Coherent MFSK detection on MIMO frequency selective channels

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Abstract— **MFSK (M-ary Frequency Shift Keying) signal has a constant envelope and is suited to power efficient nonlinear amplification. However, the application of MFSK signals to spectrally efficient and reliable MIMO (Multiple Input Multiple Output) communication schemes has not been so popular up to now. Maybe this comes from the fact that the signal processing for spatially multiplexed MIMO signal is usually done using linear matrix algebra and the application of linear signal processing techniques to nonlinear FM signals such as MFSK is difficult. In this paper, we study the coherent detection of MFSK signals based on MLD (Maximum Likelihood Detection) approach on MIMO frequency selective channels. In the proposed receiver, the receive replica signal which is frequency and phase synchronized is generated and the signal distance between the receive signal and the receive replica signal is minimized. By extending the observation length of receive signal, we improved the BER characteristics. We also reduced the complexity of MLD by using the SE (Schnorr Euchner) algorithm of Sphere Decoding to obtain ML (Maximum Likelihood) solution.**

Keywords- MIMO, MFSK, frequency selective channel, ISI canceller, MLD, Sphere Decoding, SE algorithm

I. INTRODUCTION

MFSK signal has a constant envelope property and is appropriate to be amplified by nonlinear amplifier with high power efficiency. Due to the increasing demand of high data rate and reliable data transmission, MIMO schemes with multiple transmit and receive antennas become quite popular recently. The conventional MIMO schemes process the received QAM (Quadrature Amplitude Modulation) signals using linear matrix calculation. However it has been difficult to apply the linear equalizer to the nonlinearly modulated MFSK signals, and the equalization of MFSK signal at the receiver side has been difficult when it is subjected to frequency selective channels. Accordingly there was almost no research on MIMO MFSK signaling on frequency selective channels. In [1] linear equalizer was employed before the nonlinear MFSK demodulation. Following this idea, we had already shown that the FDE (Frequency Domain Equalization) scheme using CP (Cyclic Prefix) [2] is applicable to the equalization and signal separation of MIMO MFSK signals where the FDE is done before the demodulation process of MFSK signal [3],[4]. However, the BER characteristics of MFSK receiver using FDE are not so good, because non-coherent energy detector is employed after the FDE. In order to further improve the BER, we developed the coherent detection scheme using ISI (Inter-Symbol Interference) canceller and MLD in [4],[5]. In that scheme, by generating the receive replicas using already detected symbols, the ISI components caused by the past transmit symbols were cancelled by ISI canceller. But the ISI due to the future transmit symbols and the IAI (Inter-Antenna Interference) of MIMO spatial multiplexed transmission were removed by MLD. Also zero padding is employed to circumvent the IBI (Inter Block Interference). In [6], we also proposed the iterative MFSK receiver in which by regarding the demodulated outputs from the FDE receiver as the tentative decision results, the ISI replica is generated to cancel the ISI. Then, the remaining IAI is removed by MLD. The decision results from MLD are again regarded as the improved tentative decision results and the iteration is done repeatedly up to the convergence of BER. However, all the proposed MFSK receiver using MLD in [4]~[6] do not utilize the phase memory effect of continuous phase MFSK signals. As the MFSK signal is one of CPM (Continuous Phase Modulation) signals [7], the BER characteristics are improved by enlarging the observation length of the received MFSK signal [8]. In this paper, utilizing this feature of CPM signals, we try to improve the BER characteristics of previously proposed MLD receiver in [4],[5] through enlarging the observation length of received MFSK signal. Moreover, we adopted the SE (Schnorr-Euchner) algorithm [9],[10] of Sphere Decoding to suppress the exponential increase of the complexity of MLD when the observation length is expanded. By adopting the SE algorithm, we can obtain the ML solution the same as MLD with less complexity than MLD.

This paper is organized as follows. In Section II, the BER characteristics of coherent MFSK receiver using MLD on AWGN channel are described and simulated. In Section III, the coherent MIMO MFSK receiver using ISI canceller and MLD is introduced. In Section IV, the receiver using M algorithm [11] for reducing the complexity of MLD is shown. In Section V, the SE algorithm with ML solution is described. In Section VI, the BER characteristics of coherent MIMO MFSK receiver are simulated. In Section VII, the improvement of BER by expanding the observation length is examined. The paper concludes with Section VIII.

II. COHERENT MFSK RECEIVER USING MLD

We consider the coherent demodulation of MFSK signal using MLD on AWGN channel. The MLD receiver generates the candidate receive replica signal which is frequency and phase synchronized to the receive signal. The distance between the receive signal and the receive replica signal is minimized. The receive replica signal with the shortest distance to the receive signal is the transmit signal candidate to be detected. We firstly check the BER dependence of coherent MFSK demodulator on the observation length of receive signal. On AWGN channel, by denoting the lowpass equivalent transmit signal sequence having N symbol length, the receive signal and the receive noise as $s_i(t)$, $i=1,\cdots,K$, $r(t)$ and $n(t)$ respectively, we obtain

$$r(t)=s_i(t)+n(t) \tag{1}$$

By using the sampling function (basis function)

$$\psi_l(t),\, l=1,\cdots,J,\; \int_{-\infty}^{+\infty}\psi_l(t)\psi_l^*(t)=1,\; \int_{-\infty}^{+\infty}\psi_l(t)\psi_m^*(t)=0,\, l\neq m \tag{2}$$

$s_i(t),\, r(t),\, n(t)$ in (1) are expressed as

$$s_i(t)=\sum_{l=1}^{J}s_{i/l}\psi_l(t),\; r(t)=\sum_{l=1}^{J}r_l\psi_l(t),\; n(t)=\sum_{l=1}^{J}n_l\psi_l(t) \tag{3}$$

where $s_{i/l}, r_l, n_l, l=1,\cdots,J$ is the sampling value at time l. The squared distance between the transmit signal $s_i(t)$ and the receive signal $r(t)$ is calculated as

$$d^2\{r(t),s_i(t)\}=\int_0^{NT}\left|r(t)-s_i(t)\right|^2dt=\sum_{l=1}^{J}\left|r_l-s_{i/l}\right|^2 \tag{4}$$

where T and NT denote the one symbol length and the block length of transmit signal sequence $s_i(t)$ respectively, and $NT=J\Delta t$ with Δt being the sampling interval. Also the squared distance $d^2\{s_i(t),s_j(t)\}=d_{ij}^2$ between two transmit signals $s_i(t)$ and $s_j(t)$ is calculated as

$$d_{ij}^2=\int_0^{NT}\left|s_i(t)-s_j(t)\right|^2dt=\sum_{l=1}^{J}\left|s_{i/l}-s_{j/l}\right|^2 \tag{5}$$

Based on the minimum distance criterion, if $s_i(t)$ is sent and

$$\int_0^{NT}\left|r(t)-s_i(t)\right|^2dt\geq\int_0^{NT}\left|r(t)-s_j(t)\right|^2dt \tag{6}$$

is satisfied, then the erroneous signal $s_j(t),\, j\neq i$ is detected. When the decision variable D is defined as

$$D=\int_0^{NT}\left|r(t)-s_i(t)\right|^2dt-\int_0^{NT}\left|r(t)-s_j(t)\right|^2dt \tag{7}$$

if $D>0$, then the erroneous $s_j(t)$ is detected instead of the correct transmit signal $s_i(t)$. Considering the CPM signal with the constant envelope, i.e., $\left|s_i(t)\right|^2=\left|s_j(t)\right|^2$, this error event probability is derived with some manipulation as

$$\Pr\{\text{error}\}=\Pr\{D>0\}=Q\left(\sqrt{d_{ij}^2/(4N_0)}\right) \tag{8}$$

$$\text{where}\quad Q(x)=\int_x^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)dy \tag{9}$$

and N_0 is the one sided power spectral density of white Gaussian noise process $n(t)=n_x(t)+jn_y(t)$ with

$$E\{n_x(t_1)n_x(t_2)\}=E\{n_y(t_1)n_y(t_2)\}=N_0\delta(t_1-t_2) \tag{10}$$

As the transmit signal sequence $s_i(t)$ is represented as

$$s_i(t)=A\exp(j\phi_i(t))=\sqrt{(2E)/T}\exp(j\phi_i(t)) \tag{11}$$

with A, $\phi_i(t)$ and E being the magnitude, the phase and the symbol energy of $s_i(t)$ respectively, the squared distance is evaluated as

$$d_{ij}^2=\int_0^{NT}\left|s_i(t)-s_j(t)\right|^2dt=\frac{4E}{T}\int_0^{NT}\{1-\cos\Delta\phi(t)\}dt \tag{12}$$

where $\Delta\phi(t)=\phi_i(t)-\phi_j(t)$ is the phase difference.

Next, we calculate the actual signal distance of MFSK. As an example, we consider the case of $NT=2T\,(N=2)$ for 2FSK. In this case, there are total four information bit sequences of 00,01,10,11. Corresponding to each of them, we denote the transmit signals as $s_{00}(t),s_{01}(t),s_{10}(t)$ and $s_{11}(t)$ respectively. The squared distance is calculated, for example

$$\begin{aligned} d_{01,10}^2&=\int_0^{2T}\left|s_{01}(t)-s_{10}(t)\right|^2dt=\frac{4E_b}{T}\int_0^{2T}\{1-\cos\Delta\phi(t)\}dt\\ &=4E_b\cdot 2\left[1-\frac{\sin 2\pi h}{2\pi h}\right],\; h=(f_2-f_1)T=2\Delta f T \end{aligned} \tag{13}$$

where $E_b=E$, h and Δf denotes the bit energy, the modulation index and the frequency deviation, respectively. At the receiver side, by providing the four receive candidate signals of $s_{00}(t),s_{01}(t),s_{10}(t)$ and $s_{11}(t)$, the receiver decides the transmit signal under the criterion which minimizes the squared distance between the receive signal and the candidate receive replica signal (MLD criterion). Using the idea of union bound, the BER P_{b1} of the first bit in two bit sequence and P_{b2} of the second bit are derived as follows.

$$\begin{aligned} P_{b1}&<\frac{1}{2}\left\{Q\left(\sqrt{\frac{d_{00,10}^2}{4N_0}}\right)+Q\left(\sqrt{\frac{d_{00,11}^2}{4N_0}}\right)+Q\left(\sqrt{\frac{d_{01,10}^2}{4N_0}}\right)+Q\left(\sqrt{\frac{d_{01,11}^2}{4N_0}}\right)\right\}\\ &=Q\left(\sqrt{\left[1-\frac{\sin 2\pi h}{2\pi h}+(1-\cos 2\pi h)\right]\cdot\frac{E_b}{N_0}}\right)+\frac{1}{2}Q\left(\sqrt{\left[2-\frac{\sin 4\pi h}{2\pi h}\right]\cdot\frac{E_b}{N_0}}\right)\\ &\quad+\frac{1}{2}Q\left(\sqrt{\left[2\left[1-\frac{\sin 2\pi h}{2\pi h}\right]\right]\cdot\frac{E_b}{N_0}}\right) \end{aligned} \tag{14}$$

$$\begin{aligned} P_{b2}&<\frac{1}{2}\left\{Q\left(\sqrt{\frac{d_{00\to 01}^2}{4N_0}}\right)+Q\left(\sqrt{\frac{d_{00\to 11}^2}{4N_0}}\right)+Q\left(\sqrt{\frac{d_{01\to 00}^2}{4N_0}}\right)+Q\left(\sqrt{\frac{d_{01\to 10}^2}{4N_0}}\right)\right\}\\ &=Q\left(\sqrt{\left[1-\frac{\sin 2\pi h}{2\pi h}\right]\cdot\frac{E_b}{N_0}}\right)+\frac{1}{2}Q\left(\sqrt{\left[2-\frac{\sin 4\pi h}{2\pi h}\right]\cdot\frac{E_b}{N_0}}\right)\\ &\quad+\frac{1}{2}Q\left(\sqrt{\left[2\left[1-\frac{\sin 2\pi h}{2\pi h}\right]\right]\cdot\frac{E_b}{N_0}}\right) \end{aligned} \tag{15}$$

The average BER P_b is upper bounded by $(P_{b1}+P_{b2})/2$. Commuter simulation has been done to check the accuracy of those BER bounds. We let $h=0.715$ as the modulation index and $J=32$ as the sample number during $2T$ duration, i.e., $2T=J\Delta t=32\Delta t$.

Computer simulation results are shown in Fig.1. From Fig.1, we know the theoretical upper bounds approximate the simulated BER's well. In two bit sequence transmission, the BER of the first bit P_{b1} is better than the second bit P_{b2}. This observation is peculiar to CPM signaling and is brought by the phase memory effect of CP-2FSK. Accordingly, it is expected for 2FSK that the BER of the first bit or the second bit is improved as the block length (observation length) NT increases to $3T, 4T, 5T\cdots$. To verify this BER improvement, we made the similar simulation like in Fig.1 for the block length $5T$. The simulation results are shown in Fig.2. From Fig.2, we know that the BER is improved in the order of 1st, 2nd, 3rd 4th and 5th bit. We also observe that the BER of the 1st bit is

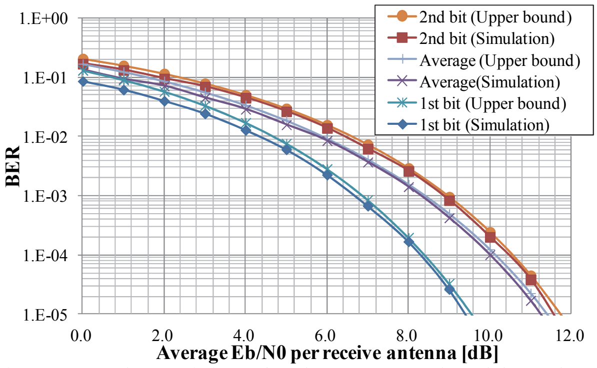

Fig.1. BER characteristics of coherent MLD demodulator for CP (Continuous Phase)-2FSK with the block length of $2T$ on AWGN channel

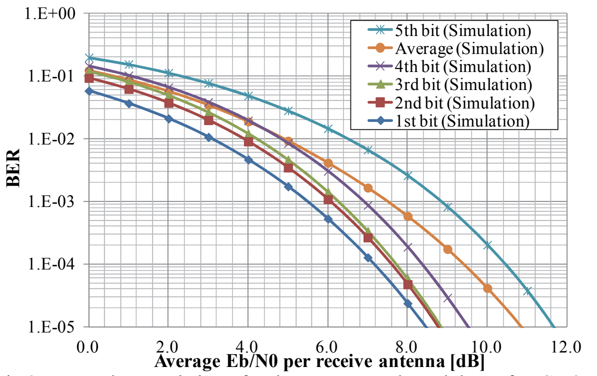

Fig.2. BER characteristics of coherent MLD demodulator for CP-2FSK with the block length of $5T$ on AWGN channel

far better than the one in Fig.1, whereas the 5th bit is almost equal to the 2nd bit in Fig.1. The theoretical BER of the 1st bit of coherent MFSK detection is given in [8] and the BER results in Fig.1 and Fig.2 coincide with [8]. The BER of the 1st bit can be improved in accordance with the block length NT, but the BER improvement saturates at around $5T$. So no further improvement is expected for the block length longer than $5T$.

III. COHERENT MFSK DEMODULATOR USING ISI CANCELLER AND MLD ON MIMO FREQUENCY SELECTIVE CHANNELS

In Fig.3, we show the block diagram of transmitter and receiver of coherent MIMO MFSK demodulation scheme using ISI canceller and MLD [5]. At the transmitter side,

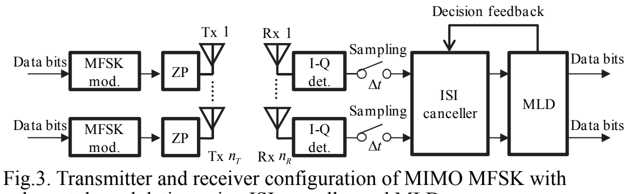

Fig.3. Transmitter and receiver configuration of MIMO MFSK with coherent demodulation using ISI canceller and MLD

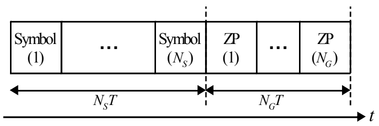

Fig.4. Transmit block structure in MFSK

after the MFSK modulated data symbols, zero symbols having the length of N_GT shown in Fig.4 are inserted (Zero Padding; ZP). By using ZP, the IBI caused by multipath delayed waves between the successive blocks is prevented. Accordingly, the length of ZP N_GT must be longer than the maximum delay time of delayed waves. The initial phase of the head symbol in the transmit block is always set to zero. This phase reset is needed to phase-synchronize the candidate replica signal generated at the receiver with the receive signal in coherent MFSK demodulator. At the receiver side, using I-Q (Inphase-Quadrature) detection, the complex baseband receive signal is obtained. By sampling the complex baseband signal at the rate of $2c$ samples per one symbol ($T = 2c\Delta t$), we get the discrete time signal with the sampling interval of Δt. The sampling interval Δt is taken so as to satisfy the sampling theorem. The sampled discrete time signal is then fed to the ISI canceller. We consider the case where the maximum delay time of the channel is $l\Delta t$ and $l\Delta t \le (L-1)T$, where l and L are integer number and $(L-1)$ means the number of data symbols influenced by the delayed wave. Due to the multipath delay, a transmit symbol during $kT \le t < (k+1)T$ spreads over the duration of $kT \le t < (k+L-1)T$ at the receiver. On the other hand, the receive symbols over $kT \le t < (k+L-1)T$ includes the transmit symbols during $\{k-(L-1)\}T \le t < (k+L-1)T$, i.e., $(2L-1)$ transmit symbols. Accordingly, to determine a transmit symbol during $kT \le t < (k+1)T$, we have to observe total $(2L-1)$ receive symbol length at the receiver. To generate all the receive replicas having $(2L-1)$ symbol length for MLD search, it requires $M^{(2L-1)n_T}$ times calculation where M is the modulation level of MFSK and n_T is the number of transmit antennas. When L and n_T are large, this exponential increase of $M^{(2L-1)n_T}$ becomes a problem of complexity. To reduce the number of searches of MLD over $(2L-1)$ symbols, we employ the ISI canceller to cancel the ISI components caused by the transmit symbols over the past duration $\{k-(L-1)\}T \le t < (k-1)T$ already detected at the receiver. This makes the MLD search over L symbols during $kT \le t < (k+L-1)T$ and reduce the number of MLD search to M^{Ln_T}. When the erroneous detection happens in somewhere in the block, the error propagation occurs after the erroneous symbol due to the erroneous ISI cancellation. But the error propagation is obviously limited within a transmit block.

Next, we consider to detect the 1st symbol $(k=1)$ of a transmit block with the length of N_ST. By employing the ZP, there is no need of ISI cancellation for the 1st symbol in the transmit block. The influence of the 1st transmit symbol spreads over L receive symbols. Accordingly, in order to detect the 1st symbol, all the $1 \sim L$ receive symbols are needed at the receiver. We denote those receive signals over L symbol duration as $\boldsymbol{Y}_1, \cdots, \boldsymbol{Y}_L$. For those receive signals, we generate the receive replica signals $\boldsymbol{Y}'_1, \cdots, \boldsymbol{Y}'_L$ assuming that the transmit signals are $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_L$. This relation is expressed as

$$\begin{pmatrix} \boldsymbol{Y}'_L \\ \vdots \\ \boldsymbol{Y}'_l \\ \vdots \\ \boldsymbol{Y}'_1 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \boldsymbol{y}'_{L,2c} \\ \vdots \\ \boldsymbol{y}'_{L,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{y}'_{l,2c} \\ \vdots \\ \boldsymbol{y}'_{l,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{y}'_{1,2c} \\ \vdots \\ \boldsymbol{y}'_{1,1} \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \boldsymbol{h}_0 & \cdots & \boldsymbol{h}_p & \cdots & \boldsymbol{h}_{J_d-1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h}_0 & \cdots & \boldsymbol{h}_p & \cdots & \boldsymbol{h}_{J_d-1} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{0} & \cdots & \cdots & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{h}_0 & \boldsymbol{h}_1 \\ \boldsymbol{0} & \cdots & \cdots & \cdots & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{h}_0 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \boldsymbol{x}_{L,2c} \\ \vdots \\ \boldsymbol{x}_{L,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{x}_{l,2c} \\ \vdots \\ \boldsymbol{x}_{l,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{x}_{1,2c} \\ \vdots \\ \boldsymbol{x}_{1,1} \end{pmatrix} \end{pmatrix} \tag{16}$$

$$\boldsymbol{Y}'_l = \begin{pmatrix} \boldsymbol{y}'_{l,2c} & \cdots & \boldsymbol{y}'_{l,n} & \cdots & \boldsymbol{y}'_{l,1} \end{pmatrix}^T, \ \boldsymbol{y}'_{l,n} = \begin{pmatrix} y'^{(1)}_{l,n} & y'^{(2)}_{l,n} \end{pmatrix}^T \tag{17}$$

$$\boldsymbol{h}_p = \begin{pmatrix} h_p^{(1,1)} & h_p^{(1,2)} \\ h_p^{(2,1)} & h_p^{(2,2)} \end{pmatrix}, \quad p = 0, \cdots, J_d - 1 \tag{18}$$

$$\boldsymbol{x}_{l,n} = \begin{pmatrix} x_{l,n}^{(1)} \\ x_{l,n}^{(2)} \end{pmatrix}, \quad l = 1, \cdots, L, \quad n = 1, \cdots, 2c \tag{19}$$

where the number of transmit and receive antennas are set to $n_T = 2$ and $n_R = 2$, respectively. $y'^{(1)}_{l,n}$ in (17), for example, means the n-th sample of the l-th receive symbol from the receive antenna 1. $h_p^{(1,2)}$ denotes the complex channel gain of the p-th delay wave from the transmit antenna 2 to receive antenna 1. $x_{l,n}^{(2)}$ the n-th sample of the l-th transmit symbol from the transmit antenna 2. The delay profile of the complex channel gain $h_p^{(j,i)}, p = 0, \cdots, J_d - 1$ from the transmit antenna i to receive antenna j is depicted in Fig.5.

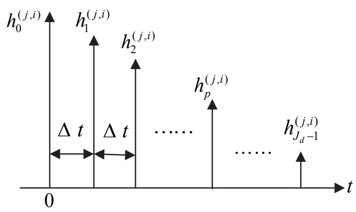

Fig.5. Delay profile of the channel

The 1st transmit signal $\boldsymbol{X}_1$ is determined to minimize the squared distance between the receive sequence $\boldsymbol{Y}_1, \cdots, \boldsymbol{Y}_L$ and the receive replica sequence $\boldsymbol{Y}'_1, \cdots, \boldsymbol{Y}'_L$ by varying the candidate transmit sequence $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_L$.

$$\boldsymbol{X}_1 = \underset{\boldsymbol{X}_1, \cdots, \boldsymbol{X}_L}{\arg\min} \left[\left\| \left(\boldsymbol{Y}_L, \cdots, \boldsymbol{Y}_1 \right)^T - \left(\boldsymbol{Y}'_L, \cdots, \boldsymbol{Y}'_1 \right)^T \right\|^2 \right] \tag{20}$$

where $\boldsymbol{X}_l = \begin{pmatrix} \boldsymbol{x}_{l,2c} & \cdots & \boldsymbol{x}_{l,1} \end{pmatrix}^T, l = 1, \cdots, L$.

Like the 1st symbol, we also consider to detect the k-th symbol in the transmit block. In order to cancel the ISI components due to the past transmit symbols over $k - (L-1) \sim k-1$ symbol duration, we make the receive replica sequence $\boldsymbol{Y}'_k, \cdots, \boldsymbol{Y}'_{k+L-1}$ using the already detected results of $\boldsymbol{X}_{k-(L-1)}, \cdots, \boldsymbol{X}_{k-1}$ as

$$\begin{pmatrix} \boldsymbol{Y}'_{k+(L-1)} \\ \vdots \\ \boldsymbol{Y}'_k \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \boldsymbol{y}'_{k+(L-1),2c} \\ \vdots \\ \boldsymbol{y}'_{k+(L-1),1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{y}'_{k,2c} \\ \vdots \\ \boldsymbol{y}'_{k,1} \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \boldsymbol{h}_0 & \cdots & \boldsymbol{h}_p & \cdots & \boldsymbol{h}_{J_d-1} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{h}_0 & \cdots & \boldsymbol{h}_p & \cdots & \boldsymbol{h}_{J_d-1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{h}_0 & \cdots & \boldsymbol{h}_p & \cdots & \boldsymbol{h}_{J_d-1} & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} & \boldsymbol{h}_0 & \cdots & \boldsymbol{h}_p & \cdots & \boldsymbol{h}_{J_d-1} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \boldsymbol{x}_{k+(L-1),2c} \\ \vdots \\ \boldsymbol{x}_{k+(L-1),1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{x}_{k,2c} \\ \vdots \\ \boldsymbol{x}_{k,1} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \boldsymbol{x}_{k-(L-1),2c} \\ \vdots \\ \boldsymbol{x}_{k-(L-1),1} \end{pmatrix} \end{pmatrix} \tag{21}$$

where already detected results of $\boldsymbol{X}_{k-(L-1)}, \cdots, \boldsymbol{X}_{k-1}$ have the fixed values. The k-th transmit signal $\boldsymbol{X}_k = \begin{pmatrix} \boldsymbol{x}_{k,2c} & \cdots & \boldsymbol{x}_{k,1} \end{pmatrix}^T$ is determined so as to minimize the squared distance between the receive sequence $\boldsymbol{Y}_k, \cdots, \boldsymbol{Y}_{k+(L-1)}$ and the receive replica sequence $\boldsymbol{Y}'_k, \cdots, \boldsymbol{Y}'_{k+(L-1)}$ in (21) by varying the candidate transmit sequence $\boldsymbol{X}_k, \cdots, \boldsymbol{X}_{k+(L-1)}$.

$$\boldsymbol{X}_k = \underset{\boldsymbol{X}_k, \cdots, \boldsymbol{X}_{k+(L-1)}}{\arg\min} \left[\left\| \left(\boldsymbol{Y}_{k+(L-1)}, \cdots, \boldsymbol{Y}_k \right)^T - \left(\boldsymbol{Y}'_{k+(L-1)}, \cdots, \boldsymbol{Y}'_k \right)^T \right\|^2 \right] \tag{22}$$

The subtraction in (22) means that the MLD search is done with respect to the transmit sequence $\boldsymbol{X}_k, \cdots, \boldsymbol{X}_{k+(L-1)}$ of L transmit symbol duration over the present time k to the future time $k + (L-1)$ where the past ISI components are cancelled using the already detected results of $\boldsymbol{X}_{k-(L-1)}, \cdots, \boldsymbol{X}_{k-1}$. In that meaning, we call the operation of (22) as {ISI canceller + MLD}.

IV. APPLICATION OF M-ALGORITHM

The {ISI canceller + MLD} in III needs M^{Ln_T} times searches for receive replica generation, which causes the exponential increase with respect to L. To reduce the complexity of MLD, we can utilize the M-algorithm [11]. The squared distance metric in the right hand side of (22) can be written as

$$\left\| \left(\boldsymbol{Y}_{k+(L-1)}, \cdots, \boldsymbol{Y}_k \right)^T - \left(\boldsymbol{Y}'_{k+(L-1)}, \cdots, \boldsymbol{Y}'_k \right)^T \right\|^2 = \sum_{l=k}^{k+(L-1)} \left\| \boldsymbol{Y}_l - \boldsymbol{Y}'_l \right\|^2 \tag{23}$$

where the squared distance is decomposed into accumulated L terms sum of individual squared distance metric $\left\| \boldsymbol{Y}_l - \boldsymbol{Y}'_l \right\|^2$. Accordingly, when increasing l from k to $k + (L-1)$ one by one, at each step of l, we can restrict the number of candidate transmit signals of $\boldsymbol{X}_{k+(l-1)}$ to M_c under the criterion of minimizing the accumulated squared metric. By using the M-algorithm, the number of total searches reduces to $(1 + (L-1)M_c)M^{n_T}$ achieving the linear increase with respect to L. However, M-algorithm only gives the quasi-ML solution.

V. APPLICATION OF SPHERE DECODING (SE ALGORITHM)

The SE algorithm [9], [10], one of the Sphere Decoding algorithms, achieves the ML solution like in MLD,

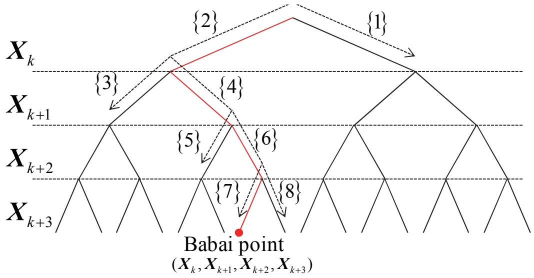

Fig.6. Tree structure of SE algorithm (2FSK, number of transmit and receive antenna; $n_T = n_R = 1$, $L = 4$)

while greatly reducing the complexity compared with MLD. We illustrate the SE algorithm briefly in Fig.6. On the MLD tree structure in Fig.6, the SE algorithm firstly searches the path in depth direction (in time axis direction) using M-algorithm $(M_c = 1)$ described in IV. The quasi-optimum path obtained through M-algorithm is depicted as {2}{4}{6}{7} in Fig.6. The quasi-ML solution of transmit sequence is called Babai point. We set the path metric corresponding to Babai point to C^2 (initial squared radius). The tree search is again done in the depth direction of Fig.6 to satisfy the accumulated path metric less than C^2. For example, if the branch metric of {1} exceeds C^2, then the subsequent paths following {1} are not to be searched. Also if the accumulated path metric of {2}{3} exceeds C^2, then the paths following {3} are no more to be searched. This means that if the accumulated path metric exceeds C^2 in the tree search, there is no need to find the subsequent diverging paths following that branch. This leads to save the extra tree searching procedure. If the accumulated path metric does not exceed C^2 up to the bottom of tree, then C^2 is replaced by the new path metric and is updated. Finally, the entire tree search results in finding the ML solution with reduced complexity.

VI. BER CHARACTERISTICS OF COHERENT MIMO MFSK RECEIVER ON FREQUENCY SELECTIVE CHANNELS

Computer simulation is made for the coherent MFSK receiver using ISI canceller with MLD, M-algorithm or SE algorithm on MIMO frequency selective channels. We also show the BER of the receiver with FDE and energy detection in [3],[4]. The simulation condition is shown in Table I. The delay profile between each transmit and receive antenna is shown in Fig.7.

Fig.8 shows the BER characteristics of 2×2 4FSK. From Fig.8, we know that the BER of {ISI canceller + MLD} is improved by 12 dB at BER=10^{-5} compared with the {FDE + energy detection}. The {ISI canceller + M-algorithm} receiver exhibits almost the same BER as the {ISI canceller+ MLD} when the parameter M_c is greater than 4. The {ISI canceller + SE algorithm} shows exactly the same BER as the {ISI canceller + MLD}.

Fig.9 compares the number of branch metric calculation versus average E_b / N_0 among MLD, M-algorithm and SE algorithm. From Fig.9, we observe that the number of branch metric calculation in SE algorithm decreases greatly

TABLE I SIMULATION CONDITION FOR COHERENT MIMO MFSK RECEIVER

Modulation	4FSK
Modulation index	h=0.715
Number of transmit and receive antenna	2×2
Channel between each Tx and Rx antenna	Quasi-static Rayleigh fading with equal power 16 delay paths
Equalization and signal separation at receiver	•{FDE (MMSE criterion) + energy detection} •ISI canceller + MLD • or M-algorithm • or SE algorithm
Number of samples in a symbol	$2c$=16
Symbol duration	T
Interval of delay waves	$T/16(=\Delta t)$
Maximum delay time	$15T/16$
CP or ZP length	T
Transmit block length $N_s T$	$16T$
FFT points for FDE ($=N_s \times 2c$)	16×16=256

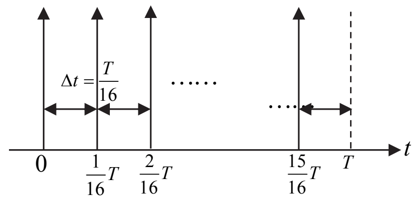

Fig.7. Delay profile between each transmit and receive antenna

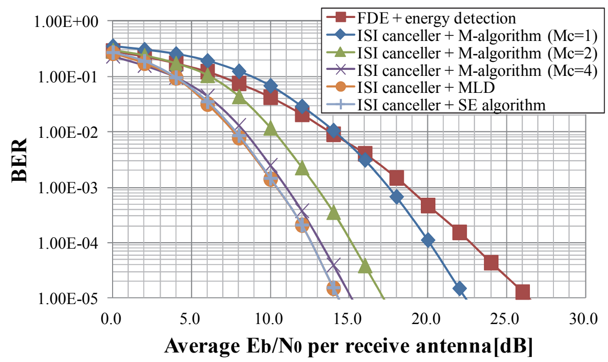

Fig.8. BER characteristics of coherent 4FSK receiver on MIMO (2×2) frequency selective channel

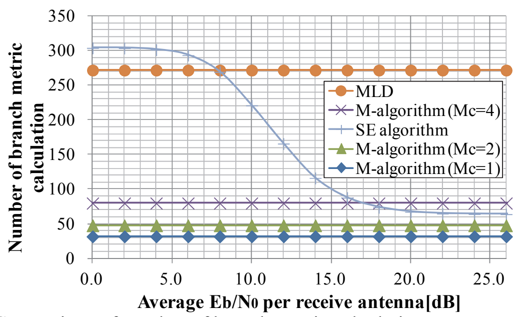

Fig.9. Comparison of number of branch metric calculation

in high E_b / N_0 region and the SE algorithm is quite useful for reducing the complexity of MLD.

VII. BER IMPROVEMENT THROUGH LONGER OBSERVATION LENGTH

The receiver with {ISI canceller + MLD} made the MLD search over L symbol duration $k \sim k+(L-1)$ as shown in (22) to determine the transmit signal $\boldsymbol{X}_k$ at time k. This is because the transmit signal $\boldsymbol{X}_k$ spreads over L symbol duration due to the delayed waves. Accordingly, on the AWGN channel, $\boldsymbol{X}_k$ is determined through only one symbol duration with $L=1$. However, as discussed in II, for the coherent detection of MFSK, the BER of the 1st symbol in the transmit block is improved by a few dB through the observation length of $4T \sim 5T$ rather than one symbol duration. Therefore, we extend the observation length in MLD to $N_w T = (L+w)T,\ (w = 0,1,2,\cdots)$ and aim to improve the BER characteristics, where $w=0$ corresponds to the usual receiver simulated in VI. Extending the symbol duration $N_w T$ for MLD causes the increasing number of replica search growing exponentially as $M^{(L+w)\cdot n_T}$. However, we solve this problem by using the Sphere Decoding SE algorithm. We made the simulation when the symbol length of MLD is extended to $N_w T$. The simulation condition is the same as in Table I, but with the extended MLD length $N_w T = (L+w)T,\ (L=2,\ w=0,1,2,3)$. The BER characteristics are shown in Fig.10. By extending the observation length of MLD from $N_w T = 2T\,(w=0)$ to $N_w T = 5T\,(w=3)$, we get the BER improvement about 2.4 dB at BER=10^{-5}. We also show the number of branch metric calculation in SE algorithm in Fig.11. From Fig.11, we see that for high E_b/N_0 region larger than 8 dB, the number of branch metric calculation is sufficiently reduced by using the SE algorithm.

VIII. CONCLUSIONS

In this paper, we discussed the coherent detection of MFSK signal on MIMO frequency selective channels. By generating the frequency and phase synchronized replica signal at the receiver, the transmit signal which minimizes the squared distance between the receive signal and the receive replica signal is determined with the exhaustive search through MLD. In this MLD search, the ISI components due to the past symbols are cancelled using already detected results. The exponentially growing complexity of MLD is greatly reduced by using the Sphere Decoding SE algorithm which achieves the same ML solution as MLD. By utilizing the phase memory of MFSK and extending the observation length of MLD at the receiver, we achieved the further BER improvement by a few dB while reducing the complexity of MLD using SE algorithm. As future studies, the coherent detection of CPM signals other than MFSK, such as GFSK, will be considered on MIMO frequency selective channels.

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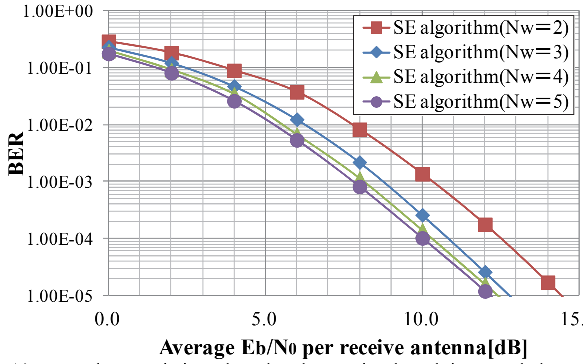

Fig.10. BER characteristics when the observation length is extended

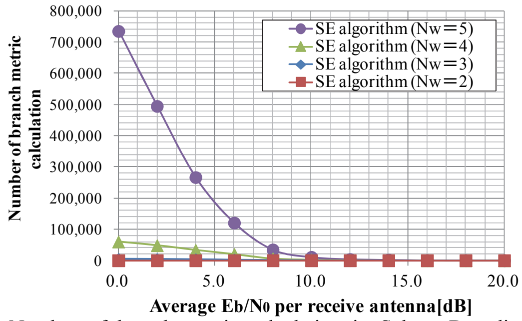

Fig.11. Number of branch metric calculation in Sphere Decoding SE algorithm when the observation length is extended

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