

# The Background Noise Estimation in the ELF Electromagnetic Wave Data Using Outer Product Expansion with Non-linear Filter

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**SUMMARY** This paper proposes a background noise estimation method using an outer product expansion with non-linear filters for ELF (extremely low frequency) electromagnetic (EM) waves. We proposed a novel source separation technique that uses a tensor product expansion. This signal separation technique means that the background noise, which is observed in almost all input signals, can be estimated using a tensor product expansion (TPE) where the absolute error (AE) is used as the error function, which is thus known as TPE-AE. TPE-AE has two problems: the first is that the results of TPE-AE are strongly affected by Gaussian random noise, and the second is that the estimated signal varies widely because of the random search. To solve these problems, an outer product expansion based on a modified trimmed mean (MTM) is proposed in this paper. The results show that this novel technique separates the background noise from the signal more accurately than conventional methods.

**key words:** background noise reduction, tensor product expansion, electromagnetic wave, signal processing

## 1. Introduction

Multivariate analysis is well known as a powerful tool for extraction of the features of multi-sensor signals. Major second-order statistical techniques like principal component analysis (PCA) can produce some of the global features of various data. However, signal separation using PCA is difficult.

A tensor product expansion (TPE) [1], [2] can approximate an  $m$ -variable function as a sum of the products of  $m$  single variable functions (SVFs). This technique has been applied to non-linear system identification [3] and 3D image processing [4], and has produced substantial results. The tensor is calculated by minimization of the L2-norm between the input vector and the sum of products of the SVFs. Assuming that an input signal expressed using a 2D matrix is composed of the background noise and the local signals, signal separation using the TPE is difficult since the local signals are treated as outliers. The main cause of this prob-

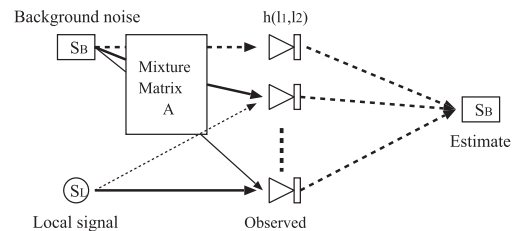


Fig. 1 The TPE-AE model.

lem is that the separated signal is strongly affected by the local signals due to the L2-norm. To achieve signal separation based on robustness, the L1-PCA was proposed by Ke [5]. This technique uses the weighted median method and convex programming methods. To calculate the outer product that minimizes the L1-norm, we have shown that a tensor product expansion with absolute error (TPE-AE) [6] is effective for separation of the local signals from the background noise; the former are observed in only a few signals (Fig. 1). The TPE-AE also uses the L1-norm, on which outliers have little influence, to derive the outer product and the bias component from the observed data. However, we need to define the optimal variance of a random number in a Monte Carlo simulation (MCS), since a large variance does not give us the correct solutions, while smaller numbers yield the local minimum. In addition, MCS cannot estimate the background noise uniformly because of random search.

To solve these problems, an outer product expansion using a median filter (the median method) has been proposed [8]. We showed that the median method can estimate the background noise more uniformly than TPE-AE. However, in both TPE-AE and the median method, the background noise, which is observed concurrently with local signals, is not estimated accurately due to an effect of the random noise.

In this paper, we propose a signal separation technique based on the modified trimmed mean (MTM) method, which is an iterative algorithm used to overcome the weaknesses of TPE-AE and the median method. TPE-AE and the median method both focus on the minimization problem of the L1-norm between the input matrix and its outer product. The MTM method does not have any of these criteria. Results have shown that the MTM method can separate the background from the local signal without any shrinkage of

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either component.

## 2. Algorithm for Background Noise Estimation

### 2.1 TPE-AE

We assume that the observed signal consists of two source signals, where one is observed in most signals (i.e., the background noise) and the other is seen in just a few signals (i.e., a local signal). In this case, the latter signal can be considered to be an outlier. It is only necessary to extract either a few outliers or the background noise from the observed signals (Fig. 1), while a major blind source separation method such as independent component analysis often estimates  $u$  sources from  $u$  input signals.

To separate the background noise from a local signal, the L1-norm was used as a criterion to calculate the outer product [5], [6]. Let  $h(l_1, l_2)$  be an input 2D matrix consisting of observation signals, where the tensor of the 2D matrix calculated by TPE-AE is given as:

$$J = \sum_{l_1=1}^{q_1} \sum_{l_2=1}^{q_2} |h(l_1, l_2) - (f_1(l_1)f_2(l_2) + f_3(l_2))| \quad (1)$$

where  $J$  is the error function,  $f_1(l_1)f_2(l_2)$  and  $f_3(l_2)$  are approximation terms for the AC and DC components, respectively.  $l_1$  represents the time,  $l_2$  indicates the index of the sensor,  $q_1$  is the length of the signal, and  $q_2$  is the number of observation signals. Conventional research shows that  $f_1(l_1)$ ,  $f_2(l_2)$  and  $f_3(l_2)$  which yield a minimum  $J$  give us the background noise that is included in the input matrix  $h(l_1, l_2)$ . Using TPE-AE, the estimated background noise is represented by  $f_1(l_1)f_2(l_2) + f_3(l_2)$ . Thus, the local signal can be calculated by subtracting  $f_1(l_1)f_2(l_2) + f_3(l_2)$  from  $h(l_1, l_2)$ .

### 2.2 Median Method

The median is applied to minimize the L1-norm shown in (1). A median filter is generally used to reduce the impulse noise in image data (Fig. 2). Assume that  $f_2(l_2)$ ,  $f_3(l_2)$  have constant values. We calculate the optimum  $f_1(l_1)$  to minimize the criterion of (1). The absolute error  $J_{f_1}$  at  $l_1 = m$  is given as:

$$J_{f_1} = \sum_{l_2=1}^{q_2} |h(m, l_2) - (f_1(m)f_2(l_2) + f_3(l_2))|. \quad (2)$$

Consider the data set of  $X = (x_1, x_2, \dots, x_{q_2})$ . The middle value of  $X$  is the point  $\hat{x}$  that minimizes the following L1-norm [7]:

$$\sum_{i=1}^{q_2} |x_i - \hat{x}|. \quad (3)$$

From this property,  $f_1(m)f_2(l_2) + f_3(l_2)$ , which yields minimum  $J_{f_1}$ , is given by the middle value of  $h(m, l_2)$ . In this case,  $J_{f_1}$  depends on the  $f_1(m)$  since  $f_2(l_2)$ ,  $f_3(l_2)$  are fixed.

$f_1(m)$ , which yields  $J_{f_1} = 0$  at  $l_2 = n$ , is expressed as:

$$f_1(m) = (h(m, n) - f_3(n))/f_2(n). \quad (4)$$

Then, we get  $q_2$  solutions from (2) and (4). From these property, the middle value of  $f_1(m)$  decreases  $J_{f_1}$  in (2). Therefore, we have the following updated expression:

$$f_1(l_1) = \text{med}_{l_2}((h(l_1, l_2) - f_3(l_2))/f_2(l_2)) \quad (5)$$

where  $\text{med}_{l_2}()$  outputs the median in the window  $l_2$  with a width of  $q_2$ . Similarly, updated algorithms for  $f_2(l_2)$ ,  $f_3(l_2)$  are obtained using the following expressions.

$$f_2(l_2) = \text{med}_{l_1}((h(l_1, l_2) - f_3(l_2))/f_1(l_1)) \quad (6)$$

$$f_3(l_2) = \text{med}_{l_1}(h(l_1, l_2) - f_1(l_1)f_2(l_2)). \quad (7)$$

Expressions (5)–(7) give the background noise through extensive trials using a median. First,  $f_1(l_1)$ ,  $f_2(l_2)$ ,  $f_3(l_2)$  are initialized using a small random number. The outer product used to estimate the background noise is then found via the following steps:

- I. Estimate  $f_3(l_2)$  using (7)
- II. Estimate  $f_2(l_2)$  using (6)
- III. Estimate  $f_1(l_1)$  using (5)

Termination criteria for the iterative steps given above must be decided. The iteration time is referred to as  $K_{MED}$ . The effectiveness of this iterative approach is confirmed by using artificial signals [8].

### 2.3 MTM Method

Trimmed means were proposed as a compromise between the moving averages and the medians [9], [10]. The MTM defines the amount of trimming of the signals, depending on the current window. The input data, which are closer than a given distance  $R$  from the middle value, are averaged by the MTM filter (Fig. 2). The calculation of the MTM filter is given as follows:

$$\text{MTM}(x_{l_1}) = \frac{1}{|I_{l_1}|} \sum_{i \in I_{l_1}} x_{l_1+i} \quad (8)$$

$$I_{l_1} = \{i = -k, \dots, k : |x_{l_1+i} - \hat{\mu}_{l_1}| \leq R\}$$

$$\hat{\mu}_{l_1} = \text{med}(x_{l_1-k}, \dots, x_{l_1+k}), l_1 \in \mathbf{Z}.$$

If  $R = 0$ , (8) outputs the median, while  $R = \infty$  gives the moving average. For  $R$ , the median absolute derivation

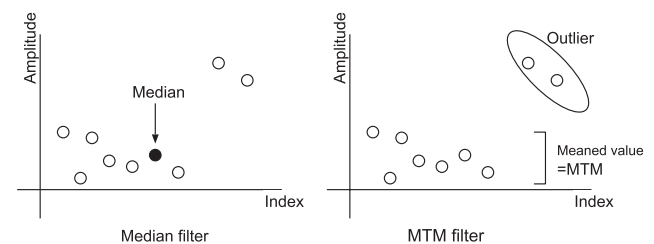


Fig. 2 Non-linear filters.

(MAD) or the standard deviation are often used to achieve a robust estimate. This paper applies the MAD, which is expressed as:

$$R = 2c_n \cdot \text{med}(|x_{l_1-k} - \hat{\mu}_{l_1}|, \dots, |x_{l_1+k} - \hat{\mu}_{l_1}|). \quad (9)$$

$c_n$  is the important factor for definition of the trimming threshold  $R$ .  $c_n$  is often defined at 1.5 to trim the outlier. The MTM provides a reliable and accurate estimation since the outliers, which are at the extremes of the sorted list, are not averaged.

We assume that  $\text{MTM}_{l_i}()$  outputs the MTM using (8) in a window  $l_i$  of width  $q_i$ . First,  $f_1(l_1)$ ,  $f_2(l_2)$ ,  $f_3(l_2)$  are initialized using a small random number. The updated expressions for  $f_1(l_1)$ ,  $f_2(l_2)$ ,  $f_3(l_2)$  based on the MTM are expressed as:

$$f_1(l_1) = \text{MTM}_{l_2}((h(l_1, l_2) - f_3(l_2))/f_2(l_2)) \quad (10)$$

$$f_2(l_2) = \text{MTM}_{l_1}((h(l_1, l_2) - f_3(l_2))/f_1(l_1)) \quad (11)$$

$$f_3(l_2) = \text{MTM}_{l_1}(h(l_1, l_2) - f_1(l_1)f_2(l_2)). \quad (12)$$

The outer product, which approximates the background noise, is calculated using an iterative algorithm based on (10)–(12). The iteration time for this algorithm is referred to as  $K_{MTM}$ . We call this iterative algorithm the MTM method.

### 3. Artificial Signal Separation

#### 3.1 Artificial Signal

The simulation results from the MTM method demonstrate the method's effectiveness for background noise reduction. A simple artificial signal based on known functions is applied to both the conventional and proposed methods. The artificial signal is composed of background noise and a local signal, which are generated using a sine wave and a block pulse, respectively. The separability conditions of TPE-AE were confirmed in [6]. Table 1 lists the conditions for the input matrix  $h(l_1, l_2)$  shown in Fig. 3, where  $l_1$  represents the index of the time courses, and  $l_2$  is the index of artificial signals A-E (0 to 4). The vertical axis in Fig. 3 shows the amplitude of the artificial signal, and the horizontal axis indicates the cycle of the sine wave ( $l_1/576$ ). In the input matrix, two of the signals include a local signal in cycles 7 to 10.

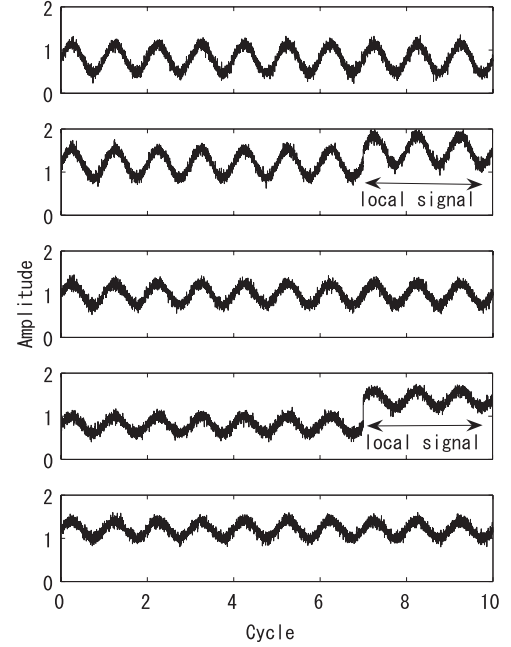
#### 3.2 Definition of SNR

In conventional research, only one of the strengths of the zero-mean Gaussian noise is applied as the observation noise. It would be interesting to see how the outer product expansions are able to reduce noise interference. For that purpose, different levels of Gaussian noise are added to the artificial signal. The noise levels are expressed in terms of the signal-to-noise ratio (SNR) in decibels (dB). We assume that  $h$  and  $\hat{h}$  are the input signal without the observation noise and its averaged value, respectively. The SNR is represented as follows:

**Table 1** The artificial signal conditions.

|   | Background noise                        | Block pulse |
|---|---|-------------|
| A | $\sin(2\pi/576)/3.0 + N(0.8, \sigma^2)$ | none        |
| B | $\sin(2\pi/576)/3.0 + N(1.2, \sigma^2)$ | 0.3         |
| C | $\sin(2\pi/576)/4.0 + N(1.0, \sigma^2)$ | none        |
| D | $\sin(2\pi/576)/5.0 + N(0.8, \sigma^2)$ | 0.6         |
| E | $\sin(2\pi/576)/5.0 + N(1.2, \sigma^2)$ | none        |

$N(\mu, \sigma^2)$  means that Gaussian noise,  $\mu$  is the mean, and  $\sigma^2$  is the variance



**Fig. 3** 5 artificial signals for the outer product expansion  $\sigma = 0.05$ .

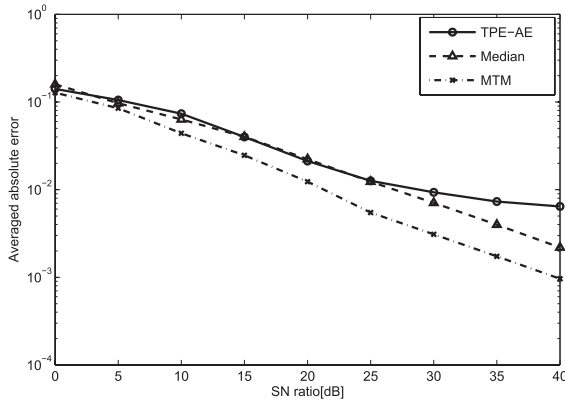
$$\text{SNR} = 10 \log \frac{(h - \hat{h})^2}{N(\mu, \sigma^2)^2} \quad (13)$$

where  $N(\mu, \sigma^2)$  is the additive white Gaussian noise (AWGN), for which the mean and the variance are  $\mu$  and  $\sigma^2$ , respectively. The TPE-AE, the median method and the MTM method are then all applied to the noisy artificial signal. Averaged absolute errors for various noise levels are calculated using these methods.

#### 3.3 Estimation Results

Based on empirical results, the number of iterations required for the median and MTM methods ( $K_{MED}$ ,  $K_{MTM}$ ) is 10, and  $c_n$  is 1.7 [9]. The number of iterations required for TPE-AE is defined as 1000.

Detailed results are shown in Fig. 4 for 3 background noise reduction processes performed by imposing various numbers for the SNR used in the AWGN. Figure 4 shows the mean-value absolute errors for 100 trials using different initial numbers. The vertical axis shows the error, while the horizontal axis shows the SNR. The error is defined as the absolute averaged error between the estimated signal



**Fig. 4** Results of noise reduction of artificial signals: SNR vs. averaged absolute error for the background signals.

$(f_1(l_1)f_2(l_2) + f_3(l_2))$  and the actual background noise (sine wave).

From Fig. 4, the error of the MTM method is shown to be lower than that of TPE-AE and of the median method in the range from 0 dB to 40 dB. This is attributed to the fact that the MTM method can estimate the background noise from signals that include the local signals.

#### 4. Background Noise Reduction in Extremely Low Frequency EM Waves

##### 4.1 Electromagnetic Wave Observation

The features of the EM wave data to evaluate the denoising performance of the median and MTM methods are expressed here.

Attention is currently being paid to the EM waves emitted from the earth's crust in advance of the occurrence of earthquakes and volcanic activity [11]. The main focus of this interest is the EM waves in the extremely low frequency (ELF) band [12]. We have observed the EM wave radiation in the ELF band (223 Hz) as represented by the east-west, north-south, and vertical magnetic field components at about forty observation stations located in Japan. Each observation station captures the magnetic field components and averages them over 6 s intervals. A total of 14,400 data points are thus collected per day for each component. In this paper, we use this data averaged over 150 s for convenience (i.e. 576 points per day per component). Since various noise components, including sensor noise, man-made noise, atmosphere noise and magnetosphere noise are present in the captured data, the raw data must be processed very accurately.

Anomalous EM wave emissions in the ELF band that have been reported as portents of an impending earthquake have been attributed to changes that are occurring in the earth's crust. Our research is directed towards identifying the seismic radiation that occurs before great earthquakes [12], [13]. Accurate precursor phenomena for earthquakes are required to enable extraction of the precursor emissions.

However, the EM wave data include undesired components that are associated with noise. EM waves observed in the ELF band consist of two kinds of components. The first is global radiation, which has daily trends and shows similarities across all stations. The global radiation in the observed data, such as that caused by fluctuations in the magnetosphere or the ionized layer and by lightning radiation from the tropics, becomes the background noise. The other is the local radiation, which is observed in just a few sensors. The EM waves emitted from the Earth's crust and from lightning in the near field are observed as local signals. If the background noise in the EM waves can be estimated, then the seismic radiation will be detected adequately. The purpose of our EM wave analysis is to reduce the background noise included as part of the observed data. The analysis is an important task to enable extraction of the anomalous radiation that is related to great earthquakes.

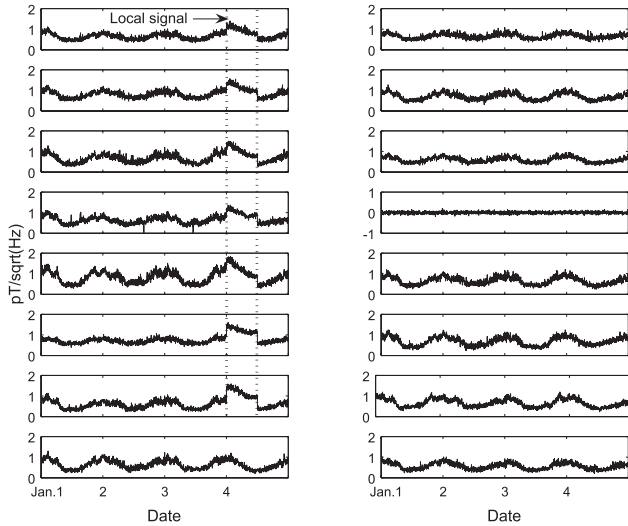
##### 4.2 Performance for Different Levels of Noise

In this section, we show how to evaluate the denoising performance of the outer product expansions for EM wave analysis. The absolute-value difference of the artificial local signal and the denoised signal is applied to evaluation of the denoising performance, since it is difficult to know the true background noise in an ELF EM wave.

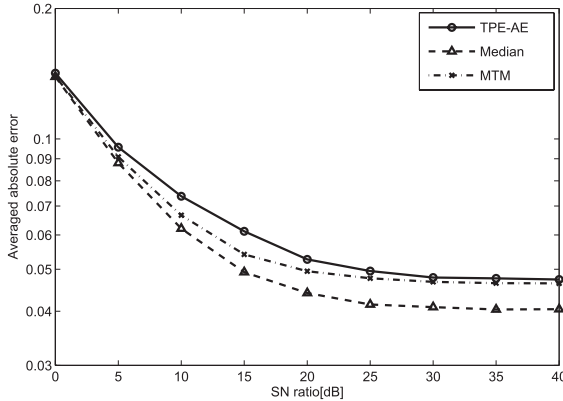
The input matrix used to apply the background noise reduction is composed of east-west components that were captured in the period from January 1st to 4th, 2001, at 16 antennas. The input data with a single missing value shown in Fig. 5, which contain a few anomalous signals, represent the EM data that include the artificial local signal expressed by the block pulse over half a day. The vertical axis of Fig. 5 represents the density of the magnetic flux ( $\text{pT}/\sqrt{\text{Hz}}$ ), and the horizontal axis indicates time.  $\text{pT}/\sqrt{\text{Hz}}$  indicates the strength of a magnetic field in pico Tesla normalized by the square root of a frequency. The right-hand side origin is set to 23:57:30 on January 4th, 2001. Daily trends can be observed in Fig. 5; the signal is weak in the daytime and strong at night. It is known that this trend is observed for all observation sites during a year [12]. This trend forms the dominant background noise in the ELF band EM wave. In addition, the input signals from the 1st to the 7th rows in the left column contain the artificial rectangular wave from 0:00 to 12:00 on January 4th. The averaged amplitude of 7 rectangular waves is defined as  $A$  assuming the weak seismic radiation. 7 amplitude values are generated by uniform random numbers  $[0, 1)$ . The mean value is adjusted to  $A = 0.45$  using the subtraction.

The denoising performance for Gaussian noise is shown here. The evaluation is described using the following steps:

1. Input signal is calculated using the EM data, block pulse and AWGN.
2. Estimate the background noise using 3 outer product expansions.



**Fig. 5** 16 EM waves with artificial local signals ( $A = 0.45$ ).



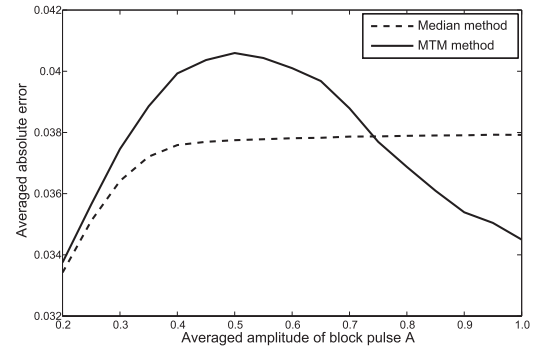
**Fig. 6** Effectiveness of the outer product expansions for EM waves: SNR vs. averaged absolute error for local signals.

3. Calculate the estimated local signal from the estimated background noise.
4. Calculate the absolute-value difference between the local signal and the estimated local signal (i.e. the absolute error).

To avoid the effects of the random numbers, 100 trials using the above steps were processed while using different initial numbers for each method and trial. The numbers of iterations required for the median and MTM methods and for TPE-AE are the same as those in Sect. 3.3.

Detailed results are shown in Fig. 6 for 3 background noise reductions performed by imposing various SNR (see Sect. 3.2). Figure 6 shows the mean-value absolute errors for 100 trials. The vertical axis shows the averaged absolute error, while the horizontal axis indicates the SNR. From Fig. 6, the errors of the median method are fewer than those of both TPE-AE and the MTM method from 0 dB to 40 dB. This is attributed to the fact that TPE-AE and the MTM method often give us poor results.

Figure 6 shows that the median method gives us a min-



**Fig. 7** Local signal estimation from EM waves: Averaged block pulse amplitude  $A$  vs. averaged absolute error for the block pulse.

imum absolute error under the various SNR (i.e., SNR = 0, 5, 10, ..., 40).

#### 4.3 Performance for Block Pulse

In this section, the denoising performance for various block pulse strengths is indicated. The steps taken for the evaluation are represented by following steps:

1. Input a signal consisting of EM data and a block pulse.
2. Estimate the background noise using the 3 methods.
3. Calculate the local signal from the estimated background noise.
4. Calculate the absolute-value difference between the local signal and the estimated local signal.

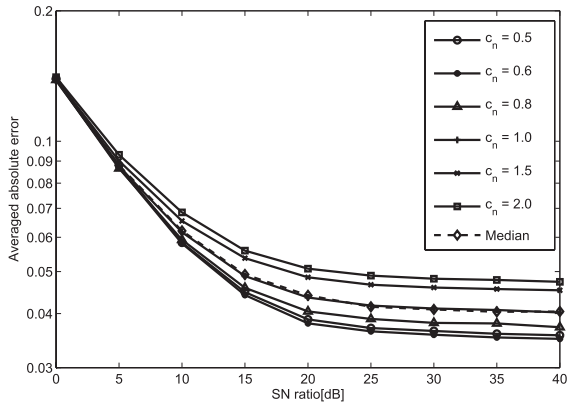
To avoid the effects of the initial values, 100 trials using the above steps were processed using different initial numbers for each method and block pulse strength.

To see how the outer product expansions are able to reduce noise interference, different levels of the artificial local signal are added to the EM data. The averaged amplitude  $A$  of a block pulse for 7 observation signals is defined as 0.2 to 1.0, assuming that the seismic radiation is continuous. The median method and the MTM method are then also applied to the noisy EM data. The averaged absolute errors are calculated for the different artificial signals. Based on an empirical result, the parameters for the median and MTM methods ( $K_{MED}$ ,  $K_{MTM}$ ) are set at 10, and  $c_n$  is defined to be 1.5 to exclude the outliers, which have an amplitude of  $3 \times \text{MAD}$  or more away from the median [9].

Detailed results are shown in Fig. 7 for two background noise reductions performed by imposing various numbers for the amplitude  $A$  used in the artificial local signal. Figure 7 shows the mean-value absolute errors from 100 trials using different initial numbers. The vertical axis shows the averaged absolute error, while the horizontal axis indicates the averaged amplitude  $A$ .

From Fig. 7, the averaged absolute errors of the median method are lower than those of the MTM method in the  $0.2 \leq A \leq 0.7$  range, while the MTM method yields the better results for  $0.75 \leq A$ . These results indicate that the median method gives the minimum absolute error for





**Fig. 8** Absolute error of the median method and the MTM method for various  $c_n$ .

lower-level local signals, while the MTM method can reduce the background noise in the EM wave more accurately than the median method at larger local signal levels. This is attributed to the fact that the MTM method gives us poor results for lower  $A$  since the MTM method cannot trim the effects of the local signal due to a large  $c_n$ .

## 5. MTM Thresholding

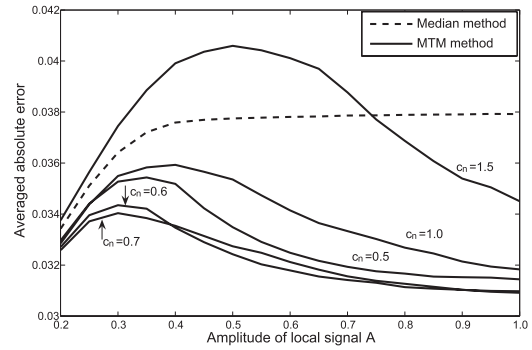
Figure 7 has shown that the median method can accurately estimate the background noise in ELF EM waves. However, Sect. 3 indicates that the MTM method can separate the background noise from a local signal more correctly than the median method. It is important to see how the median and MTM methods estimate the background noise using certain parameters. To achieve a more accurate background noise estimation, a suitable threshold  $R$  should be applied to the MTM method.

### 5.1 Suitable Threshold for Gaussian Noise

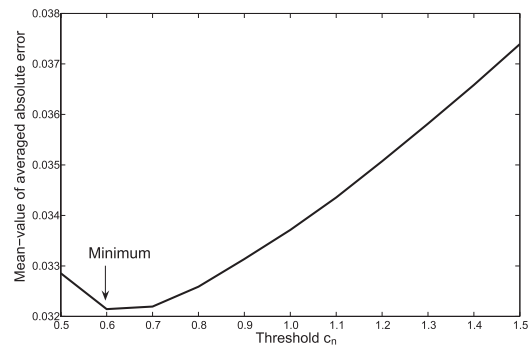
If the threshold  $R$ , which is not influenced by the random noise, is defined, then the background noise estimation accuracy will be improved. From (9),  $R$  can be adjusted using  $c_n$ . The performance of the MTM method with  $c_n = 0.5, 0.6, 0.8, 1.0, 1.5, 2.0$  and that of the median method are shown in Fig. 8. This figure shows the mean-value absolute errors for 100 trials using different initial numbers. The vertical and horizontal axes are the same as those shown in Fig. 6. From Fig. 8, the averaged absolute errors of the MTM method with  $c_n = 0.5, 0.6, 0.8$  are lower than the corresponding errors of the median method from 0 dB to 40 dB, while other thresholds yield poor results. It is also clear that the use of  $c_n = 0.6$  yields a better background noise reduction performance. When compared with the median method, the absolute error of the MTM method is reduced by  $5 \times 10^{-3}$  at 40 dB.

### 5.2 Suitable Thresholds for Various Block Pulse Levels

The performances of the MTM method with  $c_n =$



**Fig. 9** Absolute error of the median method and the MTM method for various  $c_n$ : Amplitude of the block pulse  $A$  vs. averaged absolute error for the block pulse.



**Fig. 10** Averaged absolute error for various values of  $A$ : Averaged absolute error for the block pulse vs.  $c_n$ .

0.5, 0.6, 0.7, 1.0, 1.5 and the median method are shown in Fig. 9. This figure shows the mean-value absolute errors for 100 trials. The vertical and horizontal axes are the same as those used in Fig. 7. From Fig. 9, the errors for the MTM method with  $c_n = 0.6, 0.7$  are lower than those with the other parameters and the median method for all values of the amplitude  $A$ . To show the optimum  $c_n$ , the averaged error values for all  $A$  at  $c_n$  are shown in Fig. 10. The vertical axis denotes the mean value of the averaged absolute error for all  $A$ , and the horizontal axis indicates  $c_n$ . From Fig. 10, it is also clear that  $c_n = 0.6$  yields better performance for background noise reduction in EM waves.

## 6. Concluding Remarks

We have demonstrated the denoising performances of background noise estimation methods (i.e., the median method and the MTM method) for both an artificial signal and ELF EM wave data. The simulation results indicate that the median method provides the better local signal estimation performance for the EM wave data. The other results show that the MTM method, when using a suitable trimming threshold, can estimate the background noise more accurately than the median method. These results show that the MTM method, when used with a small threshold, provides accurate background noise reduction for EM data analysis. Our future work will address extension of the MTM method to

the signal separation with time delays or the separation of 3 or more sources.

## References

- [1] D.O'Leary and S. Peleg, "Digital image compression by outer product expansion," *IEEE Trans. Commun.*, vol.CON-31, no.3, pp.441-444, March 1983.
- [2] T. Saito, T. Komatsu, H. Harashima, and H. Miyagawa, "Still picture coding by multi-dimensional outer product expansion," *IEICE Trans. Commun. (Japanese Edition)*, vol.J68-B, no.4, pp.547-548, April 1985.
- [3] N. Toda and S. Usui, "A method for structural identification of non-linear system via tensor product expansion for symmetric function," *IEICE Trans. Fundamentals (Japanese Edition)*, vol.J73-A, no.1, pp.51-58, Jan. 1990.
- [4] J. Murakami, T. Gouriki, and Y. Tadokoro, "Detection of discontinuous frame from image sequence by 3-d tensor product expansion method," *Proc. Joint Technical Conf. Circuits/Systems, Computers and Communications*, vol.2, pp.638-642, July 1993.
- [5] Q. Ke and T. Kanade, "Robust L1 norm factorization in the presence of outliers and missing data by alternative convex programming," *Proc. IEEE CVPR 2005*, pp.739-746, June 2005.
- [6] A. Itai, H. Yasukawa, I. Takumi, and M. Hata, "Global noise estimation based on tensor product expansion with absolute error," *IEICE Trans. Fundamentals*, vol.E90-A, no.4, pp.778-783, April 2007.
- [7] Y. Dodge and V. Rousson, "Multivariate L1 mean," *Metrika*, vol.49, no.2, pp.127-134, Sept. 1999.
- [8] A. Itai and H. Yasukawa, "Improvement of accuracy for background noise estimation method based on TPE-AE," *IEEJ Trans. EIS*, vol.127, no.12, pp.2086-2087, 2007.
- [9] Y.H. Lee and S.A. Kassam, "Generalized median filtering and related non-linear filtering techniques," *IEEE Trans. Acoust. Speech Signal Process.*, vol.ASSP-33, no.3, pp.672-683, 1987.
- [10] T. Bernholt, R. Fried, U. Gather, and I. Wegener, "Modified repeated median filters," *Statistics and Computing*, vol.16, no.2, pp.177-192, 2006.
- [11] M.B. Gokhberg, V.A. Morgunovm, T. Yoshino, and I. Tomizawa, "Experimental measurements of electromagnetic emissions possibly related to earthquakes in Japan," *Journal of Geophysical Research*, vol.87, pp.7824-7829, 1982.
- [12] M. Hata, I. Takumi, S. Adachi, and H. Yasukawa, "An analytical method to extract precursor from noisy atmospherics," *Proc. European Geophysical Society XXV*, NH014, pp.25-29, 2000.
- [13] I. Takumi, S. Murakami, A. Shimura, M. Hata, and H. Yasukawa, "Clustering of EM radiation source based eigenvector," *Proc. Int. Geoscience and Remote Sensing Symposium, INT1 -D17 -01*, pp.2614-2616, 2002.



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