Performance Improvement by Two-step Search Method in DOA Estimation Based on Compressed Sensing

Toshiya Nasu, Nobuyoshi Kikuma, and Kunio Sakakibara
Dept. of Electrical and Mechanical Engineering
Nagoya Institute of Technology, Nagoya 466-8555, Japan

Abstract – **Sparse reconstruction and compressed sensing are increasingly applied to DOA estimation of radio waves. This paper proposes a two-step search method in which ISTA and conjugate gradient method are combined effectively. The DOA estimation accuracy of the proposed method is evaluated by computer simulation.**

Index Terms — **Array antenna, DOA estimation, compressed sensing, ISTA, conjugate gradient method.**

1. Introduction

Recently, much attention is paid to applying signal-processing techniques called compressed sensing to DOA (Direction-Of-Arrival) estimation of radio waves by array antenna. As one of the algorithms, ISTA (Iterative Shrinkage-Thresholding Algorithm) [1]–[3] employing the steepest gradient method is used popularly. On the other hand, the conjugate gradient (CG) method [3], [4] is conventionally known as an optimization algorithm with fast convergence. In this paper, we propose the combined use of the two algorithms which is called two-step search method, and we examine the estimation accuracy and the computation time of the proposed method by computer simulation.

2. DOA Estimation Based on Compressed Sensing

We consider L waves are incident on a K-element, half-wavelength spaced uniform linear array as shown in Fig. 1. In compressed sensing, the array received signal vector $\boldsymbol{x} \in \mathbb{C}^{K\times 1}$ at a sampling time is expressed as

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{A} \in \mathbb{C}^{K\times N}$ is the extended mode matrix, $\boldsymbol{s} \in \mathbb{C}^{N\times 1}$ is the extended signal vector, $\boldsymbol{n}$ is the internal noise vector, and $N(\gg L)$ is the number of divisions in the estimation angle range $(-90^\circ \sim 90^\circ)$. N pieces in the angle range are called angle bins. In (1), $\boldsymbol{s}$ is the sparse vector in which only L elements are non-zero and the other ones are all zero. This unknown vector $\boldsymbol{s}$ can be found by solving the following minimization problem with a condition related to sparseness of $\boldsymbol{s}$:

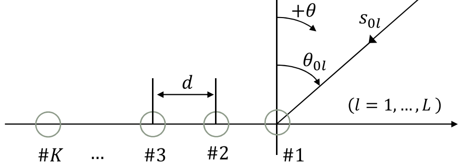

Fig. 1. Reception model (K-element uniform linear array).

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s}} \left(\frac{1}{2}\|\boldsymbol{A}\boldsymbol{s} - \boldsymbol{x}\|_2^2 + \alpha\|\boldsymbol{s}\|_1\right) \tag{2}$$

where α is a constant.

In practice, estimation $\boldsymbol{s}$ is performed by iterative updating based on the following gradient method:

$$\hat{\boldsymbol{s}}_{k+1} = \hat{\boldsymbol{s}}_k + \frac{1}{P}\boldsymbol{g}_k \tag{3}$$

$$\boldsymbol{g}_k = -\boldsymbol{A}^H(\boldsymbol{A}\boldsymbol{s}_k - \boldsymbol{x}) \tag{4}$$

where $\hat{\boldsymbol{s}}_k$ is the estimate of $\boldsymbol{s}$ at the k-th iteration, $\boldsymbol{g}_k$ is the search direction vector, and P is Lipschitz constant equal to maximum eigenvalue of $\boldsymbol{A}^H\boldsymbol{A}$. ISTA applies the following soft decision threshold function to $\hat{\boldsymbol{s}}_{k+1}$ [1]:

$$\eta_{\alpha/P}(x) = \begin{cases} \left(\dfrac{|x| - \alpha/P}{|x|}\right)x & |x| \geq \alpha/P \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

3. Two-step Search Method Using ISTA and Conjugate Gradient Method

In CG method which is one of the optimization methods, the search direction vectors $\boldsymbol{g}_0, \boldsymbol{g}_1, \cdots$ are updated so as to mutually have a conjugate relation with respect to $\boldsymbol{A}' = \boldsymbol{A}^H\boldsymbol{A}$. The vector $\boldsymbol{s}$, the search direction vector $\boldsymbol{g}$, and the residual vector $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{A}\boldsymbol{s}$ are iteratively calculated as follows:

$$\hat{\boldsymbol{s}}_{k+1} = \hat{\boldsymbol{s}}_k + \beta_k\boldsymbol{g}_k \tag{6}$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{r}_k - \beta_k\boldsymbol{A}'\boldsymbol{g}_k \tag{7}$$

$$\boldsymbol{g}_{k+1} = \boldsymbol{r}_{k+1} + \gamma_k\boldsymbol{g}_k \tag{8}$$

$$\beta_k = \frac{(\boldsymbol{r}_k, \boldsymbol{r}_k)}{(\boldsymbol{g}_k, \boldsymbol{g}_k)_{\boldsymbol{A}'}}, \ \gamma_k = \frac{(\boldsymbol{r}_{k+1}, \boldsymbol{r}_{k+1})_{\boldsymbol{A}'}}{(\boldsymbol{r}_k, \boldsymbol{r}_k)_{\boldsymbol{A}'}} \tag{9}$$

$$(\boldsymbol{a}, \boldsymbol{b})_{\boldsymbol{A}'} \equiv \boldsymbol{a}^H\boldsymbol{A}'\boldsymbol{b} \tag{10}$$

where β_k and γ_k are the update coefficients of $\boldsymbol{g}_k$ and $\boldsymbol{r}_k$, respectively.

CG method is generally known to be a linear optimization method with very fast convergence [3], [4]. However, it is difficult to apply nonlinear processing of (5) to CG method, in other words, CG method cannot realize sparseness of $\boldsymbol{s}$ easily [5]. Therefore, it is expected that the estimation accuracy of CG method can be improved by using the estimated vector $\hat{\boldsymbol{s}}$ of ISTA as the initial value of CG method. This is the two-step search method proposed in this paper.

4. Performance Analysis by Computer Simulation

Computer simulation of DOA estimation using the two-step search method by ISTA and CG method is carried out under the conditions described in Table I. The estimation accuracy (RMSE: Root Mean Squared Error), computation time, and error probability of estimation are examined comparatively for ISTA, CG, and proposed method. The convergence conditions shown in Table II are utilized for each method. Figs. 2, 3, and 4 show the estimation accuracy (RMSE), the computation time, and the error probability, respectively, as a function of SNR. In Fig. 4, success of estimation is defined as the case where the estimated angle agrees with the true DOA within permissible errors of (a) 0° and (b) ±2°. It can be seen from Figs. 2 and 4 that the two-step search method demonstrates the improved estimation accuracy. In addition, it is found from Fig. 3 that increase in the computation time of proposed method is extremely small because of very short computation time of CG method at the second step.

5. Conclusion

The DOA estimation accuracy of the two-step search method by ISTA and CG method was evaluated. From the simulation results, it is confirmed that the estimation accuracy of the proposed method is improved with almost no increase in the computation time. In the future, performance evaluation by CG method alone will be continued, and further improved algorithm will be studied.

References

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TABLE I
Simulation Conditions

Number of elements	10	Number of trials	1000
Number of incident waves	2	Input SNR	0 ~ 40 [dB]
DOA	0°, 45°	Signal power	1.0, 1.0
Upper limit of iteration	1000	Number of angle bins	181
α of ISTA	1.0		

TABLE II
Convergence Conditions

ISTA	$\frac{\|\hat{s}_k-\hat{s}_{k-1}\|_2^2}{\|\hat{s}_k\|_2^2} \leq 10^{-6}$
CG	$\|r_k\|_2 \leq 10^{-6}$

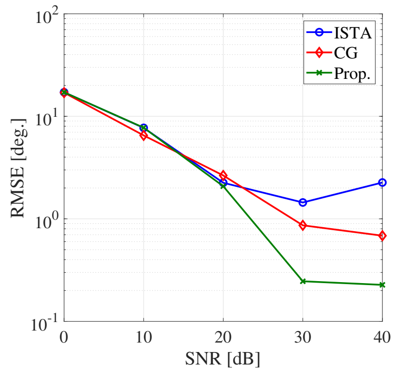


Fig. 2. RMSE of DOA estimates vs. SNR.

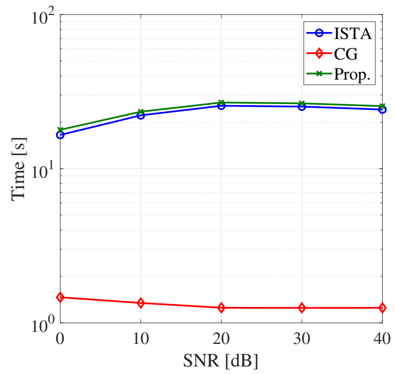
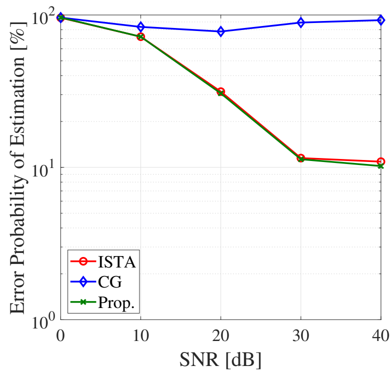
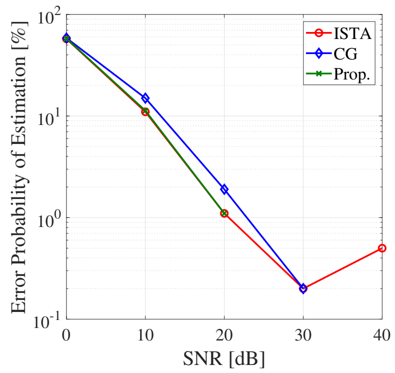


Fig. 3. Computation time vs. SNR.



(a) permissible error = 0°



(b) permissible error = ±2°

Fig. 4. Error probability vs. SNR.