A Study on Frequency Response Analysis Using Friction Model for Frictional Systems

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Abstract—This paper examines a frequency response analysis (FRA) method which aims at obtaining an precise linear plant model for a controller design, for fast and precise positioning of mechatronic systems with nonlinear friction. As well known, friction generated at linear guideways and/or bearings shows nonlinear and complicated behaviors, which causes deterioration of the motion control performance. In addition, the frequency response of the plant system dynamically changes depending on the measurement condition such as displacement and/or excitation force in the frequency domain analysis. The dynamic change of the frequency response makes it difficult to identify the actual linear plant characteristic for design of a precise simulator and/or an effective compensator.

In this study, therefore, effects of parameter identification errors due to the dynamic frequency property on the fine positioning performance are clarified by simulation analyses with a nonlinear friction model. Then, a fundamental friction model-based FRA method is introduced, and the effectiveness of the presented FRA and effects of the friction model errors are demonstrated by frequency-domain simulations.

I. Introduction

In the research field of fast-response and high-precision positioning control of mechatronic systems, friction modeling and compensation is one of the important topics for improving the control performance [1]. As an example of the mechatronic systems, table positioning devices using a linear motor or a ball screw with a rotary motor are popular and widely used in industry, and micro- and nanometer positioning accuracy with a high-speed response is strongly required to possess advantages in productivity and/or quality of products [2]. However, the friction in the mechanism, e.g., linear guideways and bearings with rolling elements, shows nonlinear and complicated behaviors in the fine positioning motion, and various kinds of unfavorable responses, e.g., steady state position error [2], [3], transient position tracking error [4], and response dispersion [5], deteriorate the positioning accuracy [6].

On the other hand, it is well-known that friction affects frequency response analysis (FRA) of the plant characteristic due to the nonlinear behavior [7], [8]. The frequency response variation causes identification errors for the linear plant characteristic (e.g., mass, moment of inertia, and resonant frequency), which makes it difficult not only to construct a precise simulator but also to design an effective compensator. Some FRA methods taking account of the nonlinear friction behavior, therefore, have been studied in literature [4], [9]. In particular, the reference [10] presents an effective FRA method with a friction model-based compensation. However, it needs experimental tuning of a friction compensation gain and larger excitation amplitude to obtain a precise linear plant property in frequency-domain, while the relation between the tuning and the excitation amplitude has not been examined in detail.

In this paper, an FRA technique using a friction model is examined as an initial consideration, to achieve a precise linear plant model for an effective controller design for fast and precise positioning control. The presented FRA method has a simple structure without a kind of the tuning gain, and aims at estimating an unique frequency response of the linear plant even if the excitation amplitude is different. Effectiveness of the FRA method and effects of the friction model errors are investigated by numerical simulations using a prototype of linear motor-driven table systems.

II. Table Positioning System

A. System Configuration

Fig. 1 shows an overview of target table positioning device as a prototype. A linear motor (Sanyo Denki DS050) drives the target table with a flexible load along two rolling ball linear guideways on a machine stand. The machine stand is supported by six leveling bolts on floor, and resonant vibration between the machine stand and the floor is excited in the fast-response positioning motion. The table position is detected by a linear scale (Mitsutoyo AT211, resolution of 0.1 μm) along the guides, and is controlled in a full-closed control manner by a DSP (SDS PCI-DSP46713, sampling time T_s of 500 μs) through an AC servo amplifier (Sanyo Denki PY0).

B. Friction Characteristic

Rolling friction is generated by deformation and slips at contact points between rolling elements and rolling ball guides, and shows quite different properties between micro and macro displacement regions: the rolling friction behaves as a nonlinear elastic component in the micro displacement region (so-called "presliding region", less than $10\sim100$ μm in general) after velocity reversal. On the other hand, the rolling elements effectively roll and the friction force is static (Coulomb friction) in the macro displacement region. Light solid lines in Fig. 2 show experimental rolling friction characteristics of the prototype, and the presliding region is about 100 μm.
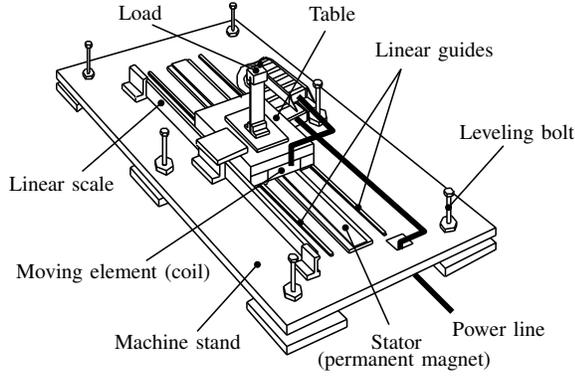
Fig. 1. Schematic configuration of prototype.

(a) Overview

(b) Presliding region after the velocity reversal
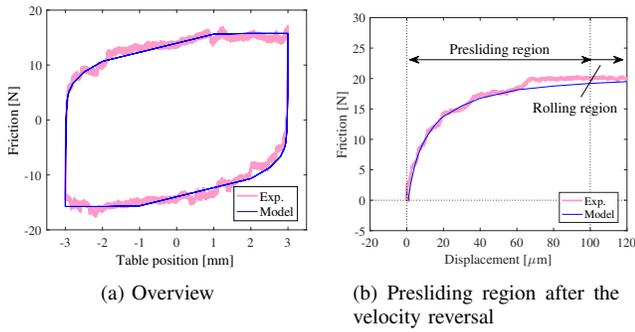
Fig. 2. Nonlinear elastic characteristics of rolling friction.

Fig. 3. Conventional FRA method.

Fig. 4. Experimental bode plots of plant.

Note that although the target positioning system has viscous friction depending on the table velocity, the effect of the viscous friction is much smaller than the rolling friction.

C. Frequency Characteristic

Fig. 3 shows a conceptual diagram of an FRA method widely used by industrial engineers, where $C(z)$ is the feedback (FB) compensator, $DFT\,Alg$ is the discrete Fourier transformation (DFT)-based frequency analysis algorithm, u_{sin} is the sinusoidal sweep signal $u_{sin} = A\sin(2\pi ft)$, r is the target position reference, y' is the table position as the plant output, and u is the motor thrust reference as the control input, respectively. The conventional method analyzes the frequency response of the plant by using u and y'.

In order to clarify the influence of the nonlinear friction on the FRA, experimental frequency characteristics of y' for u are depicted by dark dotted lines and light solid lines in Fig. 4. Here, each experimental result was measured by changing the amplitude A of u_{sin} to excite the table in different displacement regions. From the figure, the plant system shows quite different frequency characteristics depending on the excitation amplitude, i.e., gain in low frequency range less than 25 Hz, the first vibration mode around 40 Hz (machine stand), and the second vibration mode around 80 Hz (flexible load). This variation of the superficial plant characteristic is caused by the nonlinear property of the rolling friction as shown in Fig. 2 [2], [8], [10]. An equivalent block diagram of the plant can be expressed by Fig. 5, where $P(s)$ is the linear plant model,

e^{-Ls} is the dead time component (such as D/A conversion, low pass filters, and a current control system), and u_e is the effective thrust actually driving the table, respectively. $P(s)$ is mathematically formulated as follows, with consideration of a rigid mode and the first and second vibration modes:

$$P(s) = \frac{y(s)}{u_e(s)} = K_t \left(\frac{K_{t0}}{s^2} + \sum_{i=1}^{2} \frac{k_{ti}}{s^2 + 2\zeta_{ti}\omega_{ti}s + \omega_{ti}^2} \right). \quad (1)$$

Dark dotted lines and light solid lines in Fig. 6 shows bode plots of the plant model of Fig. 5 which were calculated by simulations using the conventional FRA method. Here, a friction model defined in Section V-A was applied as the friction in Fig. 5. In addition, the linear plant characteristic $P(s)$ of eq.(1) is depicted by dark broken lines. From the figure, the plant model including the rolling friction property reproduces the remarkable difference in the experimental frequency characteristics shown in Fig. 4.

III. MODEL-BASED FEEDFORWARD COMPENSATION AND ITS SUBJECT

A. 2-Degree-of-Freedom Position Control System

Fig. 7 shows a block diagram of a two-degree-of-freedom (2DoF) table position control system, where $N_{ff}(z)$ and $D_{ff}(z)$ are the feedforward (FF) compensators based on
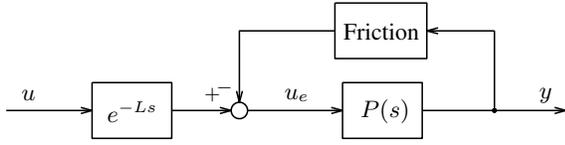
Fig. 5. Equivalent block diagram of plant with friction.

Fig. 7. Block diagram of 2DoF table position control system.

Fig. 6. Simulated bode plots of plant.
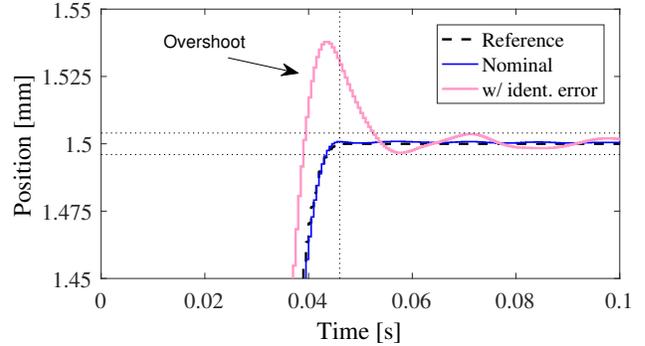
Fig. 8. Simulated response waveforms of table position.

a deadbeat control framework [11] with a design model $P_d(s)$, z^{-2} is the dead time compensator based on Smith method, $C(z)$ is the FB compensator, RFM is the rolling friction model for FF friction compensation, r is the target table position (step signal), and u_{ff} is the FF motor thrust reference, respectively. In this 2DoF control system, RFM cancels the actual friction behavior in the positioning, while the FF compensators $N_{ff}(z)$ and $D_{ff}(z)$ mainly determine the response speed and the vibration suppression capability for the target resonant system. In order to realize desired position tracking performance in the case that RFM ideally cancels the actual friction in the plant, $P_d(s)$ for the FF control design should precisely represent the frequency characteristic of the actual linear plant $P(s)$ of eq.(1), from a viewpoint of the model-based control framework.

B. Residual Vibration due to Identification Error for Resonant Mode Parameters

In order to clarify the influence of the plant identification error in the FF control design, parameters of $P_d(s)$ are identified for the frequency characteristic in the case of the smaller excitation amplitude ($A = 12$ N) in Fig. 6. Dark solid lines indicate the frequency characteristic of $P_d(s)$. From the figure, $P_d(s)$ cannot express the actual linear plant characteristic $P(s)$: the higher gain in all frequency ranges and the higher resonant frequency of the first vibration mode. In this case, the identified rigid mode gain and first resonant frequency of $P_d(s)$ became $k_0 = 177$ mm/Ns^2 and $f_1 = 41.9$ Hz, compared to $k_0 = 132$ mm/Ns^2 and $f_1 = 38.8$ Hz of

$P(s)$. Although larger excitation amplitude of the sinusoidal sweep is required in general to neglect the effect of nonlinear friction in FRA [10], it may be difficult to sweep with sufficient amplitude due to abnormal noise from the mechanism and/or restriction of the control input amplitude. This kind of identification error, therefore, would be happened in the plant parameter identification process in industrial engineering.

Fig. 8 shows simulated step response waveforms of the table position around the target position of $r = 1.5$ mm, where a dark solid line denotes a nominal response under $P(s) = P_d(s)$ and a light solid line denotes the one with the identification error. In the case with the identification error in the FF compensator design, a remarkable overshoot response appears at the settling region, which obviously degrades the target settling accuracy of ± 4 μm indicated by horizontal dotted lines. From the result of the positioning simulation, it is clear that a precise FRA technique considering the nonlinear friction phenomena is highly important to obtain the fast and precise positioning performance.

IV. PRINCIPLE OF PROPOSED FRA METHOD

In this section, fundamental principle of the proposed FRA method using a friction model is explained in detail. As shown in Fig. 5, the actual plant system can be expressed by the linear plant $P(s)$ with a dead time component e^{-Ls} and friction. In the conventional FRA method shown in Fig. 3, the motor thrust reference u and the table position y' detected by the sensor are regarded as input and output of $P(s)$. However, since u does not equal to the effective thrust for the table, u_e should be estimated to analyze $P(s)$ in principle.
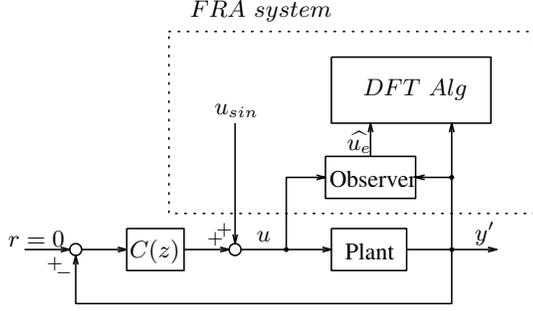
Fig. 9. Block diagram of proposed FRA method.
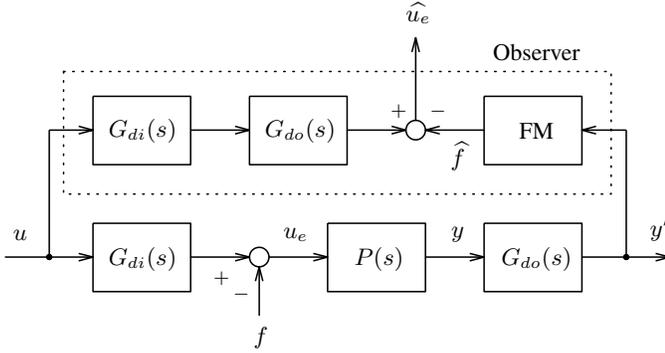
Fig. 10. Block diagram of effective control input observer.

Fig. 11. Rolling friction model with N elementary models.

Fig. 9 shows a block diagram of the proposed FRA method with an effective control input observer, while Fig. 10 shows a detailed block diagram of the observer. In the figures, $P(s)$ is the actual linear plant, $G_{di}(s)$ and $G_{do}(s)$ are the characteristics of delay components (such as a dead time, a current control system, D/A and A/D conversions, etc.) on input and output of the plant, FM is the friction model for estimating friction force, u is the control input, u_e is the effective control input, y is the position output, y' is the detected position output with the delay, f is the actual friction, $\hat{f}$ is the estimated friction, and $\widehat{u_e}$ is the estimated effective control input, respectively. The observer is composed of $G_{di}(s)$, $G_{do}(s)$, and FM, and the proposed FRA method estimates $\widehat{u_e}$ by using $\hat{f}$ calculated by FM. $\widehat{u_e}$ can be mathematically formulated as follows:

$$\widehat{u_e} = G_{di}(s)G_{do}(s)u - \hat{f}. \tag{2}$$

Here, since y' is expressed as $G_{do}(s)y$, $\hat{f}$ includes a time delay due to $G_{do}(s)$. Therefore, $\hat{f}$ can be defined as follows:

$$\hat{f} = G_{do}(s)f. \tag{3}$$

By substituting eq.(3) to eq.(2), $\widehat{u_e}$ can be expressed as follows:

$$\widehat{u_e} = G_{do}(s)\left\{G_{di}(s)u - f\right\}. \tag{4}$$

Note that $\widehat{u_e}$ includes the influence of $G_{do}(s)$ both on u and f. By using $\widehat{u_e}$ and y', the linear plant characteristic can be achieved by the following equation:

$$\widehat{P}(s) = \frac{y'}{\widehat{u_e}} = \frac{G_{do}(s)y}{G_{do}(s)\left\{G_{di}(s)u - f\right\}}. \tag{5}$$

The proposed FRA method has a simple structure and the models $G_{di}(s)$, $G_{do}(s)$, and FM should be identified before performing FRA. If the models $G_{di}(s)$, $G_{do}(s)$, and FM can precisely express the actual characteristics, then $\widehat{P}(s)$ of eq.(5) equals to $P(s)$. On the other hand, although the FRA method in [10] needs a FB compensation for the friction and the larger excitation amplitude to successfully suppress the influence of the friction in the frequency-domain analysis, the proposed method does not need a kind of the FB compensation and can achieve a same frequency response theoretically even if the excitation force variation.

V. DESIGN AND EVALUATION OF PROPOSED FRA

A. Design of FRA System

In order to construct the proposed FRA method for the target table positioning system, we introduce a rolling friction model [3], [5] and a viscous friction model as FM in Fig. 10, since the rolling friction and the viscous friction are dominant in the movement of the table during the sinusoidal sweep.

Fig. 11 shows conceptual diagram of the rolling friction model which considers rheology at contact points of friction surface by N pieces of elementary models. The rolling friction model is a kind of multiple-structure models such as an elastoplastic model [12] and Generalized Maxwell Slip model [2] that can well express friction phenomena, and the mathematical formulation is defined as follows:

$$y_i = \begin{cases} y + y_{ri} & (|y_i| < Y_{mi}) : \text{stick} \\ \text{sgn}(\frac{dy}{dt})Y_{mi} & (|y_i| = Y_{mi}) : \text{slip} \end{cases}, \tag{6}$$

$$f_i = K_i y_i + D_i \frac{dy_i}{dt}, \tag{7}$$

$$F_{mi} = K_i Y_{mi}, \tag{8}$$

$$-Y_{mi} \le y_i \le Y_{mi}, \tag{9}$$

$$-F_{mi} \leq f_i \leq F_{mi}, \tag{10}$$

$$f_{rolling} = \sum_{i=1}^{N} f_i, \tag{11}$$

where y is the displacement input corresponding to the table position, y_i is the element displacement, y_{ri} is the element displacement at velocity reversal, f_i is the element force, Y_{mi} is the maximum element displacement, F_{mi} is the maximum element force, K_i is the element elastic coefficient, D_i is the element viscous coefficient, $\mathrm{sgn}(\cdot)$ is the sign function, and $f_{rolling}$ is the rolling friction force, respectively. From eqs.(6)-(10), the elementary model generates viscoelastic friction force in the stick region and the static force with the limit stress of $\pm F_{mi}$ in the slip region. The parameters of the rolling friction model, i.e., K_i, D_i, and F_{mi}, are identified by using Back Propagation algorithm [5], while N is decided so that an evaluation function for a square error between the experimental data and the model output is minimized. In this study, $N = 20$ was chosen and the parameters were identified. Dark solid lines indicated in Figs. 2(a) and 2(b) show the characteristics of the rolling friction model, and well reproduces the experimental characteristic indicated by light lines.

The viscous friction model which simulates the static friction property for velocity is simply formulated as

$$f_{viscous} = D_v v, \tag{12}$$

where $f_{viscous}$ is the viscous friction force, $v(= dy/dt)$ is the table velocity, and D_v is the viscous friction coefficient, respectively. D_v was identified as $D_v = 0.00469$ Ns/mm by constant velocity drive experiments. By using the rolling friction model of eqs.(6)-(11) and the viscous friction model of eq.(12), $\widehat{f}$ can be calculated as

$$\widehat{f} = f_{rolling} + f_{viscous}. \tag{13}$$

B. FRA Simulations in Different Excitation Conditions

theoretical effectiveness of the proposed FRA method considering the nonlinear friction dynamics is verified by numerical simulations. In the simulations, the amplitude of the sinusoidal sweep is set as $A = 75$ and 12 N to evaluate whether the proposed FRA method can calculate the actual linear plant property $P(s)$ in the different excitation conditions. Note that the friction model of the actual plant is set as same as of the FM in the FRA system so as to verify the principle presented in Section IV in an ideal condition.

Fig. 12 shows the simulation results of the conventional FRA method and the proposed FRA method. In the figure, dark broken lines are the frequency characteristics of the actual linear plant $P(s)$ represented by eq.(1), light and dark solid lines are the estimated frequency characteristics with $A = 75$ N, and light and dark dotted lines are the ones with $A = 12$ N, respectively. In the case of the conventional FRA method, the superficial linear plant characteristic varies depending on the excitation amplitude as aforementioned in Section II-C. The proposed FRA method, on the other hand,
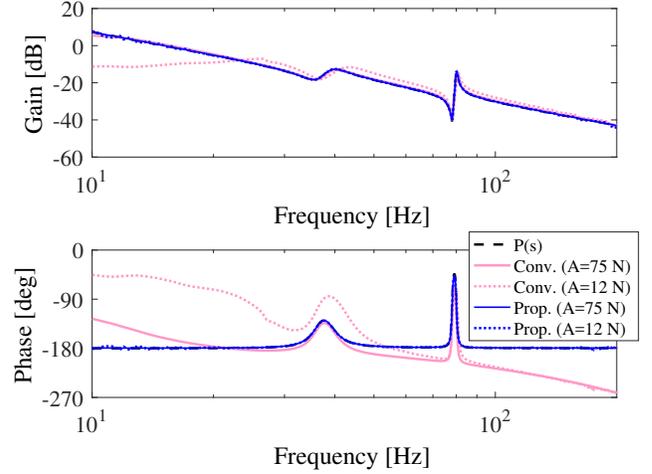
Fig. 12. FRA simulation results in different excitation conditions.
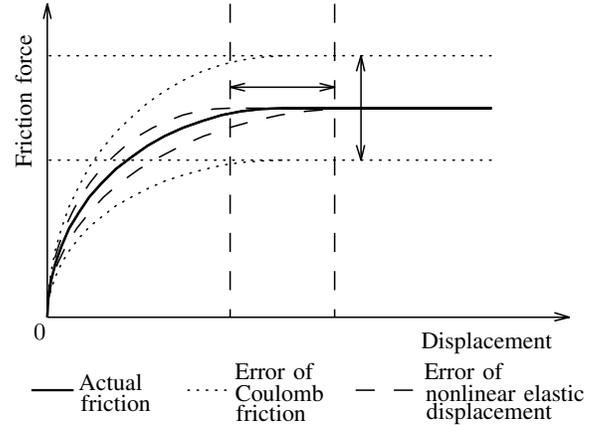
Fig. 13. Conceptual diagram of rolling friction model errors.

can precisely estimate the actual linear plant characteristic and the estimated frequency responses are almost same in the different excitation conditions. From the simulation results, the fundamental principle of the FRA presented in Section IV is successfully verified.

C. FRA Simulations for Friction Model Errors

Next, effects of the rolling friction model errors in FM are evaluated, since the proposed FRA method is a simple friction model-based estimation. In the numerical simulations, we intentionally varied the Coulomb friction force $\sum_{i=1}^{20} F_{mi}$ and the displacement of the nonlinear elastic region Y_{mi} of the rolling friction model in the FRA system by ± 10 % for the actual friction as shown in Fig. 13, with consideration of the identification errors due to parameter fluctuations and/or the mathematical expression of the friction model. The excitation signal amplitude is set as $A = 12$ N. From Fig. 14, it can be seen that the error of the Coulomb friction deteriorates the FRA performance not only in the low frequency range
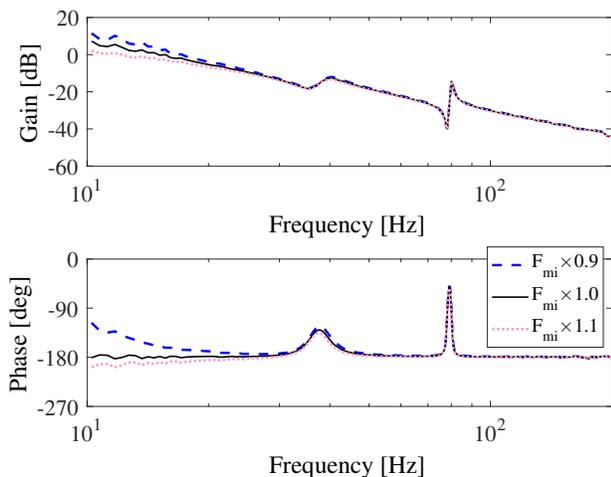
Fig. 14. FRA simulation results for friction model error in Coulomb friction.
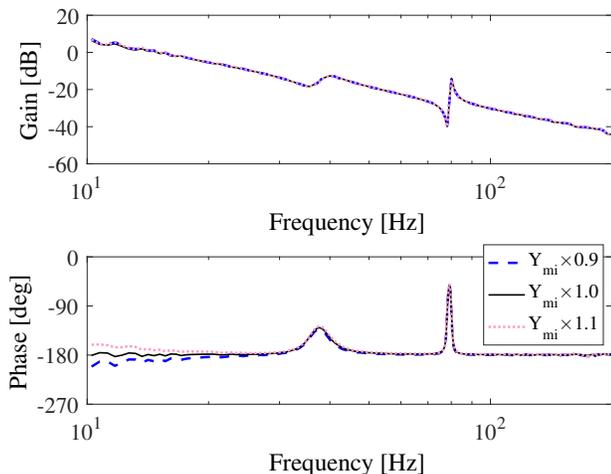
Fig. 15. FRA simulation results for friction model error in displacement of nonlinear elastic region.

but also around the first resonant mode at 40 Hz. On the other hand, from Fig. 15, although effects of the displacement of the nonlinear elastic region are smaller than the Coulomb friction, it degrades the FRA performance. Since these kinds of model errors are unavoidable in the industrial engineering in practice [10], accurate friction model identification in advance is indispensable in the proposed FRA. Effective friction model identification methods considering the variation properties shown in Figs. 14 and 15 will be studied as a future challenge.

VI. CONCLUSION

In this study, an FRA method has been presented to identify a linear plant characteristic for frictional systems. The proposed FRA method has a simple structure with a friction model and estimates time-domain plant input without influence of nonlinear friction for the FRA. By applying the proposed method to a table positioning device with rolling friction, an exact and unique linear plant characteristic can be theoretically achieved even if the superficial frequency property dynamically varies due to the rolling friction. However, at the same time, it has been confirmed that the friction model errors are sensitive to the FRA performance. As future works, the proposed FRA method will be verified by experiments using some frictional mechatronic systems. In addition to this, autonomous parameter identification and plant modeling technologies based on the presented FRA method will be examined.

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