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Shock-induced phase transitions in systems of hard spheres with attractive interactions

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Abstract Shock-induced phase transitions are studied by adopting the recently-developed theoretical framework, which is applicable for shock waves in three phases (gas, liquid, and solid), based on the system of hard spheres with mutual attractive interactions. The Rankine-Hugoniot conditions derived from the system of Euler equations with caloric and thermal equations of state are studied, and the admissibility (stability) of a shock wave is analyzed. Two typical scenarios of the shock-induced phase transitions from gas phase to solid phase are found. A scenario of shock-induced phase transitions involving three phases simultaneously near the triple point is also found.

Keywords shock wave · shock-induced phase transition · hard-sphere system with attractive interaction · Rankine-Hugoniot relations · stability of shock waves

Mathematics Subject Classification (2000) 76L05 · 82C26 · 35L67

1 Introduction

Dynamic phase transitions induced by a shock wave are quite different from equilibrium phase transitions. Several kinds of shock-induced phase transitions have been studied in the literature[1–8]; shock-induced phase transitions between gas phase and liquid phase, between liquid phase and solid phase, and between structurally-different two phases.

The authors and their co-workers have theoretically studied two kinds of shock-induced phase transitions quantitatively based on the system of the Euler equations with the caloric and the thermal equations of state for several simple models. The results can be summarized as follows: (i) Shock-induced *liquid-solid* phase transitions in hard-sphere systems

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with[9] and without[10] internal degrees of freedom. The non-polytropic case of the system was also analyzed[11]. (ii) Shock-induced *gas-liquid* phase transitions in van der Waals fluids[12]. A new type of compressive shock waves accompanying the phase transitions was also found[13]. In these studies, the mathematical theory of hyperbolic systems played important roles for analyzing the theoretical possibility of shock-induced phase transitions because it was found that the solutions of shock waves inducing phase transitions are not always admissible.

The purpose of the present paper is to make clear the theoretical possibilities of other kinds of shock-induced phase transitions by using the recently-developed framework [14] for analyzing shock waves in three phases (gas, liquid, solid) in a unified way.

In the present paper, we will consider two kinds of shock-induced phase transitions which have never been analyzed: (I) shock-induced phase transition from gas phase to solid phase (II) Shock-induced phase transition involving three phases simultaneously near the triple point.

2 Basic equations

We consider one-dimensional waves (plain waves) in three-dimensional space. In this section, we summarize the basic equations for the present analysis.

2.1 Caloric and thermal equations of state

We adopt the system modeled by the hard-spheres with mutual attractive interactions, which is proposed by Longuet-Higgins and Widom [15] and Young and Alder [16]. This is the simplest version in the perturbation theory in liquid-state physics [17–19]. The specific internal energy e and the pressures p are given by the sum of the contributions from a hard-sphere system, e^{HS} and p^{HS}, and the mean-field-theoretical corrections characterized by the constant $a(>0)$ being the strength of the attractive force between the particles. The caloric and thermal equations of state are given by

$$e = e^{\mathrm{HS}} - \frac{a\eta}{m\omega} \tag{1}$$

and

$$p = p^{\mathrm{HS}} - \frac{a\eta^2}{\omega^2} \tag{2}$$

with m, ω and η being the mass of a particle, the volume of a hard sphere, and the packing fraction ($\equiv \rho\omega/m$, where ρ is the mass density), respectively.

The contributions from a hard-sphere system e^{HS} and p^{HS} can be expressed as follows:

$$e^{\mathrm{HS}} = \frac{\mathscr{D}}{2m}k_{\mathrm{B}}T, \quad p^{\mathrm{HS}} = \frac{\eta}{\omega}k_{\mathrm{B}}T\Gamma(\eta), \tag{3}$$

where k_{B} and T are the Boltzmann constant and the absolute temperature, respectively. The degrees of freedom of a particle $\mathscr{D}$ are given by the sum of the space dimension 3 and the internal degrees of freedom f, which is assumed to be constant. Because the form of $\Gamma(\eta)$ depends on the phases, we adopt the Padé approximation (P(3,3)) for the liquid phase [20] and the results from the free volume theory for the solid phase [17], respectively. The forms $\Gamma(\eta) = \Gamma^{\mathrm{L}}(\eta)$ for liquid phase and $\Gamma(\eta) = \Gamma^{\mathrm{S}}(\eta)$ for solid phase are given by

$$\begin{aligned}
\Gamma^{\mathrm{L}}(\eta) &\equiv 1 + \frac{4\eta + 1.016112\eta^2 + 1.109056\eta^3}{1 - 2.245972\eta + 1.301008\eta^2},\\
\Gamma^{\mathrm{S}}(\eta) &\equiv 1 + \frac{1}{\left(\sqrt{2}\pi/6\eta\right)^{1/3} - 1}.
\end{aligned} \tag{4}$$

We assume the liquid/solid coexistence states in the range of the packing fraction between $\eta_{\mathrm{L}}^{\mathrm{HS}}$(=0.4946; freezing point) and $\eta_{\mathrm{S}}^{\mathrm{HS}}$(=0.5564; melting point). The form of $\Gamma(\eta)$ for the coexistence phase is given by $\Gamma(\eta) = \Gamma^{\mathrm{CO}}(\eta)(\equiv \eta_{\mathrm{L}}^{\mathrm{HS}}\Gamma^{\mathrm{L}}(\eta_{\mathrm{L}}^{\mathrm{HS}})/\eta = \eta_{\mathrm{S}}^{\mathrm{HS}}\Gamma^{\mathrm{S}}(\eta_{\mathrm{S}}^{\mathrm{HS}})/\eta)$.

It is useful to introduce the following dimensionless quantities:

$$\hat{T} = T/T_{\mathrm{cr}}, \quad \hat{p} = p/p_{\mathrm{cr}}, \quad \hat{e} = (m/(\omega p_{cr}))\,e,$$

where T_{cr} and p_{cr} are, respectively, the temperature and the pressure at the critical point. From the critical conditions; $(\partial p/\partial\eta)_T = 0$ and $(\partial^2 p/\partial\eta^2)_T = 0$, the explicit expressions are given by $T_{\mathrm{cr}} = A_T a/(k_{\mathrm{B}}\omega)$ and $p_{\mathrm{cr}} = A_p a/\omega^2$ with $A_T \sim 0.09421$ and $A_p \sim 0.004401$. By adopting these dimensionless variables, the thermal and caloric equations of state (1) and (2) can be rewritten in the forms which are independent of the material-dependent constant a (*law of corresponding states*).

Let us obtain the phase diagram in which different phases are separated by the coexistence curves. These curves are determined by the two equality conditions between the two states in different phases; the equality of the specific Gibbs free energy $g(\equiv e - Ts + p/\rho$ with s being the specific entropy) and the equality of the pressure with a common temperature T^*. We can obtain the specific entropy s by inserting the caloric and thermal equations of state (1) and (2) to the Gibbs relation ($\mathrm{d}s = (1/T)\mathrm{d}e + (p/T)\mathrm{d}(1/\rho)$).

After the calculations[9,10,12], we obtain the conditions for the gas/liquid coexistence curve as follows:

$$\begin{aligned}
&2(\eta_{\mathrm{L}} - \eta_{\mathrm{G}}) - A_T\hat{T}^*\left(\int_{\eta_{\mathrm{G}}}^{\eta_{\mathrm{L}}} \frac{\Gamma^{\mathrm{L}}(\eta)}{\eta}\mathrm{d}\eta + \Gamma^{\mathrm{L}}(\eta_{\mathrm{L}}) - \Gamma^{\mathrm{L}}(\eta_{\mathrm{G}})\right) = 0,\\
&\frac{A_T}{A_p}\hat{T}^*\eta_{\mathrm{L}}\Gamma^{\mathrm{L}}(\eta_{\mathrm{L}}) - \frac{\eta_{\mathrm{L}}^2}{A_p} = \frac{A_T}{A_p}\hat{T}^*\eta_{\mathrm{G}}\Gamma^{\mathrm{L}}(\eta_{\mathrm{G}}) - \frac{\eta_{\mathrm{G}}^2}{A_p},
\end{aligned} \tag{5}$$

where η_{G} and η_{L} are the packing fraction on the coexistence curve on the gas-phase side and on the liquid-phase side, respectively. Note that $\Gamma(\eta) = \Gamma^{\mathrm{L}}(\eta)$ for both gas phase and liquid phase.

The coexistence curves of gas / solid and liquid / solid are determined by the following relations:

$$\begin{aligned}
&2(\eta_{\mathrm{S}} - \eta_{\mathrm{G,L}}) - A_T\hat{T}^*\left[\int_{\eta_{\mathrm{G,L}}}^{\eta_{\mathrm{L}}^{\mathrm{HS}}} \frac{\Gamma^{\mathrm{L}}(\eta)}{\eta}\mathrm{d}\eta + \int_{\eta_{\mathrm{S}}^{\mathrm{HS}}}^{\eta_S} \frac{\Gamma^{\mathrm{S}}(\eta)}{\eta}\mathrm{d}\eta\right.\\
&\left.\quad - \eta_{\mathrm{L}}^{\mathrm{HS}}\Gamma^{\mathrm{L}}(\eta_{\mathrm{L}}^{\mathrm{HS}})\left(\frac{1}{\eta_{\mathrm{S}}^{\mathrm{HS}}} - \frac{1}{\eta_{\mathrm{L}}^{\mathrm{HS}}}\right) + \Gamma^{\mathrm{S}}(\eta_{\mathrm{S}}) - \Gamma^{\mathrm{L}}(\eta_{\mathrm{G,L}})\right] = 0,\\
&\frac{A_T}{A_p}\hat{T}^*\eta_{\mathrm{G,L}}\Gamma^{\mathrm{L}}(\eta_{\mathrm{G,L}}) - \frac{\eta_{\mathrm{G,L}}^2}{A_p} = \frac{A_T}{A_p}\hat{T}^*\eta_{\mathrm{S}}\Gamma^{\mathrm{S}}(\eta_{\mathrm{S}}) - \frac{\eta_{\mathrm{S}}^2}{A_p},
\end{aligned} \tag{6}$$

where η_{S} is the packing fraction on the coexistence curve on the solid-phase side, and $\eta_{\mathrm{G,L}}$ means η_{G} or η_{L} depending on the temperature: If the temperature of the coexistence state

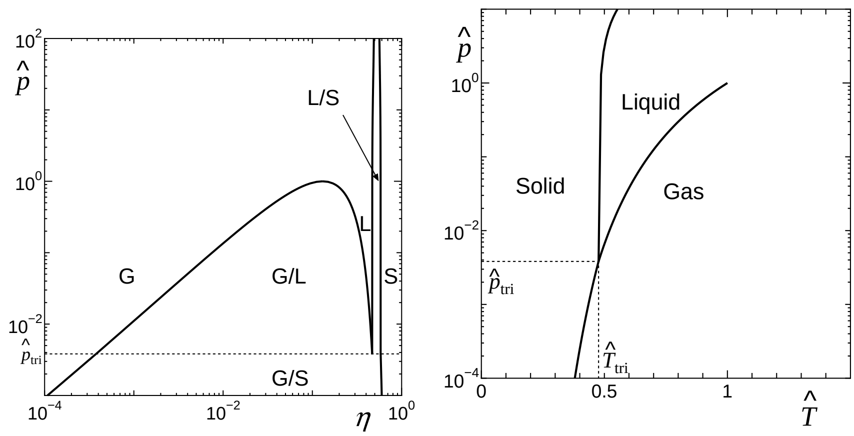

Fig. 1 Left: The phase diagram in the $\hat{p}-\eta$ plane. Right: The phase diagram in the $\hat{p}-\hat{T}$ plane. G, L, S, G / L, L / S and G / S stand for the gas, liquid, solid, gas / liquid coexistence, liquid /solid coexistence, and gas / solid coexistence phases, respectively.

is lower than the triple point temperature, the gas / solid coexistence curve is given by (6). Otherwise the conditions (6) give liquid / solid coexistence curve.

Figure 1 shows the phase diagrams in the $\hat{p}-\eta$ plane and in the $\hat{p}-\hat{T}$ plane. In the present system, three phases (gas, liquid, solid) appear and coexist at the triple point in the low-pressure region; the values of the pressure $\hat{p}_{\mathrm{tri}}$ and the temperature $\hat{T}_{\mathrm{tri}}$ at the triple point are given by $\hat{p}_{\mathrm{tri}} \simeq 0.003824$ and $\hat{T}_{\mathrm{tri}} \simeq 0.4763$.

2.2 System of the Euler equations

We adopt the system of Euler equations describing the conservation of mass, momentum and energy for a compressible fluid. The system for the one-dimensional problem can be expressed as

$$\mathbf{u}_t + \mathbf{F}_x(\mathbf{u}) = 0, \tag{7}$$

where the subscripts (time t and position x) denote partial differentiation. Here the density $\mathbf{u}$ and the flux $\mathbf{F}$ are given by

$$\mathbf{u} = \begin{pmatrix} \rho \\ \rho v \\ \rho e + \frac{1}{2}\rho v^2 \end{pmatrix}, \qquad \mathbf{F} = \begin{pmatrix} \rho v \\ \rho v^2 + p \\ \left(\rho e + \frac{1}{2}\rho v^2 + p\right) v \end{pmatrix} \tag{8}$$

with v being the velocity.

It is convenient to introduce the dimensionless quantities as follows:

$$\hat{v} = \sqrt{m/(\omega p_{cr})}\, v, \quad \hat{c} = \sqrt{m/(\omega p_{cr})}\, c,$$

where $c(= \sqrt{(\partial p/\partial \rho)_s})$ is the sound velocity.

3 Rankine-Hugoniot conditions

We analyze a plane shock wave propagating with velocity U_s in the positive x-direction. We focus on the case that the unperturbed state (the state before a shock front) is in the gas phase.

The system of Euler equations has a shock-type solution with discontinuous jumps of some physical quantities that satisfy the following Rankine-Hugoniot (RH) conditions:

$$\begin{gathered}\hat{v}_1 = \hat{c}_0 M_0 \frac{\eta_1-\eta_0}{\eta_1}, \quad \hat{p}_1 = \hat{p}_0 + \hat{c}_0^2 M_0^2 \frac{\eta_0(\eta_1-\eta_0)}{\eta_1}, \\ M_0 = \frac{1}{\hat{c}_0}\sqrt{\frac{2\eta_1}{\eta_0(\eta_1-\eta_0)}\left(\hat{p}_1 - \frac{\eta_0\eta_1(\hat{e}_1-\hat{e}_0)}{\eta_1-\eta_0}\right)},\end{gathered} \tag{9}$$

where the quantities with the subscript 0 are those in the unperturbed state, while the quantities with the subscript 1 are those in the perturbed state (the state after a shock front). $M_0(\equiv (U_s - v_0)/c_0)$ is the Mach number in the unperturbed state. In the derivation of the conditions above, due to the Galilean invariance, we have assumed $v_0 = 0$ without any loss of generality.

We have the dimensionless internal energy and sound velocity in the unperturbed state as follows:

$$\begin{gathered}\hat{e}_0 = \frac{\mathscr{D}}{2\eta_0\Gamma_0}\left(\hat{p}_0 + \frac{\eta_0^2}{A_p}\right) - \frac{\eta_0}{A_p}, \\ \hat{c}_0 = \sqrt{\frac{-2\mathscr{D}\eta_0 + A_T\hat{T}_0[2\Gamma_0^2 + \mathscr{D}(\Gamma_0 + \Gamma_0'\eta_0)]}{A_p\mathscr{D}}},\end{gathered} \tag{10}$$

where $\Gamma_0 = \Gamma^{\mathrm{L}}(\eta_0)$ and $\Gamma_0' = (\mathrm{d}\Gamma^{\mathrm{L}}(\eta)/\mathrm{d}\eta)_{\eta=\eta_0}$.

In order to obtain the expression of $\hat{e}_1$ which depends on the phase of the perturbed state, let us discuss two cases separately; the first case is that the perturbed state is in any one of the gas, liquid and solid phases and the second case is that the perturbed state is in coexistence states.

3.1 Case 1. unperturbed state: gas phase, perturbed state: gas or liquid or solid phase

When the perturbed state is in any one of the gas, liquid and solid phases, we have

$$\hat{e}_1 = \frac{\mathscr{D}}{2\eta_1\Gamma_1}\left(\hat{p}_1 + \frac{\eta_1^2}{A_p}\right) - \frac{\eta_1}{A_p}, \tag{11}$$

where $\Gamma_1 = \Gamma^{\mathrm{L}}(\eta_1)$ and $\Gamma_1 = \Gamma^{\mathrm{S}}(\eta_1)$ are, respectively, for the perturbed state in the gas or liquid phase and for the perturbed state in the solid phase.

3.2 Case 2. unperturbed state: gas phase, perturbed state: coexistence phase

Let us consider the case that the perturbed state is in the gas / liquid coexistence phase. We assume that the specific internal energy and the perturbed packing fraction can be expressed by $\hat{e}_1 = (1-\alpha)\hat{e}_{\mathrm{G}} + \alpha\hat{e}_{\mathrm{L}}$ and $(1/\eta_1) = (1-\alpha)/\eta_{\mathrm{G}} + (\alpha/\eta_{\mathrm{L}})$ with α being the ratio of the liquid phase in the coexistence state. $\hat{e}_G$ and $\hat{e}_L$ are, respectively, the dimensionless specific internal energies on the coexistence curve on the gas-phase side and on the liquid-phase side. By using these relations, we have the following expression [12]:

$$\hat{e}_1 = \frac{\mathscr{D}}{2\eta_G \Gamma^L(\eta_G)} \left(\hat{p}_1 + \frac{\eta_G^2}{A_p} \right) - \frac{\eta_G}{A_p} - \frac{\eta_L}{A_p} + \frac{\eta_G \eta_L}{A_p \eta_1}. \tag{12}$$

When the perturbed state is a liquid / solid coexistence state, η_G and η_L in the above expression should be replaced by η_L and η_S. When the perturbed state is a gas / solid coexistence state, only η_L should be replaced by η_S. The RH conditions from the equations (9) , (10) and (12) can be solved with the coexistence conditions (5) or (6).

4 Admissibility condition

According to the theory of hyperbolic systems, not every solution of the RH conditions is admissible (stable). We need a rule to select perturbed states $\mathbf{u}_1$ forming admissible shock waves with a given unperturbed state $\mathbf{u}_0$. The suitable rule is given by the Liu condition [21–23] which includes the well-known Lax condition[24] as a special one. The Liu condition asserts that *a shock wave is admissible if and only if*

$$U_s(\mathbf{u}_0, \tilde{\mathbf{u}}) \leq U_s(\mathbf{u}_0, \mathbf{u}_1) \ \ \forall \tilde{\mathbf{u}} \in \mathscr{H}(\mathbf{u}_0) \text{ between } \mathbf{u}_0 \text{ and } \mathbf{u}_1,$$

where $\mathscr{H}(\mathbf{u}_0)$ *is the set of solutions of the RH conditions for a given unperturbed state* $\mathbf{u}_0$. The shock velocity U_s above may be replaced by the Mach number M_0 because these quantities are proportional to each other.

If the Liu condition is not satisfied, we have the so-called *shock splitting* phenomenon; the initial shock eventually splits into a combination of shock and rarefaction waves. [25–27]

5 Shock-induced phase transitions

We analyzed the RH conditions and depicted the RH curves for many unperturbed states in the gas phase with many f. The admissibility of a shock wave was also analyzed based on the Liu condition.

We have successfully confirmed the validity of the present analysis, which can be regarded as the unified version of the previous analyses, by the fact that our theory can explain all previous theoretical prediction of shock-induced phase transitions; the results of shock-induced *liquid-solid* phase transitions in the hard-sphere systems and also the results of shock-induced *gas-liquid* phase transitions in the van der Waals systems.

Since shock waves in three phases can be analyzed in a unified way by using the present theoretical framework, we can also analyze other kinds of dynamic phase transitions. In this section the theoretical possibilities of two kinds of shock-induced phase transitions is made clear. The first one is (I) shock-induced phase transitions from gas phase to solid phase. The second one is (II) shock-induced phase transitions involving three phases simultaneously, which can be analyzed through studying the RH curves passing the triple point.

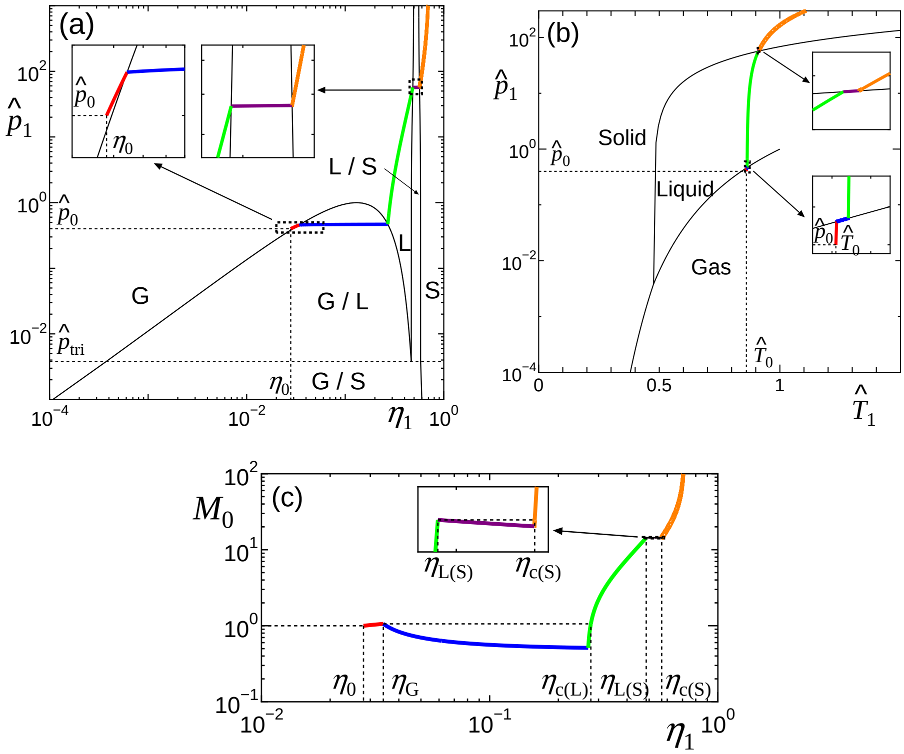

Fig. 2 The RH curves (thick lines) and the coexistence curves (thin lines) are shown in the (a) $\hat{p}_1 - \eta_1$ plane, (b) $\hat{p}_1 - \hat{T}_1$ plane and (c) $M_0 - \eta_1$ plane. $f = 2000$, $\eta_0 = 0.028$, and $\hat{p}_0 = 0.4$. A shock wave with the perturbed packing fraction in the ranges $\eta_0 < \eta_1 < \eta_{\mathrm{G}}$, $\eta_{c(\mathrm{L})} < \eta_1 < \eta_{\mathrm{L(S)}}$ and $\eta_1 > \eta_{c(\mathrm{S})}$ is admissible.

5.1 Two scenarios of shock-induced phase transitions from gas phase to solid phase

We found the following two typical scenarios for shock-induced phase transition from gas phase to solid phase: (S1) With the increase of the strength of a shock wave, the phase of the perturbed state changes from gas phase to liquid phase, and lastly to solid phase. (S2) With the increase of the shock strength, the phase of the perturbed state changes from gas phase to solid phase directly.

Figure 2 shows a typical example of the RH curves of the scenario (S1), where $f = 2000$, $\eta_0 = 0.028$, and $\hat{p}_0 = 0.4$. We adopted the perturbed packing fraction η_1 as the shock strength. It is noticeable that there exist also the perturbed coexistence states. From the Liu condition, we see in Fig. 2 that only a shock wave with the perturbed packing fraction such that $\eta_0 < \eta_1 < \eta_{\mathrm{G}}$, $\eta_{c(\mathrm{L})} < \eta_1 < \eta_{\mathrm{L(S)}}$ and $\eta_1 > \eta_{c(\mathrm{S})}$ is stable, while a shock wave with the perturbed packing fraction such that $\eta_{\mathrm{G}} < \eta_1 < \eta_{c(\mathrm{L})}$ and $\eta_{\mathrm{L(S)}} < \eta_1 < \eta_{c(\mathrm{S})}$ is unstable. Therefore we conclude that a shock wave can stably induce the phase transitions from gas phase to solid phase if the shock strength is so strong as to satisfy the condition $\eta_1 > \eta_{c(\mathrm{S})}$.

Figure 3 is an example of the RH curves of the scenario (S2), where $f = 50000$, $\eta_0 = 0.0002$, and $\hat{p}_0 = 0.00198$. From the Liu condition, we see that only a shock wave with the perturbed packing fraction such that $\eta_0 < \eta_1 < \eta_{\mathrm{G}}$ and $\eta_1 > \eta_{c(\mathrm{S})}$ is stable. Therefore we

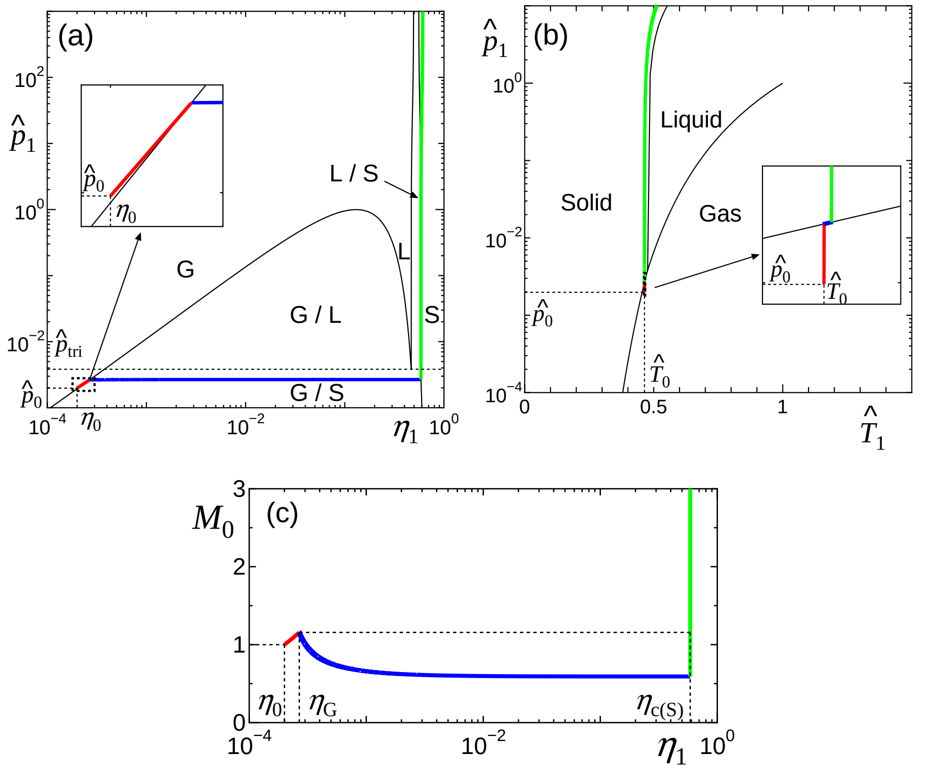

Fig. 3 The RH curves (thick lines) and the coexistence curves (thin lines) are shown in the (a) $\hat{p}_1 - \eta_1$ plane, (b) $\hat{p}_1 - \hat{T}_1$ plane and (c) $M_0 - \eta_1$ plane. $f = 50000$, $\eta_0 = 0.0002$, and $\hat{p}_0 = 0.00198$. A shock wave with the perturbed packing fraction in the ranges $\eta_0 < \eta_1 < \eta_{\rm G}$ and $\eta_1 > \eta_{c(\rm S)}$ is admissible.

conclude that there is a stable shock wave of the scenario (S2) if the shock strength satisfies the condition $\eta_1 > \eta_{c(\rm S)}$.

In order to obtain the RH curves of these scenarios, the selection of a suitable unperturbed state and a suitable value of f is important. The role of the internal degrees of freedom f is qualitatively understood as follows: If the value of f is larger, a considerable portion of the energy supplied by a shock compression contributes to the internal motions, and therefore the rise of temperature is smaller. The RH curve of the system with larger internal degrees of freedom is more likely than that of the system with less value of f to cross the coexistence curve.

For obtaining RH curves of (S1), the unperturbed states must be very near the gas-liquid coexistence curve on the gas-phase side and also the value of f must be larger than $O(10^1)$. For obtaining RH curves of (S2), the unperturbed states must be very near the gas-solid coexistence curve on the gas-phase side and also the value of f must be larger than $O(10^3)$. Detailed prescription of these selection is left for the next study. There exist materials satisfying the necessary values of f for (S1). We adopted the large value of 2000 as the value of f in Fig. 2 only for showing the characteristics of the RH curves clearly. On the other hand, the experimental observation of the scenario (S2) seems to be impossible, or at least extremely difficult. We consider this point again in a concluding remark below.

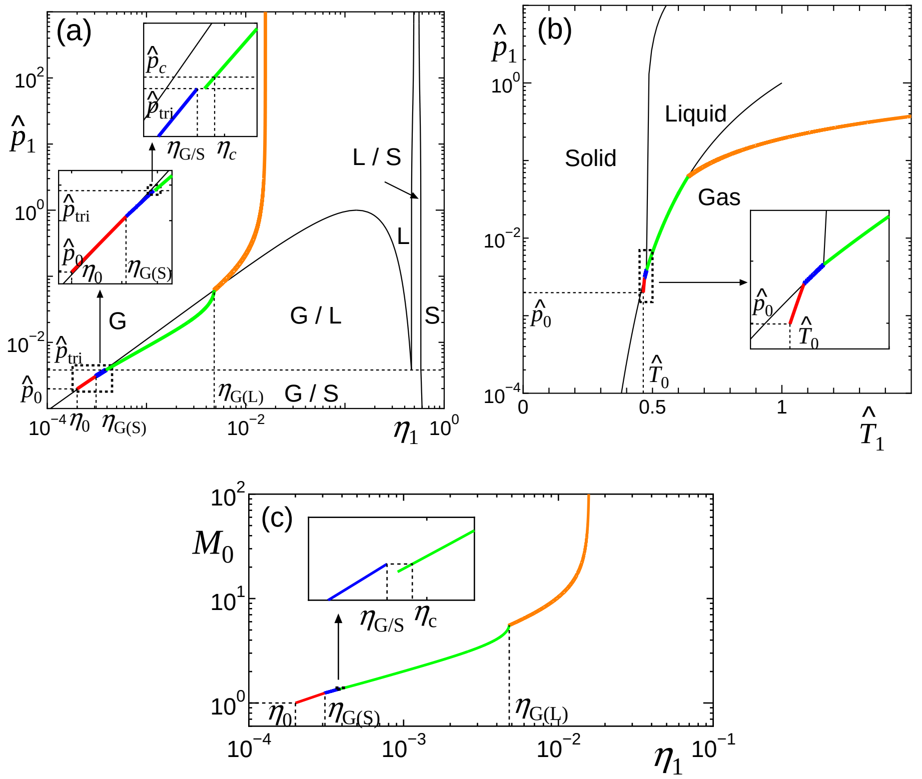

Fig. 4 The RH curves (thick lines) and the coexistence curves (thin lines) are shown in the (a) $\hat{p}_1 - \eta_1$ plane, (b) $\hat{p}_1 - \hat{T}_1$ plane and (c) $M_0 - \eta_1$ plane. $f = 80$, $\eta_0 = 0.0002$, and $\hat{p}_0 = 0.00198$. A shock wave with the perturbed packing fraction in the ranges $\eta_0 < \eta_1 < \eta_{\mathrm{G/S}}$ and $\eta_1 > \eta_c$ is admissible, where $\eta_{\mathrm{G/S}}$ is the packing fraction when the pressure in the gas /solid coexistence perturbed state equals to the triple point pressure. $\hat{p}_c$ is the perturbed pressure when the perturbed packing fraction equals to η_c.

5.2 Shock-induced phase transitions involving three phases simultaneously near the triple point

We also found that there exists a scenario for shock-induced phase transitions involving three phases simultaneously near the triple point: (T) With the increase of the strength of a shock wave, the phase of the perturbed state changes from gas phase to gas/solid coexistence phase firstly until the perturbed pressure reaches the pressure at the triple point. If the strength of a shock increases more, the phase of the perturbed state changes from gas/solid coexistence phase to gas/liquid coexistence phase, and lastly to gas phase. Even though the RH curves across the gas/solid coexistence curve on the gas-phase side, the RH curves can enter the gas/liquid coexistence state passing the triple point.

An example is shown in Fig. 4, where $f = 80$, $\eta_0 = 0.0002$ and $\hat{p}_0 = 0.00198$. Note that the perturbed packing fraction η_1 and the unperturbed Mach number M_0 change discontinuously at the point that the perturbed pressure equals to the pressure at the triple point. In other words, the limiting value of η_1 and M_0 as the perturbed pressure $\hat{p}_1$ tends to the triple point pressure $\hat{p}_{\mathrm{tri}}$ from *above* are different from the ones as $\hat{p}_1$ tends to $\hat{p}_{\mathrm{tri}}$ from *below*.

The solution of a shock wave with the perturbed packing fraction such that $\eta_0 < \eta_1 < \eta_{\mathrm{G/S}}$ and $\eta_1 > \eta_c$ satisfy the Liu condition, where $\eta_{\mathrm{G/S}}$ is the packing fraction when the pressure in the gas /solid coexistence perturbed state equals to the triple point pressure. We conclude that a shock wave can stably propagate when the perturbed state is in the gas / solid coexistence phase and that a stable shock wave can propagate when the perturbed state is in the gas / liquid coexistence phase except for the case that the perturbed pressure is very near the triple point pressure.

In order to obtain RH curves of the scenario (T), the unperturbed states must be very near the gas-solid coexistence curve on the gas-phase side and also the value of f must be larger than $O(10^1)$. We expect the experimental observation of the scenario (T).

6 Summary and concluding remarks

Shock-induced phase transitions were analyzed by using the recently-developed theoretical framework which consists of the Euler equations with the caloric and the thermal equations of state for the system of hard spheres with attractive interactions. It is found that there can exist two typical scenarios (S1) and (S2) of the shock-induced phase transitions from gas phase to solid phase. We have also found a possible scenario (T) of the shock-induced phase transition involving three phases simultaneously near the triple point.

Lastly the concluding remarks are summarized as follows:

(I) We found that the internal degrees of freedom f should be larger than $O(10^1)$ for the scenarios (S1) and (T), and $O(10^3)$ for the scenario (S2). The shock waves in some real gases with $f = O(10^1)$ were studied for analyzing shock-induced phase transition from gas phase to liquid phase[1]. We highly expect experiments of shock waves in the system with larger f from the viewpoint of shock-induced phase transitions.

(II) Concerning shock-induced phase transition involving three phases simultaneously near the triple point, theoretical possibility of only one scenario (T) is made clear in the present paper. It is interesting to study further in order to make clear whether another possible scenarios of RH curves passing the triple point can exist or not. The detail will be reported in the forthcoming paper.

(III) In the present paper the admissibility of a shock wave was analyzed based on the Liu condition. However, strictly speaking, the Liu condition can not be applied to nonsmooth or discontinuous RH curves. Numerical analysis to solve the system of the Euler equations with the thermal and the caloric equations of state directly is important for confirming the validity of the present analysis. This study is left for future subject.

(IV) The present analysis can be easily extended to be more realistic by adopting a non-polytropic hard-sphere system as a reference system[11].

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