_d}\mathbf{B}_{N_d}^H] \\ &\mathbf{B}' = \begin{bmatrix} \mathbf{B}_1\mathbf{\Lambda}_{1,1}^{1/2} & \left(\mathbf{B}_1^H\right)^{-1}\mathbf{h'}_{11}^H\mathbf{h'}_{12} & \cdots & \left(\mathbf{B}_1^H\right)^{-1}\mathbf{h'}_{11}^H\mathbf{h'}_{1N_d} \\ \mathbf{0} & \mathbf{B}_2\mathbf{\Lambda}_{2,2}^{1/2} & \cdots & \left(\mathbf{B}_2^H\right)^{-1}\mathbf{h'}_{22}^H\mathbf{h'}_{2N_d} \\ \mathbf{0} & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{N_d}\mathbf{\Lambda}_{N_d,N_d}^{1/2} \end{bmatrix} \mathbf{\Gamma}_1^{1/2}\mathbf{\Gamma}_2^{1/2} \end{aligned} \tag{23}$$

where $\mathbf{\Lambda}_{1,k}^{1/2} = diag^{1/2}(\lambda_1^k)$. From the triangular matrix $\mathbf{B}'$ in (23), BS generates the transmit signal with the same expression in (15), where the DPC technique is used to subtract the interference in advance among DU's.

Consequently, we give the effective SINR of i-th data stream at k-th DU as

$$SINR_{JD}^k = \frac{p_{1,i}^k p_{2,i}^k \lambda_{1,i}^k \omega_{2,i}^k}{p_{2,i}^k \sigma_1^2 \omega_{2,i}^k + \sigma_2^2} \tag{24}$$

where $\omega_{2,i}^k = \left|b_{i,i}^k\right|^2$, $i = 1, \cdots, n_d^k$ and $b_{i,i}^k$ is the diagonal entry of $\mathbf{B}_k$ in (23).

The achievable rate of system can be expressed as

$$\begin{aligned} R_{JD} &= \frac{1}{2}\max\left[\sum_{k=1}^{N_d}\sum_{i=1}^{n_d^k}\log_2(1 + SINR_{JD}^k)\right] \\ &= \frac{1}{2}\max_{\substack{p_{1,i}^k, p_{2,i}^k \\ k=1,\cdots,N_d}}\left[\sum_{k=1}^{N_d}\sum_{i=1}^{n_d^k}\log_2\left(1 + \frac{p_{1,i}^k p_{2,i}^k \lambda_{1,i}^k \omega_{2,i}^k}{p_{2,i}^k \sigma_1^2 \omega_{2,i}^k + \sigma_2^2}\right)\right] \end{aligned}$$

subjected to

$$\begin{aligned} &tr(\mathbf{F}\mathbf{H}_1\mathbf{H}_1^H\mathbf{F}^H) + tr(\mathbf{F}\mathbf{F}^H)\sigma_1^2 \le P_2 \\ &\sum_{k=1}^{N_d}\sum_{i=1}^{n_d^k} p_{1,i}^k \le P_1, p_{1,i}^k \ge 0, p_{1,i}^k \in \mathbf{p}_1^k \end{aligned} \tag{25}$$

The optimization in (17) and (25) can be transformed into a conventional geometric program [11], [24].

3.3 Effective User Selection and Analysis of Complexity

In the previous works, the DPC technique based QRD is employed to remove the interferences among DU's, where each receive antenna independently detects the signal at each DU. This situation is equivalent to the case of single receive antenna for each DU. However, in this case, the achievement of multiuser diversity consumes the great calculation load, because we have to consider all the possible DU combinations for the effective user selection. Even for each combination, the power distribution with WF scheme and the optimal covariance structure for DPC lead to further computational burden due to the nonconvex optimization problem [19]. However, in this paper, we know the successive cancellation scheme can easily obtain the optimal covariance structures and has no need to consider of power distribution for each case of DU combination.

Though the design method for the AF relay case can easily achieves the optimal transmit weight (covariance structures) because of the successive cancellation scheme, the RS cannot obtain the multiuser diversity due to the inseparability of $\mathbf{H}_G$. The joint weight design in the non-AF relay case obtains the multiuser diversity at RS by generating the transmit weights at BS and RS. For the optimization in (17) and (25), since the operation of exhaustive search in geometric program is highly costly, without loss of generality, in this paper we assume the equal power allocation strategy at BS.

We focus on the effective user selection for the joint weight design in the non-AF relay case. Most of the user selection algorithms are based on the estimation of capacity rate for each DU through the capacity of channel matrix of each DU or the Frobenius norm [20], [21]. However, the estimation based on the capacity leads to the great computational load due to the operation of SVD. The selection based on Frobenius norm cannot accurately estimate the system rate because the achievable rate of each DU is not monotonically increasing with respect to its Frobenius norm $||\mathbf{H}_k||_F = \sqrt{Tr(\mathbf{H}_k^H\mathbf{H}_k)}$. In this section, we propose a suboptimal but effective user selection algorithm.

Let $U = \{1, \cdots, k, \cdots, K\}$ denotes the congregation of all DU's and N_d is the number of effective DU's.

Definition 1. For two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{l \times 1}$ with the descending order components $x_1 \geq x_2 \geq \cdots \geq x_l \geq 0$ and $y_1 \geq y_2 \geq \cdots \geq y_l \geq 0$, we adopt the following notation.

If $\sum_{i=1}^{l} x_i > \sum_{i=1}^{l} y_i$; $i = 1, \cdots, l-1$, we say that the vector $\mathbf{x}$ weakly majorizes the vector $\mathbf{y}$ and denote $\mathbf{x} \succ \mathbf{y}$.

If $\sum_{i=1}^{l} x_i = \sum_{i=1}^{l} y_i$ is added, we say that the vector $\mathbf{x}$ majorizes the vector $\mathbf{y}$ and write $\mathbf{x} \succeq \mathbf{y}$ [22].

Proposition 1. For the k-th DU in (23), the vector $\lambda_2^k \in \mathbb{R}^{n_d^k \times 1}$ composed of eigenvalues of $\mathbf{H}_{2,k}^H \mathbf{H}_{2,k}$ majorizes the vector composed of diagonal entries of $\mathbf{h'}_{kk}^H \mathbf{h'}_{kk}$, i.e.,

$$\lambda_2^k \succ [(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k} \tag{26}$$

where $[(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k} \in \mathbb{R}^{n_d^k \times 1}$ is the vector composed of diagonal entries of $\mathbf{h'}_{kk}^H \mathbf{h'}_{kk}$. The proof of (26) is detailed in Appendix A.

Assuming that the transmit data streams for each DU are independently encoded and decoded, the sum achievable rate of multiuser MIMO downlink relay system is simply the summation of rate for each DU. In the case of fixed relay, since the capacity rate $C_{JD}^k(\lambda_2^k)$ of k-th DU is Schur-convex with respect to λ_2^k under the given transmit power at RS, $C_{JD}^k(\lambda_2^k)$ is monotonically increasing with respect to the majorization order. If $\lambda_2^p \succ \lambda_2^q$ $(p, q = 1, \cdots, N_d)$, then $C_{JD}^p(\lambda_2^p) \geq C_{JD}^q(\lambda_2^q)$. In addition, at high SNR for each DU, the achievable rate $R_{JD}^k(\omega_2^k)$ of k-th DU can reaches the capacity $C_{JD}^k(\lambda_2^k)$, where $\omega_2^k \in \mathbb{R}^{n_d^k \times 1}$ $(\omega_{2,i}^k \in \omega_2^k)$ is the vector composed of diagonal entries of $\mathbf{B}_k$. Consequently, we have the following relationship

$$R_{JD}^k(\omega_2^k) \approx C_{JD}^k(\lambda_2^k) \geq R_{JD}^k([(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k}) \tag{27}$$

Expression (27) is proved in Appendix B. Therefore, we can estimate the achievable rate $R_{JD}^k(\omega_2^k)$ of k-th DU by the lower bound $R_{JD}^k([(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k})$ of $C_{JD}^k(\lambda_2^k)$. Returning to Eq. (21), for the k-th DU, using the transmit weight obtained from the operation of (20), we select the k-th DU by comparing the majorization order of $[(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k}$, $k = 1, \cdots, N_d$. Compared with the conven-tional user selection methods, obviously we can not only avoid the great calculation load due to SVD operation but also obtain more accurate estimate of the rate for each DU. The user selection algorithm is shown in Table 1.

Compared with the conventional user selection methods, obviously we can avoid not only the great calculation load due to SVD operation but also obtain more accurate estimate of the rate for each DU.

We compared the computational complexities of proposed joint design method, conventional DPC method based on QRD [11], proposed design method for AF relay, block diagonalization method in [14], and channel inversion method. Since the cancellation of interferences between DU's and the user selection are implemented at BS, and the computational load at DU's of the above schemes do not vary largely, we only focus on the total computational load at BS for calculating each transmit vector $\mathbf{X} \in \mathbb{C}^{N_s \times 1}$ (Generally, we assume the generation of transmit weights and user selection at RS are implemented at BS [6], [7], [13]). The computational complexity is counted by the number of flops. A flop is defined to be a real floating point operation, e.g., a real addition, multiplication, or division is counted as one flop [26].

The total computational complexities for calculating each transmit vector $\mathbf{X} \in \mathbb{C}^{N_T \times 1}$ are derived as on Table 2.

Table 2 shows the computational complexities of the proposed joint design for non-AF relay $\Phi_{\text{Joint Design}}$, conven-tional DPC scheme [11] $\Phi_{\text{DPC with QRD}}$, design for AF relay case $\Phi_{\text{Design for AF}}$, block diagonolization scheme in [14] $\Phi_{\text{Block Diagonalization}}$, and channel inversion scheme

Table 1 User selection algorithm.

Step 1: Initialization
Let $U = \{1, 2, \cdots, K\}$, $\Theta = \varnothing$;
Define $\mathbf{H}_{2,U} = [\mathbf{H}_{2,1}^T \ \mathbf{H}_{2,2}^T, \cdots \ \mathbf{H}_{2,K}^T]^T$;
Step 2: 1st DU
For $k = 1 : K$
$\quad \mathbf{\Delta}_k = diag(\mathbf{H}_{2,k}^H \mathbf{H}_{2,k})$;
End
$\quad i = \arg \underset{\mathbf{\Delta}_k}{majorize}(\mathbf{\Delta}_k)$, $k \in U$;
$\quad \mathbf{H}_{2,i} = \mathbf{U}_{2,i} \mathbf{\Lambda}_{2,i} [\mathbf{V}_{2,i}^1 \ \ \mathbf{V}_{2,i}^0]^H$, $\mathbf{F}_{i,k} \in \mathbf{V}_{i,k}^1$;
$\quad \Theta = \Theta \bigcup i$;
$\quad U = U \backslash i$;
Step 3: k -th DU
For $k = 2 : N_d$
$\quad \widehat{\mathbf{H}}_{2,k} = \mathbf{H}_{2,\Theta}$, where $\mathbf{H}_{2,\Theta} = [\mathbf{H}_{2,\theta_1}^T, \cdots, \mathbf{H}_{2,\theta_k}^T, \cdots]^T, \theta_k \in \Theta$;
$\quad \widehat{\mathbf{H}}_{2,k} = \mathbf{U}_{2,k} \mathbf{\Lambda}_{2,k} [\mathbf{V}_{2,k}^1 \ \ \mathbf{V}_{2,k}^0]^H$, $\mathbf{F}_k \in \mathbf{V}_{2,k}^0$;
$\quad \mathbf{\Delta}_k = diag(\mathbf{F}_k^H \mathbf{H}_{2,i}^H \mathbf{H}_{2,i} \mathbf{F}_k), i \in U$, $i = \arg \underset{\mathbf{\Delta}_k}{majorize}(\mathbf{\Delta}_k)$, $k \in U$;
$\quad \Theta = \Theta \bigcup i$;
$\quad U = U \backslash i$;
End
$\mathbf{H}_2 = \mathbf{H}_{2,\Theta}$

Table 2 Computation complexity.

$\Phi_{\text{Joint Design}}$	$\sum_{k=2}^{N_d} \left[2N_r \left(\sum_{j=1}^{k-1} n_d^j \right)^2 - 2\left(\sum_{j=1}^{k-1} n_d^j \right)^3 / 3 \right] + \sum_{k=1}^{K} [2n_d^k (n_d^k - 1) + 6 \left(n_d^k \right)^3]$ $+20N_r^3 + 13N_s^2 + 41N_s + 6$
$\Phi_{\text{DPC with QRD}}$	$\left(13\,{}_K C_{N_d} / 3 + 20\right) N_s^3 + \left(10 - {}_K C_{N_d}\right) N_s^2$ $+\left(29\,{}_K C_{N_d} + 12\right) N_s + 6\,{}_K C_{N_d}$
$\Phi_{\text{Design for AF}}$	$\sum_{k=2}^{N_d} \left[2N_r \left(\sum_{j=1}^{k-1} n_d^j \right)^2 - 2\left(\sum_{j=1}^{k-1} n_d^j \right)^3 / 3 \right] + \sum_{k=1}^{K} [2n_d^k (n_d^k - 1) + 6 \left(n_d^k \right)^3]$ $+19N_s^3 + 15N_s^2 + 39N_s + 6$
$\Phi_{\text{Block Diagonalization}}$	${}_K C_{N_d} \left[\sum_{k=1}^{N_d} \left(2N_r (N_{N_{\Sigma_d}} - n_d^k)^2 - 2(N_{N_{\Sigma_d}} - n_d^k)^3 / 3 \right) \right] + 20N_s^3$ $+\left(3\,{}_K C_{N_d} + 10\right) N_s^2 + \left(29\,{}_K C_{N_d} + 2\right) N_s + 6\,{}_K C_{N_d}$
$\Phi_{\text{Channel Inversion}}$	$\left({}_K C_{N_d} + 20\right) N_s^3 + \left(3\,{}_K C_{N_d} + 10\right) N_s^2 + \left(29\,{}_K C_{N_d} + 2\right) N_s + 6\,{}_K C_{N_d}$

$\Phi_{\text{Channel Inversion}}$. For example, when $n_d^k = 1$, $N_d = 4$ and $K = 4$ (No effective user selection), the computational complexities of those schemes are 1773, 1850, 1733, 1874 and 1682 flops, respectively. It is noticed that though the channel inversion scheme consumes the least computa-tional complexity compared with the other four schemes, overall, computational loads of these schemes doo not change largely. However, when the effective user selection is employed, i.e., $n_d^k = 1$, $N_d = 4$ and $K = 6$($_K C_{N_d}$ means N_d effective DU's are selected from K DU's), the computational loads of those schemes are 4604, 12988, 4564, 10476 and 8508 flops, respectively. Obviously, the proposed schemes with successive elimination method can greatly reduce the computational load compared with the other schemes. In addition, in the case of multiple receive antennas at each DU, it is also noticed that the proposed schemes in this paper does not consume large computa-tional load compared with the block diagonolization scheme in [14].

4. Computer Simulation

In this section, computer simulation results are presented to evaluate the performance of proposed transmit schemes. The channel model is assumed to be a flat Rayleigh (i.i.d.) fading and the coefficient γ $(= d_1/d_2)$ is the ratio of the distance d_1 (between the BS and RS) to the distance d_2 (between the RS and the DU's). The total transmit power at BS and RS are assumed to be equal, i.e., $P_1 = P_2 = P$. Firstly, we consider the case of $4 \times 4 \times [1, 1, 1, 1]$ ($N_s = 4$; $N_r = 4$; $n_d^k = 1$) in multiuser MIMO relay downlink system, where the average achievable rate is determined over 1000 realizations of $\mathbf{H}_1$ and $\mathbf{H}_2$. To evaluate the performance of the proposed scheme, we also give the theoretical upper bound of rate $C(\mathbf{H}_1)/2$ which is derived in Appendix C and similar to the one in [10].

In Fig. 2, we compared the achievable sum rate of the system among the proposed joint design method, DPC method based on QRD at RS in [11], proposed design for AF relay case, block diagonalization method in [14], and channel inversion method at RS. Computer simulations show that, at high SNR's, the joint weight design method obtains almost the same sum rate as the DPC based on QRD in [11] under the condition of given DU. Since the joint design obtains the optimal DU order by employing the DU selection algorithm, it obtains the slight advantage to the conventional DPC method based on QRD in [11] with no concern to the optimal covariance structure. This agrees with the conclusion that the achievable rate is dependent on the order of DU at low SNR [19]. As shown in Fig. 2, the joint design for non-AF relay is closer to the upper bound compared with the AF relay. This is because the weight design for AF relay has no multiuser diversity at RS. In addition, by comparing the proposed schemes with the linear weight design methods, i.e., the BD method at RS [14] and the channel inversion method at RS, the simulation results also support the conclusion that the nonlinear method, i.e., DPC technique, outperforms the linear method, such as BD, etc.

Then we consider the case of $4\times4\times[2, 2]$ ($N_s = 4$; $N_r = 4$; $n_d^{(k)} = 2$) multiuser MIMO downlink relay system. Under the condition of effective DU selection, as shown in Fig. 3, we can obtain the better rate performance, i.e., $N_d = 2$ is selected from the users of $K = 4$. Both cases for AF relay and joint design for non-AF relay improve the performance by about 1 dB at the achievable rate of 8 bits/s/Hz. It is also noticed that either the design for AF relay or the joint design for non-AF relay can obtain the better rate preference compared with the linear block diagonalization method.

Next we show the average BER performance of proposed transmit scheme and compare it with the BD method in Fig. 4. Here QPSK is used to modulate the transmit signal, the modulo operation is denoted as $x_k = [\bmod(real(s_k + \tau_k) + M/2, M) - M/2] + j\times[\bmod(imag(s_k + \tau_k) + M/2, M) - M/2]$ where $M = 4$ in the case of QPSK. It is noticed that the similar observations to the achievable rate are also reflected on the average BER. As shown in Fig. 4, with the equal power allocation at BS and RS, the proposed schemes obtain the better BER performance than the conventional linear BD method, since the effective channel gain $b_{i,i}^k$ can be obtained

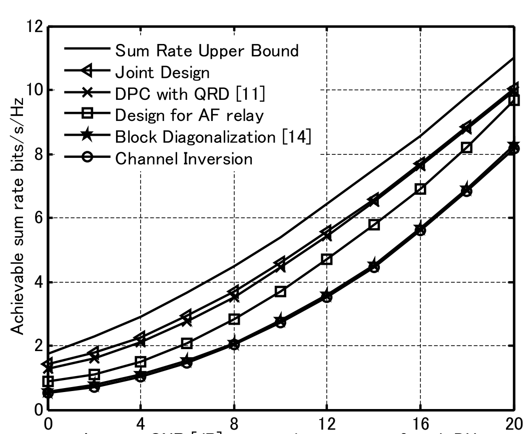

Fig. 2 Achievable sum rate performance with the proposed transmit weight design ($K = 4$; $N_d = 4$; $n_d^k = 1$; $\gamma = 1$).

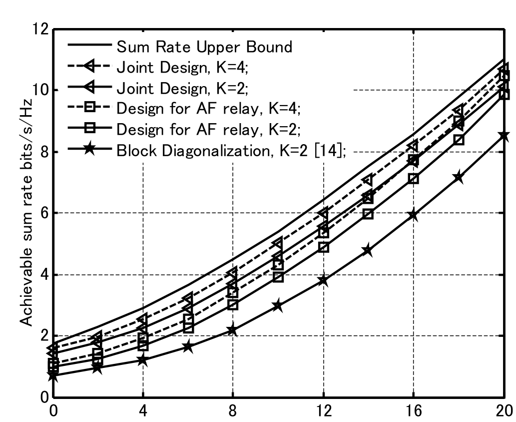

Fig. 3 Achievable sum rate performance with effective DU selection (K = 2,4; $N_d = 2$; $n_d^k = 2$; $\gamma = 1$).

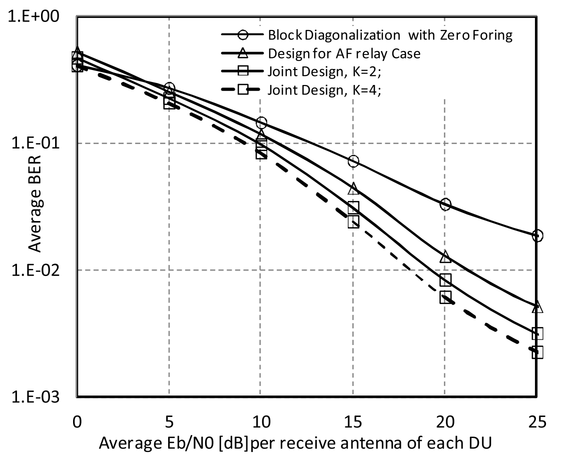

Fig. 4 BER performance with effective DU selection ($K = 2, 4$; $N_d = 2$; $\gamma = 1$).

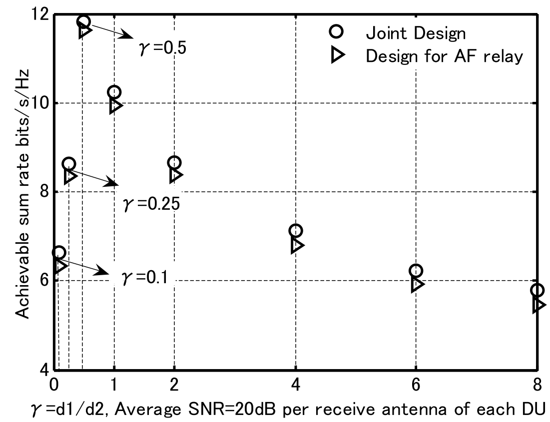

Fig. 6 Achievable sum rate performance with different relay locations for free space propagation at average SNR=20 dB per receive antenna in each DU ($K = 2$; $N_d = 2$).

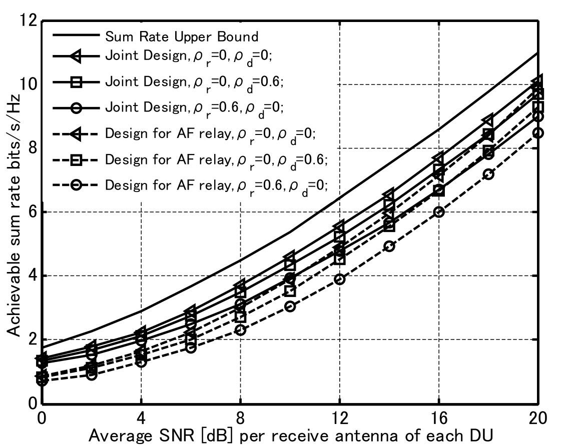

Fig. 5 Achievable sum rate performance in the presence of antenna correlation at RS and DU's ($K = 2$; $N_d = 2$; $\gamma = 1$).

as large as possible during the design of transmit weight. It is also noticed that the joint design for non-AF relay is superior to the design for AF relay by 2.5 dB at BER=1.E-02. This is because of multiuser diversity at RS. In addition, the system rate performance can be even improved when the effective DU's selection algorithm is employed. That is, when $N_d = 2$ is selected from $K = 4$, the BER performance can be improved by about 1 dB at BER=1.E-02.

Next we consider the case of antenna correlation, where ρ_r and ρ_d denote the correlation coefficients between antennas at RS and DU's respectively. Simulation results in Fig. 5 show that the correlations at RS or DU's degrade the rate performance. For the same condition of correlation coefficients, the performance with joint weight design having the diversity at RS gets the advantage over the AF relay. It is also noticed that the correlation at RS has more adverse effects on the system performance than DU's. At the rate of 8 bits/s/Hz, $\rho_r = 0.6$ degrades the performance by about 2 dB compared with the case of no correlation, while the case of correlation at DU degrades the performance by only about 1 dB. This is because the correlation at RS degrades the performance for both the first link $\mathbf{H}_1$ and the second link $\mathbf{H}_2$, while the correlation at DU only degrades the performance for the second link $\mathbf{H}_2$.

We briefly study the effect of location of RS on the whole system rate performance. We assume that the flat Rayleigh fading $\mathbf{H}_1$ and $\mathbf{H}_2$ are subjected to the free space propagation condition where the channel gain is inversely proportional to the squared distance between the transmitter and receiver. Let us define $d_0(d_0 = d_1 + d_2)$ as the distance between BS and the centre of DU's, then $\mathbf{H}_1$ and $\mathbf{H}_2$ can be modelled as $\mathbf{H}_1 = (1+\gamma)^2\mathbf{H}/4\gamma^2$ and $\mathbf{H}_2 = (1+\gamma)^2\mathbf{H}/4$ respectively, where $\mathbf{H} = \mathbf{H}_1|_{\gamma=1} = \mathbf{H}_2|_{\gamma=1}$ is the reference channel matrix of flat Rayleigh (i.i.d.) fading when $\gamma = 1$. In Fig. 6, it is noticed that the system rate in case of $\gamma = 0.5$ is larger than those of $\gamma = 0.25$ and $\gamma = 1$, that is, the achievable rate of relay system firstly increases and then decreases when RS moves from BS to the centre of DU's. The maximum system rate lies between 0.25 and 1. Since the maximum value of system rate depends on many factors including power allocation at BS and RS, and to find the maximum rate is beyond the scope of this paper, we will not study it in detail. In addition, Fig. 6 shows that the substantial system rate loss is experienced as the RS is located far from the BS (i.e., from $\gamma = 1$ to $\gamma = 8$) This is because of the amplification of noise from the first link.

5. Conclusion

In this paper, we studied the transmit weight design for the downlink of multiuser MIMO relay system, where each DU is equipped with multiple receive antennas. A novel transmit weight design is implemented for the cancellation of interferences among DU's. Firstly, we focus on the weight design for the AF relay scheme, but despite of its low computational feature, it cannot obtain the multiuser diversity at RS. Therefore we next proposed the joint weight design method for non-AF relay with phase adjustment function, where the user selection algorithm with low complexity is employed at the RS to obtain the multiuser diversity. From computer simulation results we have verified that the joint weight design method for non-AF relay, where DPC transmit technique is also used to ensure the achievable rate, obtains the better rate performance compared with the weight design for the AF relay and the other conventional methods such as DPC based on QRD, BD and inverse channel methods.

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Appendix A

A.1 Proof of expression in (26)

For the Definition 1, the necessary and sufficient condition for weak majorization is

$$\mathbf{x} \succeq \mathbf{y} \text{ if and only if } \mathbf{y} = \mathbf{\Phi x} \tag{A$\cdot$1}$$

where $\mathbf{\Phi}$ is the stochastic matrix [22, pp.36–38, Thm.2.C.1]. For the k-th DU, the eigenvalue decomposition of $\mathbf{H}_{2,k}^H\mathbf{H}_{2,k}$ is written as $\mathbf{H}_{2,k}^H\mathbf{H}_{2,k} = \mathbf{Q}_{2,k}\mathbf{\Lambda}_{2,k}\mathbf{Q}_{2,k}^H$ where $\mathbf{\Lambda}_{2,k} = diag(\lambda_2^k)$. It is known that $\mathbf{h'}_{kk}^H\mathbf{h'}_{kk}$ has the same eigenvalue as $\mathbf{H}_{2,k}^H\mathbf{H}_{2,k}$, its eigenvalue decomposetion can be expressed as $\mathbf{h'}_{kk}^H\mathbf{h'}_{kk} = \mathbf{Q'}_{2,k}\mathbf{\Lambda}_{2,k}\mathbf{Q'}_{2,k}^H$. So the diagonal element $(\mathbf{h'}_{kk}^H\mathbf{h'}_{kk})_{ii}$ is expressed as

$$(\mathbf{h'}_{kk}^H\mathbf{h'}_{kk})_{ii} = \sum_{j=1}^{n_d^k} q'_{ij}q'^H_{ij}(\mathbf{\Lambda}_{2,k})_{jj} = \sum_{j=1}^{n_d^r} \varphi_{ij}(\mathbf{\Lambda}_{2,k})_{jj} \tag{A$\cdot$2}$$

where $(\mathbf{\Lambda}_{2,k})_{jj}$ is the j-th eigenvalue of $\mathbf{H}_{2,k}^H\mathbf{H}_{2,k}$ or $\mathbf{h'}_{kk}^H\mathbf{h'}_{kk}$, q'_{ij} is the entry of $\mathbf{Q'}_{2,k}$ and $\varphi_{ij} = q'_{ij}q'^H_{ij}$ is the entry of $\mathbf{\Phi}$. Since $\mathbf{Q'}_{2,k}$ is unitary matrix and φ_{ij} is doubly stochastic, we have

$$[(\mathbf{h'}_{kk}^H\mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k} = \mathbf{\Phi}\lambda_2^k \tag{A$\cdot$3}$$

Form (A·1), we obtain $\boldsymbol{\lambda}_2^k \succ [(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk})_{ii}]_{i=1}^{n_d^k}$.

Appendix B: Derivation of Achievable Rate $R_{JD}^k(\omega_2^k)$

For the k-th DU, we have the relationships as follows

$$\mathbf{B}_k = chol(\mathbf{h'}_{kk}^H \mathbf{h'}_{kk}),\ \prod_{i=1}^{n_d^k} \lambda_{2,i}^k = \det(\mathbf{H}_{2,k}^H \mathbf{H}_{2,k}) = \prod_i^{n_d^k} \omega_{2,i}^k \quad \text{(A·4)}$$

where $\lambda_{2,i}^k \in \boldsymbol{\lambda}_2^k$ and the arithmetic means of $1/\lambda_{1,i}^k \omega_{2,i}^k$ and $1/\lambda_{1,i}^k \lambda_{2,i}^k$ are expressed as

$$\Delta_a = \frac{1}{n_d^k} \sum_{i=1}^{n_d^k} \frac{1}{\lambda_{1,i}^k \omega_{2,i}^k},\ \Delta'_a = \frac{1}{n_d^k} \sum_{i=1}^{n_d^k} \frac{1}{\lambda_{1,i}^k \lambda_{2,i}^k} \quad \text{(A·5)}$$

with $\lambda_{1,i}^k \in \boldsymbol{\lambda}_1^k$ and the geometric mean

$$\Delta_g = \left(\prod_{k=1}^{n_d^k} 1/\lambda_{1,i}^k \omega_{2,i}^k \right)^{1/n_d^k} = \left(\prod_{i=1}^{n_d^k} 1/\lambda_{1,i}^k \lambda_{2,i}^k \right)^{1/n_d^k} \quad \text{(A·6)}$$

The achievable rate of k-DU with WF scheme can be denoted as

$$R_{JD}^k(\omega_2^k) \approx \frac{1}{2} \max \left[\sum_{i=1}^{n_d^k} \log_2(\mu p_s \lambda_{1,i}^k \omega_{2,i}^k / \sigma^2) \right] \quad \text{(A·7)}$$

where $p_s = P_1/N_s$ and $\sigma^2 = P_2 \sigma_1^2/[\lambda_{1,i}^k p_s + \sigma_2^2]$ and σ^2 is processed by the same method as in [11].

The factor μ solves

$$\sum_{i=1}^{n_d^k} [\mu - \sigma^2/p_s \lambda_{1,i}^k \omega_{2,i}^k]_+ = p_2^k, \quad \sum_{k=1}^{N_d} p_2^k = P_2 \quad \text{(A·8)}$$

where p_2^k denotes the transmit power allocated for the k-th DU at RS.

Then we find μ to satisfy (A·8) with $p'^k_2 < \infty$. In (A·8) we assume $\mu > \sigma^2/p_s \lambda_{1,i}^k \omega_{2,i}^k$ and it becomes

$$\begin{aligned} &\sum_{i=1}^{n_d^k} \mu - \sum_{i=1}^{n_d^k} \sigma^2/p_s \lambda_{1,i}^k \omega_{2,i}^k = p'^k_2 \\ &\Rightarrow \sum_{i=1}^{n_d^k} \mu = p'^k_2 + \sum_{i=1}^{n_d^k} \sigma^2/p_s \lambda_{1,i}^k \omega_{2,i}^k \end{aligned} \quad \text{(A·9)}$$

By letting $\mu_0 = p'^k_2/n_d^k + \Delta_a$, we obtain

$$\sum_{i=1}^{n_d^k} \mu_0 = \sum_{i=1}^{n_d^k} \left(\frac{p'^k_2}{n_d^k} + \Delta_a \right) = p'^k_2 + \sum_{i=1}^{n_d^k} \Delta_a = p'^k_2 + \sum_{i=1}^{n_d^k} 1/\lambda_{1,i}^k \omega_{2,i}^k \quad \text{(A·10)}$$

The formula (A·10) proves the existence of μ_0. So for all $p'^k_2 < p_2^k$, (A·7) can be expressed as

$$\begin{aligned} R_{JD}^k(\omega_2^k) &\approx \frac{1}{2} \sum_{i=1}^{n_d^k} \left\{ \log_2 \mu p_s \lambda_{1,i}^k \omega_{2,i}^k / \sigma^2 \right\}_+ \\ &= \frac{1}{2} \sum_{i=1}^{n_d^k} \left\{ \log_2 \left(p_2^k/n_d^k + \Delta_a \right) p_s \lambda_{1,i}^k \omega_{2,i}^k / \sigma^2 \right\}_+ \\ &= \frac{1}{2} \left\{ \log_2 \left(p_2^k/n_d^k + \Delta_a \right)^{n_d^k} p_s^{n_d^k} \prod_{k=1}^{n_d^k} \lambda_{1,i}^k \omega_{2,i}^k / \sigma^{2n_d^k} \right\}_+ \\ &= \frac{1}{2} \left\{ \log_2 \left[\left(p_2^k/n_d^k + \Delta_a \right) p_s / \Delta_g \sigma^2 \right]^{n_d^k} \right\}_+ \\ &= \frac{n_d^k}{2} \left\{ \log_2 \left[\left(p_2^k/n_d^k + \Delta_a \right) p_s / \Delta_g \sigma^2 \right] \right\}_+ \end{aligned} \quad \text{(A·11)}$$

In the high SINR condition, we have

$$\begin{aligned} R_{JD}^k(\omega_2^k) &\overset{(a)}{\approx} \frac{1}{2} \max \left[\sum_{i=1}^{n_d^k} \log_2(\mu p_s \lambda_{1,i}^k \omega_{2,i}^k / \sigma^2) \right] \\ &= \frac{n_d^k}{2} \left\{ \log_2 \left[\left(p_2^k/n_d^k + \Delta_a \right) p_s / \Delta_g \sigma^2 \right] \right\}_+ \\ &\approx \frac{n_d^k}{2} \left\{ \log_2 \left[\left(p_2^k/n_d^k \right) p_s / \Delta_g \sigma^2 \right] \right\}_+ \\ &\approx \frac{n_d^k}{2} \left\{ \log_2 \left[\left(p_2^k/n_d^k + \Delta'_a \right) p_s / \Delta_g \sigma^2 \right] \right\}_+ \\ &\overset{(b)}{=} \frac{1}{2} \max \left[\sum_{i=1}^{n_d^k} \log_2(\mu p_s \lambda_{1,i}^k \lambda_{2,i}^k / \sigma^2) \right] \\ &\approx C_{JD}^k(\boldsymbol{\lambda}_2^k) \end{aligned} \quad \text{(A·12)}$$

It is noticed that $R_{JD}^k(\omega_2^k)$ does not depend on the terms Δ_a and Δ'_a. Therefore, from (a) and (b) in (A·12), we have

$$\begin{aligned} &\lim_{p_2^k \to \infty} \left[R_{JD}^k(\omega_2^k) - C_{JD}^k(\boldsymbol{\lambda}_2^k) \right] \\ &= \frac{n_d^k}{2} \lim_{p_{2,k} \to \infty} \left\{ \log_2 \left[\left(p_2^k/n_d^k + \Delta_a \right) p_s / \left(p_2^k/n_d^k + \Delta'_a \right) p_s \right] \right\}_+ \\ &= 0 \end{aligned} \quad \text{(A·13)}$$

Appendix C: Derivation of upper Bound Rate for the Whole System

It is known that the receive signal vector $\mathbf{Y}_d \in \mathbb{C}^{N_{\Sigma_d} \times 1}$ at DU's depends on $\mathbf{Y}_r$ but is conditionally dependent on $\mathbf{X}$. We notice this relation as

$$\mathbf{X} \to \mathbf{Y}_r \to \mathbf{Y}_d \quad \text{(A·14)}$$

From the above Markov chain, we obtain the following inequality [23, pp.34–35]

$$I(\mathbf{X}; \mathbf{Y}_d) \leq I(\mathbf{X}; \mathbf{Y}_r) \quad \text{(A·15)}$$

Accordingly the upper bound of achievable rate can be expressed as

$$\begin{aligned} R_{upper-bound} &= \frac{1}{2} \max \left[I(\mathbf{X}; \mathbf{Y}_d) \right] \leq \frac{1}{2} \max \left[I(\mathbf{X}; \mathbf{Y}_r) \right] \\ &= \frac{1}{2} C(\mathbf{H}_1) \end{aligned} \quad \text{(A·16)}$$

where the coefficient of 1/2 means two time slots are consumed and the relationship of $I(\mathbf{X}; \mathbf{Y}_r) = C(\mathbf{H}_1)$ is detailed in [23, pp.263–299], [25].

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