Robust Load States Estimation against Mechanical Parameters Variation of a Two-Mass System using Acceleration-aided Dynamic Kalman Filter

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***Abstract*—This paper presents a robust load states estimation method of a linear motor driven table system using an acceleration-aided dynamic Kalman filter. The system is composed of a table and a load connected via an elastic beam attached on the table, therefore the system is modeled as a two-mass system. In order to improve the positioning performance at load-side, acquisition of the load position is important. However it might be difficult to place position sensors on the load from the viewpoint of cost saving and available space. In this research, a small-size and low-cost MEMS accelerometer is installed on the load in order to measure load acceleration. Load acceleration and table displacement information are utilized to the acceleration-aided dynamic Kalman filter in order to estimate the load position. Moreover, to achieve robust estimation capability against mechanical parameter variations of the system, the extended Kalman filter is configured in order to estimate mechanical parameters together with the load states. The effectiveness of the proposed estimation method is verified by numerical simulations.**

I. Introduction

In high performance mechatronic systems such as machine tools, inspection equipments, and surface mount equipments, it is important to improve positioning performance at load side in order to achieve high quality production and cost saving [1], [2]. To compensate for vibrations excited by a low stiffness and nonlinearities, a full-closed position controller is constructed which is using load side position information together with sensors at actuator side and so on. However this approach leads to additional costs, and it might be not applicable, in case that new sensors do not fit in available spaces.

Recently, using low resolution sensors with Kalman filter[3] for improvement of positioning and/or motion control performance are reported[4]–[12]. Authors have also reported the load state estimation methodology for linear motor driven table system using Kalman filter and a MEMS accelerometer[13], [14]. The proposed method can estimate load states of the rigid mass system well, however it is difficult to apply to systems with elastic mechanism. In particular, resonance frequency change may deteriorate estimation performance of the proposed method.

In this paper a robust load states estimation method of a linear motor driven table system using an acceleration-aided dynamic Kalman filter is proposed. The system is composed of a table and a load connected via an elastic beam attached on the table, therefore the system is modeled as a two-mass system. A small-size and low-cost MEMS accelerometer is installed on the load in order to measure load acceleration. Load acceleration and table displacement information are utilized to the acceleration-aided dynamic Kalman filter in order to estimate the load position[15]. Moreover, in order to achieve robust estimation capability against mechanical parameter variations, the extended Kalman filter is configured in order to estimate mechanical parameters together with the load states.

The effectiveness of the proposed estimation method is verified by numerical simulations.

II. System Structure and System Modeling

A. Linear Motor Driven Table System

In this paper, a linear motor-driven table system is performed as a control target. Fig. 1 shows a picture of an experimental setup, where the table is driven by the linear motor installed on the machine stand. A weight, as load, is attached on the top of the elastic beam which is setting on the table in order to reproduce mechanical vibrations seen often in mechatronic systems. The table displacement is measured by a linear encoder of which sensor resolution of 0.1 μm, and a MEMS accelerometer is attached on the load to measure load acceleration. For validation, a laser displacement meter is mounted on a tripod to measure displacement and velocity of tip of load.

As stiffness of the elastic beam is a finite value, the control system is modeled as a two-mass system. The motion equations

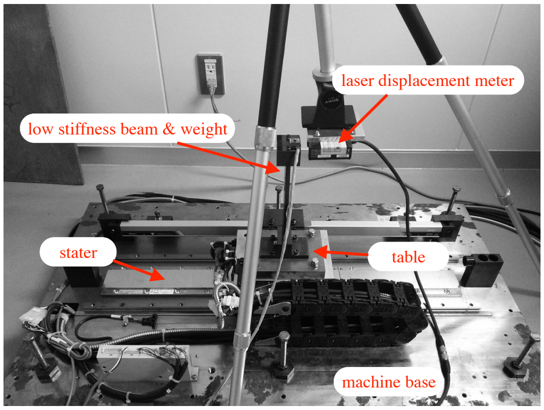


Fig. 1: Picture of experimental setup.

TABLE I: Model parameters.

table mass	M	15.5	kg
load mass	m	1.0	kg
spring constant	k	2.44×10^5	N/m
viscosity	c	9.05	N/(m/s)

of the two-mass system are expressed as follows:

$$M\ddot{x}_1 = u - k(x_1 - x_2) - c(\dot{x}_1 - \dot{x}_2), \quad (1)$$

$$m\ddot{x}_2 = k(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2), \quad (2)$$

where, M; table mass, m; load mass, k and c; spring constant and viscosity of the beam, x_1 and x_2; table and load displacement and u; control input, respectively. Model parameters, listed in Table I, are identified by curve fitting techniques based on FRF of the system.

B. MEMS accelerometer

A MEMS accelerometer (Analog Devices, ADXL335 3-Axis accelerometer[16]) is attached on the top of the load, where Table II shows the specifications of the MEMS accelerometer. The accelerometer can measure the load acceleration between ± 3 g and its typical sensitivity is 300 mV/g at supplied voltage of 3.0 V. The sensitivity is affected by its supplied voltage as ratio metric and also be affected ambient temperature from 270 mV/g to 330 mV/g. The provided acceleration includes 0 g offset. Its typical value is 1.5 V at supplied voltage of 3.0 V, and fluctuate from 1.35 V to 1.65 V due to supplied voltage and ambient temperature. The bandwidth can be chosen by attached capacitor at sensor outputs from 0 Hz to 1500 Hz.

III. Acceleration-aided dynamic Kalman filter

A. Dynamic system for Kalman filter

In order to achieve robust load states estimation against mechanical parameter variations of the two-mass system, an acceleration-aided dynamic Kalman filter (a^{2}d-KF) is utilized, where the continuous time plant system used for a^{2}d-KF is assumed as Fig. 2, in which input of the system is the load acceleration a_m, while output of the system is the table position

TABLE II: Specifications of ADXL335.

Parameter	Min	Typ	Max	Unit
Measurement Range	±3	±3.6		g
Sensitivity	270	300	330	mV/g
Sensitivity Change Due to Temp.		0.01		%/°C
0 g Voltage	1.35	1.5	1.65	V
0 g Offset vs. Temp.		±1		mg/°C
Bandwidth		1500		Hz

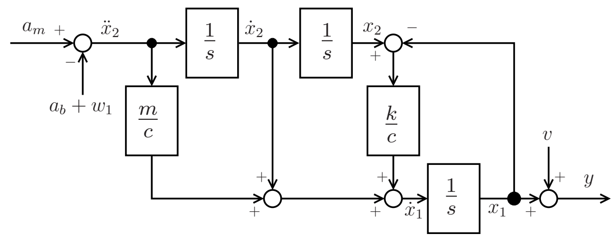


Fig. 2: Block diagram of continuous time plant model.

y. Now let the measured acceleration a_m includes the real acceleration a $(= \ddot{x}_2)$ and the bias a_b and noises w_1 as:

$$a_m = a + a_b + w_1, \quad (3)$$

which is rewritten as:

$$\ddot{x}_2 = a_m - a_b - w_1. \quad (4)$$

The bias is modeled as constant value for simplicity as:

$$\dot{a}_b = w_2. \quad (5)$$

Substituting eq.(4) for eq.(2), the state space model, in which input is load acceleration and output is table position, can be determined as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}a_m + \boldsymbol{G}\boldsymbol{w}, \quad (6)$$

$$y = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}a_m + \boldsymbol{H}\boldsymbol{w} + v, \quad (7)$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \dot{x}_2 & a_b \end{bmatrix}^T, \boldsymbol{w} = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T,$$

$$\boldsymbol{A} = \begin{bmatrix} -k/c & k/c & 1 & -m/c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} m/c \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} -m/c & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\boldsymbol{D} = \boldsymbol{H} = \boldsymbol{0}$$

The process noise w_1, w_2 and the observation noise v are white Gaussian random process with zero mean and variance σ_a^2, σ_b^2, σ_R^2, respectively, and they do not depends on each other.

B. Discretization of dynamic system

Selecting the proper discretized method would be the important point to realize better estimation performance, since the discrete time plant system is used as a design model for the Kalman filter. the system describe as eqs. (6) and (7) has resonance characteristics and does not include zero-order hold (ZOH) mechanism like A/D converter. Additionally, for the proper estimation of the Kalman filter, it is important to predict the next sampling state accurately (the innovation sequence is a white noise if the filter runs optimally). Thus using Tustin method with pre-warping is preferred than using the other methods. By using Tustin method with pre-warping, eqs. (6) and (7) are discretized as:

$$\boldsymbol{x}_{n+1} = \boldsymbol{A}_d\boldsymbol{x}_n + \boldsymbol{B}_d a_{m,n} + \boldsymbol{G}\boldsymbol{w}_n, \tag{8}$$

$$y_n = \boldsymbol{C}_d\boldsymbol{x}_n + \boldsymbol{D}_d a_{m,n} + \boldsymbol{H}\boldsymbol{w}_n + v_n, \tag{9}$$

$$\begin{aligned}
\boldsymbol{A}_d &= (\boldsymbol{I} - \frac{\Delta}{2}\boldsymbol{A})^{-1}(\boldsymbol{I} + \frac{\Delta}{2}\boldsymbol{A}),\\
\boldsymbol{B}_d &= \Delta(\boldsymbol{I} - \frac{\Delta}{2}\boldsymbol{A})^{-1}\boldsymbol{B},\ \boldsymbol{C}_d = \boldsymbol{C}(\boldsymbol{I} - \frac{\Delta}{2}\boldsymbol{A})^{-1},\\
\boldsymbol{D}_d &= \frac{\Delta}{2}\boldsymbol{C}(\boldsymbol{I} - \frac{\Delta}{2}\boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D},\\
\boldsymbol{G}_d &= \Delta(\boldsymbol{I} - \frac{\Delta}{2}\boldsymbol{A})^{-1}\boldsymbol{G},\\
\boldsymbol{H}_d &= \frac{\Delta}{2}\boldsymbol{C}(\boldsymbol{I} - \frac{\Delta}{2}\boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{H},\\
\Delta &= \frac{\tan(\pi f T_s)}{\pi f}
\end{aligned}$$

where, f; resonance frequency of 80 Hz and T_s ; sampling time of 500 μs.

C. Extended Kalman filter

Mechanical parameter variations deteriorate estimation performance of the load states. In particular, resonance frequency mismatch may cause large estimation error. In this research, the load mass, m, and spring constant, k, therefore, are considered as variation factors. Under the assumption that the viscosity, c, is constant, variables k_c and m_c are determined as follows:

$$k_c = \frac{k}{c}, \quad m_c = \frac{m}{c}. \tag{10}$$

In the proposed Kalman filter, mechanical parameter, k_c, is estimated in order to match resonance frequency of the system, resulting in the load states can be estimated well.

Now define the unknown parameter $\boldsymbol{\theta}$ as:

$$\boldsymbol{\theta} = k_c. \tag{11}$$

The discretized state space model based on eqs.(6) and (7) is described as:

$$\boldsymbol{x}_{n+1} = \boldsymbol{A}_d(\boldsymbol{\theta})\boldsymbol{x}_n + \boldsymbol{B}_d(\boldsymbol{\theta})a_{m,n} + \boldsymbol{G}_d(\boldsymbol{\theta})\boldsymbol{w}_n, \tag{12}$$

$$y_n = \boldsymbol{C}_d(\boldsymbol{\theta})\boldsymbol{x}_n + \boldsymbol{D}_d(\boldsymbol{\theta})a_{m,n} + \boldsymbol{H}_d(\boldsymbol{\theta})\boldsymbol{w}_n + v_n \tag{13}$$

In order to estimate $\boldsymbol{\theta}$ simultaneously with $\boldsymbol{x}$, $\boldsymbol{\theta}$ is treated as time function as:

$$\boldsymbol{x}_{n+1} = \boldsymbol{A}_d(\boldsymbol{\theta}_n)\boldsymbol{x}_n + \boldsymbol{B}_d(\boldsymbol{\theta}_n)a_{m,n} + \boldsymbol{G}_d(\boldsymbol{\theta}_n)\boldsymbol{w}_n, \tag{14}$$

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n, \tag{15}$$

$$y_n = \boldsymbol{C}_d(\boldsymbol{\theta}_n)\boldsymbol{x}_n + \boldsymbol{D}_d(\boldsymbol{\theta}_n)a_{m,n} + \boldsymbol{H}_d(\boldsymbol{\theta}_n)\boldsymbol{w}_n + v_n \tag{16}$$

Now, expand the state vector as $\boldsymbol{z}_n = \begin{bmatrix} \boldsymbol{x}_n^T & \boldsymbol{\theta}_n^T \end{bmatrix}^T$, then eq.(14) can be expanded as:

$$\boldsymbol{z}_{n+1} = \boldsymbol{f}(\boldsymbol{z}_n, a_{m,n}) + \boldsymbol{\Gamma}(\boldsymbol{\theta}_n)\boldsymbol{w}_n, \tag{17}$$

$$\boldsymbol{f}(\boldsymbol{z}_n, a_{m,n}) = \begin{bmatrix} \boldsymbol{A}_d(\boldsymbol{\theta}_n)\boldsymbol{x}_n + \boldsymbol{B}_d(\boldsymbol{\theta}_n)a_{m,n} \\ \boldsymbol{\theta}_n \end{bmatrix},$$

$$\boldsymbol{\Gamma}(\boldsymbol{\theta}_n) = \begin{bmatrix} \boldsymbol{G}_d(\boldsymbol{\theta}_n) \\ \boldsymbol{0} \end{bmatrix}$$

Here, $\boldsymbol{A}_d(\boldsymbol{\theta})$, $\boldsymbol{B}_d(\boldsymbol{\theta})$, $\boldsymbol{C}_d(\boldsymbol{\theta})$ and $\boldsymbol{D}_d(\boldsymbol{\theta})$ in eqs.(12) and (13) are differentiable with $\boldsymbol{\theta}$, thus define the following equations.

$$\boldsymbol{F}(\hat{\boldsymbol{z}}_{n,n}, a_{m,n}) = \begin{bmatrix} \boldsymbol{A}_d(\hat{\boldsymbol{\theta}}_n) & \boldsymbol{F}_{1,2,n} \\ 0 & \boldsymbol{I} \end{bmatrix}, \tag{18}$$

$$\boldsymbol{\eta}(\hat{\boldsymbol{z}}_{n,n-1}) = \begin{bmatrix} \boldsymbol{C}_d(\hat{\boldsymbol{\theta}}_{n-1}) & \boldsymbol{\eta}_{2,n} \end{bmatrix}, \tag{19}$$

$$\boldsymbol{F}_{1,2,n} = \frac{\partial[\boldsymbol{A}_d(\boldsymbol{\theta})\boldsymbol{x} + \boldsymbol{B}_d(\boldsymbol{\theta})a_m]}{\partial\boldsymbol{\theta}^T},$$

$$\boldsymbol{\eta}_{2,n} = \frac{\partial \boldsymbol{C}_d(\boldsymbol{\theta})\boldsymbol{x}}{\partial\boldsymbol{\theta}^T} \tag{20}$$

while $\boldsymbol{F}(\hat{\boldsymbol{z}}_{n,n}, a_{m,n})$ and $\boldsymbol{F}_{1,2,n}$ are partial differential for $\boldsymbol{z} = \hat{\boldsymbol{z}}_{n,n}$ and $a_m = a_{m,n}$, and also $\boldsymbol{\eta}(\hat{\boldsymbol{z}}_{n,n-1})$ and $\boldsymbol{\eta}_{2,n}$ are partial differential for $\boldsymbol{z} = \hat{\boldsymbol{z}}_{n,n-1}$.

Above all, define as $\boldsymbol{C}_d(\hat{\boldsymbol{\theta}}_n) = \boldsymbol{C}_{e,n}$, $\boldsymbol{D}_d(\hat{\boldsymbol{\theta}}_n) = \boldsymbol{D}_{e,n}$, $\boldsymbol{H}_d(\hat{\boldsymbol{\theta}}_n) = \boldsymbol{H}_{e,n}$, $\boldsymbol{\Gamma}(\hat{\boldsymbol{\theta}}_n) = \boldsymbol{\Gamma}_{e,n}$, $\boldsymbol{F}(\hat{\boldsymbol{z}}_{n,n}, a_{m,n}) = \boldsymbol{F}_{e,n}$, $\boldsymbol{\eta}(\hat{\boldsymbol{z}}_{n,n-1}) = \boldsymbol{\eta}_{e,n}$, then the covariance matrix $\boldsymbol{Q}_n$ and $\boldsymbol{R}_n$ at the n^{th} sample can be describe as:

$$\begin{aligned}
\boldsymbol{Q}_n &= \boldsymbol{\Gamma}_{e,n}\boldsymbol{E}\begin{bmatrix} \boldsymbol{w}\cdot\boldsymbol{w}^T \end{bmatrix}\boldsymbol{\Gamma}_{e,n}^T\\
&= \boldsymbol{\Gamma}_{e,n}\begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{pmatrix}\boldsymbol{\Gamma}_{e,n}^T
\end{aligned} \tag{21}$$

$$\begin{aligned}
\boldsymbol{R}_n &= \sigma_R^2 + \boldsymbol{H}_{e,n}\boldsymbol{E}\begin{bmatrix} \boldsymbol{w}\cdot\boldsymbol{w}^T \end{bmatrix}\boldsymbol{H}_{e,n}^T\\
&= \sigma_R^2 + \boldsymbol{H}_{e,n}\begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{pmatrix}\boldsymbol{H}_{e,n}^T
\end{aligned} \tag{22}$$

Eventually, extended Kalman filter which estimate the state vector $\hat{\boldsymbol{z}}_{n,n}$ at the n^{th} sample is constructed as:

Estimation:

$$\boldsymbol{P}_{n,n-1} = \boldsymbol{F}_{e,n-1}\boldsymbol{P}_{n-1,n-1}\boldsymbol{F}_{e,n-1}^T + \boldsymbol{Q}_{n-1} \tag{23}$$

$$\hat{\boldsymbol{z}}_{n,n-1} = \boldsymbol{f}(\hat{\boldsymbol{z}}_{n-1,n-1}, a_{m,n-1}) \tag{24}$$

Update:

$$\boldsymbol{K}_n = \boldsymbol{P}_{n,n-1}\boldsymbol{\eta}_{e,n}^T\left(\boldsymbol{\eta}_{e,n}\boldsymbol{P}_{n,n-1}\boldsymbol{\eta}_{e,n}^T + \boldsymbol{R}_n\right)^{-1} \tag{25}$$

$$\hat{\boldsymbol{z}}_{n,n} = \hat{\boldsymbol{z}}_{n,n-1} + \boldsymbol{K}_n(y - \boldsymbol{C}_{e,n}\hat{\boldsymbol{z}}_{n,n-1} - \boldsymbol{D}_{e,n}a_{m,n}) \tag{26}$$

$$\boldsymbol{P}_{n,n} = \left(\boldsymbol{I} - \boldsymbol{K}_n\boldsymbol{\eta}_{e,n}\right)\boldsymbol{P}_{n,n-1} \tag{27}$$

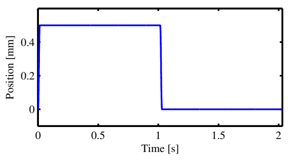
The initial covariance matrix $\boldsymbol{P}_{0,0}$ is defined as eq.(28).

$$\boldsymbol{P}_{0,0} = \mathrm{cov}\left(\boldsymbol{z}_0 - \hat{\boldsymbol{z}}_{0,0}\right), \tag{28}$$

where the $\boldsymbol{P}_{0,0}$ is determined with trial and error process to obtain better estimation performance.

The variance of measurement noise σ_R^2 can be defined as:

$$\sigma_R^2 = \rho^2/12 \tag{29}$$



Position [mm]
0.4
0.2
0
0
0.5
1
1.5
2
Time [s]

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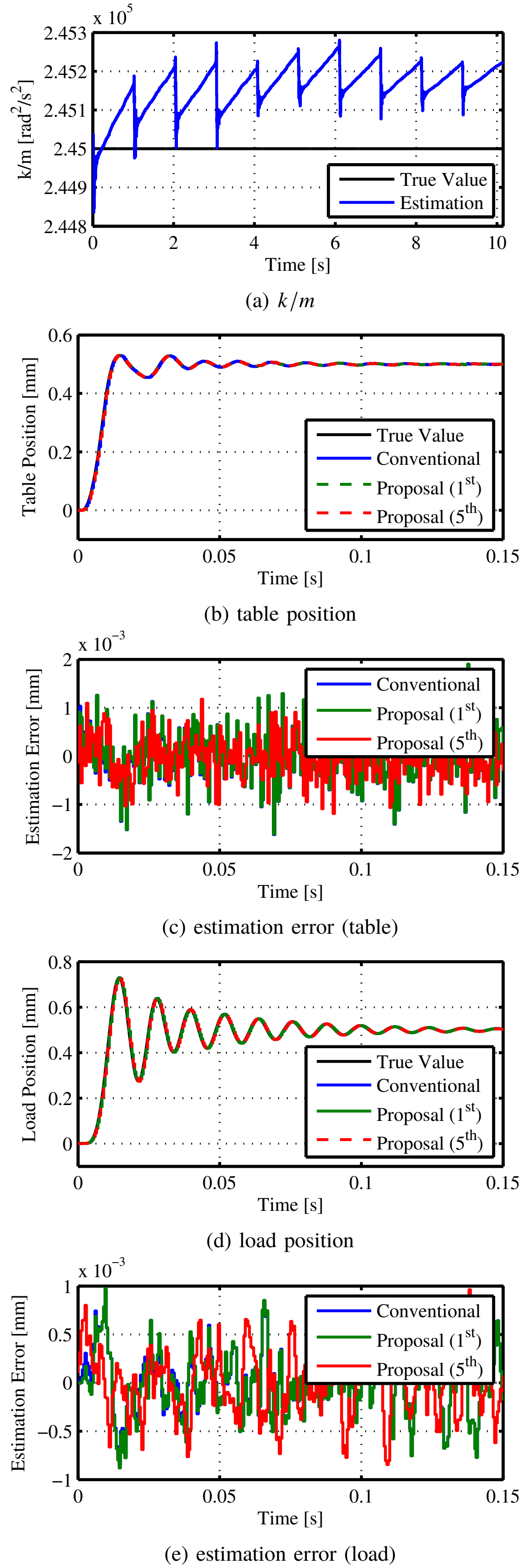

(a) k/m

(b) table position

(c) estimation error (table)

(d) load position

(e) estimation error (load)

Fig. 6: State estimation results (nominal)

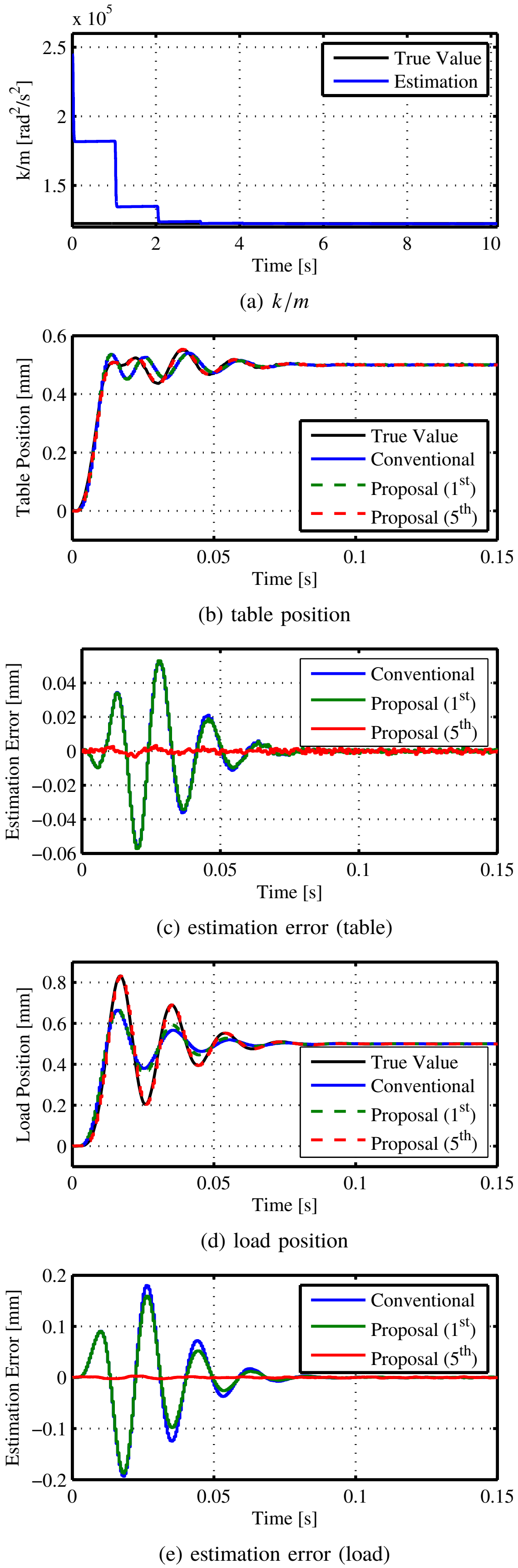

Fig. 7: State estimation results (error for m)

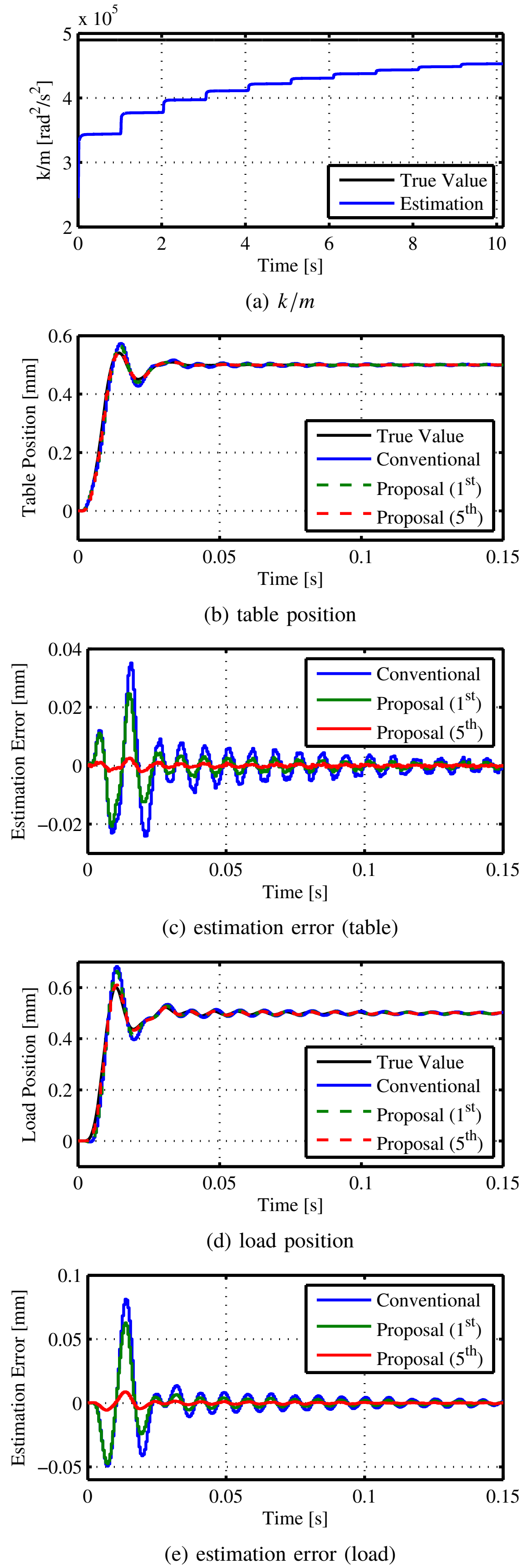

Fig. 8: State estimation results (error for k)