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Localization of Near-Field Sources Using Compressed Sensing

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Abstract – **This paper focuses on the localization of the near-field sources by using compress sensing with array antenna. A novel step-by-step method is proposed for estimating the angle and distance of near-field source. Estimation accuracy of source location by the proposed method is evaluated by computer simulation.**

Index Terms — **Array antenna, near-field source, compressed sensing, source-location estimation.**

1. Introduction

It is important to know the locations of radio sources or reflection points in mobile communications and radio sensing. For localization of the near-field sources, a signal model must be constructed based on the feature that the incident wave fronts are spherical [1], [2]. In addition, multiple snapshots of array antenna are generally required to employ high-resolution algorithms such as MUSIC [2]. On the other hand, it is known that compressed sensing can estimate the DOA (Direction-Of-Arrival) of radio waves with one snapshot of array antenna [3]. In this paper, we introduce compressed sensing into localization of the near-field sources, and we propose a method to obtain estimates of angle and distance of each source step by step. By computer simulation, the estimation performance of the proposed method is examined.

2. Signal Model

The array configuration is a uniform linear array of K elements as shown in Fig. 1. In this figure, the element spacing is d, and the first element of the array is the phase reference point. We have L near-field sources, and the position of the lth source ($l = 1, 2, \cdots, L$) is represented by the distance $r_{1,l}$ from the reference point and the angle $\theta_{1,l}$ at the reference point measured from the broadside direction of the array. Then, array input vector $\boldsymbol{x}(t)$, mode matrix $\boldsymbol{A}_0$ and mode vector $\boldsymbol{a}(\theta_{1,l}, r_{1,l})$ of the lth source are expressed as follows:

$$\boldsymbol{x}(t) = \boldsymbol{A}_0\boldsymbol{s}_0(t) + \boldsymbol{n}(t) \ \in \mathbb{C}^{K\times 1} \tag{1}$$

$$\boldsymbol{A}_0 = [\boldsymbol{a}(\theta_{1,1}, r_{1,1}), \cdots, \boldsymbol{a}(\theta_{1,L}, r_{1,L})] \ \in \mathbb{C}^{K\times L} \tag{2}$$

$$\boldsymbol{a}(\theta_{1,l}, r_{1,l}) = \left[1, \frac{r_{1,l}}{r_{2,l}}\exp(-j\tau_{2,l}), \cdots, \frac{r_{1,l}}{r_{K,l}}\exp(-j\tau_{K,l})\right]^T \tag{3}$$

$$r_{k,l} = r_{1,l}\sqrt{1 + \left(\frac{kd}{r_{1,l}}\right)^2 - \frac{2kd\sin\theta_{1,l}}{r_{1,l}}} \tag{4}$$

$$\tau_{k,l} = \frac{2\pi}{\lambda}(r_{k,l} - r_{1,l}) \quad (k = 1, \cdots, K) \tag{5}$$

where $\boldsymbol{s}_0(t) \in \mathbb{C}^{L\times 1}$ is a signal waveform vector, and $\boldsymbol{n}(t)$ is an internal noise vector. Also, $r_{k,l}$ is the distance between the lth source and the kth element, $\tau_{k,l}$ is the phase difference of the lth source at the kth element from the reference point, and λ is wavelength. In this paper, we assume the number of sources is 1 ($L = 1$) for simplicity.

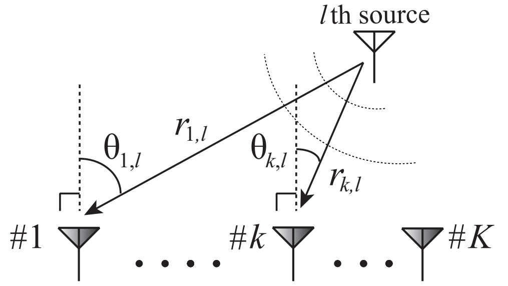

Fig. 1. Signal model (uniform linear array of K elements).

3. Principle of Compressed Sensing

(1) Sparse Reconstruction of Input Vector

In compressed sensing, one snapshot of the array input vector is reconstructed as follows [3]:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \tag{6}$$

where $\boldsymbol{A}$ is an extended mode matrix, $\boldsymbol{s}$ is an extended signal waveform vector. $\boldsymbol{A}$ and $\boldsymbol{s}$ have forms of $\boldsymbol{A} \in \mathbb{C}^{K\times N_1}$ and $\boldsymbol{s} \in \mathbb{C}^{N_1\times 1}$ respectively for angle estimation, and have forms of $\boldsymbol{A} \in \mathbb{C}^{K\times N_2}$ and $\boldsymbol{s} \in \mathbb{C}^{N_2\times 1}$ respectively for distance estimation. Here, N_1 is the number of divisions (bins) of the estimated angle range, and N_2 is the number of divisions (bins) of the estimated distance range. When $N_1, N_2 \gg L$, the vector $\boldsymbol{s}$ becomes a sparse vector in which only L elements are nonzero and the others are zero. When determining this unknown vector $\boldsymbol{s}$ by compressed sensing, we normally attach a sparsity condition to the vector $\boldsymbol{s}$.

(2) FISTA

The problem of obtaining the unknown vector $\boldsymbol{s}$ with a sparsity condition is expressed as follows [3]:

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s}}\left(\frac{1}{2}\|\boldsymbol{A}\boldsymbol{s} - \boldsymbol{x}\|_2^2 + \alpha\|\boldsymbol{s}\|_1\right) \tag{7}$$

where $\|\cdot\|_p$ is l_p norm, and α is a constant. In this paper, α is equal to 1. FISTA (Fast Iterative Shrinkage-Thresholding Algorithm) [4] is one of compressed sensing algorithms solving (7). Specifically, FISTA can be iteratively represented by the following equations:

$$s_{m+1} = S_{\alpha/P}\left(v_{m+1} - \frac{1}{P}A^H(Av_{m+1} - x)\right) \tag{8}$$

$$v_{m+1} = s_m + \left(\frac{t_m - 1}{t_{m+1}}\right)(s_m - s_{m-1}) \tag{9}$$

$$t_{m+1} = \frac{1 + \sqrt{1 + 4t_m^2}}{2} \tag{10}$$

$$S_{\alpha/P}(x) = \begin{cases} \left(\frac{|x|-\alpha/P}{|x|}\right)x & |x| \geq \alpha/P \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where s_m is the estimated value of s at the mth iteration and P is Lipschitz constant equal to maximum eigenvalue of $A^H A$ [3].

4. Localization of Near-Field Sources Using Compressed Sensing

In creating the near-field mode matrix A, source distance information is required for angle estimation, and source angle information is required for distance estimation. Therefore, angle and distance of the lth source are estimated by the following step-by-step procedure.

Step 1: We set the phase reference at the array center $p = (K + 1)/2$. Using the far-field mode matrix, the angle $\theta_{p,l}$ at the array center is estimated by compressed sensing with N_1 angle bins.

Step 2: The phase reference is changed to the first element, and the distance $r_{1,l}$ is estimated by compressed sensing using the near-field mode matrix. In this case, the angle of the reference element $\theta_{1,l}$ is calculated from the angle at the array center estimated at Step 1, and distance range in the obtained angle is divided into N_2 distance bins.

Step 3: The angle $\theta_{1,l}$ is estimated again by compressed sensing using the near-field mode matrix with the distance estimated in Step 2.

Step 4: The distance $r_{1,l}$ is estimated again by compressed sensing using the near-field mode matrix with the angle estimated in Step 3.

By estimating angle and distance again in Step 3 and Step 4 respectively, we aim at reducing estimation errors generated in Step 1 and Step 2.

5. Performance Analysis by Computer Simulation

Computer simulation of near-field source localization is carried out under the conditions described in Table I. We examine the change of the estimation accuracy from Step 1 to Step 4 of the proposed method. RMSE (Root Mean Square Error) is used for evaluation of estimation accuracy.

Figs. 2 and 3 show RMSEs of angle estimate and distance estimate, respectively, with change of source distance. It is seen from Fig. 2 that the estimation accuracy of angle in Step 3 is improved over that in Step 1. This means that the estimation error caused by using the far-field mode matrix in Step 1 is reduced in Step 3. In addition, the closer to array antenna the source is, the larger the error reduction effect is. For distance estimation, there is no appreciable difference between Step 2 and Step 4 as found from Fig. 3.

6. Conclusion

We have evaluated the localization performance of near-field source using the proposed method based on compressed sensing. As a result of computer simulation, it is shown that our step-by-step procedure can improve significantly the estimation accuracy of source angle while the estimation accuracy of source distance is unchanged. As a future work, we will try to enhance the estimation accuracy of source distance.

References

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TABLE I
Simulation Conditions

Array configuration	Uniform linear array
Number of elements	11
Element spacing	0.5λ
Number of sources	1
Source angle	30°
Source distance	$10\lambda \sim 40\lambda$
Number of snapshots	1
Number of trials	500
Input SNR	30dB
Division number of angle region	361 (−90° ~ 90°)
Division number of distance region	44 ($7\lambda \sim 50\lambda$)
Upper limit of iteration	1000 (angle), 10000 (distance)

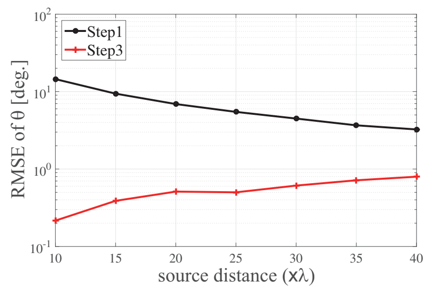

Fig. 2. RMSE of angle estimate vs. source distance.

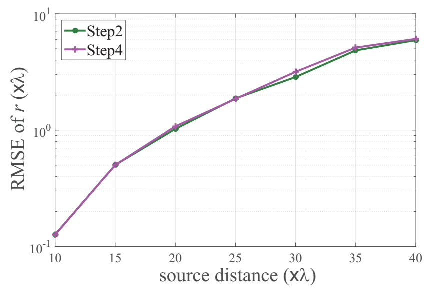

Fig. 3. RMSE of distance estimate vs. source distance.