A Diversity Order Design of Linearly Precoded MU-MIMO Downlink System

Nobuaki Shimakawa, Yasunori Iwanami
Department of Electrical and Mechanical Engineering
Nagoya Institute of Technology
Nagoya, Japan
28413082@stn.nitech.ac.jp, iwanami@nitech.ac.jp

***Abstract*— Recently, Muti-User MIMO (MU-MIMO) downlink system which uses multiple antennas at Base Station (BS) and accommodates multiple users with multiple receive antennas attracts much attention. It can achieve a high channel capacity when the channel state information is known both at BS and each User Equipment (UE). There exist linear and non-linear MU-MIMO downlink schemes, but the linear scheme is considered more easily to be implemented and adjusted. Linear Block Diagonalization (BD) has been proposed in order to cancel the Inter-User Interference (IUI), and combined with the Eigenbeam-Space Division Multiplexing (E-SDM). In this paper, by noting that the block size of each user channel matrix in BD becomes (number of receive antennas of each UE) × (nullity of the matrix constituted by other users), we design the BD + E-SDM scheme based on the down link quality of each UE, i.e., the diversity order. We made simulations for some examples and show the effectiveness of design method based on the diversity order of each UE.**

Keywords*— MU-MIMO, block diagonalization, E-SDM, nullity, diversity order

I. Introduction

Recently, spatially multiplexed MU-MIMO downlink system in which a Base Station (BS) can transmit multiple streams to each User Equipment (UE) without IUI attracts much attention [1]-[7]. In MU-MIMO downlink communication, the Channel State Information (CSI) between BS and each UE has to be shared at both of them, but by increasing the number of transmit antennas at BS, the frequency efficiency and the channel capacity of downlink can arbitrary be improved. Transmit signals from BS to each UE located at different position are spatially multiplexed and the IUI received at each UE is pre-excluded by the precoding at BS. As representative methods for excluding the IUI at BS, there exist BD [2] and Channel Inversion (CI) [3] both categorized as linear schemes, and Dirty Paper Coding (DPC) [4], Tomlinson-Harashima Precoding (THP) [5] and Vector Perturbation (VP) [6],[7] as nonlinear schemes. Although nonlinear schemes attain high downlink channel capacity, the complexity is high and the deign method is difficult. The linear CI method has a problem on increasing transmit power and the restriction of sum rate characteristics. The linear BD method consumes much degree of freedom on making nulls, but it eliminates IUI completely and it matches the E-SDM [8] very well. Hence the BD + E-SDM seems practical solution for the MU-MIMO downlink.

In this paper, we investigate the MU-MIMO downlink system in which both BS and UE's have multiple antennas. The BS eliminates the IUI using BD and transmits multiple streams to each UE using E-SDM for the block matrix in the block diagonalized channel matrix. We note that the size of the block matrix for UE in the block diagonalized matrix becomes (number of receive antennas of UE) × (nullity of the channel matrix in which channel matrix of UE is excluded from the entire channel matrix). When the channel is i.i.d. flat Rayleigh fading, as the diversity order of the first eigen mode in E-SDM for the block channel matrix of UE is also given by the above product, we propose a design method in MU-MIMO down link in which the diversity order of the first eigen mode is designated. We made simulations for some design examples and show the effectiveness of the proposed design method.

The paper is organized as follows. In Section II, the MU-MIMO down link model is introduced. In Section III, we propose the design method of MU-MIMO down link based on the receive quality of UE, i.e., the diversity order of 1st eigen mode channel of UE. In Section IV, we show some examples of design method and clarify the BER characteristics. The paper is concluded with Section V with the most important results and future work. In appendix, we derive the theoretical BER in BD+E-SDM when the number of transmit stream is one.

II. MU-MIMO Down Link Model

We show the transmission model of MU-MIMO down link in Fig.1. We denote the number of transmit antennas at BS and the number of UE's as N_T and N_u respectively. When the number of receive antennas of UE k $(k=1,\cdots,N_u)$ is n_{Rk}, the total number of receive antennas of all UE's are given by $N_R=\Sigma_{k=1}^{N_u} n_{Rk}$. Denoting the channel matrix for UE k as $\boldsymbol{H}_k$ ($n_{Rk}\times N_T$), the entire channel matrix from BS to UE's is expressed as

$$\boldsymbol{H}=[\boldsymbol{H}_1^T \quad \boldsymbol{H}_2^T \quad \cdots \quad \boldsymbol{H}_k^T \quad \cdots \quad \boldsymbol{H}_{N_u}^T]^T \tag{1}$$

We denote the matrix $\tilde{\boldsymbol{H}}_k$ in which the channel matrix $\boldsymbol{H}_k$ of UE k is excluded from the entire channel matrix $\boldsymbol{H}$ as

$$\tilde{\boldsymbol{H}}_k=[\boldsymbol{H}_1^T \quad \cdots \quad \boldsymbol{H}_{k-1}^T \quad \boldsymbol{H}_{k+1}^T \quad \cdots \quad \boldsymbol{H}_{N_u}^T]^T \tag{2}$$

with the size of $(N_R-n_{Rk})\times N_T$. By making the Singular Value Decomposition (SVD) on $\tilde{\boldsymbol{H}}_k$, we get

$$\tilde{\boldsymbol{H}}_k=\tilde{\boldsymbol{U}}_k\tilde{\Sigma}_k\left[\tilde{\boldsymbol{V}}_k^1 \quad \tilde{\boldsymbol{V}}_k^0\right]^H \tag{3}$$

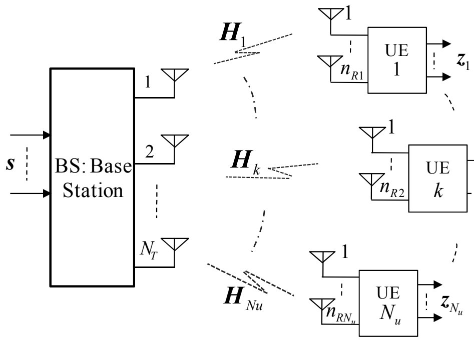

Fig. 1. MU-MIMO down link model

where $\tilde{\boldsymbol{V}}_k^0$ is the null space of $\tilde{\boldsymbol{H}}_k$ and it orthogonalizes $\boldsymbol{H}_{k'}$ $(k' \neq k)$, i.e., $\boldsymbol{H}_{k'}\tilde{\boldsymbol{V}}_k^0 = \boldsymbol{0}$ $(k' \neq k)$. Using the null space $\tilde{\boldsymbol{V}}_k^0$ for each UE, the entire channel matrix $\boldsymbol{H}$ is block diagonalized as

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{H}_1 \\ \vdots \\ \boldsymbol{H}_k \\ \vdots \\ \boldsymbol{H}_{N_u} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{V}}_1^0 & \cdots & \tilde{\boldsymbol{V}}_k^0 & \cdots & \tilde{\boldsymbol{V}}_{N_u}^0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}_1 & \cdots & \boldsymbol{0} \\ \vdots & \ddots \quad \boldsymbol{D}_k \quad \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{D}_{N_u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_1\tilde{\boldsymbol{V}}_1^0 & \cdots & \boldsymbol{0} \\ \vdots & \ddots \quad \boldsymbol{H}_k\tilde{\boldsymbol{V}}_k^0 \quad \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{H}_{N_u}\tilde{\boldsymbol{V}}_{N_u}^0 \end{bmatrix} \tag{4}$$

The number of column vectors of $\tilde{\boldsymbol{V}}_k^0$ becomes the nullity $n\ell_k$ of $\tilde{\boldsymbol{H}}_k$. The nullity of a matrix is given by the difference between the number of columns and the rank of a matrix. The nullity $n\ell_k$ of matrix $\tilde{\boldsymbol{H}}_k$ is expressed as

$$n\ell_k = nullity(\tilde{\boldsymbol{H}}_k) = N_T - rank(\tilde{\boldsymbol{H}}_k) \tag{5}$$

As the rank of a matrix is given by the minimum between the number of rows and columns, the rank of $\tilde{\boldsymbol{H}}_k$ is expressed as

$$rank(\tilde{\boldsymbol{H}}_k) = \min(N_R - n_{Rk}, N_T) \tag{6}$$

If the nullity $n\ell_k \geq 1$, then the null space $\tilde{\boldsymbol{V}}_k^0$ exists and the BD becomes available. Accordingly, the condition for realizing BD is expressed as

$$n\ell_k = N_T - rank(\tilde{\boldsymbol{H}}_k) = N_T - N_R + n_{Rk} > 0 \tag{7}$$

where we set $rank(\tilde{\boldsymbol{H}}_k) = \min(N_R - n_{Rk}, N_T) = N_R - n_{Rk}$ in (6). In some literature, there is a description that the condition for realizing BD is $N_T \geq N_R$, but correctly the condition of $n\ell_k = N_T - N_R + n_{Rk} > 0$ must be satisfied. Therefore even in the case of $N_T < N_R$, the BD is realizable. For example, for the parameters of N_T=4, $N_u = 2$, $n_{R1} = n_{R2} = 3$, $N_R = 6$, the nullity of $n\ell_1 = n\ell_2 = 4 - 6 + 3 = 1 > 0$ and the BD is realizable for N_T=4<$N_R = 6$ [2]. Finally we know that the size of block channel matrix $\boldsymbol{D}_k$ in (4) is given by $n_{Rk} \times n\ell_k$.

III. Design Method of MU-MIMO Down Link Based on Receive Quality of UE

We assume that the channel matrix $\boldsymbol{H}$ represents the quasi-static flat Rayleigh fading where each element of $\boldsymbol{H}$ follows the i.i.d. complex Gaussian random variable. From the section II, in the block diagonalized matrix $\boldsymbol{B}$ in (4), the size $n_{Rk} \times n\ell_k$ of the block matrix $\boldsymbol{D}_k$ for the UE k is determined by the number of receive antennas n_{Rk} of UE k and the nullity $n\ell_k$ of the matrix $\tilde{\boldsymbol{H}}_k$. The diversity order of the 1st eigen mode channel having the channel gain of λ_{max} is given by the product of number of rows and columns of block channel matrix $\boldsymbol{D}_k$ [9]. Hence, the diversity order n_{dk} of the 1st eigen mode channel of the block channel matrix $\boldsymbol{D}_k$ for the UE k becomes $n_{dk} = n_{Rk} \times n\ell_k$. It is also known that the diversity order of the 2nd eigen mode channel is given by $(n_{Rk} - 1) \times (n\ell_k - 1)$ and the diversity order of the i-th eigen mode channel decreases as $(n_{Rk} - (i-1)) \times (n\ell_k - (i-1))$. Using those findings, we propose the design method of MU-MIMO down link based on the diversity order of the stream of UE indicating the receive quality.

A. Design Method of MU-MIMO Down Link 1

We consider the case where the number of transmit antennas of BS N_T is fixed, the number of active UE's is detected and known at BS and also the number n_{Rk} of receive antennas at each UE is known at BS. In this case, the total number of receive antennas of all UE's is $N_R = \Sigma_{k=1}^{N_u} n_{Rk}$. For the block channel matrix $\boldsymbol{D}_k$ with the size of $n_{Rk} \times n\ell_k$, from (7) the nullity of $n\ell_k$ is given by

$$n\ell_k = N_T - N_R + n_{Rk} \tag{8}$$

Accordingly, the diversity order n_{dk} of the 1st eigen mode channel for UE k is expressed as

$$n_{dk} = n_{Rk} \times n\ell_k \tag{9}$$

When there are some inactive UE's, the number of total receive antennas of all UE's N_R decreases and the nullity for UE k $n\ell_k$ in (8) increases.

B. Design Method of MU-MIMO Down Link 2

We consider the case where the number N_u of all UE's is determined and the UE i has the minimum number of receive antennas $n_{Ri} = n_{R\min}$ among UE's. When the desired diversity order of the 1st eigen mode channel of UE i is denoted as $n_{d\,min}$, it holds $n_{d\min} = n\ell_i \times n_{R\min}$ and $n\ell_i = n_{d\,min} / n_{R\,min} = n\ell_{min}$. If $n\ell_{min}$ is designated, the number of transmit antennas at BS is determined from (7) as

$$N_T = n\ell_{min} + N_R - n_{R\,min} \tag{10}$$

where the size of block channel matrix of $\boldsymbol{D}_i$ of UE i having the minimum number of receive antennas $n_{R\,min}$ is given by $n_{R\,min} \times n\ell_{min}$. On the other hand, the nullity of UE k $n\ell_k = nullity(\tilde{\boldsymbol{H}}_k)$ other than the UE i is expressed as $n\ell_k = N_T - N_R + n_{Rk} > n\ell_{\min}$, which means $n\ell_k > n\ell_{\min}$. The size $\boldsymbol{D}_k$ of block channel matrix of UE k is given as $n_{Rk} \times n\ell_k$. In this way, by designating the minimum diversity order $n_{d\min} = n\ell_i \times n_{R\min}$ of the 1st eigen mode channel of UE i, the diversity orders of other UE's other than UE i can have larger ones. This is the design method in which the number of transmit antennas N_T at BS is finally determined based on the requirement of UE i with the worst receive quality.

C. Design Method of MU-MIMO Down Link 3

There exists the case where the number of transmit antennas N_T at BS is smaller than the number of total receive antennas N_R of all UE's, i.e., $N_T < N_R$, under the positive nullity condition $n\ell_k = N_T - N_R + n_{Rk} > 0$ which enables the BD as stated in II. If the following condition

$$N_T = n\ell_{\min} + N_R - n_{R\min}, \quad n\ell_k = N_T - N_R + n_{Rk} > 0, \quad N_T < N_R \tag{11}$$

is satisfied, then the BD+E-SDM transmission is available even though $N_T < N_R$.

IV. Examples of Design Method of MU-MIMO Down Link

We show some examples for evaluating the design methods *A~C*. The BER characteristics are simulated by MATLAB to verify the diversity order of each UE in Fig.1. The abscissa of BER characteristic is taken as the transmit SNR [10]. The transmit SNR is defined as the ratio of the total transmit power P to the receive noise power σ_n^2 per a receive antenna of each UE and is given by

$$(S/N)_{\text{transmit}} = P/\sigma_n^2 \tag{12}$$

where the variance of complex Gaussian random variable h_{ij} in the channel matrix $\boldsymbol{H}$ in (1) is set to $E\{|h_{ij}|^2\} = 1$.

A. Example of Design Method of MU-MIMO Down Link 1

When the number of transmit antennas at BS $N_T = 6$, the number of UE's $N_u = 3$ and number of receive antennas of UE is all equal with $n_{R1} = n_{R2} = n_{R3} = 1$, the number of total receive antennas becomes $N_R = n_{R1} + n_{R2} + n_{R3} = 3$. We call it BD+E-SDM $6\times(1,1,1)$ model. In this case, the channel matrix $\boldsymbol{H}$ is 3×6 and $\tilde{\boldsymbol{H}}_k$ becomes 2×6. For the UE's $k = 1 \sim 3$, the nullity of $\tilde{\boldsymbol{H}}_k$ is calculated as $n\ell_k = 6-2 = 4,\ k = 1,2,3$. The size of block channel matrix $\boldsymbol{D}_k$ is given by $n_{Rk} \times n\ell_k = 1\times 4$, $k = 1 \sim 3$ and the diversity order n_{dk} with 1 stream transmission to UE's $k = 1 \sim 3$ becomes equal and is given by $n_{dk} = n_{Rk} \times n\ell_k = 1\times 4 = 4$, $k = 1 \sim 3$.

In Fig.2, the BER characteristic of BD+E-SDM $6\times(1,1,1)$ model with QPSK (2bps/Hz) modulation and 1 stream transmission is shown as the orange circle. From Fig.2, the diversity order of 4 is observed for UE 1~3, because the BER falls by 10^{-2} between 15~20 dB of transmit SNR. Likewise, the BER results of BD+E-SDM $3\times(1,1,1)$ and $6\times(1,1,1,1)$ are also shown in Fig.2.

For BD+E-SDM $3\times(1,1,1)$, the nullity becomes $n\ell_k = 1$ and the size of block matrix $\boldsymbol{D}_k$ is given by $n_{Rk} \times n\ell_k = 1\times 1$. Thus the diversity order is calculated as $n_{dk} = n_{Rk} \times n\ell_k = 1$ and we can confirm it from the slope in Fig.2. When comparing $6\times(1,1,1)$ with $3\times(1,1,1)$, by using more antennas at BS, $6\times(1,1,1)$ can achieve the higher nullity of 4 rather than 1 leading to the communication quality improvement of UE.

Next, BD+E-SDM $6\times(1,1,1,1)$ model is also shown in Fig.2. In this case, the number of UE's $N_u = 4$, the nullity becomes $n\ell_k = N_T - N_R + n_{Rk} = 6-4+1 = 3,\ k = 1\sim 4$ and the size of $\boldsymbol{D}_k$ is given by 1×3. When comparing $6\times(1,1,1,1)$ with $6\times(1,1,1)$, the increase of UE's causes the decrease of nullity from 4 to 3 and we can find the BER degradation in Fig.2.

Also in Fig.2, we can confirm that the simulated BER coincides exactly with the theoretical BER. The theoretical BER is derived in appendix.

B. Example of Design Method of MU-MIMO Down Link 2

When $N_u = 3$ and $n_{R1} = 3$, $n_{R2} = 2$, $n_{R3} = 1$, it becomes

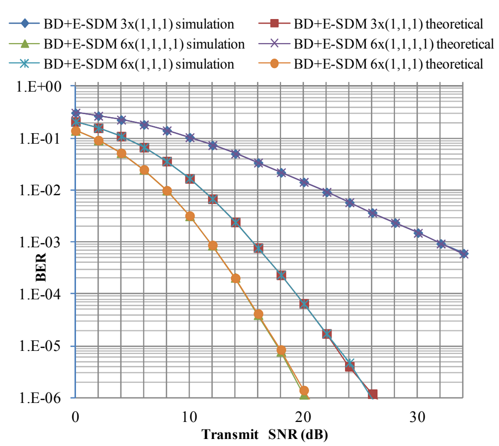

Fig. 2. BER characteristics of BD+E-SDM ($3\times(1,1,1)$, $6\times(1,1,1,1)$, $6\times(1,1,1)$, Modulation:QPSK)

$N_R = n_{R1} + n_{R2} + n_{R3} = 6$. In this case, the number of receive antenna $n_{R3} = 1$ of UE 3 is minimum, thus we can say $n_{R\min} = n_{R3} = 1$. If the desired diversity order of the 1st eigen mode channel of UE3 is $n_{d\min} = 3$, from the relation of $n_{dk} = n_{Rk} \times n\ell_k$, it holds $n\ell_{\min} = n_{d\min}/n_{R\min} = 3$. Hence, N_T is calculated as $N_T = n\ell_{min} + N_R - n_{R\,min} = 3+6-1 = 8$ and the size of $\boldsymbol{D}_3$ for UE 3 becomes 1×3. The nullity of UE 2 is given by $n\ell_2 = nullity(\tilde{\boldsymbol{H}}_2) = N_T - N_R + n_{R2} = 8-6+2 = 4$ and the size of $\boldsymbol{D}_2$ becomes 2×4. The diversity order of the 1st eigen mode channel of UE 2 is calculated as $n_{d2} = n_{R2} \times n\ell_2 = 2\times 4 = 8$. Likewise, the nullity of UE 1 is given by $n\ell_1 = nullity(\tilde{\boldsymbol{H}}_1) = N_T - N_R + n_{R1} = 8-6+3 = 5$ and the size of

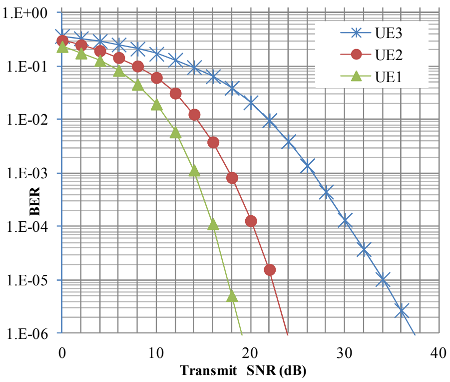

Fig. 3. BER characteristics of BD+E-SDM ($8\times(3,2,1)$, Fixed transmission rate of 6(bps/Hz), Modulation formats are optimally selected from 64QAM, 16QAM and QPSK.)

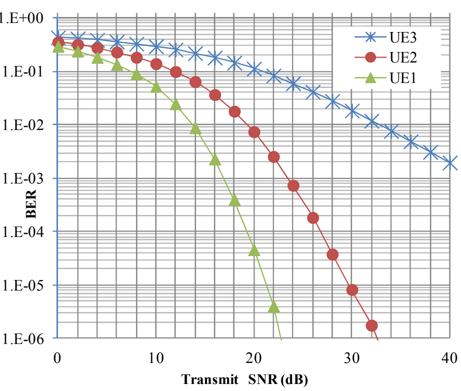

Fig. 4. BER characteristics of BD+E-SDM ($6\times(3,2,1)$, Fixed transmission rate of 6(bps/Hz), Modulation formats are optimally selected from 64QAM, 16QAM and QPSK.)

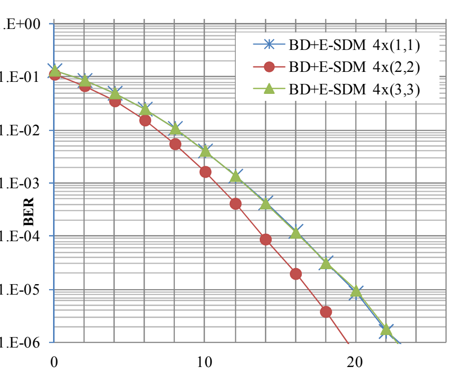

Fig. 5. BER characteristics of BD+E-SDM (Modulation:QPSK)

$\boldsymbol{D}_1$ becomes 3×5. The diversity order of the 1st eigen mode channel of UE 1 is calculated as $n_{d1} = n_{R1}\times n\ell_1 = 3\times5 = 15$.

Fig.3 shows the BER characteristics of UE's 1~3 in BD+E-SDM $8\times(3,2,1)$ model. The BER of UE 3 marked by blue $*$ transmits only 1 stream using the 1st eigen mode and is equivalent to BD+MRT (Maximum Ratio Transmission) [11]. The diversity order of 3 is observed because the BER falls by about 10^{-3} during 25dB～35dB of transmit SNR. On the other hand, the BER of UE2 and UE1 marked by red circle and green triangular respectively are the average BER of all streams where UE 2 and UE 1 transmit 2 and 3 streams using 2 and 3 eigen values respectively. As stated before, when multiple streams are transmitted in parallel using E-SDM, the optimum selection of modulation formats and the optimum power assignment to each stream are done to make the average BER of all streams small as possible [8]. For UE 3, UE 2 and UE 1, under the constraint of constant bit rate of 6(bps/Hz), the combination of modulation formats are selected optimally from 64QAM, 16QAM and QPSK. Also for selected modulation formats to multiple streams, the optimum transmit power assignment to each stream has been done. But for UE 3, there is no selection on modulation formats and the entire transmit power is assigned to the one stream of 64QAM. In addition, UE 1,UE 2 and UE 3 share the total transmit power from BS equally, i.e., 1/3 of total transmit power.

In Fig.3, if the desired diversity order of the 1st eigen mode channel of UE 3 is changed from $n_{d3} = 3$ to 1, then it becomes $N_T = 6$. The BER characteristics are shown in Fig.4. We can confirm that the diversity order of UE 3 is actually 1 from Fig.4.

C. Example of Design Method of MU-MIMO Down Link 3

We consider the case where $N_u = 2$, $n_{R1} = 3$, $n_{R2} = 3$ and $N_R = 6$. In this case, from the relation $N_T = n\ell_{\min} + N_R - n_{R\min}$, when $n\ell_{\min} = 1$, it holds $N_T = 1+6-3 = 4$ and $N_T = 4 < N_R = 6$. In this $4\times(3,3)$ model, the size of block channel matrix of $\boldsymbol{D}_k$ of UE k becomes $n_{Rk}\times n\ell_k = 3\times1$ and 1 stream transmission becomes available. The diversity order becomes $n_{d1} = n_{d2} = 3\times1 = 3$. In Fig.5, the BER characteristics of $4\times(3,3)$ model are shown. For comparison, the BER characteristics o $4\times(1,1)$ and $4\times(2,2)$ are also shown in Fig.5. The nullity of UE in $4\times(1,1)$ model is calculated as $n\ell_k = N_T - N_R + n_{Rk} = 4-2+1 = 3$ and the size of block channel matrix of $\boldsymbol{D}_k$ becomes 1×3. Hence the diversity order becomes $1\times3 = 3$ which is equal to the one of $4\times(3,3)$. From Fig.5, we see that the BER's of $4\times(3,3)$ and $4\times(1,1)$ coincide. On the other hand, the nullity of UE in $4\times(2,2)$ model is calculated as $n\ell_k = N_T - N_R + n_{Rk} = 4-4+2 = 2$ and the size of block channel matrix of $\boldsymbol{D}_k$ becomes 2×2, thus the diversity order of 4 is obtained for the 1st eigen mode channel. We consider the reason why $4\times(2,2)$ has better BER characteristics than $4\times(3,3)$ even though $4\times(2,2)$ has less receive antennas than $4\times(3,3)$ at each UE. From the nullity equation of $n\ell_k = N_T - N_R + n_{Rk}$, if the number of receive antennas at each UE is increased by i' under the constraint of $N_T =$ constant, then the nullity $n\ell_k$ is calculated as

$$n\ell_k = N_T - (N_R + i'N_u) + (n_{Rk} + i') = N_T - N_R + n_{Rk} - i'(N_u - 1) \quad (13)$$

From (13), it is found that if i' is increased, the nullity $n\ell_k$ decreases. Therefore, we can say that when the number of transmit antennas N_T is constant, increasing the number of receive antennas at each UE leads to the decrease of nullity of each UE. As the size of block channel matrix is given by $\boldsymbol{D}_k = n_{Rk}\times n\ell_k$, the decrease of $n\ell_k$ degrades the communication quality in some cases.

V. CONCLUSIONS

In this paper, we investigated the MU-MIMO down link system in which BS and UE's have multiple transmit and receive antennas respectively. The IUI among UE's is pre-excluded at BS by using BD. Then the E-SDM is applied to the block channel matrix of UE to transmit multiple streams. By utilizing the fact that the size of block matrix of UE in the Block Diagonalized channel matrix becomes (number of receive antennas of UE) × (nullity of the channel matrix in which the rows of UE are excluded from the entire channel matrix), we proposed the design method of MU-MIMO down link system where the diversity order of the 1st eigen mode channel of UE is designated. As a result, we have shown by increasing the number of transmit antennas at BS, we can arbitrary improve the diversity order of eigen mode channel of UE. For some examples and through computer simulations, we verified the effectiveness of our design method. For future works, adaptive scheduling algorithms for the change of UE parameters will be considered.

APENDIX
DERIVATION OF THEORETICAL BER IN FIG.2

We derive the theoretical BER's of BD+E-SDM $3\times(1,1,1)$, $6\times(1,1,1,1)$ and $6\times(1,1,1)$ in Fig.2. The sizes of block matrix $\boldsymbol{D}_k$ of $3\times(1,1,1)$, $6\times(1,1,1,1)$ and $6\times(1,1,1)$ are given by 1×1, 1×3 and 1×4 respectively, where one stream is transmitted using E-SDM. One stream transmission in E-SDM is referred to as Maximum Ratio Transmission (MRT) [11] and is equivalent to the Maximum Ratio Combining (MRC) with transmit and receive weights.

We firstly derive the transmit weight $\boldsymbol{v}\,(n_T\times1)$ and receive weight $\boldsymbol{w}\,(n_R\times1)$ in MRT. The transmit signal x is multiplied by the weight $\boldsymbol{v}$ and transmitted to the channel through n_T transmit antennas. The transmit power is expressed as

$$(1/2)\cdot E\{\|\boldsymbol{v}x\|^2\} = \|\boldsymbol{v}\|^2\,E\{|x|^2\}/2 = (1/2)\cdot E\{|x|^2\} = P_x \quad (13)$$

where $\|\boldsymbol{v}\|^2 = 1$ and P_x is the transmit power. At the receiver side, the receive weight $\boldsymbol{w}$ is multiplied by the received signal

$$\hat{x} = \boldsymbol{w}^H\boldsymbol{H}\boldsymbol{v}x + \boldsymbol{w}^H\boldsymbol{n} \quad (14)$$

where $\boldsymbol{H}$ is the channel matrix in MRT. The receive SNR in (14) is given by

$$\frac{S}{N} = \frac{P_x}{\sigma_n^2}\frac{|\boldsymbol{w}^H\boldsymbol{H}\boldsymbol{v}|^2}{\boldsymbol{w}^H\boldsymbol{w}} \quad (15)$$

where $(1/2)E\{\boldsymbol{n}\boldsymbol{n}^H\} = \sigma_n^2\boldsymbol{I}$ and $\boldsymbol{I}$ is the $n_R\times n_R$ identity matrix.

Using Lagrange multiplier method, we derive the weights $\boldsymbol{v}$ and $\boldsymbol{w}$ which maximize the receive SNR. By fixing the value of numerator in (15), we minimize the value of denominator in (15). We consider the real function h

$$h = \boldsymbol{w}^H \boldsymbol{w} + \mathrm{Re}\left\{\lambda^* \left(\boldsymbol{w}^H \boldsymbol{H}\boldsymbol{v} - g\right)\right\} + \xi\left(\boldsymbol{v}^H \boldsymbol{v} - 1\right) \quad (16)$$

where λ and ξ are the Lagrange multipliers. The constraint conditions in (16) are $\boldsymbol{w}^H \boldsymbol{H}\boldsymbol{v} = g$ and $\|\boldsymbol{v}\|^2 = 1$. By partially differentiating h with respect to $\boldsymbol{w}^*$ and $\boldsymbol{v}^*$, we get

$$\begin{cases} \dfrac{\partial h}{\partial \boldsymbol{w}^*} = \boldsymbol{w} + \dfrac{\lambda^* \boldsymbol{H}\boldsymbol{v}}{2} = \boldsymbol{0} \\ \dfrac{\partial h}{\partial \boldsymbol{v}^*} = \dfrac{\lambda}{2}\left(\boldsymbol{w}^T \boldsymbol{H}^*\right)^T + \xi\boldsymbol{v} = \dfrac{\lambda}{2}\boldsymbol{H}^H \boldsymbol{w} + \xi\boldsymbol{v} = \boldsymbol{0} \end{cases} \quad (17)$$

From (17), we have

$$\boldsymbol{w} = -\lambda^* \boldsymbol{H}\boldsymbol{v}/2, \quad \boldsymbol{v} = -\lambda \boldsymbol{H}^H \boldsymbol{w}/(2\xi) \quad (18)$$

From (18), we get

$$\boldsymbol{w} = \frac{|\lambda|^2}{4\xi}\boldsymbol{H}\boldsymbol{H}^H \boldsymbol{w}, \quad \boldsymbol{v} = \frac{|\lambda|^2}{4\xi}\boldsymbol{H}^H \boldsymbol{H}\boldsymbol{v} \quad (19)$$

By substituting $\eta = 4\xi / |\lambda|^2$ into (19), the transmit and receive weights satisfy

$$\begin{cases} \boldsymbol{H}^H \boldsymbol{H}\boldsymbol{v} = \eta\boldsymbol{v} \\ \boldsymbol{H}\boldsymbol{H}^H \boldsymbol{w} = \eta\boldsymbol{w} \end{cases} \quad (20)$$

From (20), it is seen that $\boldsymbol{v}$ is the eigen vector corresponding to the eigen value η of $\boldsymbol{H}^H \boldsymbol{H}$ and $\boldsymbol{w}$ is the eigen vector corresponding to the eigen value η of $\boldsymbol{H}\boldsymbol{H}^H$. From (18) and $\|\boldsymbol{v}\|^2 = \boldsymbol{v}^H \boldsymbol{v} = 1$, we also have

$$\boldsymbol{w}^H \boldsymbol{w} = \|\boldsymbol{w}\|^2 = \xi \quad (21)$$

From (15), the receive SNR is expressed as

$$\frac{S}{N} = \frac{P_x}{\sigma_n^2} \cdot \frac{\left|\boldsymbol{w}^H \boldsymbol{H}\boldsymbol{H}^H \boldsymbol{w}\right|^2}{\eta\xi^2} \quad (22)$$

As $\boldsymbol{w}^H \boldsymbol{H}\boldsymbol{H}^H \boldsymbol{w} = \eta\xi$, the receive SNR is finally obtained as

$$S/N = (P_x/\sigma_n^2) \cdot \eta \quad (23)$$

Accordingly, the receive SNR is maximized when the eigen value η has its maximum value $\eta_{\max}$ of $\boldsymbol{H}^H \boldsymbol{H}$ and $\boldsymbol{H}\boldsymbol{H}^H$ and the weights $\boldsymbol{v}$ and $\boldsymbol{w}$ are the eigen vectors corresponding to $\eta_{\max}$. This ends the derivation of MRT weights.

Using (23), the BER of user k in MRT transmission with QPSK signaling is given by

$$P_b = \frac{1}{2}erfc\left(\frac{1}{2} \cdot \frac{S}{N}\right) = \frac{1}{2}erfc\left(\frac{1}{2}\eta_{\max}\frac{P_k}{\sigma_n^2}\right) \quad (22)$$

where $erfc(\ )$ is the complementary error function.

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REFERENCES

[1] H. S Quentin, B. P. Christian, A. Lee Swindlehurst, M. Hardt, “An introduction to the multi-user MIMO downlink,” IEEE communications Magazine, vol.42, Issue 10, pp.60-67, Oct. 2004.

[2] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, “Zero forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” IEEE Trans. Sig. Processing, vol.52, no.2, pp.461-471, Feb.2004.

[3] T.Haustein, C.von Helmolt, E.Jorswieck, V.Jungnickel, V.Pohl, “Performance of MIMO systems with channel inversion,” IEEE 55th VTC Spring, vol.1, pp. 35 – 39, 2002.

[4] S. Vishwanath, N. Jindal, and A. Goldsmith, “Duality, achievable rates, and sum-rate capacity of MIMO broadcast channels,” IEEE Trans. Inform. Theory, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.

[5] Veljko Stankovic and Martin Haardt, “Successive optimization Tomlinson-Harashima precoding (SO THP) for multi-user MIMO systems,” IEEE International Conference on Acoustics, Speech, and Signal Processing, Proceedings (ICASSP '05), vol.3, pp.iii/1117-iii/1120, March 2005.

[6] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, “A vector-perturbation technique for near-capacity multi antenna multiuser communication-part I: Channel inversion and regularization,” IEEE Trans. Commun., vol. 53, pp.195-202, Jan. 2005.

[7] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, “A Vector-Perturbation Technique for Near-Capacity Multi antenna Multiuser Communication—Part II: Perturbation,” IEEE Trans. Commun., vol. 53, pp. 195–202, Jan. 2005.

[8] K. Miyashita, T. Nishimura, T. Ohgane, Y. Ogawa, Y. Takatori, and K. Cho, “Eigenbeam-Space Division Multiplexing (E-SDM) in a MIMO Channel,” IEICE Technical Report，RCS2002−53，pp.13−18，May, 2002.

[9] A. Paulraj, R. Nabar and D. Gore, Introduction to Space Time wireless Communication, Cambridge University Press, 2008.

[10] J. K. Cavers, “Single-User and Multiuser Adaptive Maximum Ratio Transmission for Rayleigh Channel,” IEEE Trans. on Vehicular Technology, Vol.49, No.6, pp.2043-2050, Nov. 2000.

[11] T. K. Y. Lo,“Maximum Ratio Transmission,” IEEE Trans. Commun., vol.47, no.10, pp.1458-1461, Oct. 1999.