DEVELOPMENT OF TIME-OF-DAY USER EQUILIBRIUM TRAFFIC ASSIGNMENT MODEL CONSIDERING TOLL LOAD ON EXPRESSWAYS

Yasuhiro UJII
Doctoral Student
Graduate School of Engineering
Nagoya Institute of Technology
Gokisochou, Showa-ku, Nagoya,
4668555 Japan
Fax: +81-52-735-5492
E-mail: ujii@keik1.ace.nitech.ac.jp

Motohiro FUJITA
Research Professor
Graduate School of Engineering
Nagoya Institute of Technology
Gokisochou, Showa-ku, Nagoya,
4668555 Japan
Fax: +81-52-735-5492
E-mail: fujita@doboku2.ace.nitech.ac.jp

Yukimasa MATSUMOTO
Associate Professor
Department of Civil Engineering
Meijo University
1-501,Shiogamaguchi,Tenpaku-ku,
Nagoya, 468-8502 Japan
Tel: +81-52-832-1151,
Fax: +81-52-832-1178,
E-mail: matumoto@meijo-u.ac.jp

Hiroshi MATSUI
Professor
Department of Civil Engineering
Meijo University
1-501,Shiogamaguchi,Tenpaku-ku,
Nagoya, 468-8502 Japan
Tel: +81-52-832-1151,
Fax: +81-52-832-1178,
E-mail: matsui@meijo-u.ac.jp

Abstract : In this paper we propose an extended time-of-day user equilibrium assignment model for an urban road network including expressways with toll load. In the time-of-day traffic assignment, it is significant to modify semi-dynamically a part of traffic demand remained on each path at the end of a study period. The extended model proposed in this paper is able to modify the traffic demand in considering expressways with toll load especially. In the model the user equilibrium conditions are also based on both the travel time for arterial road users and the generalized travel time considering the toll load for expressway users. This new model is applied to a real-scale road network and the results show good accuracy and practicability.

Key Words: user equilibrium, time-of-day traffic assignment, expressway

1. INTRODUCTION

Recently, in a field of traffic flow estimation for urban road network, user equilibrium assignment models (M.J. Beckmann. et al. 1956) are very significant because it is based on the behavioral principle (Wordrop principle) that each driver chooses the root which minimizes the travel time from origin to destination, and thus it is very logical. Furthermore it is necessary for the transportation planning in some particular hours, such as rush hours in morning, to estimate hourly traffic flow for urban road network including expressways with toll load. Therefore, we develop and extend the time-of-day user equilibrium assignment model to the urban road network including expressways with toll load.

In the previous paper (Fujita M. et al. 1988) we have already proposed a time-of-day user equilibrium assignment model. We call this model TUE. TUE is based on Wordrop principle, and TUE is able to semi-dynamically estimate time-of-day traffic flow by each study time period (such as one hour unit) as considering the traffic volume remained at the end of each time period. That is, TUE surely has some remained traffic volume on path at the end of each study period because TUE divides continuous traffic demand for one-day unit into several traffic demands for each study time period.

In this paper, we propose an extended TUE model including expressways with toll load on urban road network. We call this model TUEE. TUEE adopts a diversion function that determines the ratio of traffic demand passing expressways between each OD pair in order to accurately estimate time-of-day traffic flow, especially traffic flow on expressways. In this TUEE the user equilibrium conditions can be based on both the travel time for arterial road users and the generalized travel time considering the toll load for expressway users. This model is applied to Nagoya metropolitan road network.
In this application of TUEE to the network, we examine the accuracy of assignment results with not only traffic flows of arterial roads and expressways, but also travel time. Because to examine the accuracy of travel time is very important for evaluation of transportation policy, however, estimation value of travel time has not been almost examined in the previous papers (Fujita M. et al. 2001).

There are many researches and applications of the traffic assignment considering expressway with toll. Generally, the assignment model considering expressway with toll have been applied to basic user equilibrium assignment with the generalized travel time including toll. In order to estimate efficiently the traffic volume on expressway links especially, some practical research institutes such as expressway public corporations in Japan have developed several incremental assignment models combined with diversion function that can estimate the ratio of expressway demand (for example: Nagoya Expressway public corporation 1996). However, these models are not based on the user equilibrium theory.

(Matsui H. et al. 2000) have researched on the model integrated the diversion function into the user equilibrium assignment. This model was developed as a traffic assignment model applied to traffic demand in day-long unit, but did not consider the remained traffic volume in time-of-day traffic assignment.

On the other hand, the TUE model in the previous paper (Fujita M. et al. 1988) did not clearly consider the toll load for expressway. The modification of remained traffic volumes in TUE must be calculated by using real travel time without toll, since the remained volume is in physical quantity. However, the user equilibrium condition especially for expressway paths should be based on the generalized travel time including toll load. The TUE has not considered the relationship between the remained traffic volume and the generalized travel time.

(Akamatsu T. et al. 1989) have proposed a semi-dynamic TUE that the remained traffic volume was considered by link. However this model did not use diversion functions and thus did not clearly consider the route choice behavior between expressway path with toll and arterial path without toll.

There are no researches for TUE that clearly consider the route choice behavior between expressway path and arterial path and examine the accuracy of results with not only traffic flows but also travel time. In this paper we will formulate and examine TUEE model in consideration of several problems mentioned above.
Therefore, the contents of this research are as follows.In Chapter 2 we explain the assumptions and the modification method for the remained traffic volume for TUEE. In Chapter 3 we propose the model structure of TUEE and formulate the model. In Chapter 4 we explain link-performance functions and diversion functions for some particular hours. We apply TUEE to Nagoya metropolitan road network and examine the accuracy of the model.

2. ASSUMPTIONS OF TUEE AND THE METHOD OF OD-FLOW MODIFICATION

TUEE has two assumptions as follows. The TUE in the previous paper also set the same assumptions.
Assumption 1: We divide a day-long period into small study time periods. Each study time period size(T) should be longer than the longest trip length in a study area.
Assumption 2: Each OD flow is uniformly generated from each origin during a time period and distributed on path connecting each OD pair.

The TUE surely has some remained traffic volume on path at the end of each study period. The TUEE developed in this paper adopts a method of OD-flow modification developed in the TUE in order to consider the remained traffic volume at the end of each study period.
The OD-flow modification method modifies demand flow (OD-flow) so that the remained volume of links on each path at the end of a study period are leveled out through the path.
Therefore, the OD flow g_{rs}^{n} between OD pair r-s during time period n, that is modified in order to semi-dynamically consider the remained volume, is as follows; (See Appendix.1 and Figure 6 for understanding the modification of the remained traffic volume in detail.)

$$g_{rs}^{n} = q_{rs}^{n-1} + G_{rs}^{n} - \frac{C_{rs}^{n}}{2T} G_{rs}^{n} \tag{1}$$

where ,
g_{rs}^{n} : the modified OD flow between OD pair r-s during time period n.
C_{rs}^{n} : the minimum travel time between OD pair r-s during time period n
G_{rs}^{n} : OD flow between OD pair r-s during time period n
T : length of study time period
q_{rs}^{n-1} : the remained flow during time period n -1(constant value during time period n)

The equation (1) means that the remained flow (q_{rs}^{n-1}) during the previous time period ‘n-1’ is added into the present OD-flow(G_{rs}^{n}) and the remained flow($G_{rs}^{n} C_{rs}^{n} / 2T$) during the present time period ‘n’ is subtracted from G_{rs}^{n} . In section.4(3)(d), we’ll confirm the effect of the OD-flow modification above in real road network.
The TUEE will be formulated based on this OD-flow modification method.

3. FORMULATION OF TUEE

In the TUEE, we define two types of minimum paths between each OD pair, “arterial path” and “expressway path”. The arterial path is the minimum path that consists of arterial roads without toll and takes the minimum travel time between each OD pair on only arterial road network without toll. The expressway path is the minimum path that necessarily utilizes expressways with toll and takes the minimum generalized travel time considering toll on the whole study network including arterial roads and expressways between each OD pair. In this paper, we define that expressways are always roads with toll load. Similarly, OD flows for two types of the minimum paths are called arterial OD flow and expressway OD flow respectively.

(1) Model structure and assumptions

a) Model structure

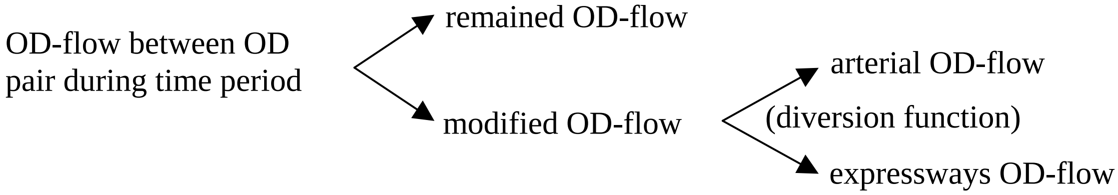

Figure 1 Tree-structure of TUEE model

There are two steps for our model to estimate OD flows as shown in Figure 1.
In the first step, we divide OD flow between an OD pair during time period n into the modified OD flow assigned in the time period n and the remained OD flow used in the next time period n+1, according to the OD-flow modification method. In the second step, we divide the modified OD flow into the arterial OD flow and the expressway OD flow by using diversion function. This diversion function is the demand function that estimates the rate of expressway OD flow in the modified OD flow by using two travel times of a real travel time for arterial path and a generalized travel time for expressway path as shown in equation (2a, 2b).

Since this model is formulated as a UE assignment with Variable Demand (M.J. Beckmann. et al. 1956, Sasaki M. et al. 1987, Fujita M. et al. 1988), we can obtain a set of equilibrium solutions such as modified OD flow during time period n, remained OD flow, expressway OD flow, arterial OD flow and link flows. Then, solution for the arterial path and OD flow satisfies the user equilibrium condition on the basis of real travel time. Solution for the expressway path and OD flow satisfies the user equilibrium condition based on the generalized travel time including toll load.
In the solution, the rate of expressway OD flow in the modified OD flow corresponds to the rate estimated from diversion function. Solution for the remained OD flow corresponds to the flow estimated from the third term of Equation (1).

On the other hand, we tried to develop another type of TUEE model that the tree-structure was inverted from the tree-structure in Figure 1. That is, in another model, we first divide an OD flow into an expressway OD flow and an arterial OD flow by using the diversion function. Then, we divide each OD flow of expressway and arterial, into the modified OD flow and the remained OD flow. As a result of considering two tree-structures of TUEE above, we have been able to only formulate the model of Figure 1. However, we will examine that the modified OD flows for two model structures above are almost the same if proper parameters in the TUEE formulation of Figure 1 are estimated in the next Chapter, because it should be satisfied that the models of two tree-structures which provide the same solution.

b) Time of day diversion function for TUEE
The time-of-day diversion function that estimates the rate of expressway OD flow in the modified OD flow is given by

$$Q_{rs}^{en} = \frac{1}{\exp(-\theta_{rs}(C_{rs}^{an} - C_{rs}^{en}) + \psi_{rs}) + 1} g_{rs}^{n} \quad (2a)$$

$$Q_{rs}^{an} = g_{rs}^{n} - Q_{rs}^{en} \quad (2b)$$

where,
Q_{rs}^{an} : arterial OD flow between OD pair r-s during time period n.

Q_{rs}^{en} : expressway OD flow between OD pair r-s during time period n
g_{rs}^{n} : the modified OD flow between OD pair r-s during time period n.
C_{rs}^{an} : the minimum travel time of arterial path between OD pair r-s during time period n
C_{rs}^{en} : the minimum generalized travel time considering toll of expressway path between OD pair r-s during time period n
θ_{rs}, ψ_{rs} : parameter between OD pair r-s

(2) Formulation of TUEE

We now show that the TUEE considering OD-flow modification method and time-of-day diversion function can be formulated as a following Bechmann-Type UE assignment problem when we make use of the Wardrop's principle as an assignment principle. We formulate the TUEE under the conditions in a) and b) as equations (3a) and (3b). We'll prove the formulation after the equations (3a) and (3b), the equation (3a) means as follows. The first term is related to Wardrop's principle. The second term and the third term are related to the time-of-day diversion function (equation (2)). And the fourth term is related to the OD-flow modification method (equation (1)). The equation (3b) means constraint conditions on urban road network.

$$\min . Z = \sum_a \int_0^{x_a^n} t_a(\omega) d\omega + \sum_r \sum_s \frac{1}{\theta_{rs}} \{ Q_{rs}^{en} (\ln (Q_{rs}^{en} / g_{rs}^{n}) + \psi_{rs}) \}$$
$$+ \sum_r \sum_s \frac{1}{\theta_{rs}} Q_{rs}^{an} \ln (Q_{rs}^{an} / g_{rs}^{n}) - \frac{1}{b} \sum_r \sum_s \int_0^{g_{rs}^{n}} \frac{2T}{G_{rs}^{n}} (q_{rs}^{n-1} + G_{rs}^{n} - z - \frac{G_{rs}^{n}}{2T} a) dZ \qquad (3a)$$

s.t

$$\sum_k f_{rsk}^{en} - Q_{rs}^{en} = 0 \qquad \forall n, r, s$$
$$\sum_k f_{rsk}^{an} - Q_{rs}^{an} = 0 \qquad \forall n, r, s$$
$$x_a^n = \sum_{k \in K} \sum_{rs \in \Omega} (\delta_{ak}^{enrs} f_{rsk}^{en} + \delta_{ak}^{anrs} f_{rsk}^{an}) \qquad \forall n, a$$
$$g_{rs}^{n} - Q_{rs}^{en} - Q_{rs}^{an} = 0 \qquad \forall n, r, s \qquad (3b)$$
$$f_{rsk}^{en} \geq 0, f_{rsk}^{an} \geq 0, x_a^n \geq 0, Q_{rs}^{en} \geq 0, Q_{rs}^{an} \geq 0, g_{rs}^{n} \geq 0$$

where,
x^a :link flow on link a during time period n
$t_a(\bullet)$: link-cost function of link a
f_{rsk}^{an} : path flow on arterial path k connecting OD pair r-s during time period n
f_{rsk}^{en} : path flow on expressway path k connecting OD pair r-s during time period n
δ_{ak}^{anrs} : indicator variable
1: if link a is on arterial path k between OD pair r-s during time period n
0: otherwise
δ_{ak}^{enrs} : indicator variable
1: if link a is on expressway path k between OD pair r-s during time period n
0: otherwise
G_{rs}^{n} : OD flow between OD pair r-s during time period n, which departs from origin during time period n
a, b : parameters of averaged travel time (refer to next section), constant values

By using the Lagrange function of minimization problem above, we can obtain the optimality conditions. Equation(3) is almost the same formulation (Sasaki M. et al. 1987) as the minimization problem of general UE assignment with Variable Demand. Therefore, By applying Kuhn-Tucker conditions with regard to path flows, f_{rsk}^{an} and f_{rsk}^{en}, to the Lagrange function, we can obtain equilibrium solutions that satisfy both user equilibrium conditions for the real travel time on arterial path and for the generalized travel time on expressway path. By applying Kuhn-Tucker conditions with regard to Q_{rs}^{an} and Q_{rs}^{en} , we can induce the diversion function of equation(2). By applying Kuhn-Tucker conditions with respect to g_{rs}^{n} , the modified OD flow between OD pair r-s during time period n is given by

$$g_{rs}^{n} = q_{rs}^{n-1} + G_{rs}^{n} - \frac{a + bS_{rs}^{n}}{2T} G_{rs}^{n} \tag{4}$$

where,

$a + bS_{rs}^{n}$: averaged real travel time for the arterial path and the expressway path between OD pair r-s during time period n. Where, S_{rs}^{n} is given by

$$S_{rs}^{n} = -\frac{1}{\theta} \ln(\exp(-\theta C_{rs}^{en} - \psi_{rs}) + \exp(-\theta C_{rs}^{an})) \tag{5}$$

By applying Kuhn-Tucker conditions to the Lagrange function, we can obtain g_{rs}^{n} in Equation (4) including ($a + bS_{rs}^{n}$). IF we provide proper parameters with a and b, the ($a + bS_{rs}^{n}$) is able to be adjusted to the average of real travel time (without toll) with respect to both arterial and expressway paths between an OD pair. The modification for remained flow should be made by real travel time without toll even on expressway path because the remained flow at the end of present time period never relates to the toll load for expressway.

Therefore, Equation (4) for TUEE corresponds to Equation (1) for TUE. In next Chapter, we will show the propriety about adopting the averaged real travel time to estimate modified OD flow and the method to provide parameters with a and b. Therefore, when we provide proper parameters, a and b, and solve TUEE model, we can obtain a set of modified OD flow, the remained traffic volume, arterial road OD flow, expressway OD flow and link flow. Furthermore, when the parameter θ is set to ∞, this model of Equation (3) can be adapted to a road network that has only arterial road without toll. Additionally, it is proved that TUEE has a unique solution in the previous paper (Fujita M. et al. 2001).

(3) parameters(a , b) for the averaged real travel time

a) Concept for estimating parameters

S_{rs}^{n} of Equation(5) is the expected perceived travel time with respect to the generalized travel time for both arterial and expressway path. In the third term of the right-hand side of Equation (4), remained OD flow during present time period n is subtracted from OD flow, G_{rs}^{n}. Since the remained OD flow is calculated as physical quantity, the ($a + bS_{rs}^{n}$) of Equation (4) must be real travel time without toll. Now, the third term of the right-hand side of Equation (4) would be divided into two remained OD flows on arterial path and expressway path as follows:

$$-\frac{a + bS_{rs}^{n}}{2T} G_{rs}^{n} =$$

$$-\frac{(C_{rs}^{en} - C_{rs}^{d})}{2T} \frac{\exp(-\theta_{rs} C_{rs}^{en} - \psi_{rs})}{\exp(-\theta_{rs} C_{rs}^{en} - \psi_{rs}) + \exp(-\theta_{rs} C_{rs}^{an})} G_{rs}^{n} - \frac{C_{rs}^{an}}{2T} \frac{\exp(-\theta_{rs} C_{rs}^{an})}{\exp(-\theta_{rs} C_{rs}^{en} - \psi_{rs}) + \exp(-\theta_{rs} C_{rs}^{an})} G_{rs}^{n} \tag{6}$$

$$= -\frac{1}{2T} G_{rs}^{n} \times C_{rs}^{n}$$

where,

$$C_{rs}^{n} = \frac{(C_{rs}^{en} - C_{rs}^{d})\exp(-\theta_{rs}C_{rs}^{en} - \psi_{rs}) + C_{rs}^{an}\exp(-\theta_{rs}C_{rs}^{an})}{\exp(-\theta_{rs}C_{rs}^{en} - \psi_{rs}) + \exp(-\theta_{rs}C_{rs}^{an})} \tag{7}$$

C_{rs}^{d} : toll for expressway path between OD pair r-s

C_{rs}^{n} : averaged real travel time with respect to expressway path and arterial path , weighted with ratio of OD flows. Therefore, the parameter a,b must be given by

$$a + bS_{rs}^{n} \cong C_{rs}^{n} \tag{8}$$

If the parameter a,b are satisfied with Equation(8), then the inverse of tree-structure of Figure 1 must be the same modification of OD flows as the structure of Figure 1. Because the remained flow of inverse tree-structure explained in section 3(1) a) is calculated by using the right hand side of Equation (6).
Therefore, we will obtain the same solution under both the tree-structures of models.

b)Method to estimate parameter

S_{rs}^{n} in Equation(8) is defined based on the generalized travel time including toll. On the other hand, C_{rs}^{n} in Equation (7) is defined based on real travel time without toll. Therefore, although it may be difficult that the S_{rs}^{n} perfectly corresponds to the C_{rs}^{n}, we consider the method estimating parameters that approximately make correspondence between S_{rs}^{n} and C_{rs}^{n} for practical use.
When we transform the Equation (2a) of diversion function, C_{rs}^{en} is given by

$$C_{rs}^{en} = \frac{1}{\theta}(\ln(\frac{Q_{rs}^{an}}{Q_{rs}^{en}}) - \psi) + C_{rs}^{an} \tag{9}$$

By using Equation (9), we can obtain $a + bS_{rs}^{n}$ and C_{rs}^{n} from Equation (5) and (7) as follows

$$a + bS_{rs}^{n} = a + b(C_{rs}^{an} - \frac{1}{\theta}\ln(\frac{Q_{rs}^{en}}{g_{rs}^{n} - Q_{rs}^{en}} + 1)) \tag{10}$$

$$C_{rs}^{n} = -\frac{Q_{rs}^{en}}{\theta_{rs}g_{rs}^{n}}(\ln\frac{Q_{rs}^{en}}{g_{rs}^{n} - Q_{rs}^{en}} + \psi_{rs}) - \frac{Q_{rs}^{en}}{g_{rs}^{n}}C_{rs}^{d} + C_{rs}^{an} \tag{11}$$

Here, Figure 2 shows the relationship between S_{rs}^{n} and C_{rs}^{n} with respect to the changes of rate of expressway OD flow (Q_{rs}^{en} / g_{rs}^{n}) when L=20km, C_{rs}^{an}=30 minutes and C_{rs}^{d}=10 minutes. From the Figure 2, the difference between S_{rs}^{n} and C_{rs}^{n} is not so large, but trends to increase as increasing of expressway OD flow rate.
Here, we set b = 1 in order to use the same C_{rs}^{an} in both Equation (10) and (11) because the C_{rs}^{an} has the biggest influence on the averaged real travel time of C_{rs}^{n}.
When b=1, the parameter of a that satisfies Equation (8) is given by

$$a = -\frac{Q_{rs}^{en}}{g_{rs}^{n}\theta_{rs}}(\ln(\frac{Q_{rs}^{en}}{g_{rs}^{n} - Q_{rs}^{en}}) + \psi_{rs}) - \frac{Q_{rs}^{en}}{g_{rs}^{n}}C_{rs}^{d} + \frac{1}{\theta_{rs}}\ln(\frac{Q_{rs}^{en}}{g_{rs}^{n} - Q_{rs}^{en}} + 1) \tag{12}$$

From Equation (12), we can obtain the parameter a when the expressway OD flow rate, (Q_{rs}^{en} / g_{rs}^{n}), is given. And the parameter a has the characteristics shifting the function ($a + S_{rs}^{n}$) up and down. The deference of the $a + bS_{rs}^{n}$ and the C_{rs}^{n} must be reduced if we set proper parameter a and shift the function ($a + S_{rs}^{n}$) appropriately. Although the OD flow rate is given after assignment calculation, it is necessary for the parameter a to be given as an initial value before assignment calculation. Thus, we consider a method of providing proper initial parameter a in Chapter 4. Consequently, we have to examine this method by applying it to a real-scale road network in Chapter 4.

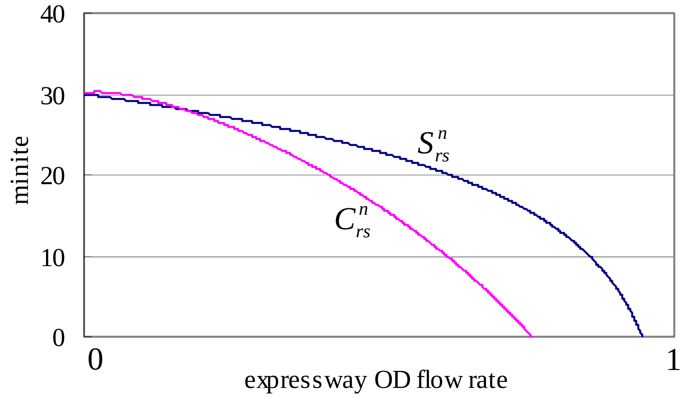

Figure 2 Relationships of S_{rs}^n and C_{rs}^n with respect to expressway OD flow rate

4. **APPLICATION TO NAGOYA METROPOLITAN ROAD NETWORK**

(1) Link- cost function including expressways

In this paper we adopt a BPR type of link-cost function including toll load. The link-cost function is given by

$$t_a = t_{a0}\left\{1+\alpha\left(\frac{x_a}{C_a}\right)^{\beta}\right\} + tc_a / \gamma \qquad (13)$$

The second term of right-hand side above is the term of toll load for expressways. The second term is removed when the link-cost function is applied to arterial road (without toll load).
where,

t_a : travel time of link a	t_{a0} : travel time of link a at $x_a = 0$
x_a : link flow on link a	C_a : capacity of link a
γ : value of travel time(yen/min)	tc_a : toll load of link a (yen)
α, β : parameters	

We utilize the parameters estimated in the previous paper (Fujita M. et al. 2000) into the above parameters of t_{a0} and α, β. The value of travel time γ is set to 65 yen/minute because it was obtained by the questionnaire survey. Since the expressways in urban area have a uniform toll, we load the toll at only each on-ramp link. The toll of expressways in the suburbs is given by using the Equations (14) and (15).

Tomei expressway and Meishin expressway: (The number of data : 29, R = 0.99)

$$y = 32.85x + 144.04 \qquad (14)$$

where, y: toll(yen), x: distance of expressway link (km)

Higashi-meihan expressway : (The number of data : 7, R = 0.99)

$$y = 30.22x + 140.78 \qquad (15)$$

Moreover, the toll is estimated in consideration of the ratio of large-size car to all cars and the average of tolls of each type of cars.

(2) **Time of day diversion function**

We adopt Equations (16) and (17) estimated by using regression analysis (Fujita M. et al. 2001), for the parameters θ, ψ of diversion function in Equation (2a, 2b).

$$\theta(L_{rs}) = cL_{rs}^{\ d} \tag{16}$$

$$\psi(L_{rs}) = u \ln(L_{rs}) + v \tag{17}$$

where

L_{rs} : distance between OD pair r-s (km)

c, d, u, v : regression coefficients

Table 1 shows the regression coefficients (c, d, u and v) of the $\theta(L_{rs}), \psi(L_{rs})$ during morning hours (6:00am-11:00am) and the correlation coefficients.

	c	d	u	V	Correlation coefficient
Am 6:00-9:00	5.266	-1.334	-0.468	3.120	0.954
Am 9:00-11:00	2.353	-0.918	-0.066	0.820	0.946

Table 1 Parameters of Time of day diversion function

(3) The analysis and consideration

TUEE model is now demonstrated through the application to Nagoya metropolitan road network. The network is the road network with 279 centroids, 1241 nodes and 4209 links. Hourly OD matrices are given by the survey in 1996, which was rectified the master tape of person trip survey executed in 1991. According to the assumption 1, the size of study time period, T, is set to 120 minutes because some travel times between OD pairs on the study network exceed 60 minutes. However, we compare the assignment result of TUEE(T=60) that set T to 60 minutes, with the TUEE(T=120) above. We start the sequence of time-of day assignments from 5:00 A.M in each case of TUEE.

In the comparison of two TUEE models (T=60 and T=120), hourly link flows assigned in TUEE (T=60) are summed up as two-hourly link flows. The number of ground counts for the comparisons are 128 data on arterial roads and 41 data on expressways obtained from road traffic census executed in 1994. The accuracy of assignment result will be examined by using the RMS errors between estimated link flows and observed link flows.

a) The convergence of objective function

Figure 3 shows the convergence of objective function. From the figure the objective function smoothly converges. It almost converges around twenty iterations.

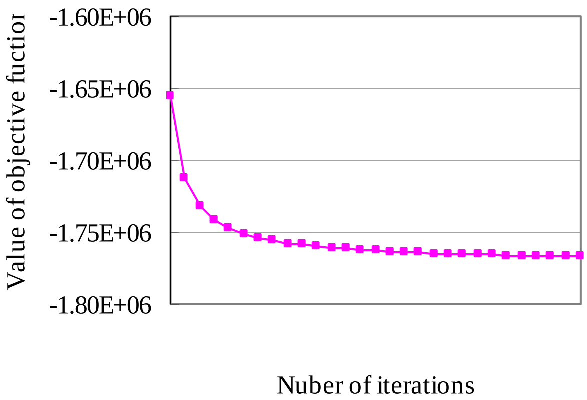

Figure 3 the convergence of the value of objective function

b) Comparisons in differences of initial values and sizes of study time period

As explained in Chapter 3(3), we examine the accuracy of TUEE by difference of the method providing parameters, a and b, and the size of study time period, T, through applications to the road network.

We examine the following two initial settings for parameters, a and b.

Initial setting-1: We uniformly set the remained flows of arterial path and expressway path in each time period. Thus we calculate parameter a in Q_{rs}^{en} / g_{rs}^{n} = 1/2. We set b = 1 according to Chapter3 (3)(b).

Initial setting-2 : This setting method is as follows: Firstly, we obtain the C_{rs}^{d}, C_{rs}^{en} and C_{rs}^{an} by calculating the minimum path algorithm under the condition that all link flows on a study network are set to 0. Secondly, we estimate the parameter a by using Equation (12) and Equation (2) substituted the C_{rs}^{d}, C_{rs}^{en} and C_{rs}^{an} for.

Table 2 shows the comparison of the accuracy of estimated link flows in two initial settings through the TUEE (study time period: 7:00am-9:00am,T=120minutes). Similarly, Table 3 shows the comparison of the accuracy in two initial settings through the TUEE (study time period: 7:00am-9:00am,T=60minutes).

From the tables, the model of T=120 has better accuracy than the model of T=60 in both initial settings. This result indicates that the model of T=120 has more validity than the model of T=60 toward the modification of remained flows because some travel times between OD pairs on the study road network exceed 60 minutes.

The comparison of two initial settings in Table 2 and 3 also shows that the model of initial setting-2 has more accuracy than the model of initial setting-1 in both cases of T=120 and T=60. This result indicates that the initial setting-2 is able to estimate the ratio of expressway OD flow and the averaged real travel time, C_{rs}^{n}, as taking account of the characteristics of each OD pair such as distance and travel time in the initial calculation.

Initial setting-1 has less accuracy than initial setting-2. However, in comparison with the difference of accuracy between the cases of T=120 and T=60, the difference of accuracy between two initial settings is very small. Therefore the difference of initial settings dose not much affects the accuracy of TUEE. From the result above, the analysis below for TUEE is examined under the conditions of T=120 and initial setting-2.

Table 2 Comparison of TUEE during peak hours (RMS errors)
<T= 120 minutes>

	Expressways	Arterial roads	sum
Initial setting-1	509.1	1468.7	1977.8
Initial setting-2	518.8	1423.1	1941.9

Table 3 Comparison of TUEE during peak hours (RMS errors)
<T= 60 minutes>

	Expressways	Arterial roads	sum
Initial setting-1	643.4	1814.1	2457.5
Initial setting-2	630.5	1776.8	2407.3

c) The analysis of TUEE

Table 4 shows comparisons between estimated link flows and observed link flows by the RMS errors. Figure 4 shows the relationships between estimated and observed link flows during the time period, 7:00am-9:00am. We'll show that TUEE has good accuracy more than the conventional model (basic TUE).

Table 4 The assigned result of TUEE during peak hours (RMS errors)

<T= 120 minutes>

	Expressways	Arterial roads	sum
7:00-9:00	518.8	1423.1	1941.9
9:00-11:00	412.5	874.6	1287.1
sum	931.3	2297.7	3229

<T= 60 minutes>

	Expressways	Arterial roads	sum
7:00-9:00	630.5	1776.8	2407.3
9:00-11:00	512.4	1008.8	1521.2
sum	1142.9	2785.6	3928.5

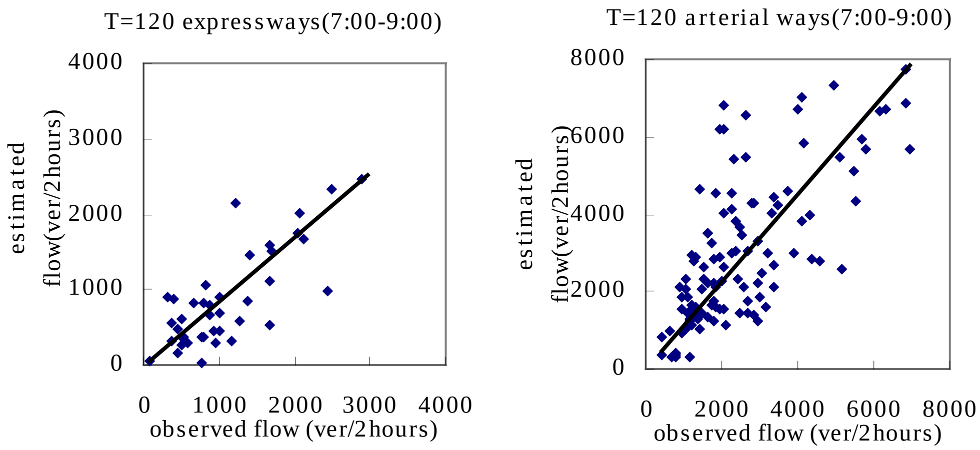

Figure 4 The relationships between observed and estimated link flows for TUEE (T=120)

d) The effect of OD-flow modification method and diversion function

In order to confirm the effect of OD-flow modification and the effect of diversion function, we now compare the assigned result of the basic model (basic TUE) that dose not adopt the OD-flow modification method and the diversion function, with the assigned result of Table 4. The result of the basic TUE is shown in Table 5.

The comparison of basic TUE (Table 5, T=120) and TUEE(Table 4, the model of T=120) indicate that RMS errors of the basic TUE is 1.5 times as large as RMS errors of TUEE. From the results we would confirm the effects of OD-flow modification method and diversion function.

Table 5 The assigned result of the basic TUE during peak hours (RMS errors)

	Expressways	Arterial roads	sum
7:00-8:59	995.9	1919.5	2915.4
9:00-10:59	717.3	1087.9	1805.2
sum	1713.2	3007.4	4720.6

e) Examination of travel time

Figure 5 shows the relationships between observed and estimated travel times on paths (expressways path: 6 data, arterial path: 32 data). These data have been obtained by executing floating survey in the study network.

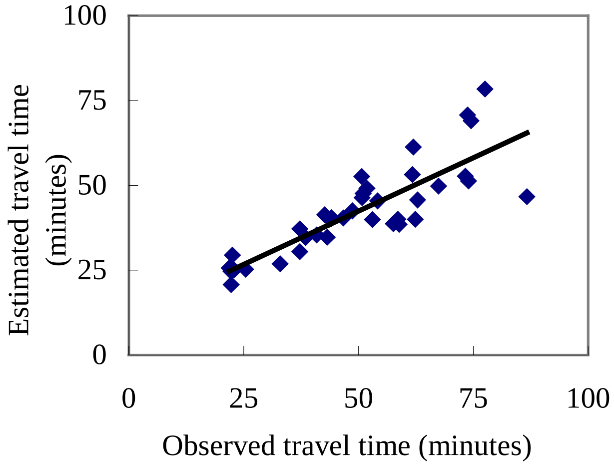

Figure 5 Relationships between observed and estimated travel times on paths

5. CONCLUSION

Through this paper we obtained the following results.

1) We formulated the TUEE model integrating the diversion function that determined the ratio of traffic demand passing expressways between each OD pair and could clearly consider the route choice behavior between expressway path and arterial path.
It was shown that TUEE would obtain a set of the equilibrium solutions which was satisfied with the OD flow ratio of diversion function between expressways path and arterial roads path, the user equilibrium principle of each paths and OD-flow modification method which is considered the remained traffic volume during present time period between the OD pair.
2) We would consider the method that estimates parameters to calculate remained traffic volumes. The validity of this method was confirmed through applications to the network.
3) The comparison of results of basic TUE and TUEE indicated that the effects of the OD-flow modification method and the diversion function were very high to estimation accuracy.
4) In the application of TUEE to the network, the accuracy of travel time was examined. It was shown that this model could also estimate travel times for paths with good accuracy. However, it is needed that several characteristics of network like link-cost functions are improved in order to become better accuracy.

Appendix.1 (OD-flow modification method for time-of-day traffic assignment)
We explain about the OD-flow modification method. We now suppose that path flow u_{rsk}^n on path k connecting OD pair r-s during time period n has been given by some ordinary traffic assignment technique in this paper we will use the assignment technique based on the UE principle. Not all path flows, u_{ik}^n, can pass through all links on path k by the end of time period n because some part of them still remain on that path at that time from assumption 2. Thus the section volume, $Y1_{ik}^n(j)$, which cannot pass the starting point of the j-th link by that time can be expressed as follows:

$$Y1_{ik}^n(j) = u_{ik}^n t_{ik}^n(j-1)/T \tag{18}$$

$t_{ik}^n(j-1)$ is the travel time from the origin to the end point of the previous link j-1 on path k between OD pair i. If link j is the last link on path k (;whose order number we will denote by m), then the distribution of $Y1_{ik}^n(j)$ along path k can be expressed by a triangle distribution with the height of $u_{ik}^n t_{ik}^n(m)/T$ as illustrated in Figure 1. Therefore, the section volume, $Y_{ik}^n(j)$, that can pass the starting point of the j-th link on path k connecting OD pair r-s during time period n is given by

$$Y_{ik}^{n}(j) = Y1_{ik}^{n-1}(j) + u_{ik}^{n} - Y1_{ik}^{n}(j) \tag{19}$$

The first term of the right-hand side of the above equation means the section volume that can not pass through the j-th link on path k during the previous time period n-1. The difference between the second term and the third term means the section volume that can pass through the j-th link on path k during the present time period n.

In the OD-flow modification method, the triangular distribution of $Y1_{ik}^{n}(j)$ is replaced by a rectangular distribution with the height of $u_{ik}^{n} t_{ik}^{n}(m)/2T$, as shown in Figure 6, which means that we level out $Y1_{ik}^{n}(j)$ along the path. Errors due to this replacement become the value of zero at the center of the path, the maximum negative value at the starting point of the path and the maximum positive value at the end of the path as shown in Figure 6. However, the adjustment in which $Y1_{ik}^{n-1}(j)$ in the previous time period n-1 is added and $Y1_{ik}^{n}(j)$ in the present time period n is subtracted, will function so as to make these errors smaller. We have already examined and verified the propriety of this replacement through several applications to the actual network in the previous papers. When we put

$$v_{ik}^{n} = u_{ik}^{n} t_{ik}^{n}(m)/2T \tag{20}$$

v_{rsk}^{n} means the remained volume on the path k connecting OD pair r-s during time period n. Since v_{ik}^{n} is independent of the link number, we only have to modify the path flow u_{ik}^{n}. And each travel time along the path, $t_{ik}^{n}(m)$, is equivalent to the minimum travel time , C_{rs}^{n}, between OD pair r-s during time period n because the OD-flow modification method is based on the assumption of the UE assignment principle. We take a sum of v_{ik}^{n} with respect to the paths, then

$$q_{rs}^{n} = \sum_{k} v_{ik}^{n} = \sum_{k} u_{ik}^{n} C_{rs}^{n}/2T = G_{rs}^{n} C_{rs}^{n}/2T \tag{21}$$

G_{rs}^{n} is the OD flow between OD pair r-s during time period n.

Therefore, considering remained volume in the OD-flow modification method becomes to modify OD-flow, G_{rs}^{n}, by using the remained flow q_{rs}^{n}. As a result, a modified OD flow for TUE between OD pair is given by the equation (1) in Chapter 2.

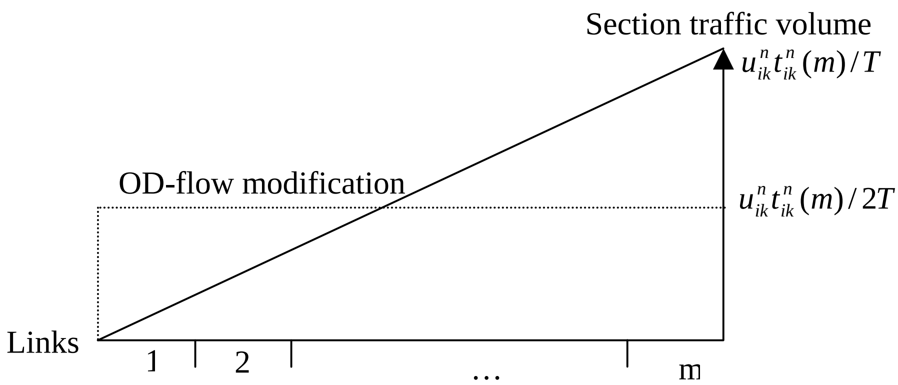

Figure 6 Some remained traffic volume on each link at the end of a study period

<u>Appendix.2</u> (calculation proceed)

In TUE we will use a calculation proceed based on the Frank-Wolf method (Evans S.P. ,1976)

Step 0 : Choose a time period during which demand flows would be fewest over a study network as an initial time period. Set n=1, $q_{rs}^{n-1} = 0$.

Step 1 : Set m=1. Find an initial feasible flow pattern $\{x_a^{n(1)}\}, \{g_{rs}^{n(1)}\}$

Step 2 : Compute the trip time between OD pair by $\{x_a^{n(1)}\}$. Find minimum expressways path and minimum arterial path, compute $\{C_{rs}^{an}\}$ and $\{C_{rs}^{en}\}$.

Step 3 : Compute the modified OD flow $g_{rs}^{n'}$ between OD pair r-s by EQS(4).

Step 4 : Compute $\{Q_{rs}^{en}{}'\}$ expressways OD flow between OD pair by diversion function.

Step 5 : Compute $\{x_a^{n}{}'\}$ using the all-or-nothing method under the condition that $\{Q_{rs}^{en}{}'\}$ is assigned to the minimum expressways path and $\{g_{rs}^{n}{}' - Q_{rs}^{en}{}'\}$ is assigned to the minimum arterial path.

Step 6 : By setting

$$g_{rs}^{n(m+1)} = \alpha g_{rs}^{n}{}' + (1-\alpha) g_{rs}^{n(m)}$$

$$Q_{rs}^{en(m+1)} = \alpha Q_{rs}^{en}{}' + (1-\alpha) Q_{rs}^{en(m)}$$

$$x_a^{n(m+1)} = \alpha x_a^{n}{}' + (1-\alpha) x_a^{n(m)}$$

find the optimal move size $\alpha^{(m)}$ and $\{ x_a^{n(m+1)}, Q_{rs}^{en(m+1)}, g_{rs}^{n(m+1)} \}$ that can minimize the objective function(3a) by one-demensional minimization.

Step 7 : $\{ x_a^{n(m+1)}, Q_{rs}^{en(m+1)}, g_{rs}^{n(m+1)} \}$ are updated.

Step 8 : If the following inequalities are held, go to Step 9. Otherwise, set m=m+1 and go to Step 2.

$$\sum_a (x_a^{n(m+1)} - x_a^{n(m)}) C(x_a^{n(m)}) \le \varepsilon_1$$

Step 9 : IF $\{ x_a^{n(m+1)}, Q_{rs}^{en(m+1)}, g_{rs}^{n(m+1)} \}$ in the overall study period are computed, terminate. Otherwise compute q_{rs}^{n}, set m=m+1 and go to Step 1.

REFERENCES

1) M.J. Beckmann, C.B. McGuire and C.B. Winsten : Studies in the economics of transportation, Yale University Press, New Haven, 1956.
2) Nagoya Expressway Public Corporation and Nagoya Expressway Society : The positive investigation research committee report about the traffic assignment method of Nagoya expressway, 1996
3) M. Fujita and H. Matsui : Development of an user equilibrium assignment model for an urban road network including expressways, Journal of infrastructure planning and management, No.653/4-48, pp.85-94, 2000.
4) M. Fujita, H. Matsui, and S. Mizogami : Modelling of the time of day traffic assignment over a traffic network, Journal of infrastructure planning and management, No.389, pp.111-119, 1988.
5) T. Akamatsu and Y. Matsumoto : Stochastic network equilibrium model with elastic demand and its solution method, Journal of infrastructure planning and management, No.401.pp109-118,1989
6) M.Sasaki and Y. Asakura, : An optimal road network design model with variable OD travel demand, Journal of infrastructure planning and management, No.383.pp93-102, 1987
7) M. Fujita, Y. Ujii and H.Matsui : Development of Time-of-Day Traffic Equilibrium Assignment Model Including Expressways, Infrastructure planning review, Vol.18, PP563-572, 2001.
8) Evans S.P. : Derivation and Analysis of Some Models for Combining Trip Distribution and Assignment, Transportation Research 10(1), pp.37-57, 1976.